Adama Science and Technology University (ASTU)
School of Applied Natural Science
Applied Mathematics Department
Applied Mathematics II (Math 1102) Worksheet I

1. Write the first four terms of the following sequences.
   a) $\left\{m - \frac{1}{m}\right\}_{m=1}^{\infty}$ b) $\left\{\frac{k-5}{k+3}\right\}_{k=3}^{\infty}$ c) $\left\{\sqrt[n]{n}\right\}_{n=2}^{\infty}$
2. Evaluate the limits of the following sequences.
   a) $\lim_{n\to\infty}\left(3 - \frac{1}{n}\right)$ c) $\lim_{n\to\infty}\frac{1-4n^3}{2n+3}$ e) $\lim_{n\to\infty}\left(1 - \frac{3}{4n}\right)^n$
   b) $\lim_{n\to\infty}\frac{2n-3}{\sqrt{n^2+3n-5}}$ d) $\lim_{n\to\infty}\ell n\,\frac{1}{n}$ f) $\lim_{n\to\infty}\frac{2^{n-1}+3^{n-1}}{6^n}$
3. Find a formula for the general term a_n of the sequence
   a) $\left\{2,\ 4+\frac{1}{2},\ 9+\frac{1}{3},\ 16+\frac{1}{4},\ \ldots\right\}$ b) $\left\{\frac{1}{2.3}, \frac{2}{3.4}, \frac{3}{4.5}, \ldots\right\}$
4. Determine whether the sequence converges or diverges. If it converges, find its limit.
   a) $\left\{\frac{\cos n\pi}{n}\right\}_{n=1}^{\infty}$ c) $\left\{\frac{2n^3-1}{(2n-1)^3}\right\}_{n=0}^{\infty}$ e) $\left\{\int_0^{\infty} e^{-nx}\,dx\right\}_{n=0}^{\infty}$
   b) $\left\{(2^n + 3^n)^{1/n}\right\}_{n=1}^{\infty}$ d) $\left\{(-1)^n + \frac{1}{n}\right\}_{n=1}^{\infty}$ f) $\left\{\frac{2^{n-1}+3}{3^{n+2}}\right\}_{n=0}^{\infty}$
5. Check whether the sequence is bounded, monotonic and the limit exist
   a) $\left\{\frac{3n}{n+2}\right\}_{n=1}^{\infty}$ b) $\left\{\frac{2^n}{1+2^n}\right\}_{n=0}^{\infty}$
6. Determine three subsequences of
   a) $\{n\}_{n=1}^{\infty}$ b) $\left\{\frac{1}{n}\right\}_{k=1}^{\infty}$ c) $\left\{\frac{3n}{n+2}\right\}_{n=1}^{\infty}$ d) $\left\{\frac{n}{2^n}\right\}_{n=1}^{\infty}$
7. Determine the fourth partial sum S_4 of the series
   a) $\sum_{n=0}^{\infty}(n+1)$ b) $\sum_{n=0}^{\infty}\ell nn$
8. Using Divergence Test, if applicable, check whether the series diverges
   a) $\sum_{n=0}^{\infty}\frac{2^n+1}{2^{n+1}}$ b) $\sum_{n=1}^{\infty}cos\ \frac{n\pi}{2}$
9. Write the repeating decimal as a geometric series
   a) $0.0242424\ \ldots$ b) $21.16\overline{123}$ c) $0.3\overline{4}$
10. Use the Integral Test, the Comparison Test or the Limit Comparison Test to determine whether the series converges or diverges.
    a) $\sum_{n=1}^{\infty} ne^{-n^2}$ d) $\sum_{n=1}^{\infty}\frac{3^n}{2^n-1}$ g) $\sum_{n=1}^{\infty}\frac{n2^n}{4n^3+1}$
    b) $\sum_{n=1}^{\infty}\frac{\sqrt{n}+5}{n^3-2n+3}$ e) $\sum_{n=10}^{\infty}\frac{n^2}{2n^3-1}$ h) $\sum_{n=1}^{\infty}\frac{1}{n\sqrt{n^2+1}}$
    c) $\sum_{n=10}^{\infty}\frac{2n^2-1}{(3n+2)n^{\frac{4}{3}}}$ f) $\sum_{n=0}^{\infty}\frac{3\sin^2 n}{n!}$ i) $\sum_{n=2}^{\infty}\frac{\ln n}{n+1}$

11. Determine whether the series converges or diverges. Use the Ratio or Root tests.

a) $\sum_{n=1}^{\infty} \frac{n}{(n+1)e^n}$

b) $\sum_{n=0}^{\infty} \frac{3^{2n}}{2^{3n}}$

c) $\sum_{n=1}^{\infty} \frac{n}{n!}$

d) $\sum_{n=1}^{\infty} \frac{(\sqrt{5}-1)^n}{n^2+1}$

e) $\sum_{n=0}^{\infty} \left(\frac{2}{n^2+n+1} \right)^{2n}$

f) $\sum_{n=0}^{\infty} \frac{5^n}{2^n+1}$

12. Check whether the following alternating series converges or diverges.

a) $\sum_{n=0}^{\infty} (-1)^n \left(\frac{1}{n!} \right)$

b) $\sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{1}{n} \right)$

c) $\sum_{n=2}^{\infty} (-1)^n \left(\frac{\ell n\, n}{n} \right)$

d) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[5]{n^3}}$

13. Determine whether the following series diverges, or converges absolutely or converges conditionally.

a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3^n}$

b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

c) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{3n+1}$

d) $\sum_{n=0}^{\infty} \frac{\cos n\pi}{n+1}$

e) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\, \ell n\, n}$

14. Find the possible values of x for which the series converges.

a) $\sum_{n=0}^{\infty} 2 \left(\frac{x}{3} \right)^n$

b) $\sum_{n=0}^{\infty} \left(\frac{x+2}{4} \right)^n$