TP2

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EX5.1

on pose , $Y = |X|$ Donc , dx = -dy quand x < 0 et dx = dy quand x > 0

$$\mathbb{E}_X[h(x)] = \int_{-\infty}^{+\infty} h(x) f_X(x) dx$$

$$= \int_{-\infty}^{0} h(x) f_X(x) dx + \int_{0}^{\infty} h(x) f_X(x) dx$$
$$= \int_{0}^{\infty} h(-x) f_X(-x) d(-x) + \int_{0}^{\infty} h(x) f_X(x) dx$$
$$= \int_{0}^{\infty} h(y) f_X(y) d(y) + \int_{0}^{\infty} h(y) f_X(y) dy$$
$$= \int_{0}^{\infty} h(y) 2 f_X(y) dy = E_Y[h(y)]$$

Donc

$f_Y(y) = 2 f_X(y) \mathbb{I}_{y>0}$

EX5.2

EX5.3

```r
rexpoentielle <- function(N,lambda)
{
  X <- rep(0,N)
  for(i in 1:N)
  {
  u <- runif(1)
  X[i] <- -log(u)/lambda
  }
  return(X)
}

r_abs_normal <- function(N)
{
  pi <- 3.1415926
  X <- rep(0,N)
  i <- 1
  for(i in 1:N)
  {
    u <- runif(1)
    Y <- rexpoentielle(1,1)
```

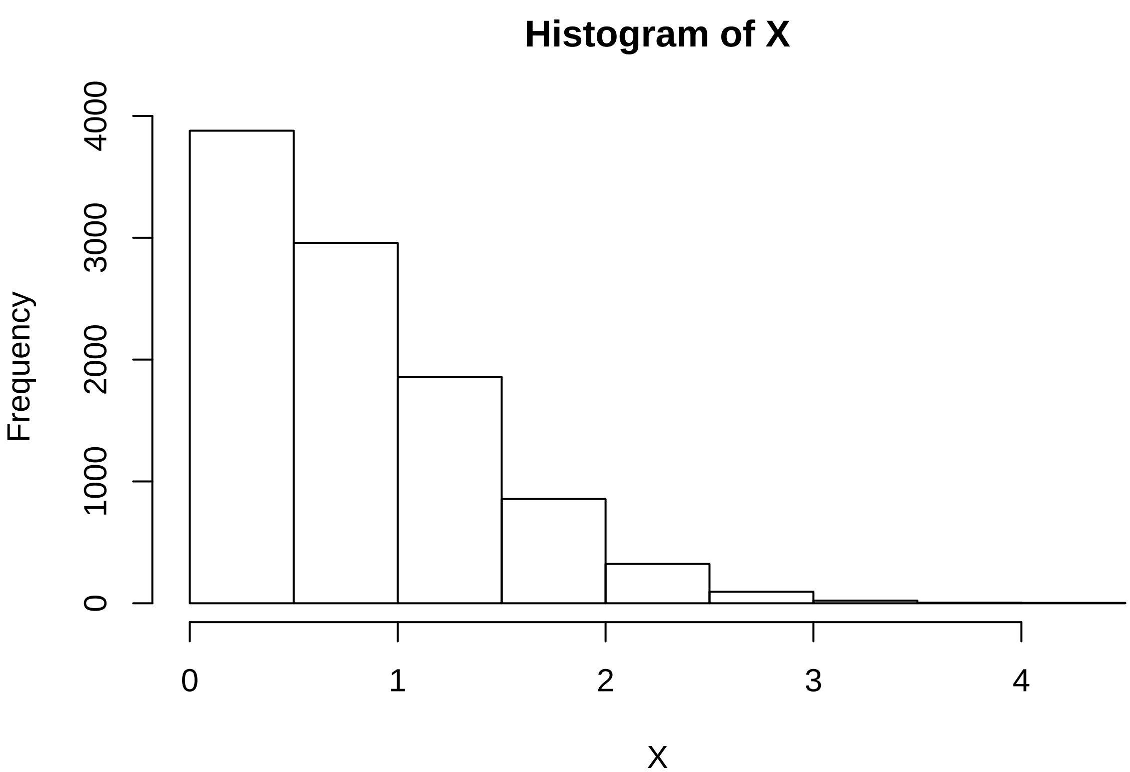
```r
    c <- sqrt(2*exp(1)/pi)
    fY <- 2/sqrt(2*pi)*exp((-1/2)*Y^2)
    gY <- exp(-Y)
    hY <- fY/(c*gY)
    while(u>hY)
    {
      u <- runif(1)
      Y <- rexpoentielle(1,1)
      c <- sqrt(2*exp(1)/pi)
      fY <- 2/sqrt(2*pi)*exp((-1/2)*Y^2)
      gY <- exp(-Y)
      hY <- fY/(c*gY)
    }
    X[i]=Y
  }
  return(X)
}

r_abs_normal(10)
```

```
##  [1] 0.80529503 1.17371905 0.03637641 0.14986678 1.39813868 1.06588321
##  [7] 0.07328164 0.25873669 2.14929340 0.39294355
```

