EE7374 – Digital Image Processing

Homework-3

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1. Modulation property of the Discrete Fourier Transform

   Answer:

$$0.5\sin\left(2\pi\frac{\xi_0}{N}m\right)$$

$$=\frac{j}{4}\left[\delta\left(u+\frac{\xi_0}{N}\right)-\delta\left(u-\frac{\xi_0}{N}\right)\right]$$

$$\therefore\ f[m,n]\times\left(0.5+0.5\sin\left(2\pi\frac{\xi_0}{N}m\right)\right)$$

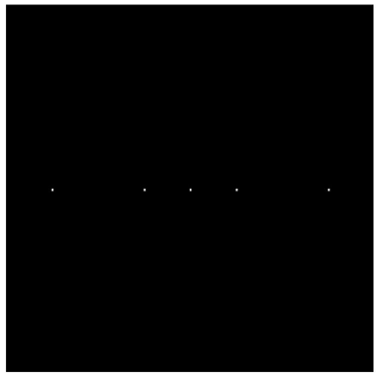
$$\overset{\mathcal{F}}{\Longleftrightarrow}\ 0.5F[k,l]+\frac{j}{4}\left[F[k,l]*\left[\delta\left(u+\frac{\xi_0}{N}\right)-\delta\left(u-\frac{\xi_0}{N}\right)\right]\right]$$

$$=0.5F[k,l]+\frac{j}{4}\left[F\left[k+\frac{\xi_0}{N},l\right]-F\left[k-\frac{\xi_0}{N},l\right]\right]$$

2. Problem 4.25 in 4th edition of Gonzalez and Woods

   (a) Suppose that the stripes of an image of the same size are four pixels wide. Sketch what the spectrum of the image would look like, including only the dc term and the two highest-value frequency terms, which correspond to the two spikes in the spectrum above.

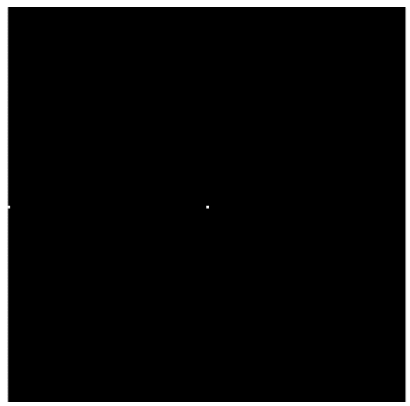
   The spectrum of the image would look like following, the distance between two highest-value and dc term should be half of the distance of original image of 2 pixels wide.



(b) Why are the components of the spectrum limited to the horizontal axis?

Because there is only periodic change in the horizontal axis. So the frequency components only limited to the horizontal axis.

(c) What would the spectrum look like for an image of the same size but having stripes that are one pixel wide? Explain the reason for your answer.



Because the frequency of one pixel wide image is twice of the two pixels wide image, so the distance between two highest-value and dc term should also be twice.

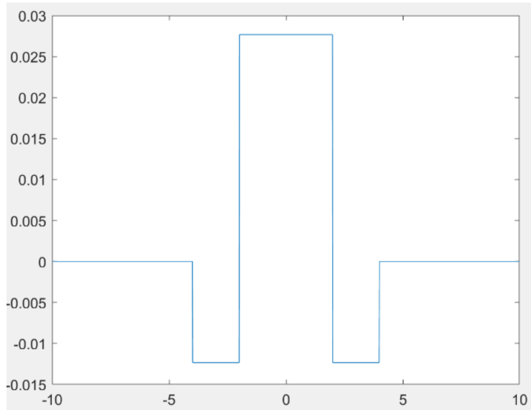
(d) Are the dc terms in (a) and (c) the same, or are they different? Explain.

They are same because is the dc terms are equivalent to the average of all the samples, which is equivalent in both (a) and (c) images.

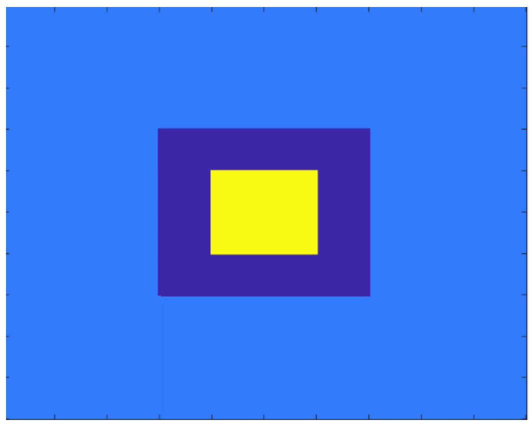
3. Guess the type of filter?

