

## Mean-field variational inference

Coordinate ascent mean-field variational inference

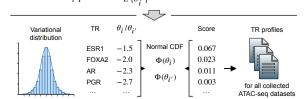
Mean-field variational family

$$\begin{split} q\left(\Theta\right) &= q\left(\mu\right) q\left(\tau^{2}\right) q\left(\sigma_{0}^{2}\right) \prod_{i=1}^{M} q\left(\sigma_{i}^{2}\right) \prod_{i=1}^{M} q\left(\theta_{i}\right) \prod_{i=1}^{M} q\left(\theta_{i'}\right) \times \\ &= \prod_{i=1}^{M} \prod_{j=1}^{J_{i}} q\left(\theta_{ij}\right) \prod_{i'=1}^{M^{c}} q\left(\theta_{i'1}\right) \prod_{i=1}^{M} \prod_{j=1}^{K_{ij}} q\left(\varepsilon_{ijk}\right) \prod_{i=1}^{M^{c}} \prod_{k=1}^{K_{i'1}} q\left(\varepsilon_{i'1k}\right) \end{split}$$

Update the expectation of factors

$$E \ [z_{ijk}] = \begin{cases} E \ [\theta_{ij}] + \phi \ (E \ [\theta_{ij}])/(1 - \Phi(E \ [\theta_{ij}])) & \text{Use coordinate ascent to find optimal:} \\ e \ [\theta_{ij}] - \phi \ (E \ [\theta_{ij}])/(\Phi(E \ [\theta_{ij}])) & q_j^*(\theta_j) \propto \exp \left\{ E_{-j} \left[ \ln \left( p \ (\theta_j \ | \ \Theta_{-j}, X \ ) \right) \right] \right\} \\ e \ [\theta_{ij}] - \phi \ (E \ [\theta_{ij}])/(\Phi(E \ [\theta_{ij}])) & \text{Use coordinate ascent to find optimal:} \\ e \ [\theta_{ij}] - \phi \ (E \ [\theta_{ij}])/(\Phi(E \ [\theta_{ij}])) & \text{Use coordinate ascent to find optimal:} \\ e \ [\theta_{ij}] - \phi \ (E \ [\theta_{ij}])/(\Phi(E \ [\theta_{ij}])) & \text{Use coordinate ascent to find optimal:} \\ e \ [\theta_{ij}] - \phi \ (E \ [\theta_{ij}])/(\Phi(E \ [\theta_{ij}])) & \text{Use coordinate ascent to find optimal:} \\ e \ [\theta_{ij}] - \phi \ (E \ [\theta_{ij}])/(\Phi(E \ [\theta_{ij}])) & \text{Use coordinate ascent to find optimal:} \\ e \ [\theta_{ij}] - \phi \ (E \ [\theta_{ij}])/(\Phi(E \ [\theta_{ij}])) & \text{Use coordinate ascent to find optimal:} \\ e \ [\theta_{ij}] - \phi \ (E \ [\theta_{ij}])/(\Phi(E \ [\theta_{ij}])) & \text{Use coordinate ascent to find optimal:} \\ e \ [\theta_{ij}] - \phi \ (E \ [\theta_{ij}])/(\Phi(E \ [\theta_{ij}])) & \text{Use coordinate ascent to find optimal:} \\ e \ [\theta_{ij}] - \phi \ (E \ [\theta_{ij}])/(\Phi(E \ [\theta_{ij}])) & \text{Use coordinate ascent to find optimal:} \\ e \ [\theta_{ij}] - \phi \ (E \ [\theta_{ij}])/(\Phi(E \ [\theta_{ij}])) & \text{Use coordinate ascent to find optimal:} \\ e \ [\theta_{ij}] - \phi \ (E \ [\theta_{ij}])/(\Phi(E \ [\theta_{ij}])) & \text{Use coordinate ascent to find optimal:} \\ e \ [\theta_{ij}] - \phi \ (E \ [\theta_{ij}])/(\Phi(E \ [\theta_{ij}])) & \text{Use coordinate ascent to find optimal:} \\ e \ [\theta_{ij}] - \phi \ (E \ [\theta_{ij}])/(\Phi(E \ [\theta_{ij}])) & \text{Use coordinate ascent to find optimal:} \\ e \ [\theta_{ij}] - \phi \ (E \ [\theta_{ij}])/(\Phi(E \ [\theta_{ij}])) & \text{Use coordinate ascent to find optimal:} \\ e \ [\theta_{ij}] - \phi \ (E \ [\theta_{ij}])/(\Phi(E \ [\theta_{ij}])) & \text{Use coordinate ascent to find optimal:} \\ e \ [\theta_{ij}] - \phi \ (E \ [\theta_{ij}])/(\Phi(E \ [\theta_{ij}])) & \text{Use coordinate ascent to find optimal:} \\ e \ [\theta_{ij}] - \phi \ (E \ [\theta_{ij}])/(\Phi(E \ [\theta_{ij}])) & \text{Use coordinate ascent to find optimal:} \\ e \ [\theta_{ij}] - \phi \ (E \ [\theta_{ij}])/(\Phi(E \ [\theta_{ij}])) & \text{Use coordinate ascent to find optimal:} \\ e \ [\theta_{ij}] - \phi \ (E \ [\theta_{ij}])$$

Similarly for all factors 
$$E\left[\theta_{ij}\right] E\left[\theta_{i}\right] E\left[\mu\right] \\ E\left[z_{ijk}\right] E\left[\theta_{i'}\right] E\left[\theta_{i'}\right] E\left[\tau^{2}\right] \\ E\left[z_{i'1k}\right] E\left[\theta_{i'}^{2}\right] E\left[\theta_{i'}^{2}\right] E\left[\sigma_{0}^{2}\right] \\ E\left[\theta_{i'1}^{2}\right] E\left[\theta_{i'}\right] E\left[\theta_{i'}\right] E\left[\sigma_{i}^{2}\right] \\ E\left[\theta_{i'1}^{2}\right] E\left[\theta_{i'}\right] E\left[\theta_{i'}\right] E\left[\theta_{i'}\right] \\ E\left[\theta_{i'1}^{2}\right] E\left[\theta_{i'}\right] E\left[\theta_{i'}\right] E\left[\theta_{i'}\right]$$
 Minimize KL divergence:  $KL\left(q_{opt}||p\left(\Theta|X\right)\right)$ 



## Integration results for TRex online portal d

