# Chi-Squared Test

*We've finally arrived at a statistical test (I bet you've been looking forward to it).*
*The* **chi-squared test** *has a scary name, but it's pretty straight-forward to do, if a bit time-consuming...*

## The Chi-Squared Test Looks at Differences in Frequencies Between Groups

An ecologist might be investigating whether there is a difference in the **frequency** of birds of three different species at two different sites.

If there is no difference between the sites you might expect results like this:

The number of birds of each species is about the same at each site.

| | Species of bird | | | |
|---|---|---|---|---|
| Site | Robin | Pied Wagtail | Chaffinch | Total |
| Site 1 | 22 | 17 | 21 | 60 |
| Site 2 | 21 | 19 | 20 | 60 |
| Total | 43 | 37 | 41 | |

If there is a very strong difference between the sites, you might get results like this:

Most of the robins are at site 2.

| | Species of bird | | | |
|---|---|---|---|---|
| Site | Robin | Pied Wagtail | Chaffinch | Total |
| Site 1 | 2 | 18 | 40 | 60 |
| Site 2 | 37 | 19 | 4 | 60 |
| Total | 39 | 37 | 44 | |

Most of the chaffinches are at site 1.

Often, your results will be somewhere between these two extremes:

There are more chaffinches at site 1, and more robins at site 2, but the numbers aren't very different.

| | Species of bird | | | |
|---|---|---|---|---|
| Site | Robin | Pied Wagtail | Chaffinch | Total |
| Site 1 | 18 | 17 | 25 | 60 |
| Site 2 | 23 | 20 | 17 | 60 |
| Total | 41 | 37 | 42 | |

The chi-squared test helps you see whether results like this are likely to be due to chance or not. It does this by **comparing** the **observed results** to the results you were **expecting** to see.

## Use the Formula to Find Chi-Squared

Here is the formula for the chi-squared test. Don't worry though — you **won't** have to learn it for the exam.

'O' is the **observed frequency** for each possibility from the table (e.g. the number of robins at site 2).

This means **chi-squared**.

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

'E' is the **expected frequency** for each possibility from the table if the null hypothesis is true. For a table with more than one row and more than one column, you work out the expected frequency for each possibility like this:

$$\text{Expected frequency} = \frac{\text{row total} \times \text{column total}}{\text{overall total}}$$

Remember, this symbol means "sum of" (see page 60).

The overall total is found by adding up the row (or column) totals.

If you only have one row or one column, then you may have to figure out your expected frequencies from other information — for example, you might know the expected ratio for certain phenotypes in a genetic cross. If all else fails, assume the total is distributed evenly between your categories.

# Chi-Squared Test

## You Also Need to Know the Degrees of Freedom of Your Data

The degrees of freedom (*df*) of your data help you to decide whether your value of chi-squared is significant or not.

If you are doing the chi-squared test on a table with just one row or column, then:

*df* = number of classes – 1

When you're testing the results of a genetic cross, this is the number of phenotypes – 1.

If you are doing the chi-squared test on a table with more than one row, then:

*df* = (number of rows – 1) × (number of columns – 1)

Don't worry about what 'degrees of freedom' means — you just need to know how to work out and use them (there's more about that below).

## The Chi-Squared Test Can Tell You if Your Results are Significant

To do a chi-squared test:

1) Work out **chi-squared** for your data using the formula.
2) Work out the **degrees of freedom** (*df*) of your data.
3) Look up the **critical value** of chi-squared for your probability level and degrees of freedom in a **critical value table**.
4) If your value for chi-squared is **equal to or greater than** (≥) the critical value, then you have a **significant result** and can **reject the null hypothesis**. If your value for chi-squared is **smaller than** (<) the critical value, then there is **no difference** between your observed and expected results and you **can't reject** the null hypothesis.
5) The **bigger** the difference between your observed and expected results, the **greater** your chi-squared value will be and so the **more likely** it will be that your results are **significant**.

A critical value table is a list of values for a test statistic (like $\chi^2$) for different probability levels and degrees of freedom. For some tests, a result is significant if your value for the test statistic is equal to or greater than the critical value. For other tests, a result is significant if your value for the test statistic is equal to or less than the critical value.

$\chi^2$ value ≥ critical value = reject the null hypothesis
$\chi^2$ value < critical value = fail to reject the null hypothesis

## Make Sure You're Using the Right Test

You can't use a chi-squared test for all data. To use a chi-squared test, check that...

