1、r,g,b 是 RGB 彩色空间沿 R,G,B 轴的单位向量，定义向量 $\mathrm{u}=\frac{\partial R}{\partial x}r+\frac{\partial G}{\partial x}g+\frac{\partial B}{\partial x}b$ 和 $\mathrm{v}=\frac{\partial R}{\partial y}r+\frac{\partial G}{\partial y}g+\frac{\partial B}{\partial y}b$，$g_{xx}, g_{yy}, g_{xy}$ 定义为这些向量的点乘：

$$g_{xx}=\mathrm{u}\cdot\mathrm{u}=\mathrm{u}^T\mathrm{u}=\left|\frac{\partial R}{\partial x}\right|^2+\left|\frac{\partial G}{\partial x}\right|^2+\left|\frac{\partial B}{\partial x}\right|^2$$

$$g_{yy}=\mathrm{v}\cdot\mathrm{v}=\mathrm{v}^T\mathrm{v}=\left|\frac{\partial R}{\partial y}\right|^2+\left|\frac{\partial G}{\partial y}\right|^2+\left|\frac{\partial B}{\partial y}\right|^2$$

$$g_{xy}=\mathrm{u}\cdot\mathrm{v}=\mathrm{u}^T\mathrm{v}=\frac{\partial R}{\partial x}\frac{\partial R}{\partial y}+\frac{\partial G}{\partial x}\frac{\partial G}{\partial y}+\frac{\partial B}{\partial x}\frac{\partial B}{\partial y}$$

推导出最大变换率方向和(x,y)点在 θ 方向上变化率的值 $\mathrm{F}(\theta)$。

解：根据梯度定义可知$\frac{\partial f}{\partial l}=\cos\theta\ \frac{\partial f}{\partial x}+\sin\theta\ \frac{\partial f}{\partial y}$

为了使梯度最大也就是使梯度值的平方值最大，即

$$\left\|\frac{\partial f}{\partial l}\right\|^2=(\sin\theta\ \frac{\partial f}{\partial x}+\sin\theta\ \frac{\partial f}{\partial y})^2$$

$$=(\frac{\partial f}{\partial x})^2\cos^2\theta+2\frac{\partial f}{\partial x}\frac{\partial f}{\partial x}\sin\theta\cos\theta\ +(\frac{\partial f}{\partial y})^2\sin^2\theta$$

根据题意可带入 g_{xx}、g_{yy}、g_{xy}，得

$$上式=g_{xx}\cos^2\theta\ +2\sin\theta\cos\theta\ g_{xy}+g_{yy}\sin^2\theta$$

根据三角变换，可得

$$上式=\frac{\cos2\theta+1}{2}g_{xx}+\sin2\theta\ g_{xy}+\frac{1-\cos2\theta}{2}g_{\mathrm{yy}}$$

所以

$$\mathrm{F}(\theta)=\frac{1}{2}(g_{xx}+g_{\mathrm{yy}})+\frac{\cos2\theta}{2}(g_{xx}-g_{\mathrm{yy}})+\sin2\theta\ g_{xy}$$

$$\frac{\partial\mathrm{F}(\theta)}{\partial\theta}=-\sin2\theta\ (g_{xx}-g_{\mathrm{yy}})+2\cos2\theta\ g_{xy}=0$$

求得

$$\theta=\frac{1}{2}tan^{-1}(\frac{2g_{xy}}{g_{xx}-g_{\mathrm{yy}}})$$