# Homework-3

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## Part-I: Deriving line equation in polar form [10 points]

In the first part of this assignment you will derive the equation for a line in polar form, given by:

$$x \cos\theta + y \sin\theta = \rho \tag{1}$$

The objective is to start from the general expression for a line in Cartesian coordinates, disclosed below:

$$a\,x + b\,y = c \tag{2}$$

Your task is to algebraically manipulate Eq(2) to get to Eq(1). The lecture slides provide clues to the derivation. **5 points**

- What is the geometric significance of terms $a, b$? (Hint: think intercepts) **3 points**
- How is $\rho$ related to $a, b, c$? **1 point**
- How is $\theta$ related to $a, b, c$? **1 point**

Problem 1.

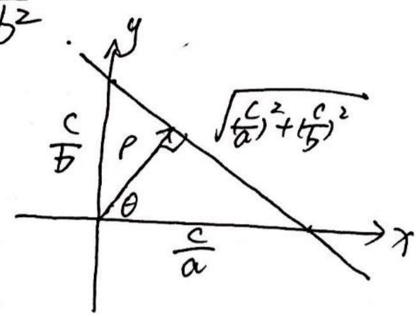
* Derivation :

General expression for line

$$ax + by = c$$

Rewrite as

$$\frac{a}{\sqrt{a^2+b^2}}x + \frac{b}{\sqrt{a^2+b^2}}y = \frac{c}{\sqrt{a^2+b^2}}.$$




The figure shows that

when $x=0$ $\quad y=\frac{c}{b}$

when $y=0$ $\quad x=\frac{c}{a}$

So the area of triangle can be calculated by

$$\frac{1}{2}\frac{c}{b}\cdot\frac{c}{a} = \frac{1}{2}\rho\cdot\sqrt{(\frac{c}{a})^2+(\frac{c}{b})^2}.$$

$$\Rightarrow \rho = \frac{c}{\sqrt{a^2+b^2}}$$

$$\cos\theta = \frac{\rho}{\frac{c}{b}} = \frac{b}{\sqrt{a^2+b^2}} \qquad \sin\theta = \frac{\rho}{\frac{c}{a}} = \frac{a}{\sqrt{a^2+b^2}}.$$

- What is the geometric significance of terms a, b?

  the horizontal intercept is $|\frac{c}{a}|$.

  the vertical intercept is $|\frac{c}{b}|$

- How is $\rho$ related to a, b, c?