1) The data is about **frequencies** for **different categories** (e.g. colours), and that the categories don't have any particular **order** (like size).
2) Each person (or animal or plant) only appears in your results **once**. Your results should not have been able to interfere with each other.

This is called categorical data.

This is called independent data.

### Worked Example 1

A species of moth has two varieties — one is dark-coloured and the other is light-coloured. A biologist wants to see if there is a difference in the frequencies of dark-coloured and light-coloured moths of this species in two different towns. He collects the data shown in the table on the right.

**Are his results significantly different from the expected results at the 5% level?**

| | Moth colour | | |
|---|---|---|---|
| Town | Dark | Light | Total |
| A | 23 | 42 | 65 |
| B | 55 | 30 | 85 |
| Total | 78 | 72 | 150 |

**1** ***Identify the null hypothesis.***

The null hypothesis is that there is no significant difference between the frequency of dark-coloured moths and the frequency of light-coloured moths in towns A and B.

**2** ***Work out what you would expect the results to be if the null hypothesis was true.***

Use the formula:

$$\text{Expected frequency} = \frac{\text{row total} \times \text{column total}}{\text{overall total}}$$

| | Moth colour | | |
|---|---|---|---|
| Town | Dark | Light | Total |
| A | $\frac{(65 \times 78)}{150} = \mathbf{33.8}$ | $\frac{(65 \times 72)}{150} = \mathbf{31.2}$ | 65 |
| B | $\frac{(85 \times 78)}{150} = \mathbf{44.2}$ | $\frac{(85 \times 72)}{150} = \mathbf{40.8}$ | 85 |
| Total | 78 | 72 | 150 |

These are the row totals.

This is the overall total.

These are the column totals.

# Chi-Squared Test

Observed frequencies:

| Town | Moth colour: Dark | Moth colour: Light |
|---|---|---|
| A | 23 | 42 |
| B | 55 | 30 |

Expected frequencies:

| Town | Moth colour: Dark | Moth colour: Light |
|---|---|---|
| A | 33.8 | 31.2 |
| B | 44.2 | 40.8 |

**3** ***Calculate chi-squared.***

Remember the formula is: $\chi^2 = \sum \frac{(O-E)^2}{E}$

**a** Find $(O - E)^2 \div E$ for each entry in the table.

| Town | Moth colour: Dark | Moth colour: Light |
|---|---|---|
| A | $\frac{(23-33.8)^2}{33.8} = \mathbf{3.45}$ | $\frac{(42-31.2)^2}{31.2} = \mathbf{3.74}$ |
| B | $\frac{(55-44.2)^2}{44.2} = \mathbf{2.64}$ | $\frac{(30-40.8)^2}{40.8} = \mathbf{2.86}$ |

Round your answers to a sensible number of decimal places. Two decimal places will normally be fine.

**b** Add these numbers up to get chi-squared.

$\chi^2 = 3.45 + 3.74 + 2.64 + 2.86 = \mathbf{12.69}$

**4** ***Work out the degrees of freedom (df).***

$df = (2-1) \times (2-1) = \mathbf{1}$

Remember: $df$ = (number of rows − 1) × (number of columns − 1) for tables with more than one row.

**5** ***Look up the critical value of chi-squared for your probability level and degrees of freedom.***

Remember, for the result to be significant, your value of $\chi^2$ needs to be greater than the critical value.

For $df = 1$ and a P value of 0.05, the critical value of chi-squared is **3.84**.

| df | Probability level (P): 0.50 | 0.20 | 0.10 | 0.05 | 0.02 | 0.01 |
|---|---|---|---|---|---|---|
| 1 | 0.46 | 1.64 | 2.71 | 3.84 | 5.41 | 6.64 |
| 2 | 1.39 | 3.22 | 4.61 | 5.99 | 7.82 | 9.21 |
| 3 | 2.37 | 4.64 | 6.25 | 7.82 | 9.84 | 11.35 |
| 4 | 3.36 | 5.99 | 7.78 | 9.49 | 11.67 | 13.28 |



12.69 is greater than 3.84, so there is a **significant difference** between the observed and expected values.

**There is a significant difference between the frequencies of light-coloured and dark-coloured moths in Town A and Town B.**

When your value of $\chi^2$ is greater than the critical value at P = 0.05, this means that the probability that the difference is due to chance is less than 5% and the null hypothesis can be rejected.