EX5.4

```r
X <- r_abs_normal(10000)
hist(X)
```

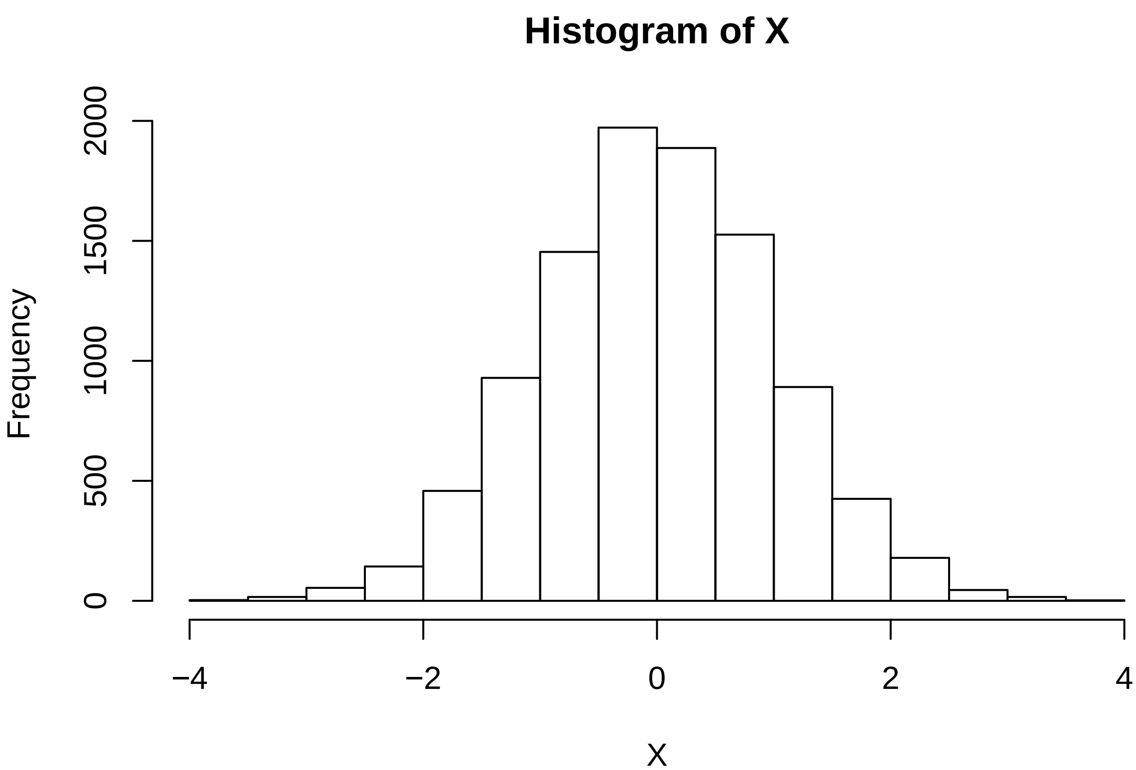


###EX5.5

EX5.6

```r
r_normal <- function(N)
{
  X <- rep(0,N)
  for(i in 1:N)
  {
    u <- runif(1)
    if(u<1/2)
    {
      X[i] <-  r_abs_normal(1)
    }
    else{
      X[i] <- -r_abs_normal(1)
    }
  }
  return (X)
}

X <- r_normal(10000)
hist(X)
```