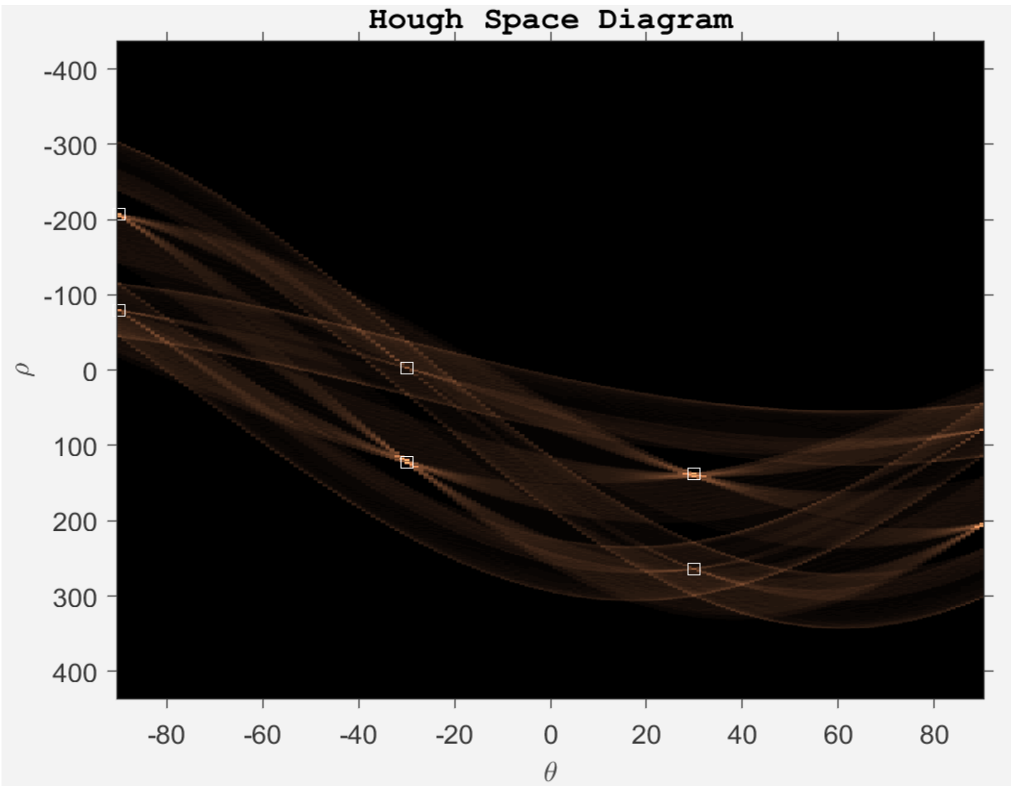
  $\rho = \frac{c}{\sqrt{a^2+b^2}}.$

- How is $\theta$ related to a, b, c?

  $\cos\theta = \frac{b}{\sqrt{a^2+b^2}} \qquad \sin\theta = \frac{a}{\sqrt{a^2+b^2}}.$

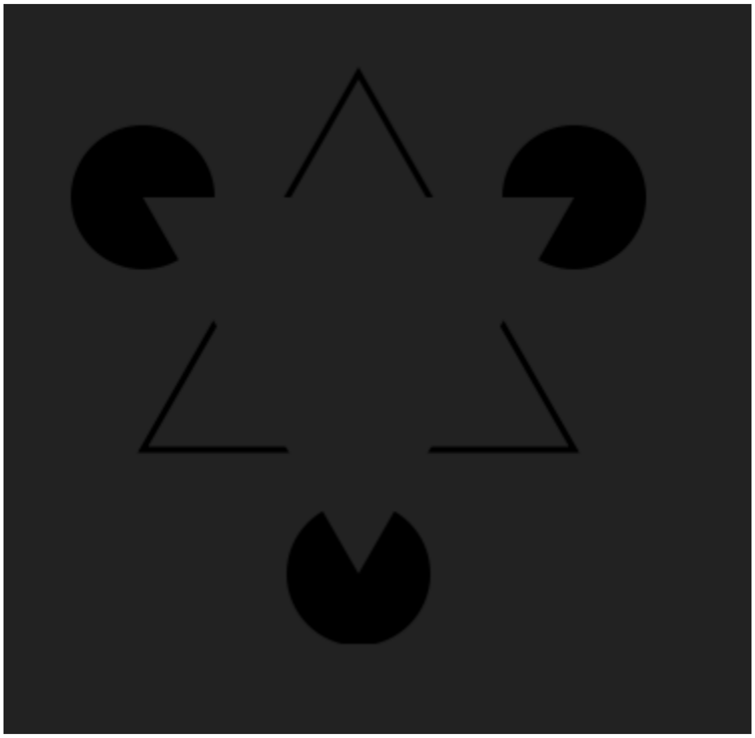
# Part-II: Hough Transform [90 points]

Screenshot of the Hough Space diagram with peaks overlaid for kanizsa.png.