## Worked Example 2

A geneticist is studying the inheritance of fruit shape and fruit colour in tomato plants. Both traits have two phenotypes. The geneticist crosses plants that are heterozygous for both traits with plants that are homozygous recessive for both traits. She thinks that this is a simple case of dihybrid inheritance, so she expects a phenotypic ratio of 1 : 1 : 1 : 1.
The results of the cross are shown in the table on the right.
**Are her results significantly different from the expected results at the 1% level?**

| Phenotype | Number of offspring |
|---|---|
| Round, red | 28 |
| Round, yellow | 18 |
| Oval, red | 19 |
| Oval, yellow | 25 |
| Total | 90 |

**1** ***Identify the null hypothesis.***

The null hypothesis is that the number of observed offspring of each phenotype will not differ significantly from what you would expect under a 1 : 1 : 1 : 1 ratio.

There's more about ratios on page 18.

# Chi-Squared Test

**2** ***Work out what you'd expect the results to be if the null hypothesis was true.***

You can calculate the expected results using the expected phenotypic ratio.

To do this, add up the parts of the expected ratio... $1 + 1 + 1 + 1 = 4$

... divide the total number of offspring by the ratio total... $90 \div 4 = 22.5$

... then multiply this number by each phenotype's part of the ratio to get the expected number for that phenotype. $22.5 \times 1 = \mathbf{22.5}$

In this cross, each part of the expected ratio is 1. Easy peasy.

The method for working out the expected results can be written down as an equation like this:

$$\text{expected results} = \frac{\text{total no. of offspring}}{\text{expected ratio total}} \times \text{part of expected ratio}$$

**3** ***Calculate chi-squared.***

**a** Find $(O - E)^2 \div E$ for each entry in the table.

Doing your calculations in a table will help to stop you from getting numbers mixed up.

| Phenotype | Observed number of offspring (O) | Part of expected ratio | Expected number of offspring (E) | (O – E) | $(O - E)^2$ | $\frac{(O-E)^2}{E}$ |
|---|---|---|---|---|---|---|
| Round, red | 28 | 1 | 22.5 | 5.5 | 30.25 | 1.34 |
| Round, yellow | 18 | 1 | 22.5 | -4.5 | 20.25 | 0.90 |
| Oval, red | 19 | 1 | 22.5 | -3.5 | 12.25 | 0.54 |
| Oval, yellow | 25 | 1 | 22.5 | 2.5 | 6.25 | 0.28 |

**b** Add these numbers up to get chi-squared.

$\chi^2 = 1.34 + 0.90 + 0.54 + 0.28 = \mathbf{3.06}$ (to 2 d.p.)

**4** ***Work out the degrees of freedom (df).***

The table only has one column, so you should work out the degrees of freedom using $df$ = number of classes – 1. (Since you're looking at the results of a genetic cross, this is the same as the number of phenotypes – 1.)

$df = 4 - 1 = \mathbf{3}$

**5** ***Look up the critical value of chi-squared for your probability level and degrees of freedom.***

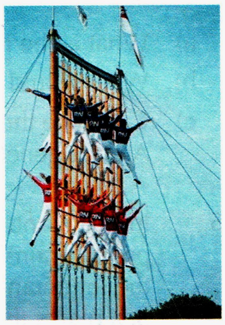
Using the table on the previous page, for a probability level of 0.01, when $df = 3$, the critical value of chi-squared is 11.35.

3.06 is less than 11.35, so there is **no significant difference** between the observed and expected values.

This means that the null hypothesis **cannot be rejected**.

**The results of this genetic cross are not significantly different to what you would expect under a 1 : 1 : 1 : 1 ratio.**

If you were to form a conclusion based on the outcome of this chi-squared test, you could say that the results support the null hypothesis (and so support the theory that this is a case of dihybrid inheritance). However, you can't say that they prove that this is a case of dihybrid inheritance.



They'd got the 'squared' part covered, but figuring out how to fit 'chi' into the statistics society's acrobatics display was proving harder than anticipated.

# Chi-Squared Test

## Practice Questions

Q1 Using the table on page 66, find the critical value of chi-squared...

a) for a probability level of 0.05 and 2 degrees of freedom.

b) when $p \leq 0.01$ and $df = 3$.

c) for a test being conducted at the 5% probability level with 4 degrees of freedom.

d) when $p \leq 0.1$ and $df = 1$.

Q2 A botanist wants to know if there is a relationship between flower colour and the likelihood of the flower being fertilised for a particular species of plant in a particular area. He collects data from each flower on 6 plants with yellow flowers and does the same for 5 plants of the same species with blue flowers to get the results in the table on the right.

