概率论 第一次作业.（P8 5,6,7,9　P15 2,4,5,8,9,11,12）

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P8 5. $\bar{A}_k$ = {至多 (k−1) 次呼唤}

$A_k - A_{k+1}$ = {恰好k次呼唤}

$\bigcup_{k=1}^{\infty} A_k$ = {至少一次呼唤}

$\bigcap_{k=0}^{\infty} A_k = \emptyset$

6. $\overline{\lim_n} A_n = \bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_k = \bigcup_{n=1}^{\infty} A_k$

$\underline{\lim_n} A_n = \bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_k = \bigcup_{n=1}^{\infty} A_k$

7. (1) 样本空间 = {正正正, 正正反, 正反正, 正反反, 反正正. 反正反, 反反正, 反反反}

A包含"正正正""正正反""正反正""正反反"

B包含"正正正""反反反"

C包含"正正正""正正反""正反正""正反反""反正正","反正反","反反正"

(2) 样本空间 $\Omega = \{(x_1, x_2) \mid x_1, x_2 \in \{1,2,\cdots,6\}\}$

$A = \{(x,x) \mid x \in \{1,2,\cdots,6\}\}$

$B = \{(2x,x) \mid x \in \{1,2,3\}\} \cup \{(x,2x) \mid x \in \{1,2,3\}\}$

$C = \{(1,5),(2,4),(3,3),(4,2),(5,1)\}$

(3) 记红球放入a盒. 黑球放入b盒. $(a,b = 1,2,3)$

样本空间 $\Omega=\{(a,b)\mid a,b\in\{1,2,3\}\}$

$A=\{(1,2),(1,3),(2,1),(3,1),(1,1)\}$

$B=\{(1,2),(2,1)\}$

$C=\{(1,2),(1,3),(2,1),(2,3),(3,1),(3,2)\}$

9. 样本空间 $\Omega=\{(x_1,x_2,x_3,x_4,x_5)\mid x_i\in\{0,1,2,\ ,9\},\ i=1,2,3,4,5\}$

E_0 必然发生 $\Rightarrow$ $E_0=\Omega$，E_6 必不发生 $\Rightarrow$ $E_6=\phi$

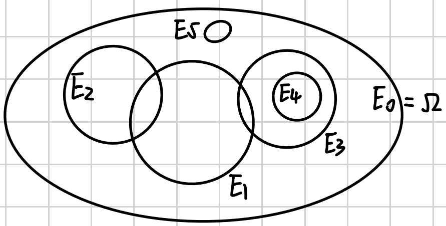
若 E_1 不发生：则 E_2. E_3 同时发生或 E_5 发生.

若 E_1 发生. 则若 E_2 不发生：E_3. E_4 恰一发生. 或仅 E_1 发生.

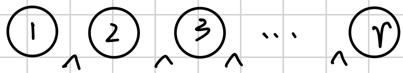
若 E_2 发生. 仅有 E_1 E_2 发生.

综上 Venn 图如下：

$E_6=\phi$



P15 2. 先考虑 (2) 取 r 个小球标号 1, 2, …, r 从左至右排列一排：



在 $(r-1)$ 个间隙中选 $(n-1)$ 处放挡板，共 $\binom{r-1}{n-1}$ 种放法

每种放法与 $x_1+x_2+\cdots+x_n=r$ 的正整数解一一对应.

故解共 $\binom{r-1}{n-1}$ 组

再考虑(1) $x_1+x_2+\cdots+x_n=r$ 的非负整数解与方程

$(x_1+1)+(x_2+1)+\cdots+(x_n+1)=r+n$ 的正整数解一一对应

故解共有 $\binom{r+n-1}{n-1}$ 组

4. 样本总数 $=\binom{2n}{2m}$ 满足"恰有k双配对"的条件: $\binom{n}{k}2^{2m-2k}$

故 $P(\text{恰有k双配对})=\dfrac{\binom{n}{k}2^{2m-2k}}{\binom{2n}{2m}}$

5. 设 $P(i,j)$ 为甲 i 个正面, 乙 j 个正面的概率 $(0\le i\le n+1,\ 0\le j\le n)$

则 $P(i,j)=\binom{n+1}{i}2^{-n-1}\cdot\binom{n}{j}\cdot 2^{-n}$

$$=\binom{n+1}{i}\binom{n}{j}2^{-2n-1}$$

甲正面比乙多概率 $P=\sum_{j=0}^{n}\sum_{i=j+1}^{n+1}P(i,j)=2^{-2n-1}\sum_{j=0}^{n}\sum_{i=j+1}^{n+1}\binom{n+1}{i}\binom{n}{j}$

