Adama Science and Technology University
College of Applied Natural Sciences
Department of Applied Mathematics
Numerical Analysis (Math3201) Final-Examination

Date: June 02, 2025 G.C
Time Allowed: 2:30 hrs.
Target Group: 3rd year SE
Max. Marks: 50%

Name: _____

ID No: _____Group: _____

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GENERAL DIRECTIONS:

- ✓ ***Make Shure that this booklet contains FIFTEEN multiple choice, TEN short answer questions and FOUR workout problems.***
- ✓ ***Using mobile phone is strictly FORBIDDEN in the examination.***
- ✓ ***Don't use pencil for writing your answer.***
- ✓ ***Check there are 11 pages including the cover page.***
- ✓ ***Any attempt of cheating during exam will result in 0 mark.***
- ✓ ***Perform your calculation to <u>FIVE DECIMAL PLACES</u>.***

For instructor's use only!

Examination part	I (15%)	II (10%)	III (25%)	Total (50%)
Marks				

Do Not Turn This Page Until You Are Told To Do So!

Part I (Multiple choices): Choose the best answer and write your choice in the space provided (Each worth 1 point)

1. In numerical analysis, a method is said to be stable if:

A. It runs in constant time
B. Small changes in input cause small changes in output
C. It always converges
D. It uses the least memory

2. Which of the following is NOT a source of error in numerical computation?

A. Truncation error
B. Rounding error
C. Syntax error
D. Discretization error

3. Which of the following best describes convergence of a numerical method?

A. The method produces results quickly
B. The method gives the exact solution in one iteration
C. The numerical solution approaches the exact solution as the number of iterations increases
D. The method uses less memory

4. In numerical integration, which of the following contributes to convergence?

A. Increasing the number of intervals
B. Decreasing the accuracy of floating-point arithmetic
C. Increasing the interval length
D. Using an explicit method

5. The order of convergence of a numerical method is defined as:

A. The number of iterations needed
B. The speed at which the number of correct digits increases
C. A measure of how fast the error reduces relative to the previous error
D. The number of steps per iteration

6. What is the main goal of interpolation?

A. To find the derivative of a function
B. To solve a differential equation
C. To minimize the error in computation
D. To estimate unknown values between known data points

7. Which of the following affects the accuracy of numerical differentiation?

A. Step size h
B. Number of iterations
C. Integration constant
D. Order of the differential equation

8. Which of the following is true about Simpson's 1/3 Rule?

A. It uses linear approximation
C. It uses parabolic approximation
B. It is less accurate than the Trapezoidal
D. It cannot be used for definite integrals

9. What is numerical integration?

A. Solving DEs numerically
C. Estimating function values
B. Approximating the area under a curve using discrete data
D. Minimizing error in equations

10. What is a key difference between a MATLAB script and a function?

A. Scripts accept inputs, functions do not
B. Functions have local variables, scripts use the workspace
C. Scripts can return outputs, functions cannot
D. Functions cannot use loops

11. What does $\boldsymbol{x = linspace(0, 1, 5)}$ produce?

A. [0, 0.2, 0.4, 0.6, 0.8]
C. [0, 1, 2, 3, 4]
B. [0, 0.25, 0.5, 0.75, 1]
D. [1, 2, 3, 4, 5]

12. Your iterative solver runs indefinitely. Which is the *most likely* cause?

A. The loop variable is not incremented.
C. A disp( ) statement is missing.
B. The termination tolerance is too strict.
D. None

13. Which of the following MATLAB commands generates the column vector $a = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$?

A. a = [1 2 3]
B. a = [1; 2; 3]'
C. a = [1 2 3]'
D. B and C

14. Which one of the following commands computes the roots of the equation $f(x) = x^3 - 2x^2 - x + 2 = 0$?

A. $f = [-2\ -1\ 2\ 1];\ roots(f)$
C. $f = [1\ -2\ -1\ 0];\ roots(f)$
B. $f = [1\ -2\ -1\ 2\ ];\ roots(f)$
D. $f = [2\ -1\ -2\ 1];\ roots(f)$

15. You get the error: **"Matrix dimensions must agree."** Which operation likely caused it?

A. $A + B$ where A & B are same-sized matr.
C. $A * B$ where A is 3×2 and B is 2×3.
B. $A .* B$ where A is 3×2 and B is 2×3.
D. A' (transpose of A).

