

First Homework: Exercise 1.1.1.2

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1. False.

Below I give a counterexample:

Suppose  $n=3$ , and we have the set of men  $M = \{m_1, m_2, m_3\}$   
and women  $W = \{w_1, w_2, w_3\}$

Let the chart show these three men's preference:

|        |                   |
|--------|-------------------|
| $m_1:$ | $w_1 > w_2 > w_3$ |
| $m_2:$ | $w_2 > w_3 > w_1$ |
| $m_3:$ | $w_3 > w_1 > w_2$ |

And the women's preference:

|        |                   |
|--------|-------------------|
| $w_1:$ | $m_2 > m_3 > m_1$ |
| $w_2:$ | $m_3 > m_1 > m_2$ |
| $w_3:$ | $m_1 > m_2 > m_3$ |

Now we suppose the following stable matching:

$$S = \{(m_1, w_1), (m_2, w_2), (m_3, w_3)\},$$

in which each man is with his best choice, while each woman is with her worst choice.

2. Obviously true

Suppose when there's a stable matching  $S'$  which doesn't include the pair  $(m, w)$

Then both  $m$  and  $w$  prefer each other than their current partner

Hence the pair  $(m, w)$  contradicts with the claim "stable"