

**Adama Science and Technology University**  
**College of Applied Natural Sciences**  
**Department of Applied Mathematics**  
**Numerical Analysis (Math3201) Mid-Examination**

**Date: March 31, 2024 G.C**  
**Time Allowed: 1:45 hrs.**  
**Max. Marks: 20%**

**Name:** \_\_\_\_\_

**ID No:** \_\_\_\_\_

**Department and Group:** \_\_\_\_\_

**Add** ☐ (If you add this course, sign here)

**GENERAL DIRECTIONS:**

- ✓ *Make Shure that this booklet contains of FIVE multiple choices, SIX short answer questions and TWO workout problems.*
- ✓ *Using mobile phone is strictly FORBIDDEN in the examination.*
- ✓ *Don't use pencil for writing your answer.*
- ✓ *Check there are 4 pages including the cover page.*
- ✓ *Any attempt of cheating during exam will result in 0 marks.*

**For instructor's use only**

Examination part	I (5%)	II (5%)	III (10%)	Total (20%)
Marks				

**Do Not Turn This Page Until You Are Told To Do So!**

**Part I (Multiple choices): Choose the best answer and circle it (Each worth 1 point)**

1. Round-off error occurs due to:
  - A. Using an approximation instead of an exact mathematical model
  - B. Limited precision in floating-point arithmetic
  - C. Errors in input data measurement
  - D. Accumulation of errors in iterative calculations
2. Which of the following numbers cannot be exactly represented in a standard floating-point system?
  - A. 0.5
  - B. 0.25
  - C. 0.1
  - D. 1.0
3. Why is machine epsilon important in numerical computations?
  - A. It determines the rounding error in floating-point arithmetic.
  - B. It defines the maximum value a floating-point number can store.
  - C. It prevents underflow and overflow errors.
  - D. All of the above
4. What does error propagation refer to in numerical computations?
  - A. The accumulation of rounding errors in a sequence of arithmetic operations.
  - B. The inability to represent small numbers due to floating-point limitations.
  - C. The process of systematically reducing numerical errors in computation.
  - D. The random occurrence of errors in computational systems.
5. For a method with  $|e_{n+1}| \approx C|e_n|^P$  if  $P = 1$  and  $C = 0.5$ , how many iterations are needed to reduce the error by  $10^{-6}$ ?
  - A. 6
  - B. 20
  - C. 50
  - D. 100

**Part II (Short Answer): Write the most simplified answer on the space provided (Each worth 1 pt.)**

6. Encode the floating point number  $-39.9$  using the IEEE standard format. Assume your computer has a single precision. **Answer:**\_\_\_\_\_
7. The percentage error in the time period  $T = 2\pi\sqrt{\frac{\ell}{g}}$  for  $\ell = 1m$  if the error in the measurement of  $\ell$  is 0.01. **Answer:**\_\_\_\_\_
8. Find the minimum number of iteration required for the Bisection Method to obtain the root of an equation  $f(x) = 0$  on  $[4, 5]$  with a tolerance of  $\varepsilon = 10^{-3}$ . **Answer:**\_\_\_\_\_
9. Using Newton-Raphson method, determine the root of the equation  $x^4 - x - 10 = 0$  correct to three decimal places by taking  $x_0 = 1.5$ . **Answer:**\_\_\_\_\_
10. Determine the root of the equation  $x^2 = 6$  with absolute error tolerance  $\varepsilon = 10^{-1}$ , using Secant method. Take  $x_0 = 3$ , and  $x_1 = 2$ . **Answer:**\_\_\_\_\_

**PART III (Work-Out Items): Work out the following questions by showing all the necessary steps clearly.**

**11.** Let  $x = \pi$  and  $\bar{x} = 3.141592653$  where  $\bar{x}$  is the approximate value of  $x$ . Then

- a) Calculate the absolute error and relative error in the approximation. **(2 points)**
- b) Determine the number of correct significant figures in  $\bar{x}$ . **(1 points)**
- c) Estimate the total absolute error and relative error in calculating the area of a circle,  $A = \pi r^2$ , with  $r = 10.0 \pm 0.1 \text{ cm}$  due to the errors in both  $\pi$  and  $r$ . Which (the uncertainty in  $r$  or  $\pi$ ) is the dominant error source in A? **(3 points)**

12. Find where the graphs of  $y = 3x - 1$  and  $y = \cos x$  on  $[0, \frac{\pi}{2}]$  intersect correct to three decimal places, using fixed point iteration method. (Take  $x_0 = 0$ ) **(4 points)**