注意到对于 j 和 $n-j$, 有:

$$\binom{n}{j}\sum_{i=j+1}^{n+1}\binom{n+1}{i}+\binom{n}{n-j}\sum_{i=n-j+1}^{n+1}\binom{n+1}{i}$$

$$=\binom{n}{j}\left[\sum_{i=j+1}^{n+1}\binom{n+1}{i}+\sum_{i=0}^{j}\binom{n+1}{i}\right]$$

$$=\binom{n}{j}\sum_{i=0}^{n+1}\binom{n+1}{i}$$

$$=\binom{n}{j}\cdot 2^{n+1}$$

故 $2P=P+P=2^{-2n-1}\sum_{j=0}^{n}\sum_{i=j+1}^{n+1}\binom{n+1}{i}\binom{n}{j}+2^{-2n-1}\sum_{j=0}^{n}\sum_{i=j+1}^{n+1}\binom{n+1}{i}\binom{n}{j}$

$$=2^{-2n-1}\sum_{j=0}^{n}\binom{n}{j}2^{n+1}=2^{-n}2^{n}=1$$

$\Rightarrow P=\frac{1}{2}$

8. 样本总数 $N = n^r$

(1) 包含样本数：$r!$ $\quad P = \dfrac{n!}{n^r}$

(2) 包含样本数：$\binom{n}{r} \cdot r!$ $\quad P = \dfrac{\binom{n}{r} r!}{n^r} = \dfrac{n!}{n^r (n-r)!}$

(3) 若 $m > r$，$P = 0$

若 $m \leq r$，样本总数 $\binom{r}{m} (n-1)^{r-m}$ $\quad P = \dfrac{\binom{r}{m} (n-1)^{r-m}}{n^r}$

(4) 若 $m < n-r$ 或 $m = n$ $\quad P = 0$

若 $n-r \leq m \leq n-1$，包含样本数 $\sum_{k=m}^{n-1} (-1)^{k-m} \binom{n}{k} (n-k)^r$

$$P = \sum_{k=m}^{n-1} (-1)^{k-m} \binom{n}{k} \left(1 - \frac{k}{n}\right)^r$$

9. (1) $P_1 = \dfrac{4}{\binom{52}{5}} = \dfrac{1}{649740}$

(2) $P_2 = \dfrac{4 \times 8}{\binom{52}{5}} = \dfrac{2}{162435}$

(3) $P_3 = \dfrac{13 \times (52-4)}{\binom{52}{5}} = \dfrac{1}{4165}$

(4) $P_4 = \dfrac{13 \times 12 \times \binom{4}{3} \times \binom{4}{2}}{\binom{52}{5}} = \dfrac{6}{4165}$

(5) $P_5 = \dfrac{\binom{13}{5} \times 4}{\binom{52}{5}} = \dfrac{33}{16660}$

(6) $P_6 + (P_1 + P_2) = \dfrac{4^5 \times 9}{\binom{52}{5}} = \dfrac{192}{54145}$

$P_6 = \dfrac{192}{54145} - (P_1 + P_2) = \dfrac{153}{43316}$

(7) $P_7 = \dfrac{13 \times \binom{4}{3} \times (1 \times 48 + 48 \times 47)}{\binom{52}{5}} = \dfrac{11}{12495}$

(8) $P_8 = \dfrac{\binom{13}{2} \times \binom{4}{2}^2 \times 44}{\binom{52}{5}} = \dfrac{198}{4165}$

(9) $P_9 = \dfrac{13 \times \binom{4}{2} \times \binom{12}{3} \times 4^3}{\binom{52}{5}} = \dfrac{1056}{2499}$

(10) $P_{10} = \dfrac{\left[\binom{13}{5} - 9\right](4^5 - 4)}{\binom{52}{5}} = \dfrac{639}{1274}$

11. $P = \dfrac{\binom{b+1}{r}}{\binom{b+r}{r}} = \dfrac{b! \cdot (b+1)!}{(b+r)!\,(b-r+1)!}$

12. (1) 若 $n > N$ $P = 0$

若 $n \leq N$，$P = \dfrac{\binom{N}{n}}{N^n}$

(2) $P = \dfrac{\binom{N+n-1}{n}}{N^n}$