(a) What is the shape of the filter? Submit a plot and/or sketch the filter shape.

In 1D, the shape of the filter is like this:

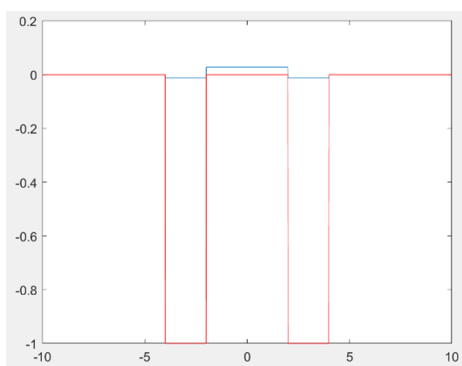


In 2D, the shape of the filter is like this:



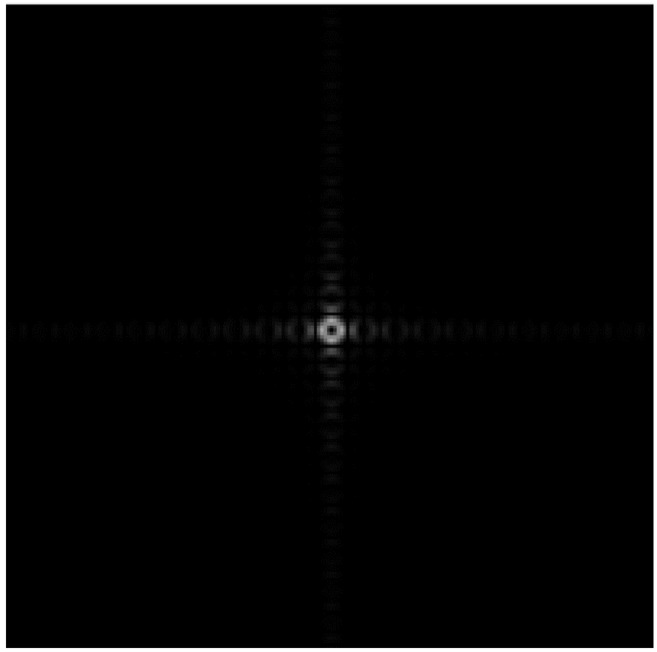
(b) What is the purpose of the scale factors $(2M+1)^{-2}$ and $(4M+1)^{-2}$?

In following image, blue filter is with scale factors while red one without. We can know that without scale factors, the filter can't pass any signal, so the purpose of the scale is to increase the threshold of filter and pass low-frequency signal.



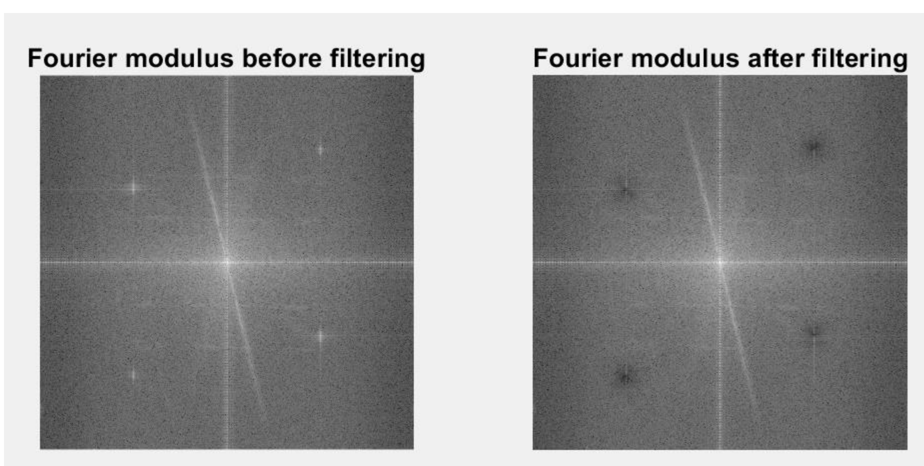
(c) Write down the expression for the frequency response of the filter obtained by computing the Fourier Transform of the filter? Submit a plot of the magnitude of the frequency response.

$$F[k,\ell] = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} \exp\left(-i2\pi\left[\frac{km}{M}+\frac{\ell n}{N}\right]\right) h[m,n]$$

