作业3：

6. 设 $u=u(x,y,z,t)$ 为 t 时刻在 (x,y,z) 处温度

设 k 为导热系数，α_0 为热交换系数，则定解问题.

泛定方程：$u_t - a^2 \Delta u = 0$

初始. $u(x,y,z,0)=100$

边界：$k \frac{\partial u}{\partial n}\Big|_\Sigma = \alpha_0 (37-u)\Big|_\Sigma$

7. $f=0$. 定解问题：

泛定方程. $u_t - a^2 \Delta u = 0$ $(a^2 = 6\times 10^{-7} m^2/s)$

初始：$u|_{t=0} = 1200$

边界：$u|_{\partial\Omega}=0$

10. 取传送带所在直线为 x 轴. 起点为原点，在区间 $[x_1,x_2]$，时间 $[t_1,t_2]$ 上

质量守恒：$\int_{x_1}^{x_2} (\rho|_{t_2} - \rho|_{t_1})\,dx = -\int_{t_1}^{t_2} dt\, a(\rho|_{x_2} - \rho|_{x_1})$

即 $\int_{x_1}^{x_2} dx \int_{t_1}^{t_2} \frac{\partial \rho}{\partial t}\,dt = -\int_{t_1}^{t_2} dt \int_{x_1}^{x_2} a\frac{\partial \rho}{\partial x}\,dx$

因 $x.t$ 均为任取. 故

泛定方程：$\rho_t + a\rho_x = 0$

初始：$\rho(x,0)=0$ $(x\geq 0)$

边界. $\rho_t(0,t) = A(1+\sin\omega t)$ $(t\geq 0)$

18. $u_t = \tilde{u}' \cdot \left(-\frac{1}{2} x t^{-\frac{3}{2}}\right)$

$u_x = \tilde{u}' \cdot \frac{1}{\sqrt{t}} \Rightarrow u_{xx} = \tilde{u}_{\xi\xi} \cdot \frac{1}{t}$

$\Rightarrow u_t - a^2 u_{xx} = 0 \Rightarrow -\frac{1}{2} x t^{-\frac{3}{2}} \tilde{u}' - a^2 \tilde{u}'' \frac{1}{t} = 0$

$\Rightarrow \tilde{u}'' + \frac{\xi}{2a^2} \tilde{u}' = 0$

转化为 $\tilde{u}'' + \frac{\xi}{2a^2} u_\xi' = 0$

$$\begin{cases} \tilde{u}\big|_{\xi=0} = 0 \\ \tilde{u}\big|_{\xi=\infty} = u_0 \end{cases}$$

$\Rightarrow$ (解 ODE) $u(x,t) = \frac{u_0}{2a\sqrt{\pi}} \int_0^{\frac{x}{t}\sqrt{t}} e^{-\frac{\eta^2}{4a^2}} d\eta$