

## Hybleland L9-Lesson 17 Solid Geometry III

### Distance and Counting-Assignment

#### Problem 1. (AMC)

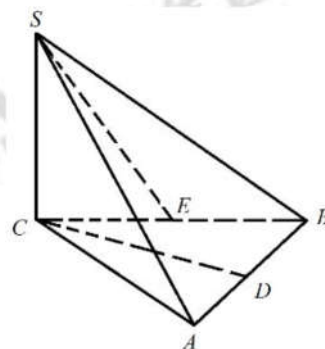
On a  $4 \times 4 \times 3$  rectangular parallelepiped, vertices A, B, and C are adjacent to vertex D. The perpendicular distance from D to the plane containing A, B, and C is

- (A)  $\frac{\sqrt{34}}{17}$       (B)  $\frac{12\sqrt{17}}{17}$       (C)  $\frac{6\sqrt{34}}{17}$       (D) 3      (E)  $\frac{17\sqrt{34}}{6}$

#### Problem 2.

In a tetrahedron with vertices S-ABC as shown,  $\triangle ABC$  is an equilateral triangle with the side length of  $4\sqrt{2}$ .  $SC=2$  and is perpendicular to  $\triangle ABC$ . E is the midpoint of BC. D is the midpoint of AB. The perpendicular distance from the line segments CD to SE is

- (A)  $\frac{2\sqrt{3}}{3}$       (B)  $\frac{3\sqrt{2}}{3}$       (C)  $\frac{2\sqrt{2}}{3}$       (D)  $\frac{\sqrt{3}}{3}$       (E)  $\frac{\sqrt{3}}{2}$



#### Problem 3.

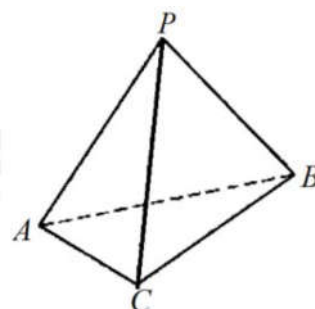
The edges of a regular tetrahedron with vertices A, B, C, and D each have length one. Find the smallest possible distance between a pair of points P and Q, where P is on the edge AB and Q is on the edge CD.

- (A)  $\frac{\sqrt{2}}{2}$       (B)  $\frac{\sqrt{3}}{2}$       (C) 2      (D) 1      (E)  $\frac{\sqrt{2}}{3}$

Problem 4.

In a tetrahedron P-ABC.  $PA = PB = PC = 2\sqrt{13}$ .  $BC = 8$ ,  $AC = 4$ , and  $AB = 4\sqrt{3}$ . The distance from point P to the plane  $\triangle ABC$  is 6. Find the distance from point B to the plane  $\triangle PAC$ .

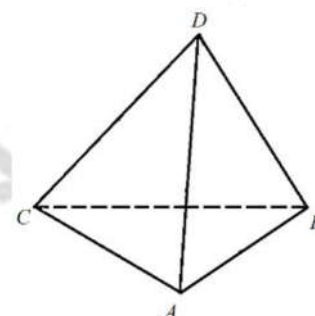
- (A)  $\frac{8\sqrt{3}}{3}$       (B)  $4\sqrt{3}$       (C) 6      (D)  $\frac{\sqrt{7}}{4}$       (E) 4



Problem 5.

In a regular tetrahedron D-ABC,  $AB = CD = 4$  and  $AC = BC = BD = 3$ . M is the midpoint of AB. N is the midpoint of CD. The perpendicular distance from N to M is

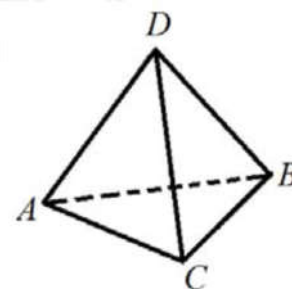
- (A)  $\frac{\sqrt{2}}{2}$       (B)  $\frac{\sqrt{3}}{2}$       (C) 2      (D) 1      (E)  $\frac{\sqrt{2}}{3}$



Problem 6.

In a tetrahedron with vertices D-ABC as shown, each of its four congruent triangular faces has the side lengths of 13, 14, and 15. The perpendicular distance from D to the plane containing A, B, and C is

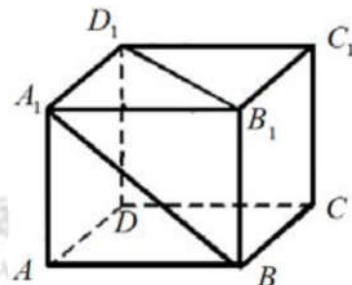
- (A)  $\frac{\sqrt{34}}{17}$       (B)  $\frac{12\sqrt{17}}{17}$       (C)  $\frac{6\sqrt{34}}{17}$       (D) 3      (E)  $\frac{17\sqrt{34}}{6}$



Problem 7.

In the adjoining figure,  $A, B, C, D, A_1, B_1, C_1, D_1$  are the vertices of a unit cube. The perpendicular distance from  $A_1B$  to  $D_1B_1$  is

- (A)  $\frac{\sqrt{3}}{2}$       (B)  $\frac{\sqrt{3}}{4}$       (C)  $\frac{2\sqrt{3}}{5}$       (D)  $\frac{\sqrt{3}}{3}$       (E)  $\frac{\sqrt{2}}{2}$



Problem 8.

AMC10B 2011 / Problem 22

A pyramid has a square base with sides of length 1 and has lateral faces that are equilateral triangles. A cube is placed within the pyramid so that one face is on the base of the pyramid and its opposite face has all its edges on the lateral faces of the pyramid. What is the volume of this cube?

- A.  $5\sqrt{2} - 7$     B.  $7 - 4\sqrt{3}$     C.  $\frac{2\sqrt{2}}{27}$     D.  $\frac{\sqrt{2}}{9}$     E.  $\frac{\sqrt{3}}{9}$

Problem 9.

AMC10A 2016 / Problem 18

Each vertex of a cube is to be labeled with an integer 1 through 8, with each integer being used once, in such a way that the sum of the four numbers on the vertices of a face is the same for each face. Arrangements that can be obtained from each other through rotations of the cube are considered to be the same. How many different arrangements are possible?

- A. 1    B. 3    C. 6    D. 12    E. 24

Problem 10.

AMC10B 2015 / Problem 20

Erin the ant starts at a given corner of a cube and crawls along exactly 7 edges in such a way that she visits every corner exactly once and then finds that she is unable to return along an edge to her starting point. How many paths are there meeting these conditions?

- A. 6   B. 9   C. 12   D. 18   E. 24

Problem 11.

AMC10A 2009 / Problem 24

Three distinct vertices of a cube are chosen at random. What is the probability that the plane determined by these three vertices contains points inside the cube?

- A.  $\frac{1}{4}$    B.  $\frac{3}{8}$    C.  $\frac{4}{7}$    D.  $\frac{5}{7}$    E.  $\frac{3}{4}$