

1. 完成课本数字图像处理第二版 116 页，习题 3.25，即拉普拉斯算子具有理论上的旋转不变性。

已知拉普拉斯算子为

$$\nabla^2 f = \frac{\delta^2 f}{\delta x^2} + \frac{\delta^2 f}{\delta y^2}$$

$$\frac{\delta f}{\delta x'} = \frac{\delta f}{\delta x} \frac{\delta x}{\delta x'} + \frac{\delta f}{\delta y} \frac{\delta y}{\delta x'} = \cos\theta \frac{\delta f}{\delta x} + \sin\theta \frac{\delta f}{\delta y}$$

$$\frac{\delta f}{\delta y'} = \frac{\delta f}{\delta x} \frac{\delta x}{\delta y'} + \frac{\delta f}{\delta y} \frac{\delta y}{\delta y'} = -\sin\theta \frac{\delta f}{\delta x} + \cos\theta \frac{\delta f}{\delta y}$$

继续求二阶导数得

$$\frac{\delta^2 f}{\delta x'^2} = \delta(\cos\theta \frac{\delta f}{\delta x} + \sin\theta \frac{\delta f}{\delta y}) / \delta x'$$

$$\frac{\delta^2 f}{\delta y'^2} = \delta(-\sin\theta \frac{\delta f}{\delta x} + \cos\theta \frac{\delta f}{\delta y}) / \delta y'$$

计算得

$$\frac{\delta^2 f}{\delta x'^2} = \cos^2\theta \frac{\delta^2 f}{\delta x^2} + \cos\theta\sin\theta \frac{\delta^2 f}{\delta x\delta y} + \cos\theta\sin\theta \frac{\delta^2 f}{\delta y\delta x} + \sin^2\theta \frac{\delta^2 f}{\delta y^2}$$

$$\frac{\delta^2 f}{\delta y'^2} = \sin^2\theta \frac{\delta^2 f}{\delta x^2} - \cos\theta\sin\theta \frac{\delta^2 f}{\delta x\delta y} - \cos\theta\sin\theta \frac{\delta^2 f}{\delta y\delta x} + \cos^2\theta \frac{\delta^2 f}{\delta y^2}$$

$$\nabla^2 f = \frac{\delta^2 f}{\delta x'^2} + \frac{\delta^2 f}{\delta y'^2} = (\cos^2\theta + \sin^2\theta) \frac{\delta^2 f}{\delta x^2} + (\cos^2\theta + \sin^2\theta) \frac{\delta^2 f}{\delta y^2} = \frac{\delta^2 f}{\delta x^2} + \frac{\delta^2 f}{\delta y^2}$$

所以拉普拉斯算子具有理论上旋转不变性。