Explain why he should not use a chi-squared test to analyse his results.

| | Fertilised? | | |
|---|---|---|---|
| Flower colour | Yes | No | Total |
| Yellow | 28 | 21 | 49 |
| Blue | 32 | 12 | 44 |
| Total | 60 | 33 | 93 |

Q3 An ecologist is interested in whether there is a significant difference in the number of butterflies of two different species present in two meadows. He collects the results in the table on the right.

a) What is the null hypothesis of this investigation?

b) Calculate the chi-squared value for these results.

c) Use the table on p. 66 to decide whether the ecologist's results are significant at the 0.01 level.

| | Species of butterfly | | |
|---|---|---|---|
| Meadow | Red Admiral | Small Blue | Total |
| A | 14 | 19 | 33 |
| B | 19 | 18 | 37 |
| Total | 33 | 37 | 70 |

Q4 A species of plant can either have blue or white flowers. If the colour of the flowers is controlled by a single gene, a cross between two heterozygous plants should give a ratio of 3 : 1 of blue to white flowers.

A geneticist wants to test this theory. She breeds two heterozygous plants then counts the frequencies of each flower colour in the offspring. She gets the results shown in the table on the right.

| Flower colour | | |
|---|---|---|
| Blue | White | Total |
| 103 | 57 | 160 |

a) What is the null hypothesis of this experiment?

b) What would you expect the results to be if the null hypothesis was true?

c) Calculate the chi-squared value for these results and compare it to the table of critical values on page 66. Are the geneticist's results significantly different from chance at the 0.05 level? What should she conclude?

Q5 In pea plants, the flowers can be purple or red and the pollen grains can be long or round. To investigate whether the inheritance of these traits is a case of dihybrid inheritance, two pea plants that are heterozygous for both flower colour and pollen shape were crossed. The results are shown in the table on the right.

If this is a case of dihybrid inheritance, the predicted ratio of purple, long : purple, round : red, long : red, round is 9 : 3 : 3 : 1. The null hypothesis for this investigation is that there's no significant difference between the observed frequencies and what you would expect under a 9 : 3 : 3 : 1 ratio.

| Phenotype | Observed frequency |
|---|---|
| Purple, long | 172 |
| Purple, round | 17 |
| Red, long | 13 |
| Red, round | 48 |
| Total | 250 |

a) Calculate the chi-squared value for these results.

b) Using the critical value table on p. 66, explain what you can conclude from these results.

**My expected interest in this test is significantly less than my observed...**

*The chi-squared test looks at the null hypothesis. This means when you do the test, you assume that any difference between the observed and expected values is down to chance. So if the observed frequencies are really different from the expected frequencies, your results are likely to be significant. That's why you'll end up with a bigger chi-squared value.*

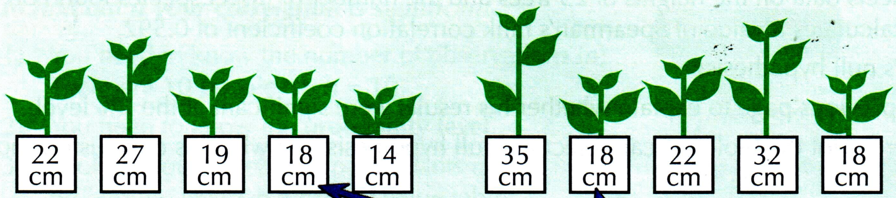
# Student's *t*-Test

***Student's t-test*** *isn't only for students, that's just the name of the guy who came up with it (or rather his pseudonym, how rock and roll). The t-test looks at whether there's a significant difference between two sets of data — go figure...*

## ***Student's t-Test*** *Tells You if the* ***Difference*** *Between* ***Means*** *is* ***Significant***

A farmer is investigating the effect of a fertiliser on the **mean** height of his crop. He divides his crop into two groups — Group 1 receives no fertiliser and Group 2 is fertilised regularly. Here are his results:

GROUP 1 GROUP 2



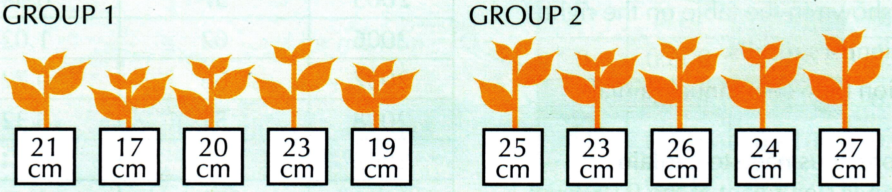

Mean = 20 cm Mean = 25 cm

Group 2 has a **higher mean height** than Group 1, but the individual heights **overlap** a lot between the groups.

