PDE数值解作业五：

1. $u_{i+1,j+1}+u_{i+1,j-1}+u_{i-1,j+1}+u_{i-1,j-1}-4u_{ij}-2h^2f_{ij}$

$=hu_x+hu_y+\frac{h^2}{2}u_{xx}+\frac{h^2}{2}u_{yy}+u_{xy}+\frac{h^3}{6}(u_{xxx}+3u_{xxy}+3u_{xyy}+u_{yyy})$

$+hu_x-hu_y+\frac{h^2}{2}u_{xx}+\frac{h^2}{2}u_{yy}-u_{xy}+\frac{h^3}{6}(u_{xxx}-3u_{xxy}+3u_{xyy}-u_{yyy})$

$-hu_x+hu_y+\frac{h^2}{2}u_{xx}+\frac{h^2}{2}u_{yy}-u_{xy}+\frac{h^3}{6}(-u_{xxx}+3u_{xxy}-3u_{xyy}+u_{yyy})$

$-hu_x-hu_y+\frac{h^2}{2}u_{xx}+\frac{h^2}{2}u_{yy}+u_{xy}+\frac{h^3}{6}(-u_{xxx}-3u_{xxy}-3u_{xyy}-u_{yyy})$

$-2h^2f_{ij}+o(h^4)=o(h^4)$

2. 令 $r_i=(i+\frac{1}{2})\Delta r$，$\theta_j=j\Delta\theta$，则 $\frac{r_{i+\frac{1}{2}}u_{i+1,j}-(r_{i+\frac{1}{2}}+r_{i-\frac{1}{2}})u_{ij}+r_{i-\frac{1}{2}}u_{i-1,j}}{r_i(\Delta r)^2}$

$+\frac{u_{i,j+1}-2u_{ij}+u_{i,j-1}}{r_i(\Delta r)^2}=f_{ij}$

对应的边界条件和 $\frac{2}{(\Delta r)^2}(u_{1,j}-u_{0,j})+\frac{4}{(\Delta r\Delta\theta)^2}(u_{0,j+1}-2u_{0,j}+u_{0,j-1})=f_{0,j}$

为二阶差分格式

3. 若取极小负数，且不为常数，则此时 $\exists i$, s.t. u_{i-1}, u_{i+1} 中存在一个比 u_i 大

$\Rightarrow L_hu_i<-(a_iu_{i-1}-b_iu_i+c_iu_{i+1})<0$ 矛盾，显然

考虑 $L_hv_i=|f_i|$ $v_0=v_N=0$，令 $w_i^{\pm}=v_i\pm u_i$

则在 $i=1,2,\cdots,N-1$ 时，$|L_hu_i|=|f_i|\le L_hv_i\Rightarrow L_hw_i^{\pm}\geq 0$

又 $i=0,N$ 时，$w_i^{\pm}=0$，故 $w_i^{\pm}\geq 0$

故 $|u_i|\le v_i$，又 $v_i\le\frac{|f_i|}{d_i}\Rightarrow\max\limits_i|u_i|\le\max\limits_i\frac{|f_i|}{d_i}$