Answer Sheet

1._____	**4. _____**	**7. _____**	**10. _____**	**13._____**
2._____	**5. _____**	**8. _____**	**11._____**	**14._____**
3._____	**6. _____**	**9. _____**	**12._____**	**15._____**

Part II (Short Answer): Write the most simplified answer on the space provided (Each worth 1 pt.)

1. Floating-point arithmetic may introduce _____ errors due to limited precision.
2. If the Gauss Seidel method is used to solve the system of linear equations

$$2x_1 + x_2 + 5x_3 = 15$$
$$2x_1 + x_2 + x_3 = 8$$
$$x_1 + 3x_2 + x_3 = 10$$

with initial guess $X^{(0)} = (x_1^0, x_2^0, x_3^0) = (0,0,0)$, then the components of $X^{(1)} = (x_1^1, x_2^1, x_3^1)$ are:

$x_1^1 =$_____ , $x_2^1 =$_____ and $x_3^1 =$_____

3. Given a linear system of equation $Ax = b$ where $A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 8 & 0 \\ -1 & 0 & 4 \end{pmatrix}$ and $b = \begin{pmatrix} 0 \\ 8 \\ 4 \end{pmatrix}$. Using Doolittle LU decomposition method, Decompose Matrix A.

L = _____ and **U =** _____

4. If $f(x) = x^2$ and $h = 1$, then $\left(\frac{\Delta^2}{E}\right) f(x) =$ _____
5. What is the result of $\mathbf{0.1 + 0.2 == 0.3}$ in MATLAB? _____(**1 / 0**)
6. What is the output of $\mathbf{3 > 2}$ && $\sim(\mathbf{4 < 5})$ in MATLAB? _____(**1 / 0**)
7. Given the following data

X	**0**	**1**	**2**	**3**	**4**
Y=f(x)	**1.0**	**1.5**	**2.2**	**3.1**	**4.6**

Then $\nabla^3 f(3) =$ _____

8. The error of interpolation at every tabulated point is equal to zero. _____(**True/False**)
9. The degree of the polynomial that interpolates a given set of ***n*** points is at most equal to ***n***. _____(**True/False**)
10. Simpson's one-third rule of integration is more accurate than Simpson's three-eighth rule of integration._____ (**True/False**)

PART III (Work-Out Items): Work out the following questions by showing all the necessary steps clearly.

1. a) Complete the **divided difference** table below. **(1 Point)**

x	$f(x)$	$f[.,.]$	$f[.,.,.]$	$f[.,.,.,.]$
-2	**2**			
		———		
-1	**-1**		———	
		———		———
0	**-2**		———	
		———		
3	**7**			

b) Derive interpolating polynomial by using **Lagrange's interpolation formula**. **(2 Points)**

c) Derive interpolating polynomial by using **Newton's divided difference formula**. **(2 Points)**

d) Approximate $f(2)$ **(1 Point)**

Use the difference table below to answer questions from 2 – 4.

x	$f(x) = lnx$	Δ	Δ^2	Δ^3	Δ^4
4.90	1.5892				
		0.0202			
5.00	1.6094		-0.0004		
		0.0198		0.0001	
5.10	1.6292		-0.0003		-0.0003
		0.0195		-0.0002	
5.20	1.6487		-0.0005		
		0.019			
5.30	1.6677				

2. **a**) Using the **appropriate Newton's difference formula**, approximate $\boldsymbol{ln\ 5.25}$. **(2 Points)**
b) Give the corresponding error by comparing your results in part (a) with the exact value. **(1 Points)**

3. **a**) Find $f'(4.90)$ and $f''(4.90)$ **(4 Points)**

b) Give the corresponding errors for each by comparing your results in part **(a)** with the exact values.

(2 Points)

4. Approximate $\int_{4.90}^{5.30} f(x)\, dx$ by using

a) Trapezoidal rule where the step length is $h = 0.10$ and compare your result with the exact value (**Hint**: $\int \boldsymbol{lnx\, dx = xlnx - x + c}$) **(3 Points)**

b) Simpson's rule where the step length is $h = 0.10$ and compare your result with the exact value. **(3 Points)**

c) Give the error bound of the approximation in (a) and to how many significant figures is your approximation correct? **(2 points)**

d) Give the error bound in the approximation in (b) and to how many significant figures is your approximation correct? **(2 points)**