

数值分析作业13 (第七章 1.2.4.7.10.11.17)

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$$1. \quad P_1 = x - \frac{(x, P_0 v(x))}{(P_0 v(x), P_0 v(x))} P_0 v(x) = x - \frac{\int_{-1}^1 x P_0 v(x) dx}{\int_{-1}^1 P_0 v(x)^2 dx} P_0 v(x) = x$$

$$\begin{aligned} P_2 &= x^2 - \frac{(x^2, P_0 v(x))}{(P_0 v(x), P_0 v(x))} P_0 v(x) - \frac{(x^2, P_1 v(x))}{(P_1 v(x), P_1 v(x))} P_1 v(x) \\ &= x^2 - \frac{\int_{-1}^1 x^2 P_0 v(x) dx}{\int_{-1}^1 P_0 v(x)^2 dx} P_0 v(x) - \frac{\int_{-1}^1 x^2 P_1 v(x) dx}{\int_{-1}^1 P_1 v(x)^2 dx} P_1 v(x) \\ &= x^2 - \frac{1}{3} \end{aligned}$$

$$\begin{aligned} P_3 &= x^3 - \frac{(x^3, P_0 v(x))}{(P_0 v(x), P_0 v(x))} P_0 v(x) - \frac{(x^3, P_1 v(x))}{(P_1 v(x), P_1 v(x))} P_1 v(x) - \frac{(x^3, P_2 v(x))}{(P_2 v(x), P_2 v(x))} P_2 v(x) \\ &= x^3 - \frac{\int_{-1}^1 x^3 P_0 v(x) dx}{\int_{-1}^1 P_0 v(x)^2 dx} P_0 v(x) - \frac{\int_{-1}^1 x^3 P_1 v(x) dx}{\int_{-1}^1 P_1 v(x)^2 dx} P_1 v(x) - \frac{\int_{-1}^1 x^3 P_2 v(x) dx}{\int_{-1}^1 P_2 v(x)^2 dx} P_2 v(x) \\ &= x^3 - \frac{3}{5} x \end{aligned}$$

$$2. \quad L_1 v(x) = 1 - x$$

$$L_2 v(x) = \frac{1}{2}(x^2 - 4x + 2)$$

$$L_3 v(x) = \frac{1}{6}(-x^3 + 9x^2 - 18x + 6)$$

$$4. \quad T_3 v(x) = 4x^3 - 3x, \quad \text{零点为 } x_1 = 0, x_2 = \frac{\sqrt{3}}{2}, x_3 = -\frac{\sqrt{3}}{2}$$

均差表:

x_i	$P(x_i)$	$-P_1'$	$=P_2'$
$-\frac{\sqrt{3}}{2}$	0.4206		
0	1	0.6690	
$\frac{\sqrt{3}}{2}$	2.3774	1.5905	0.5320

Newton 插值多项式

$$P(x) = P_0(x) + P_1(x-x_0) + P_2(x-x_0)(x-x_1)$$

$$= 0.5320x^2 + 1.1297x + 1.0002$$

5. $x_1 = -\frac{\sqrt{3}}{2}$, $x_2 = 0$, $x_3 = \frac{\sqrt{3}}{2}$

均差表:

x_i	$P(x_i)$	$-P_1'$	$=P_2'$
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$-\frac{\sqrt{3}}{2}$	0.1257		
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0	0.6931	0.6552	
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$\frac{\sqrt{3}}{2}$	1.0529	0.4155	-0.1384
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Newton 插值多项式:

$$P(x) = P_0(x) + P_1(x-x_0) + P_2(x-x_0)(x-x_1)$$

$$= -0.1384x^2 + 0.2956x + 0.6931$$

误差界估计:

$$R_2(x) = P[x_0, x_1, x_2, x](x-x_0)(x-x_1)(x-x_2)$$

$$= P(-\frac{\sqrt{3}}{2}, 0, \frac{\sqrt{3}}{2}, x)(x^2 - \frac{3}{4})x$$

$$= \sqrt{3}x^3 - \frac{3}{4}\sqrt{3}x$$

$$|R_2 v_N| = |\sqrt{3}x^3 - \frac{3}{4}\sqrt{3}x| \leq \frac{1}{12} \approx 0.0833$$

$$7. \quad p_1^*(x) = 1.1752 + 1.1036x$$

$$p_2^*(x) = 0.9963 + 1.1036x + 0.5367x^2$$

$$10 \quad s_3^*(x) = 1.5532x - 0.5622x^3$$

$$11 \quad R_{3,2}(x) = \frac{30x + 21x^2 + x^3}{30 + 36x + 9x^2}$$

$$17. \quad a=1, b=1. \quad \text{此时} \quad y = \sin \pi x + \cos \pi x$$