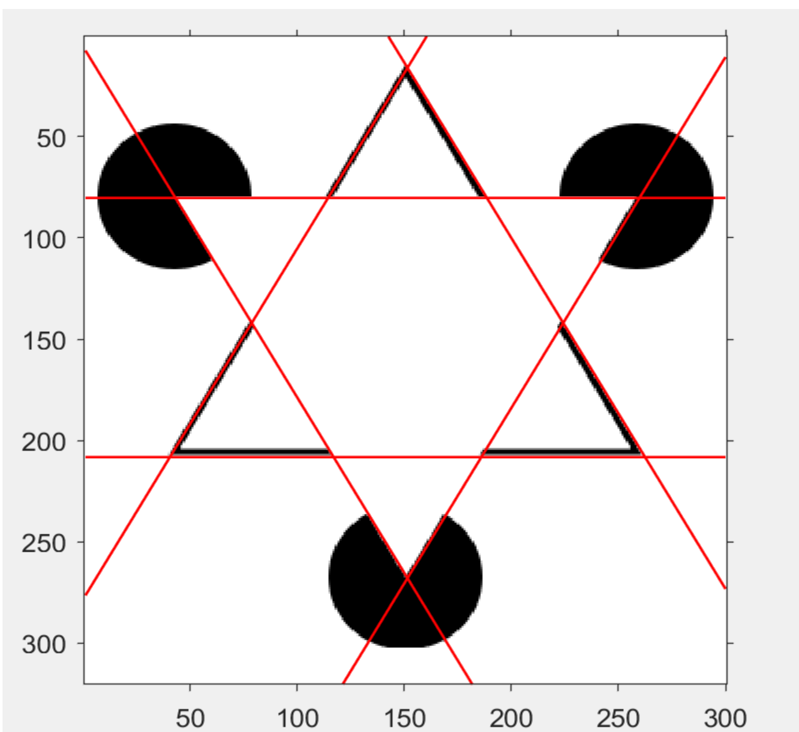



Screenshot of Hough lines overlaid on the image.

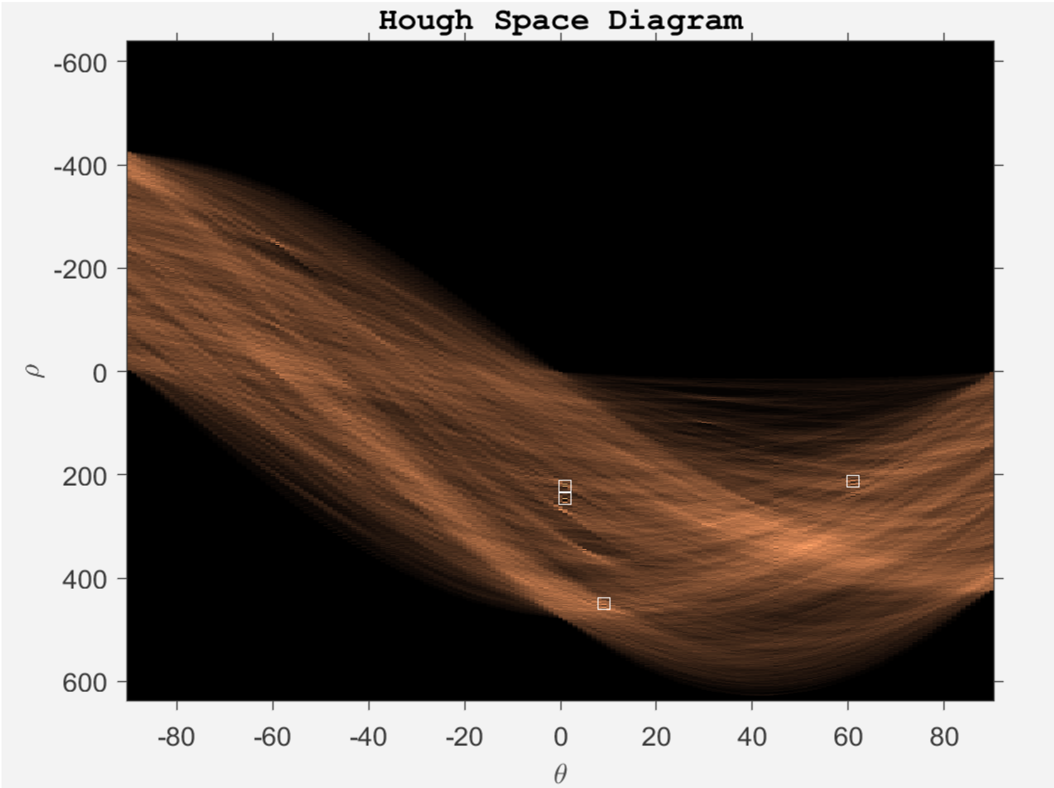
Original image:



Best fitting line:



Screenshot of the Hough Space diagram with peaks overlaid for runway.jpg.

