1. 最佳陷波滤波器进行图像恢复的过程中，为了求出调制函数 w(x,y)，使用了最小化方差的方法，公式为

$$min\sigma^2 = \frac{1}{(2a+1)(2b+1)}\sum_{s=-a}^{a}\sum_{t=-b}^{b}[\hat{f}(x+s,y+t)-\bar{f}]^2$$

其中 $\bar{f} = \frac{1}{(2a+1)(2b+1)}\sum_{s=-a}^{a}\sum_{t=-b}^{b}\hat{f}(x+s,y+t)$
由于 $\hat{f}(x,y) = g(x,y) - w(x,y)\eta(x,y)$, 并且简化 $w(x+s,y+t) = w(x,y)$
所以上式可写为

$$min\sigma^2 = \frac{1}{(2a+1)(2b+1)}\sum_{s=-a}^{a}\sum_{t=-b}^{b}([g(x+s,y+t)-w(x,y)\eta(x+s,y+t)]-[\overline{g(x,y)}-w(x,y)\overline{\eta(x,y)}])^2$$

设 $\frac{1}{(2a+1)(2b+1)}\sum_{s=-a}^{a}\sum_{t=-b}^{b} = A$, 对其求导数得

$$\frac{\delta\sigma^2}{\delta w(x,y)} = A2[g(x+s,y+t)-w(x,y)\eta(x+s,y+t)-\overline{g(x,y)}+w(x,y)\overline{\eta(x,y)}][\overline{\eta(x,y)}-\eta(x+s)(y+t)]$$

因为

$$\frac{\delta\sigma^2}{\delta w(x,y)} = 0$$

所以

$$A[g(x+s,y+t)-w(x,y)\eta(x+s,y+t)-\overline{g(x,y)}+w(x,y)\overline{\eta(x,y)}] = 0$$

所以

$$w(x,y) = \frac{1}{(2a+1)(2b+1)}\sum_{s=-a}^{a}\sum_{t=-b}^{b}[\frac{\overline{g(x,y)}-g(x+s,y+t)}{\overline{\eta(x,y)}-\eta(x+s,y+t)}]$$

为了将求和符号放入分子分母，上下同时乘以 $\overline{\eta(x,y)}+\eta(x+s,y+t)$, 得到

$$\begin{aligned}
w(x,y) =& \frac{1}{(2a+1)(2b+1)}\sum_{s=-a}^{a}\sum_{t=-b}^{b}\frac{[\overline{g(x,y)}-g(x+s,y+t)][\overline{\eta(x,y)}+\eta(x+s,y+t)]}{[\overline{\eta(x,y)}-\eta(x+s,y+t)][\overline{\eta(x,y)}+\eta(x+s,y+t)]}\\
=& \frac{\bar{g}(x,y)\bar{\eta}(x,y)+\bar{g}(x,y)\bar{\eta}(x,y)-\bar{g}(x,y)\bar{\eta}(x,y)-\overline{g(x,y)\eta(x,y)}}{\bar{\eta}^2(x,y)-\overline{\eta^2(x,y)}}\\
=& \frac{\bar{g}(x,y)\bar{\eta}(x,y)-\overline{g(x,y)\eta(x,y)}}{\bar{\eta}^2(x,y)-\overline{\eta^2(x,y)}}\\
=& \frac{\overline{g(x,y)\eta(x,y)}-\bar{g}(x,y)\bar{\eta}(x,y)}{\overline{\eta^2(x,y)}-\bar{\eta}^2(x,y)}
\end{aligned} \tag{1}$$

得证。