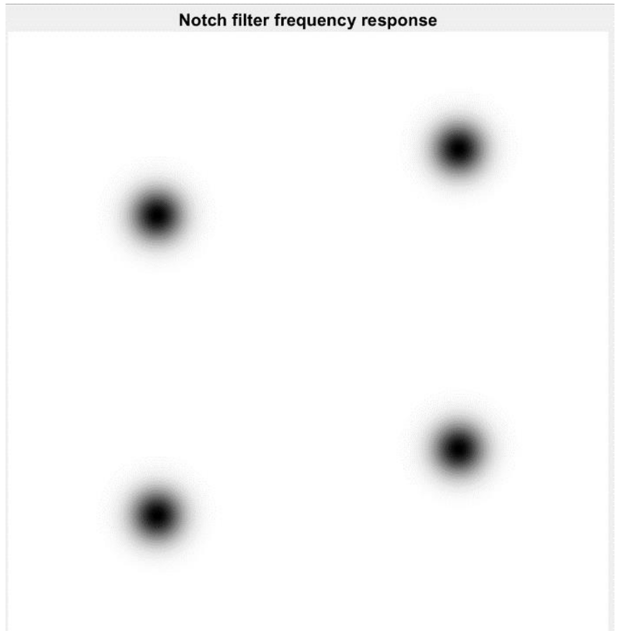


(d) What type of frequency response (low-pass, band-pass, high-pass) does the filter have?
Low-pass.

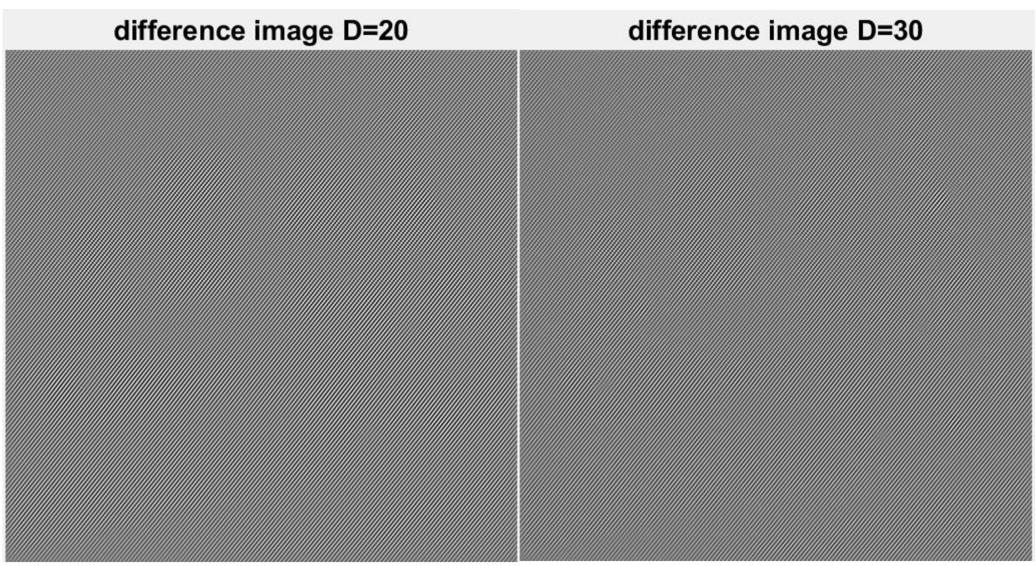
4. Suppressing Moire by notch filtering in frequency domain
   1. Completed MATLAB code for `HWK3_SuppressingMoire.m`
   2. How many center frequencies do you need to build the notch filter? What are these center frequencies?
      Because of the symmetry of Fourier transform, so we only need two center frequencies to build the notch filter, they are [313 257] and [824 257].
   3. Screenshots of the Fourier modulus of the image before and after filtering. Label each screenshot clearly. Failure to do so will result in deduction of points.