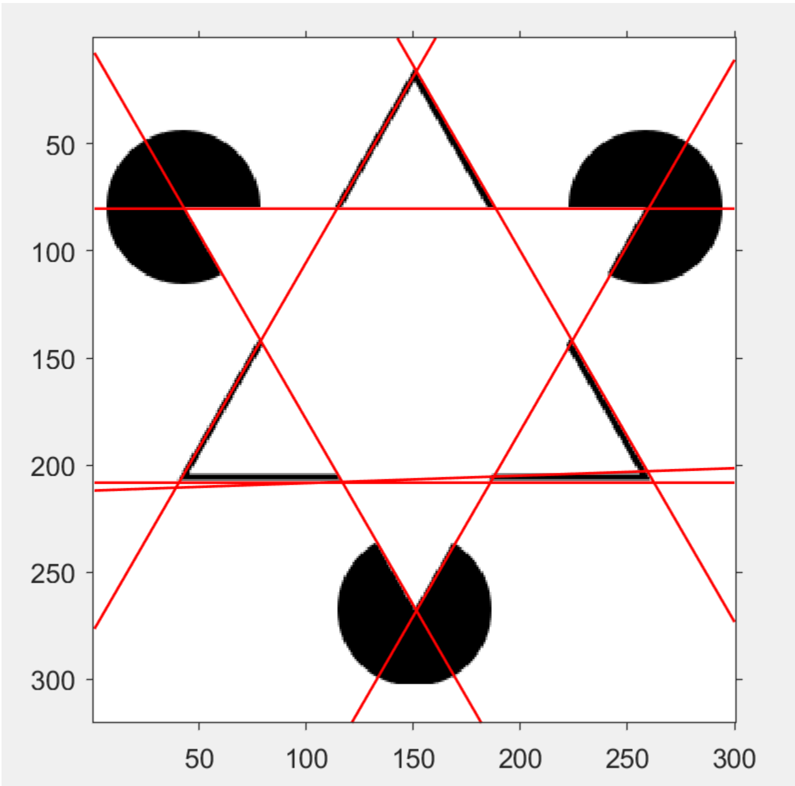



Screenshot of Hough lines overlaid on the image.

Original input image:

Best fitting line:

Change the linecount to 8, I got following picture:



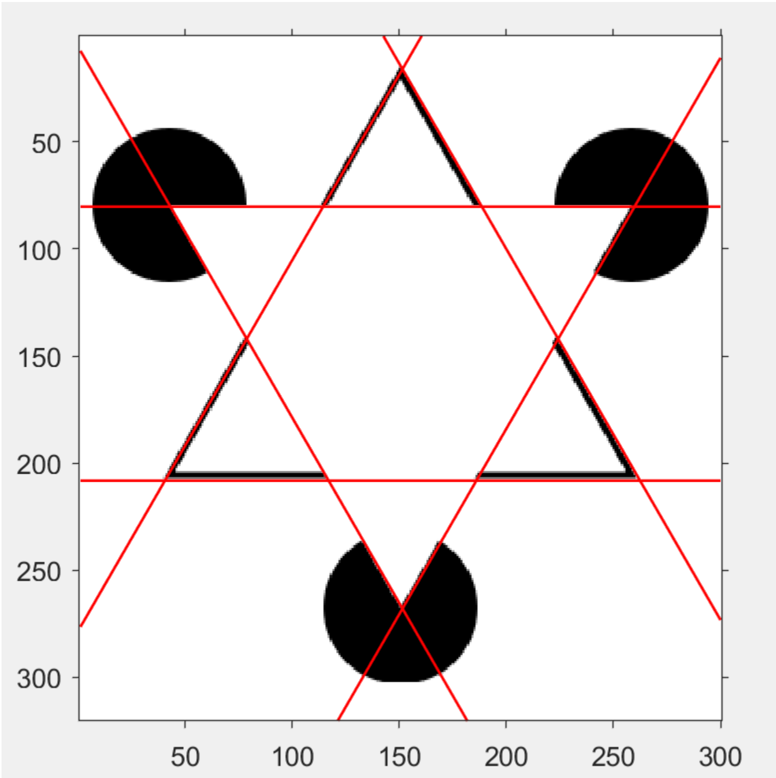

We got another 2 lines which are very close to the best fitting lines. Because there are cluster of bins with high votes, if we get bins in same cluster, the lines will be close.

