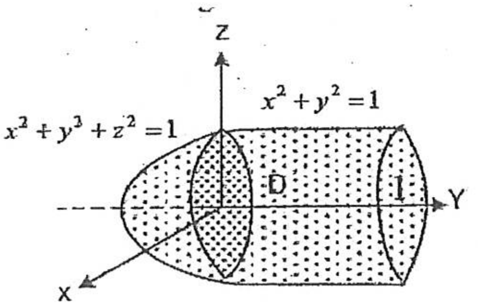
12. Find the surface area of the portion of the sphere $x^2+y^2+z^2=16$ that lies inside the cylinder $x^2-4x+y^2=0$.

13. Let $\mathcal{R}$ be the plane region between the graphs of $y=-x^2$ and $y=x^2-x$. Find the moments of $\mathcal{R}$ about the x and the y axes. Also find the centroid of $\mathcal{R}$.

14. Evaluate the iterated triple integral:

(a) $\int_0^1\int_0^1\int_{\sqrt{x^2+y^2}}^2 xyz\,dzdydx$ (b) $\int_0^2\int_0^y\int_0^{\sqrt{3}z} \frac{z}{x^2+z^2}dxdzdy$

15. Evaluate $\iiint_{\mathcal{D}} y\sqrt{1-x^2}\,dV$ where $\mathcal{D}$ is the region shown in the following figure.



16. Evaluate $\iiint_{\mathcal{D}} e^y\,dV$ where $\mathcal{D}$ is the solid region bounded by the planes $y=1$, $z=0$, $y=x$, $y=-x$ and $z=y$.

17. Find the volume of the solid region bounded above by the circular paraboloid $z=4(x^2+y^2)$ and below by the plane $z=-2$ and on the sides by the parabolic sheet $y=x^2$ and $y=x$.

18. Express the triple integral as an iterated integral in cylindrical coordinates and then evaluate it: $\iiint_{\mathcal{D}} \sqrt{z}\,dV$ where $\mathcal{D}$ is the portion of the ball $x^2+y^2+z^2\le 4$ that is in the first octant.

19. Let $\mathcal{D}$ be the solid region in the first octant bounded by the sphere $x^2+y^2+z^2=16$ and the planes $z=0$, $x=\sqrt{3}y$ and $x=y$ Evaluate $\iiint_{\mathcal{D}} \sqrt{z}\,dV$

20. Find the volume of the solid region bounded above by the sphere $x^2+y^2+z^2=4$ and below by the upper nape of the cone $z^2=x^2+y^2$.

ADAMA SCIENCE AND TECHNOLOGY UNIVERSITY

School of Applied Natural Science

Department of Applied Mathematics

Applied Mathematics-II (Math 1102) Worksheet -V

June, 2024

1. Let $\mathcal{R}$ be the region between the graphs of $y = 1 - 2x$, the $x-$ axis and the line $x = -2$. Show that $\mathcal{R}$ is simple region.

2. If $\mathcal{R}$ is the plane region between the graphs of the equations $x = y^2$ and $y = x - 2$. Show that $\mathcal{R}$ is simple.

3. Evaluate each of the following iterated double integrals.
   (a) $\int_0^1 \int_x^{x+1} xy\ dydx$, (b) $\int_1^3 \int_0^3 \frac{2}{9+x^2}\ dxdy$'

4. Evaluate $\iint_{\mathcal{R}} x(x-1)e^{xy}\ dA$ if $\mathcal{R}$ is the triangular region bounded by the lines $x = 0$, $y = 0$ and $x + y = 2$.

5. By reversing the order of integration, evaluate $\int_0^4 \int_{\sqrt{y}}^2 \cos(x^3)\ dxdy$.

6. Find the volume of the solid region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.

7. Find the area of the plane region bounded by the graphs of $y = 3 - x^2$ and $y = 2|x|$.

8. Change the integral $\int_{-3}^3 \int_0^{\sqrt{9-x^2}} \frac{1}{\sqrt{x^2+y^2}} dydx$ to an iterated integral in polar coordinates and then evaluate it.

9. Let $\mathcal{R}$ be the region bounded by the circles $r = 1$ and $r = 2$ for $0 \leq \theta \leq 2\pi$. Evaluate $\iint_{\mathcal{R}} (x^2 - y)dA$.

10. Find the volume of the solid region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$ (**Using polar coordinate**).

11. Find the area of the shaded region shown below.