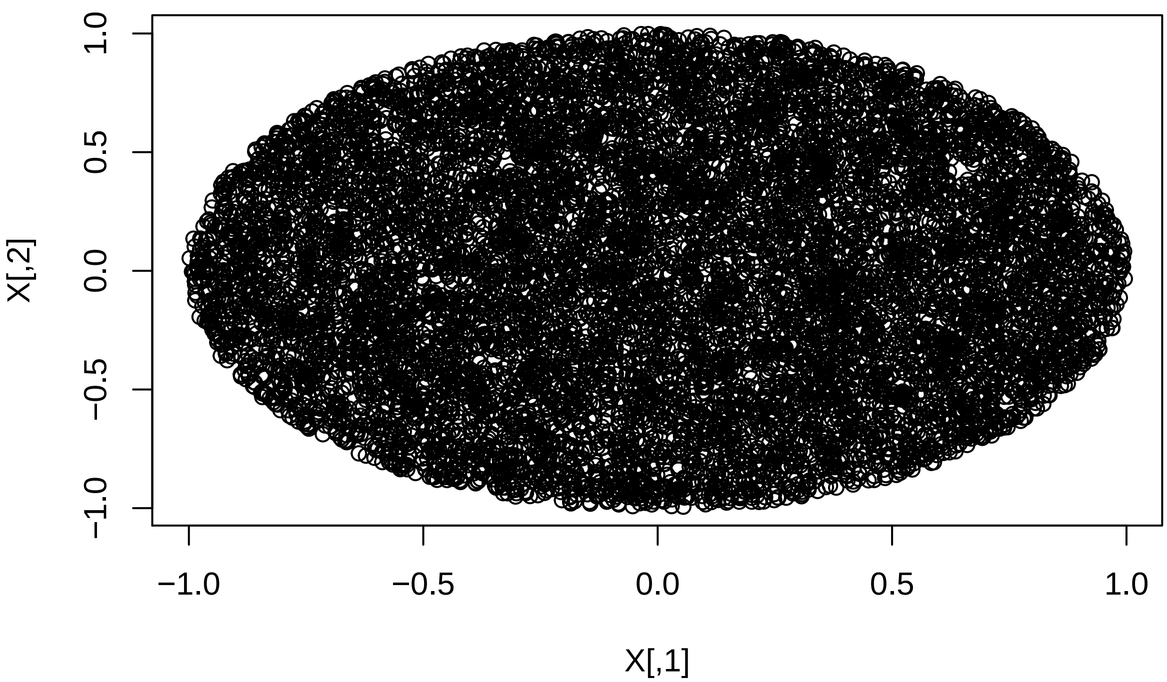
EX6.1

```r
r_unif_2dim <- function(N)
{
  U <- array(dim = c(N,2))
  for(i in 1:N)
```

```
  {
  u1 <- 2*runif(1)-1
  u2 <- 2*runif(1)-1
  while(u1^2 + u2^2 >1)
  {
  u1 <- 2*runif(1)-1
  u2 <- 2*runif(1)-1
  }
     U[i,1] <- u1
     U[i,2] <- u2
  }
  return(U)
}
```

EX6.2 (Méthode de Box-Muller)

```
X <- r_unif_2dim(10000)
plot(X)
```



EX7.1

On a $X_1 = \sqrt{R}cos(\Theta)$ et $X_2 = \sqrt{R}sin(\Theta)$

Par consequence , on peut déduire que $R = X_1^2 + X_2^2$ et $\Theta = arctan(X_2/X_1)$

On sais que $f_{(X_1,X_2)}(X) = f_{(R,\Theta)}(R,\Theta) * det(J(X)) = f_R(R) * f_\Theta(\Theta) * det(J(X))$

Donc, on dois juste calculer J(X)

$$J(X) = \begin{pmatrix} \frac{\partial R}{\partial X_1} & \frac{\partial R}{\partial X_2} \\ \frac{\partial \Theta}{\partial X_1} & \frac{\partial \Theta}{\partial X_2} \end{pmatrix}$$

$$= \begin{pmatrix} 2X_1 & 2X_2 \\ \frac{-X_2}{X_1^2+X_2^2} & \frac{X_1}{X_1^2+X_2^2} \end{pmatrix}$$

Alors , $det(J(X)) = \frac{2(X_1^2+X_2^2)}{X_1^2+X_2^2} = 2$

Finalement , on a

$$f_{(X_1,X_2)}(x) = f_R(r) * f_\Theta(\theta) * det(J(X)) = \frac{1}{2\pi} exp(-(x_1^2+x_2^2)/2) = \frac{1}{\sqrt{2\pi}} exp(-x_1^2/2) * \frac{1}{\sqrt{2\pi}} exp(-x_2^2/2)$$

ils sont independentes car $f_{(X_1,X_2)}(x) = f_{X_1}(x_1) f_{X_2}(x_2)$