The overlap between the two groups suggests that the difference in the mean heights is due to chance.

The farmer repeats his investigation using a different fertiliser.

GROUP 1 GROUP 2




Mean = 20 cm Mean = 25 cm

Again, Group 2 has a **higher mean height** than Group 1, but this time the individual heights of the groups **don't overlap**.

Although the difference between the means is the **same** in both sets of results, the heights in the second set of results are **less spread out** around the mean in both groups. This means the difference in height between the two groups in the second set of results is **less likely** to be due to chance.

You can find the **probability** that the difference is due to chance using **Student's *t*-test**. The formula you should use depends on the type of data that you have.

## *The* ***Unpaired*** *Student's t-test Uses* ***Means*** *and* ***Standard Deviations***

If you are comparing the means of **two groups of different individuals**, you need to use the **Student's *t*-test for unpaired data**. Here's the formula for calculating it. It looks pretty horrid, but it's not all that bad really....

$\bar{x}$ is the **mean** (see p. 24).

The little number refers to the data set. So $\bar{x}_2$ means the mean of data set 2.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{s_1^2}{n_1}\right) + \left(\frac{s_2^2}{n_2}\right)}}$$

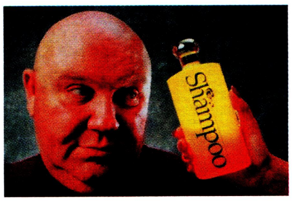
You might see the top row written as $|\bar{x}_1 - \bar{x}_2|$. The vertical lines mean 'the difference between'. They're just there to let you know that it doesn't matter whether the $t$-value is positive or negative.

$s$ is the **standard deviation** (see p. 60 for how to calculate it). You might also see it written as σ.

$n$ is the number of values in the data set.

# Student's *t*-Test

## You Need to Know **When** You Can **Use** the **Unpaired Student's *t*-Test**



Claude's jokes were getting significantly meaner.

As you know, the unpaired Student's *t*-test is used to find out if the difference between the means of two groups is significant, but to use it your data must be...

- In two categorical groups, e.g. 'treated' and 'untreated'. The groups don't have to be the same size.
- From different individuals (so it's **independent** data — see page 65). If you are comparing two sets of data from the same individuals, you should use a paired *t*-test.
- **Interval** data — this is data that is measured on a scale where each unit is the same size (e.g. length in centimetres).

## Calculate the **Degrees of Freedom** to Help Find the **Critical Value**

Once you've worked out a value for *t*, you need to compare it to a **critical value** to see whether your results are **significant**. To do this you need to know the degrees of freedom (*df*) and the **probability level**.

1) To work out the degrees of freedom (*df*) for an unpaired test, just plug the number of values (*n*) in each data set into this formula: $df = n_1 + n_2 - 2$
2) Use the degrees of freedom and the probability level to look up the critical value in the **critical value table**.
3) If the value of *t* is **greater than or equal to** the critical value, the difference between the two means is **significant** and you can **reject the null hypothesis**. If the value of *t* is **smaller than** the critical value, the difference between the two means is not **significant** and you **can't reject** the null hypothesis.

**Worked Example**

A biologist is investigating the effect of a high sugar diet on the body mass of rats. He divides fifteen rats into two groups. The control group of 7 rats is fed a normal diet for two months. The test group of 8 rats is fed a high sugar diet for two months. The biologist weighed each rat at the beginning and end of the investigation. His results are shown on the right.

**Use the unpaired Student's *t*-test to determine whether there is a significant difference between the mean weight gain of the groups at the 5% level? What can the biologist conclude from his results?**

| | Weight gain / g | |
|---|---|---|
| | Normal diet | High sugar diet |
| | 102 | 149 |
| | 95 | 122 |
| | 134 | 169 |
| | 115 | 155 |
| | 97 | 130 |
| | 142 | 173 |
| | 119 | 181 |
| | – | 165 |
| Mean | 114.9 | 155.5 |
| Standard deviation | 18.3 | 20.9 |

There's more about standard deviation and how to work it out on pages 60-61.

**1** ***Identify the null hypothesis.***

The null hypothesis is that there is no significant difference between the mean weight gain of the rats on the normal diet and the mean weight gain of the rats on the high sugar diet.

**2** ***Label the data sets and find the number of values (n) in each set.***

There are quite a few bits to this calculation so it'll help to write them all down so you know what's what.