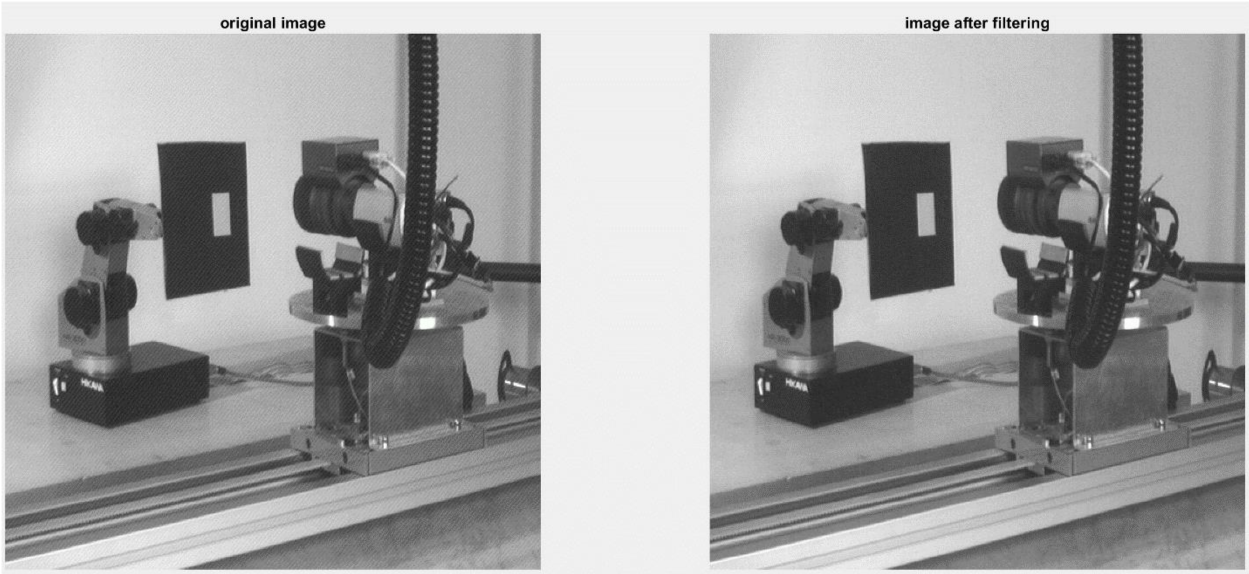
4. Screenshot of Notch filter frequency response (see example above for reference)



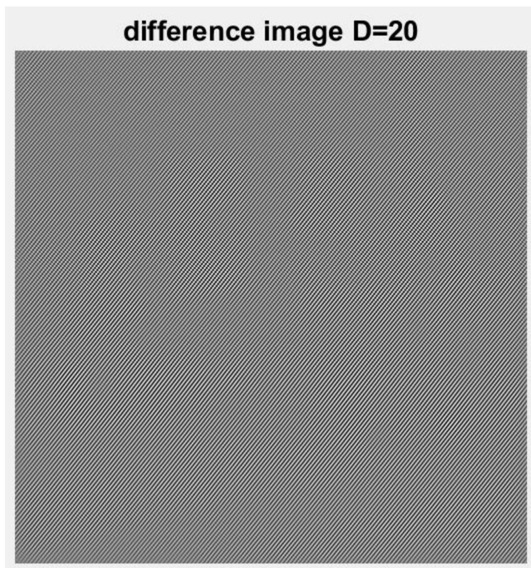
5. What value of D_0 did you choose and why? Provide as much detail as possible.
   A very small value of D_0 will fail to remove the periodic noise. A large value of D_0 will result in image detail being filtered. So we want to choose D0 as large as possible until detail being filtered.

   From the images above we can see that the periodic noise is removed very well and almost same. But when D=30 there are some details are brighter, which means detail are filtered, so 20 is better that 30. And I choosed D to be 20.
6. Screenshots of the original image and the Label each screenshot clearly. Failure to do so will result in deduction of points.



7. Screenshot of difference image and explanation for why the image makes sense.



It makes sense because it is obtained by subtracting the filtered image from the original image.

