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First Homework: Exercise 1.1, 1.2, 1.4, 1.5

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1. False.

Below I give a counterexample:

Suppose $n=3$, and we have the set of men $M=\{m_1,m_2,m_3\}$ and women $W=\{w_1,w_2,w_3\}$

Let the chart show these three men's preference:

m_1:	$w_1 > w_2 > w_3$
m_2:	$w_2 > w_3 > w_1$
m_3:	$w_3 > w_1 > w_2$

And the women's preference:

w_1:	$m_2 > m_3 > m_1$
w_2:	$m_3 > m_1 > m_2$
w_3:	$m_1 > m_2 > m_3$

Now we suppose the following stable matching:

$S=\{(m_1,w_1),(m_2,w_2),(m_3,w_3)\}$,

in which each men is with his best choice, while each women is with her worst choice.

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2. Obviously true

Suppose when there's a stable matching S' which doesn't include the pair (m,w)

Then both m and w prefer each other than their current partner

Hence the pair (m,w) contradicts with the claim "stable"

4. Since the number of students is larger than that of job oppotunities in hospitals, we let the hospitals be the active ones, which lead to the following algorithm:

Initially all $h \in H$ and $s \in S$ are free
While there is a hospital h still in need of students
    Choose such a hospital h
    Let s be the highest-ranked student in h's preference
        list to which h has not yet proposed
    If s is free then
        (h,s) become engaged
    Else s is currently engaged to h'
        If s prefers h' to h then
            h doesn't have s
        Else s prefers h to h' then
            (h,s) become engaged
            h' loses s

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Endif

Endif

Endwhile

Return the set of the engaged pairs.

Now we solve the problem by proving several conclusions

① The process ends up in finite steps

Proof: For any hospital h it will be proposed to each student once at most

So the largest possible number of loops is no larger than the number of the (h,s) pairs.

② The process ends up with a stable matching

Proof: Assume to some extends it doesn't

If it ends up with the first instability

Suppose a final pair (h,s) and a jobless student s', whom h prefers to s

If h has been proposed to s, then there's no way for s to stay jobless

But if h has not been proposed to s, how come has it been proposed to s' ?

If it ends up with the second instability

Suppose two final pairs (h,s) and (h',s'), in which h prefers s' to s and s' prefers h to h'

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Since h prefers s' to s then h should have been exposed to s', which means s' goes to a hospital no worse than h, but h' is less prefered by s. Contradictory.

According to ① and ②, we proved the existence of the stable matches. And through the algorithm we come up with above, we can find one stable match after finite loops.

5. (a) Yes. Without loss of generality, we can adjust the preference ranking by replacing all indifferences with differences randomly.
Now that we get a classical stable matching problem, we can use G-S algorithm to get a stable matching.
A strong instability exists only when the adjusted problem exists an unstable matching, which is contradictory to our previous conclusion of G-S algorithm.

(b) No. Assume an extreme example where all women treat all men equally, which means for every woman, she doesn't have a preference ranking of the men.
Then for every perfect matching, we randomly select two pairs (m_1, w_1) and (m_2, w_2), in which m_1 prefers w_2 than w_1.

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So we get a weak instable pair (m_1, w_2)