**Extra Credit:**

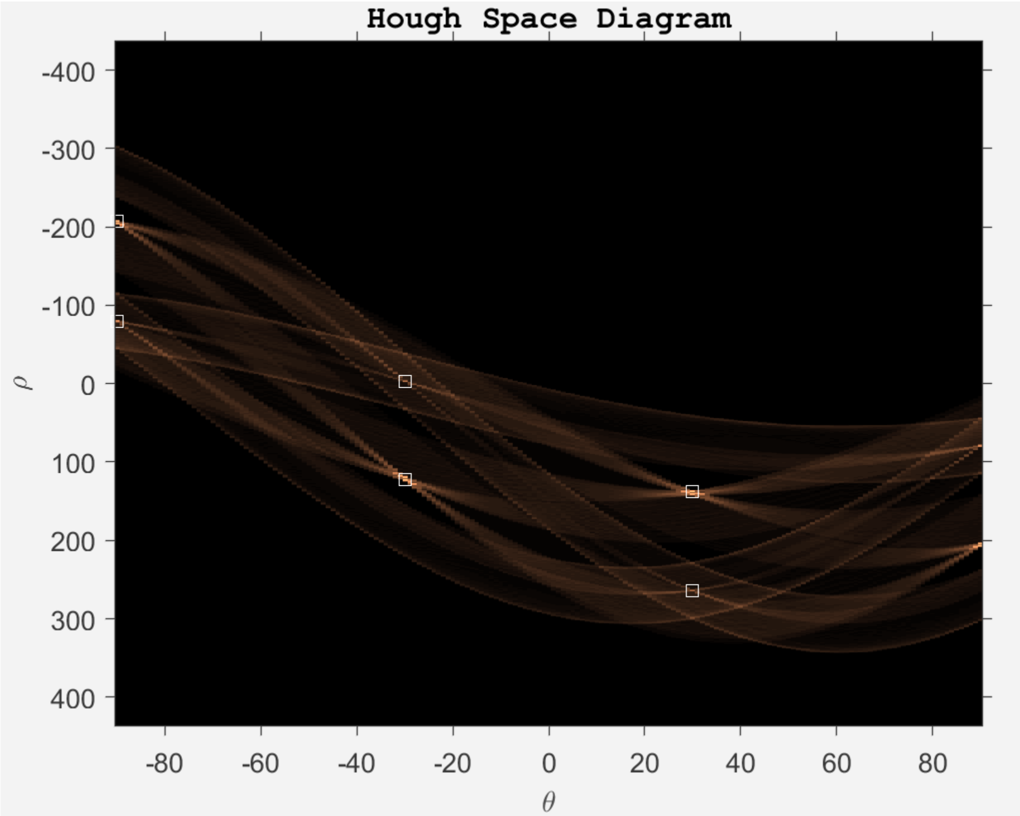
I modify the voting procedure by following code:

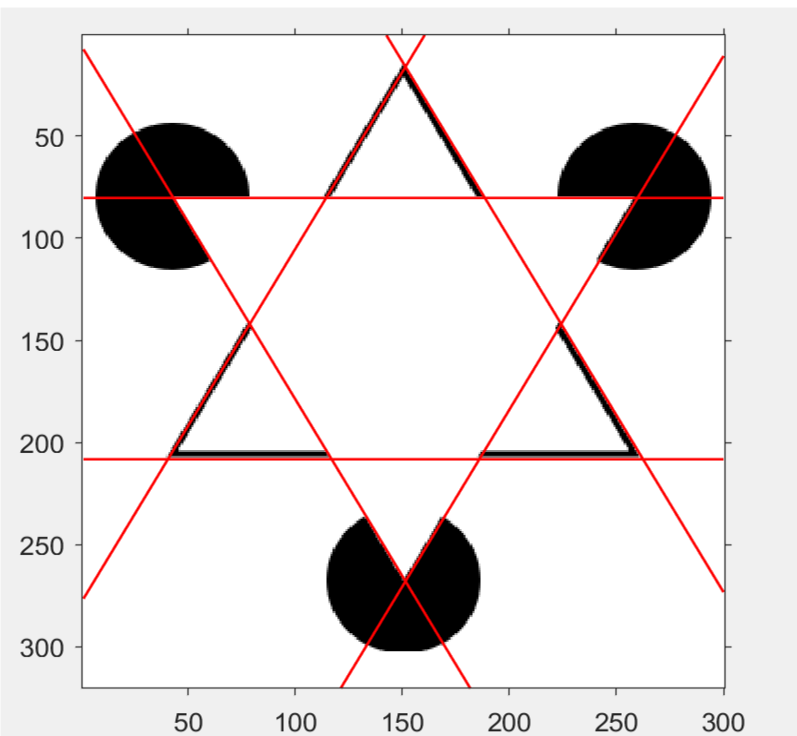
```
[Gx, Gy] = (imgradientxy(im));
            houghSpace(loc,k) =
houghSpace(loc,k)+round(sqrt(Gx(y,x)^2+Gy(y,x)^2));
```

for image 1, I got following figure with approach that votes are incremented by an integer amount that is proportional to the edge strength.