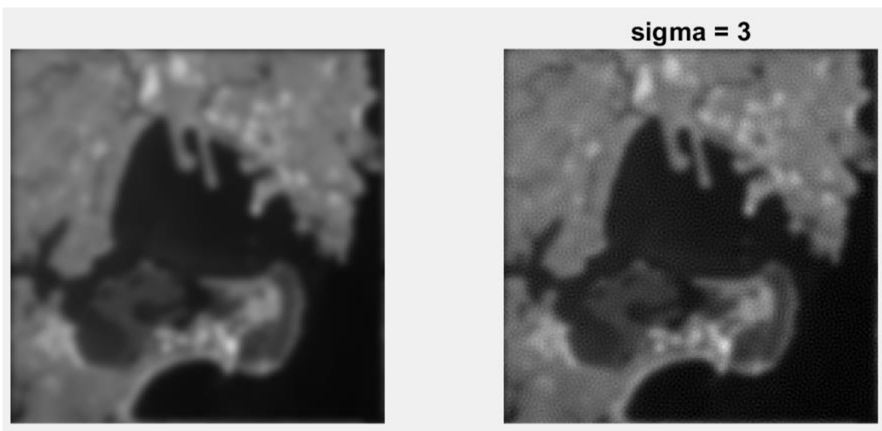
5. Image Restoration
    1. Completed MATLAB code for `HWK3_ImageRestoration.m`
    2. Explain in words the approach described in the paper "**Fast Noise Variance Estimation**". Provide as much detail as possible in your explanation. Why does the paper refer to the filters L_1, L_2 as Laplacian masks? Do you see any relation between the image derivative operators discussed in Lecture-7 and the masks L_1, L_2? If so, what is it?
    The paper presents the approach for estimating the variance of additive aero mean Gaussian noise in an image which only need to use a 3x3 mask followed by convolution with the image and then do a summation over the image after convolution.
    The paper refer to the filters Laplacian masks because it is derivative from Laplacian operator. In

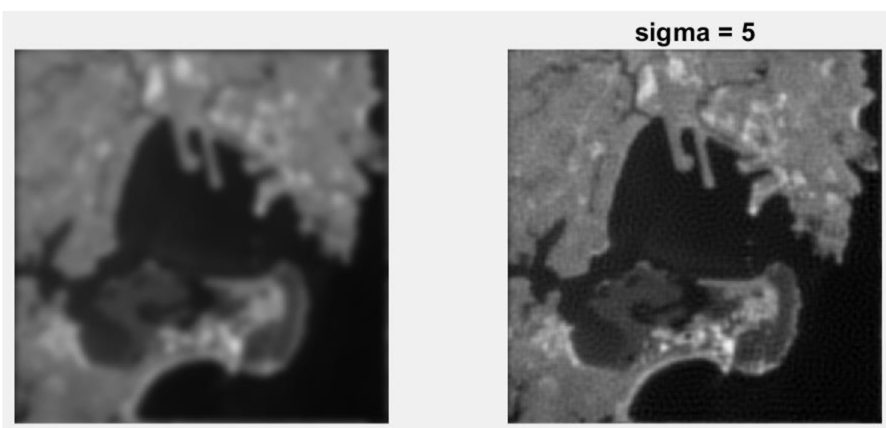
slide-23 of Lecture 7 we can see the Laplacian operator formula:

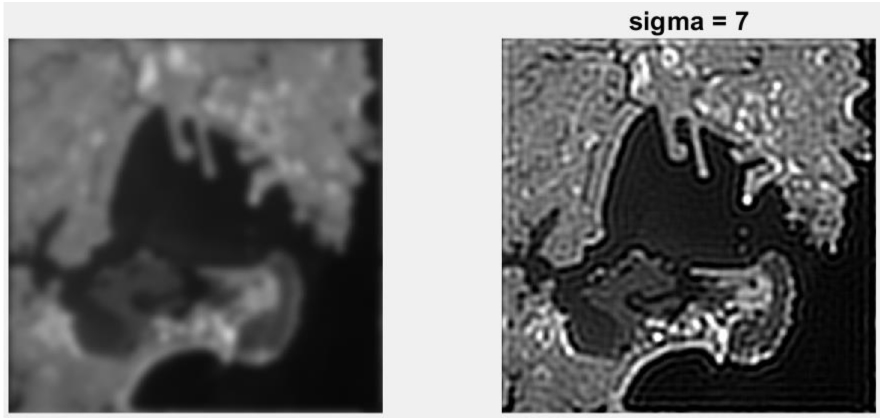
$$\nabla^2 I = I(x+1,y) + I(x-1,y) + I(x,y+1) + I(x,y-1) - 4I(x,y)$$

It shows the relation with the mask $L_1 = \begin{array}{|c|c|c|} \hline 0 & 1 & 0 \\ \hline 1 & -4 & 1 \\ \hline 0 & 1 & 0 \\ \hline \end{array}$.

3. What value of σ_b did you end up choosing to model the blur in each image? How did you choose the value? Provide as detailed an explanation as possible, including screenshots of good and bad choices for σ_b.
   I choosed sigma = 5 because a small value of sigma such as 3 will fail to remove noisy, and a big value of sigma such as 7 will filter detail.







4. Screenshots of the image before and after image restoration. Label each screenshot clearly. Failure to do so will result in deduction of points.