

2024.3.15 The Second Group Meeting

Acan Xie

March 15, 2024

1 Introduction

(1) is ideal gas law equation.

$$PV = NRT \quad (1)$$

(2) is the kinetic energy equation.

$$E = \sum_{i=1}^N \frac{|\vec{P}_i|^2}{2m} = H(p, q) \quad (2)$$

(3) is the entropy equation.

$$S = k \ln \omega \quad (3)$$

2 Strling Approximation

According to Mathematical Induction, we can prove (4)

$$N! = \int_0^\infty e^{-x} x^N dx \quad (4)$$

Laplace approximation: (5)

$$\int g(x) dx \quad (5)$$

Let $h(x) = \ln g(x)$ then we have

$$\int g(x) dx = \int e^{h(x)} dx \quad (6)$$

According to Taylor expansion, we have

$$\int e^{h(x)} dx = \int \exp(h(x_0) + h'(x_0)(x - x_0) + \frac{1}{2}h''(x_0)(x - x_0)^2) dx \quad (7)$$

If $h'(x_0) = 0$ then we have

$$\begin{aligned}\int e^{h(x)} dx &= \int \exp(h(x_0) + \frac{1}{2}h''(x_0)(x - x_0)^2) dx \\ &= e^{h(x_0)} \int \exp(-\frac{1}{2} \frac{(x - x_0)^2}{-h''(x_0)^{-1}}) dx\end{aligned}\tag{8}$$

3 Energy