Computational Probability

HW1

1. Consider the composite midpoint rule

$$\int_0^1 f(x)\,dx \approx \frac{1}{N}\sum_{n=0}^{N-1} f\left(\frac{n+1/2}{N}\right).$$

   If $f \in C^2[0,1]$, show that the error of this method is $O(h^2)$ where $h = \frac{1}{N}$.

2. Numerically compute $\int_{[-1,1]^5} exp(-x^2)\,dx$ by Monte Carlo method. Please plot the errors w.r.t. the sample sizes.