第二周第二次作业:

P39-40. 1. P(至少一个女孩|有男孩) = $\frac{P(\text{至少一个女孩且有男孩})}{P(\text{有男孩})} = \frac{6}{7}$

5. 相当于任取两球求两球同色概率.

$$P = \frac{\binom{5}{2}\times 2}{\binom{10}{2}} = \frac{20}{45} = \frac{4}{9}$$

6 P(坠落在1|未发现残骸) = $\frac{P(\text{坠在1且未发现残骸})}{P(\text{未发现残骸})}$

$$= \frac{\frac{1}{3}\times(1-\alpha_1)}{\frac{1}{3}(1+1+1-\alpha_1)} = \frac{1-\alpha_1}{3-\alpha_1}$$

P(坠落在2|未发现残骸) = P(坠落在3|未发现残骸)

$$= \frac{1}{3-\alpha_1}$$

8. 归纳法证明 $p_n = \frac{1}{2} + (c-\frac{1}{2})(2p-1)^{n-1}$

$n=1$ 成立.

若 $n=k$ 成立. 则 $n=k+1$ 时 $p_{k+1} = p_k p + (1-p_k)(1-p) = \frac{1}{2} + (c-\frac{1}{2})(2p-1)^{k-1}$

当 $n\to\infty$ 时. $|2p-1|<1$, 故 $|2p-1|^{n-1}\to 0$

从而 $p_n \to \frac{1}{2}$

9. 同上. 归纳法证明 $q_n = \frac{2}{3} + \frac{1}{3}(-\frac{1}{2})^n$

$p_n = r_n = \frac{1}{4} q^{n-1}$

10. (1) 传 n 次共有 $(r-1)^{n-1}$ 种情形, 未到甲手中 $(r-2)^{n-1}$ 种

$$P_1 = \frac{(r-2)^{n-1}}{(r-1)^{n-1}} = \left(1-\frac{1}{r-1}\right)^{n-1}$$

(2) $P = 1 \times \left(1-\frac{1}{r-1}\right) \times \left(1-\frac{2}{r-1}\right) \times \cdots \times \left(1-\frac{n-1}{r-1}\right)$

$= \frac{(r-2)(r-3)\cdots(r-n)}{(r-1)^{n-1}}$

(3) 记 p_k 为第 k 次由甲传出.

则 $p_1 = 1$. $p_2 = 0$, $p_{k+1} = \frac{1}{r-1}(1-p_k)$

$\Rightarrow p_n = \frac{1}{r}\left[1-\left(\frac{1}{1-r}\right)^{n-2}\right]$

11. P(第n次首次出现2个正面连续)

$= P$(第n次正)$\cdot P$(第(n-1)次正)$\cdot P$((n-2)次反) P(1.2 ⋯,(n-3)中无连续正)

$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times \frac{\sqrt{5}}{5}\times\left[(1+\sqrt{5})^{n-1} - (1-\sqrt{5})^{n-1}\right]\times\frac{1}{4^{n-2}}$

$= \frac{1}{2\sqrt{5}}\times\left[\left(\frac{1+\sqrt{5}}{4}\right)^{n-1} - \left(\frac{1-\sqrt{5}}{4}\right)^{n-1}\right]$

P48. 1. 3个孩子家庭中. $P(E)P(F) = \frac{1}{2}\times\frac{3}{4} = \frac{3}{8}$

$P(EF) = \frac{3}{8} = P(E)P(F) \Rightarrow E.F$ 独立

2个孩子家庭中: $P(E)P(F) = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$

$P(EF) = \frac{1}{2} \neq P(E)P(F) \Rightarrow E.F$ 不独立

7. (1) $P = (1-0.4)\times(1-0.5)\times 0.7 + (1-0.4)\times 0.5\times(1-0.7) + 0.4\times(1-0.5)\times(1-0.7)$

$= 0.36$

(2) $P = 1-(1-0.4)\times(1-0.5)\times(1-0.7) = 0.91$

10. 对 $\forall\ 1 \le i_1 < i_2 < \cdots < i_k \le n$.

$P(A_{i_1})P(A_{i_2}) \cdot P(A_{i_k})$

$$= \sum_{\substack{\{j_t\}_{1\le t\le n-k} \\ \{j_t\}\cap\{i_1,\cdots,i_k\}=\phi}} \sum_{\hat{A}_{j_t}\in\{A_{j_t},\bar{A}_{j_t}\}} P(A_{i_1})P(A_{i_2})\cdots P(A_{i_k}) \prod_{1\le t\le n-k} P(\hat{A}_{j_t})$$

$$= \sum_{\substack{\{j_t\}_{1\le t\le n-k} \\ \{j_t\}\cap\{i_1,\cdots,i_k\}=\phi}} \sum_{\hat{A}_{j_t}\in\{A_{j_t},\bar{A}_{j_t}\}} P(A_{i_1}A_{i_2}\cdots A_{i_k} \prod_{1\le t\le n-k} \hat{A}_{j_t})$$

$$= P(A_{i_1}A_{i_2}\cdots A_{i_k})$$

反之亦然. 故"$\Leftrightarrow$"得证.

11. 若 A.B 独立. 则 $P(A|B)=P(A|\bar{B})=P(A)$.

反之若 $P(A|B)=P(A|\bar{B})$ 则: $\dfrac{P(AB)}{P(B)} = \dfrac{P(A\bar{B})}{P(\bar{B})}$

$\Rightarrow P(AB)(1-P(B)) = (P(A)-P(AB))P(B)$

$\Rightarrow P(AB)=P(A)P(B)$

$\Rightarrow$ A.B indepdant

13. (1) $P(A|BC)=P(A|C) \Leftrightarrow \dfrac{P(ABC)}{P(BC)} = \dfrac{P(AC)}{P(C)}$

$\Leftrightarrow \dfrac{P(ABC)}{P(C)} = \dfrac{P(AC)}{P(C)}\dfrac{P(BC)}{P(C)}$

$\Leftrightarrow P(AB|C) = P(A|C)P(B|C)$

(2) 设 A B 独立. C 表示 A与B不等.

则 $P(A|C)P(B|C) = 0 \ (A=B)$

而 $P(AB|C)\neq 0$.

条件独立而不独立的例子乎凡