1．课本 322 页习题 7.10

(1) 此时φ_0和φ_1是一组正交基，所以有

$$a_k = \langle \varphi_k(x), f(x) \rangle$$

$$a_0 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} [3 \quad 2]^T = \frac{5\sqrt{2}}{2}$$

$$a_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} [3 \quad 2]^T = \frac{\sqrt{2}}{2}$$

所以

$$\frac{5\sqrt{2}}{2}\varphi_0+\frac{\sqrt{2}}{2}\varphi_1=\frac{5\sqrt{2}}{2}\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}^T + \frac{\sqrt{2}}{2}\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}^T=[3 \quad 2]^T$$

(2) 此时φ_0和φ_1不是一组正交基，所以有

$$a_k = \langle \widetilde{\varphi_k(x)}, f(x) \rangle$$

$$a_0 = [1 \quad -1][3 \quad 2]^T = 1$$

$$a_1 = [0 \quad 1][3 \quad 2]^T = 2$$

所以

$$\varphi_0+2\varphi_1=[1 \quad 0]^T + 2[1 \quad 1]^T=[3 \quad 2]^T$$

(3) 此时φ_0和φ_1不是一组基，所以有

$$a_0 = [1 \quad 0][3 \quad 2]^T = 3$$

$$a_1 = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} [3 \quad 2]^T = -\frac{3}{2} + \sqrt{3}$$

$$a_2 = \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} [3 \quad 2]^T = -\frac{3}{2} - \sqrt{3}$$

$$\sum a_0{}^2 + a_1{}^2 + a_2{}^2 = \frac{39}{2}$$

$$\left\| [3 \quad 2]^T \right\|^2=13$$

所以

$$\sum a_0{}^2 + a_1{}^2 + a_2{}^2 = \frac{3}{2}\left\| [3 \quad 2]^T \right\|^2$$

$$\frac{2}{3}[3\varphi_0+(-\frac{3}{2}+\sqrt{3})\varphi_1+(-\frac{3}{2}-\sqrt{3})\varphi_2]=[2 \quad 0]^T + \begin{bmatrix} \frac{1}{2}-\frac{\sqrt{3}}{3} & 1-\frac{\sqrt{3}}{2} \end{bmatrix}^T + \begin{bmatrix} \frac{1}{2}+\frac{\sqrt{3}}{3} & 1+\frac{\sqrt{3}}{2} \end{bmatrix}^T$$

$$=[3 \quad 2]^T$$