第五周第二次作业.

中文教材 P86-87 1. $P\{\xi=k\}=\frac{1}{4}$, $k=1.2.3.4$

若$\xi=1$. 则$\eta=1$

若$\xi=2$. 则$P\{\eta=1\}=P\{\eta=2\}=\frac{1}{2}$

若$\xi=3$. 则$P\{\eta=k\}=\frac{1}{3}$. $k=1.2.3$

若$\xi=4$ 则$P\{\eta=k\}=\frac{1}{4}$, $k=1.2.3.4$

故(ξ,η)的联合分布为:

(ξ,η)	(1,1)	(2,1)	(2,2)	(3,1)	(3,2)	(3,3)	(4,1)	(4.2)	(4,3)	(4.4)
P	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$

边缘分布:

$P\{\xi=k\}=\frac{1}{4}$, $k=1.2.3.4$

$P\{\eta=1\}=\frac{25}{48}$, $P\{\eta=2\}=\frac{13}{48}$, $P\{\eta=3\}=\frac{7}{48}$, $P\{\eta=4\}=\frac{1}{16}$

4. $p(x,y)=\frac{f(x+y)}{x+y}$ $x,y>0$

$f(x)$为密度函数 ⇒ 右连续 ⇒ $p(x,y)$右连续

$$\int_0^{+\infty}\int_0^{+\infty} p(x,y)\,dxdy = \int_0^{+\infty}dy\int_0^{+\infty}\frac{f(x+y)}{x+y}dx$$

$$= \int_0^{+\infty}dt\int_0^{t}\frac{f(t)}{t}dx \quad (换元.\ t=x+y)$$

$$= \int_0^{+\infty}f(t)dt = 1$$

⇒ $p(x,y)$为联合密度函数

6. $p(x,y)=cxy^2,\ 0<x<2,\ 0<y<1$ 为密度函数

(1) $\int_0^2\int_0^1 cxy^2\,dx\,dy=1 \Rightarrow \int_0^2 \frac{c}{3}x\,dx=1 \Rightarrow c=\frac{3}{2}$

(2) $P\{\min\{\xi,\eta\}<\frac{1}{2}\}=1-P\{\xi\geq\frac{1}{2},\eta\geq\frac{1}{2}\}=1-\int_{\frac{1}{2}}^{1}\int_{\frac{1}{2}}^{1} cxy^2\,dx\,dy=\frac{23}{128}$

9. $(\xi,\eta)\sim N(a_1,\sigma_1^2,a_2,\sigma_2^2,r)$

$P\{(\xi,\eta)\in D\}=1-\exp\left[-\frac{\lambda^2}{2(1-r^2)}\right]$

10. 设随机变量 $\xi\sim N(0,1)$.

则左式 $=\frac{1}{\sqrt{2\pi}}\int_{-a}^{a}e^{-\frac{x^2}{2}}dx=P\{\xi\in(-a,a)\}$

右式 $=\sqrt{1-e^{-a^2}}=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{a}e^{-\frac{x^2}{2}}dx=P\{\xi\in(-\infty,a)\}$.

故左式<右式

P92 4. (1) $p_\xi(\frac{1}{2})=\int_0^{\frac{1}{2}}p(\frac{1}{2},y)\,dy=\frac{1}{2}$

$p_\eta(y|\xi=\frac{1}{2})=\frac{p(\frac{1}{2},y)}{p_\xi(\frac{1}{2})}=2p(\frac{1}{2},y)=2\times24y(\frac{1}{2}-y)=24y-48y^2$

(2) $p_\eta(\frac{1}{2})=\int_0^{\frac{1}{2}}p(x,\frac{1}{2})\,dx=\frac{3}{2}$

$p_\xi(x|\eta=\frac{1}{2})=\frac{p(x,\frac{1}{2})}{p_\eta(\frac{1}{2})}=\frac{2}{3}p(x,\frac{1}{2})=4-8x$

5. (ξ,η) r.v. and ξ's density $p_1(x)=\lambda^2xe^{-\lambda x}\quad(x>0)$

$\eta\sim U(0,\xi)$

$p_\eta(y)=\int_y^{+\infty}p_1(x)\cdot\frac{1}{x}\,dx=\int_y^{+\infty}\lambda^2xe^{-\lambda x}\,dx=\lambda e^{-\lambda y}$

7. $p(x,y)=\frac{1+xy}{4},\ |x|<1$ 且 $|y|<1$

$p_1(x) = \int_{-1}^{1} p(x,y)\,dy = \frac{1}{2}$ 同理 $p_2(y) = \frac{1}{2}$

$p(x,y) \neq p_1(x)p_2(y)$ a.s. $\Rightarrow \xi$与η不独立

记 $\xi' = \xi^2$. $\eta' = \eta^2$. 设 (ξ', η')的联合密度函数 $p'(x,y)$, $0 \leq x, y < 1$

则 $p'(x,y) = \frac{1}{4}$. $p_1'(x) = p_2'(y) = \frac{1}{2} \Rightarrow \xi^2$与$\eta^2$不独立.

8. $P\{\xi=1\} = P\{\xi=-1\} = \frac{1}{2}$, $P\{\eta=1\} = P\{\eta=-1\} = \frac{1}{2}$

因ξ. η取值互不影响. 故ξ与η独立.

同理 ξ与$\xi\eta$独立. η与$\xi\eta$独立.

但 $P\{\xi=1, \eta=1, \xi\eta=1\} = \frac{1}{4}$

$P\{\xi=1\}P\{\eta=1\}P\{\xi\eta=1\} = \frac{1}{8}$

故这三个变量不独立.