数值分析作业9：（第六章 2.12.13.16.17.19）

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2. $L_2(x)=\sum_{i=0}^{2} f(x_i)l_i(x)$

$$=\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}f(x_0)+\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}f(x_1)+\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}f(x_2)$$

$$=10.074x^2-9.2914x+1.93566$$

$L_2(1.03)=3.05305$

$R_2(1.03)=f(1.03)-L_2(1.03)=1.14168\times10^{-4}$

$|R_2(1.03)|=|f(1.03)-L_2(1.03)|=\frac{e^{\xi}(7+3\xi)}{3!}|(1.03-x_0)(1.03-x_1)(1.03-x_2)|$

$$\le\frac{e^{1.07}(7+3\times1.07)}{3!}\times2.4\times10^{-5}\approx1.19064\times10^{-4}$$

12. 具有重节点的均差表：

x_i	$P(x_i)$	一阶	二阶	三阶
0	0			
0	0	0		
1	1	1	1	
2	2	0	$-\frac{1}{2}$	$-\frac{3}{4}$

Newton 插值多项式．

$$P(x)=P(0)+P[0,0]\cdot(x-0)+P[0,0,1](x-0)^2+P[0,0,1,2](x-0)^2(x-1)$$

$$=0+0\cdot x+1x^2-\frac{3}{4}x^2(x-1)$$

$$=\frac{7}{4}x^2-\frac{3}{4}x^3$$

Newton 形式余项:

$$R_3(x) = P[0,0,1,2,x]\,x^2(x-1)(x-2)$$

13. 具有重节点的均差表:

x_i	$P(x_i)$	一阶	二阶	三阶	四阶
1	0				
1	0	0			
1	0	0	4		
2	1	1	1	-3	
3	0	-1	-1	-1	1

Newton 插值多项式.

$$P(x) = P(1) + P[1,1](x-1) + P[1,1,1](x-1)^2 + P[1,1,1,2](x-1)^3 + P[1,1,1,2,3](x-1)^3(x-2)$$

$$= 0 + 0(x-1) + 4(x-1)^2 + (-3)(x-1)^3 + 1(x-1)^3(x-2)$$

$$= x^4 - 8x^3 + 22x^2 - 24x + 9$$

Newton 形式余项:

$$R_4(x) = P[1,1,1,2,3,x](x-1)^3(x-2)(x-3)$$

16. $S \in C^2[0,2] \Rightarrow S(1^-) = S(1^+).\ S'(1^-) = S'(1^+),\ S''(1^-) = S''(1^+)$

$$S(x) = \begin{cases} 1+2x-x^3 & x \in [0,1] \\ 2+b(x-1)+c(x-1)^2+d(x-1)^3 & x \in [1,2] \end{cases}$$

$$S'(x)=\begin{cases}2-3x^2 & x\in[0,1]\\ b+2c(x-1)+3d(x-1)^2 & x\in[1,2]\end{cases}$$

$$S''(x)=\begin{cases}-6x & x\in[0,1]\\ 2c+6d(x-1) & x\in[1,2]\end{cases}$$

$$\Rightarrow\begin{cases}2=2\\ -1=b\\ -6=2c\end{cases}$$

$$S''(0),\ S''(2)=0 \Rightarrow \begin{cases}0=0\\ 2c+6d=0\end{cases}$$

由此解得 $b=-1,\ c=-3,\ d=1$

17 $$S(x)=\begin{cases}1+Bx+2x^2-2x^3 & x\in[0,1]\\ 1+b(x-1)-4(x-1)^2+7(x-1)^3 & x\in[1,2]\end{cases}$$

$$S'(x)=\begin{cases}B+4x-6x^2 & x\in[0,1]\\ b-8(x-1)+21(x-1)^2 & x\in[1,2]\end{cases}$$

$$S''(x)=\begin{cases}4-12x & x\in[0,1]\\ -8+42(x-1) & x\in[1,2]\end{cases}$$

$S\in C^2[0,2] \Rightarrow S(1^-)=S(1^+),\ S'(1^-)=S'(1^+),\ S''(1^-)=S''(1^+)$

$$\Rightarrow\begin{cases}1+B=1\\ B-2=b\\ -8=-8\end{cases}$$

解得 $B=0,\ b=-2$

$\Rightarrow S'(0)=0,\ S'(2)=11$

19 $h_0=1,\ h_1=3,\ h_2=3$

$$\begin{cases} \mu_1=\frac{1}{4},\ \mu_2=\frac{1}{2} \\ \lambda_1=\frac{3}{4},\ \lambda_2=\frac{1}{2} \\ d_1=\frac{9}{2},\ d_2=-\frac{5}{3} \end{cases}$$

(1) $d_0=6f[-3,-3,-2]=6\left.\frac{df[x,-2]}{dx}\right|_{x=-3}=-6$

$d_3=6f[1,4,4]=6\left.\frac{df[1,x]}{dx}\right|_{x=4}=\frac{10}{3}$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 0 & 0 \\ \frac{1}{4} & 2 & \frac{3}{4} & 0 \\ 0 & \frac{1}{2} & 2 & \frac{1}{2} \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} M_0 \\ M_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} -6 \\ \frac{9}{2} \\ -\frac{5}{3} \\ \frac{10}{3} \end{bmatrix}$$

$\Rightarrow (M_0, M_1, M_2, M_3)^T = \left(-\frac{152}{31}, \frac{118}{31}, -\frac{78}{31}, \frac{272}{93}\right)^T$

代入即可

(2) $M_0=0,\ M_3=0$

$$\begin{pmatrix} 2 & \frac{3}{4} \\ \frac{1}{2} & 2 \end{pmatrix} \begin{pmatrix} M_1 \\ M_2 \end{pmatrix} = \begin{pmatrix} \frac{9}{2} \\ -\frac{5}{3} \end{pmatrix}$$

$\Rightarrow M_1=\frac{82}{29},\ M_2=-\frac{134}{87}$

代入即可