2. 根据书中对傅立叶变换的定义，证明课本 165 页上有关傅立叶变换的平移性质。

1）频移

$F(u,v)=\frac{1}{MN}\sum_{x=0}^{M}\sum_{y=0}^{N}f(x,y)e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})}$

$$F(u-u_0,v-v_0)=\frac{1}{MN}\sum_{x=0}^{M}\sum_{y=0}^{N}f(x,y)e^{-j2\pi(\frac{(ux-u_0)}{M}+\frac{(vx-v_0)}{N})}$$

$$=\frac{1}{MN}\sum_{x=0}^{M}\sum_{y=0}^{N}f(x,y)e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})+j2\pi(\frac{u_0}{M}+\frac{v_0}{N})}$$

$$=\mathcal{F}[f(x,y)e^{j2\pi(\frac{u_0}{M}+\frac{v_0}{N})}]$$

2) 时移

$f(x,y)=\frac{1}{MN}\sum_{x=0}^{M}\sum_{y=0}^{N}F(u,v)e^{j2\pi(\frac{ux}{M}+\frac{vy}{N})}$

$f(x-x_0,y-y_0)=\sum_{x=0}^{M}\sum_{y=0}^{N}F(u,v)e^{j2\pi(\frac{u(x-x_0)}{M}+\frac{v(y-y_0)}{N})}$

$=\sum_{x=0}^{M}\sum_{y=0}^{N}F(u,v)e^{j2\pi(\frac{ux}{M}+\frac{vy}{N})-j2\pi(\frac{ux_0}{M}+\frac{vy_0}{N})}$

$\mathcal{F}[f(x-x_0,y-y_0)]=F(u,v)e^{-j2\pi(\frac{ux_0}{M}+\frac{vy_0}{N})}$

3) $F(u-\frac{M}{2},v-\frac{N}{2})=\frac{1}{MN}\sum_{x=0}^{M}\sum_{y=0}^{N}f(x,y)e^{-j2\pi(\frac{ux}{M}-\frac{x}{2}+\frac{vy}{N}-\frac{y}{2})}$

$=\frac{1}{MN}\sum_{x=0}^{M}\sum_{y=0}^{N}f(x,y)e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})}e^{j\pi(x+y)}$

因为$e^{j\pi}=\cos\pi+j\sin\pi=-1$

所以 $F(u-\frac{M}{2},v-\frac{N}{2})=\mathcal{F}[f(x,y)(-1)^{(x+y)}]$

4) $f(x-\frac{M}{2},y-\frac{N}{2})=\frac{1}{MN}\sum_{x=0}^{M}\sum_{y=0}^{N}F(u,v)e^{j2\pi(\frac{ux}{M}-\frac{u}{2}+\frac{vy}{N}-\frac{v}{2})}$

$=\frac{1}{MN}\sum_{x=0}^{M}\sum_{y=0}^{N}F(u,v)e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})}e^{j\pi(u+v)}$

$=\frac{1}{MN}\sum_{x=0}^{M}\sum_{y=0}^{N}F(u,v)e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})}(-1)^{(u+v)}$