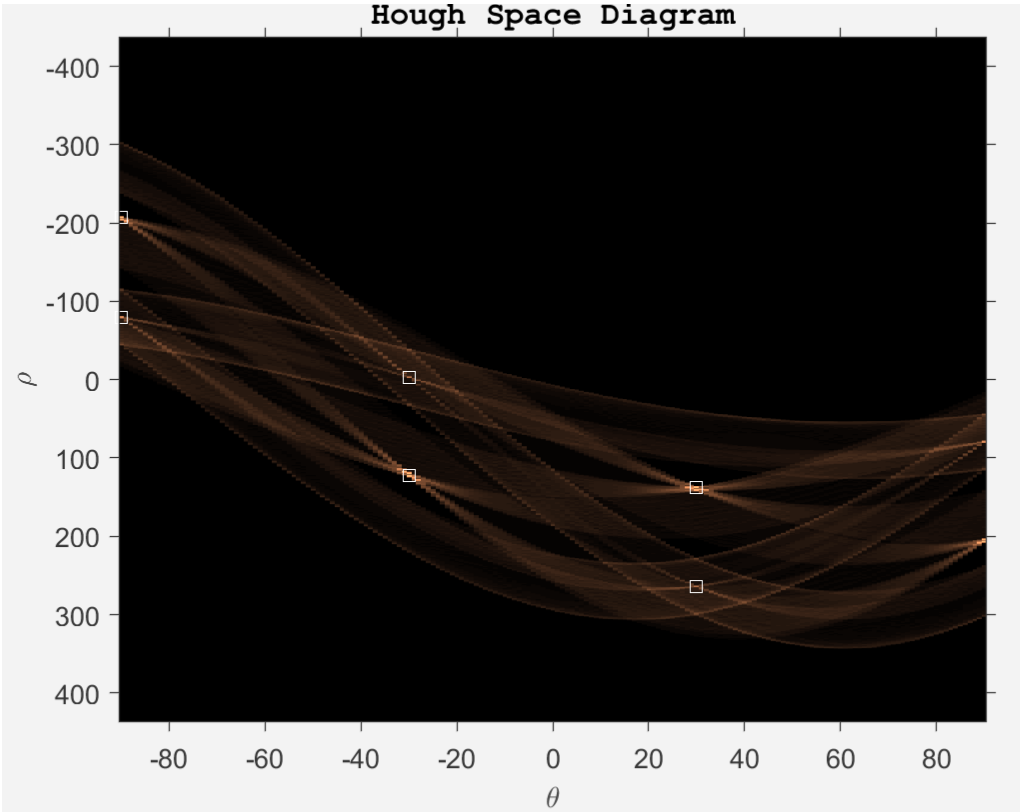


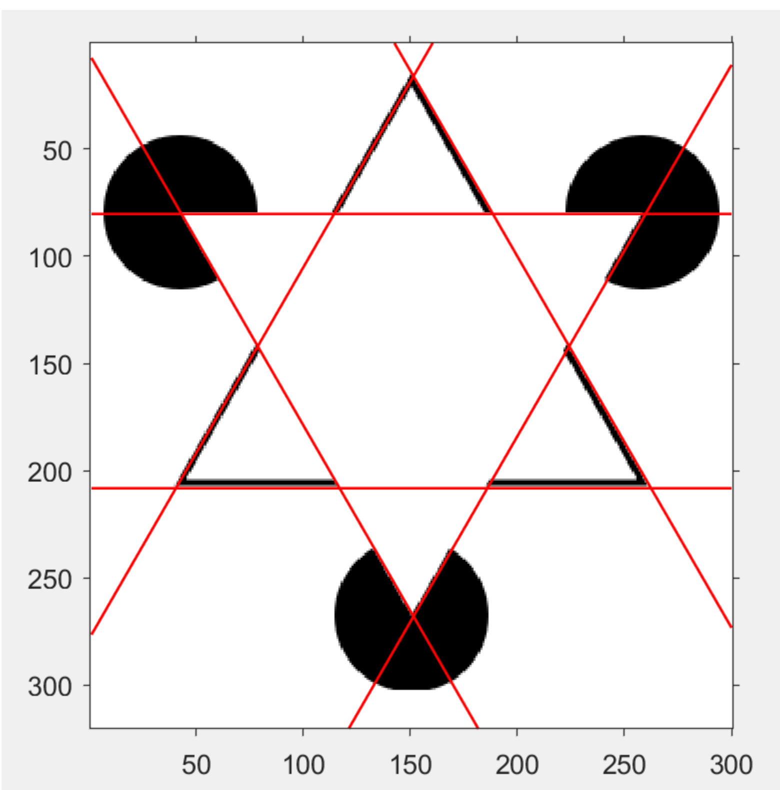

The original figure with standard Hough Transform increments the vote by 1.


Hough Space Diagram
-400
-300
-200
-100
0
100
200
300
400
ρ
-80
-60
-40
-20
0
20
40
60
80
θ

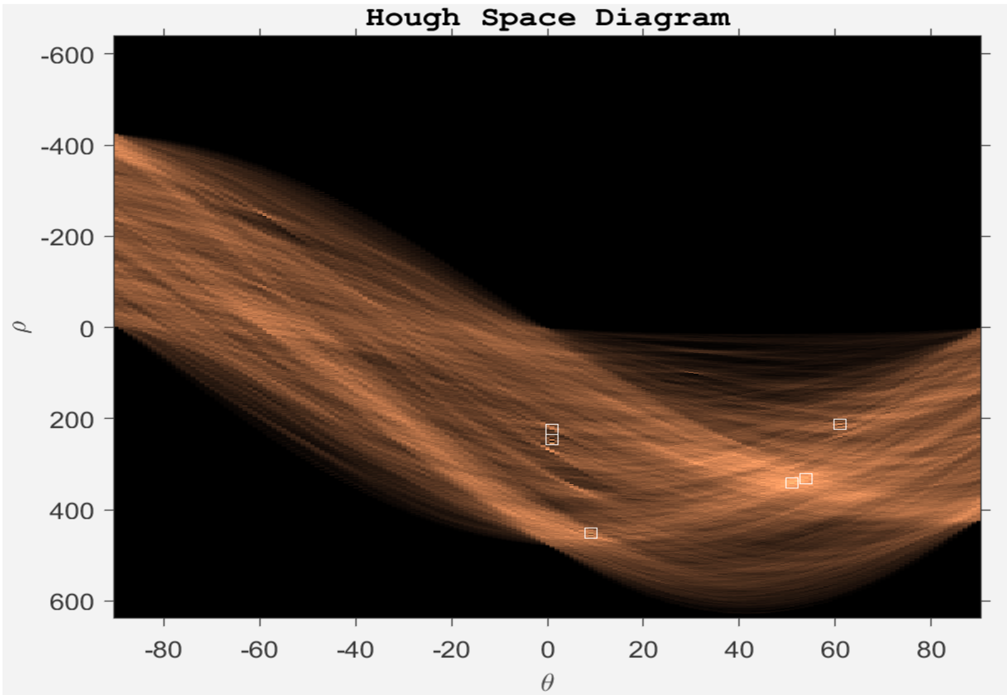

50
100
150
200
250
300
50
100
150
200
250
300

Even when I remove the filter, the Hough Transform still work.

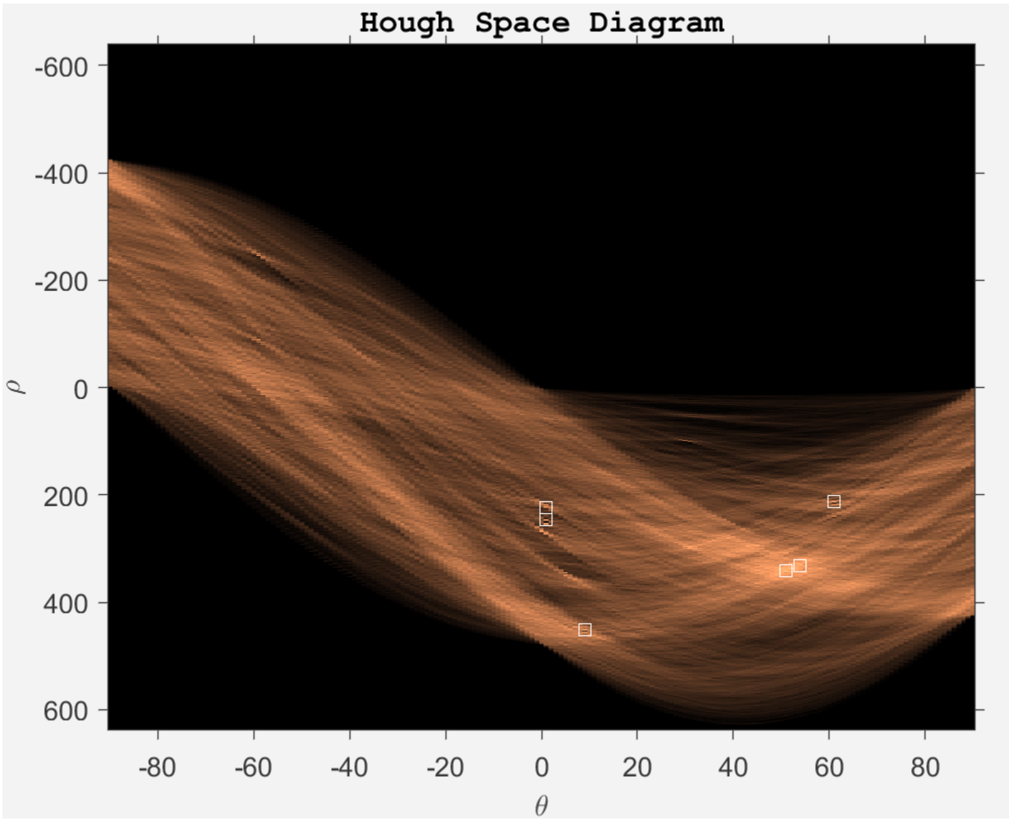


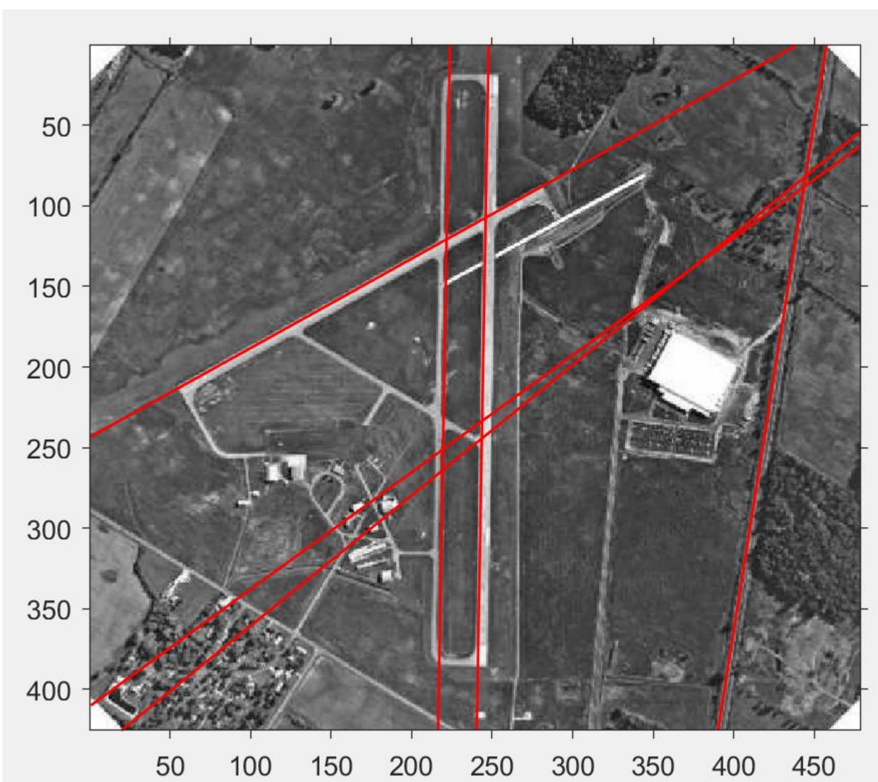



for image 2, I got following figure with approach that votes are incremented by an integer amount that is proportional to the edge strength with linecount 6.




The original figure with standard Hough Transform increments the vote by 1.



We can see that the results between two approach is very close. But when we use the approach that votes are incremented by an integer amount that is proportional to the edge strength, we even don’t need to add filter to specify the bin, maybe because in canny edge figure, gradient magnitude is very useful to identify a pixel.

Explanation for why you think the output of the Hough Transform makes sense?

It makes sense because the votes are accurate. It exclude much noise and only get important information of lines.