First, number the data sets 1 and 2.

Data set 1 = normal diet
Data set 2 = high sugar diet

It doesn't matter which way round you number the data sets, just that you stick with these numbers from now on.

You know *n* for each data set, so write that down too.

This is the number of data points in the normal diet group...

$n_1 = \mathbf{7}$ $n_2 = \mathbf{8}$

... and this is the number of data points in the high sugar group.

# Student's *t*-Test

**3** ***Calculate t.***

Remember the formula is:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{s_1^2}{n_1}\right) + \left(\frac{s_2^2}{n_2}\right)}}$$

It's easier to work out the value of $t$ in steps:

**a** Calculate $\bar{x}_1 - \bar{x}_2$. 114.9 – 155.5 = **-40.6**

**b** Calculate $s_1^2 \div n_1$. $18.3^2 \div 7 =$ **47.8**

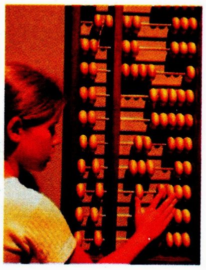
**c** Calculate $s_2^2 \div n_2$. $20.9^2 \div 8 =$ **54.6**

**d** Now plug these value into the formula.

$$\frac{-40.6}{\sqrt{47.8 + 54.6}} = -4.01 \text{ (to 2 d.p.)}$$

You can ignore the minus sign — you need to use a positive $t$ value when comparing it to the critical value.

So $t =$ **4.01**



Sybil loved her retro calculator, but it did make the *t*-test just that little bit more difficult to calculate.

Don't forget the rules of BODMAS when you're doing this calculation — you need to enter it into your calculator as 40.6 ÷ √(47.8 + 54.6) if you want your calculator to do it in the right order. Without the brackets, you'd get a completely different answer.

**4** ***Use $n_1$ and $n_2$ to work out the degrees of freedom.***

The formula for the number of degrees of freedom you should use is: $df = n_1 + n_2 - 2$

7 + 8 – 2 = **13**

**5** ***Use a critical value table to find out if the result is significant.***

The **probability level** you're interested in is 0.05.

This is the column for a P value of 0.05.

This is the row for $df = 13$.

| df | Probability level (P) | | | |
|---|---|---|---|---|
| | 0.20 | 0.10 | 0.05 | 0.02 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 |



The critical value of $t$ is **2.160**.

4.01 is **more than** 2.160 so the difference **is significant** at the 5% level.

This means that there is a 95% probability that the difference between the two means is not due to chance.

This means that the **null hypothesis can be rejected**, so you can conclude that:

**There is a significant difference between the mean weight gain of the rats on the normal diet and the mean weight gain of the rats on the high sugar diet.**

# Student's *t*-Test

## Use a **Paired t-test** When Data is From the **Same Individuals**

If you are comparing the means of two sets of data that have come from the **same individuals** (e.g. two measurements taken from the same individual before and after an event, or under two different conditions) you need to use the **paired Student's *t*-test** to see if the mean difference between the two sets is significant.

Here's the formula for calculating the **paired Student's *t*-test**:

$\bar{d}$ is the **mean of the differences** between each pair of measurements.

$$t = \frac{\bar{d}\sqrt{n}}{s_d}$$

$n$ is the number of pairs.

$s_d$ is the **standard deviation of the differences** between each pair of measurements.

The **type of data** needed to use the paired Student's *t*-test is similar to the unpaired test (see p.75), **except that**:

- the two categorical groups should include the **same individuals** (so two measurements are taken from each individual in total).
- each measurement in one group is **paired** with a measurement in the other group.
- both groups should be the **same size**.

## Work Out the **Degrees of Freedom** to Find the **Critical Value**

You need to calculate the **degrees of freedom** for the paired *t*-test. Here's how you do it:

1) Count up the number of pairs of data. This is $n$.
2) Then subtract 1. Simple

$df = n - 1$

**Worked Example**

A student was investigating the effect of exercise on heart rate. She recorded the heart rates of a group of volunteers at rest and after skipping for 3 minutes. The results are shown in the table on the right. The mean difference between the resting heart rate and heart rate after skipping was 52.5 bpm and the standard deviation of the difference was 8.2 bpm.

**Use the paired Student's *t*-test to determine whether skipping has a significant effect on heart rate at the 1% probability level.**

| Heart rate / bpm | |
|---|---|
| At rest | After skipping for 3 mins |
| 75 | 120 |
| 62 | 126 |
| 70 | 114 |
| 71 | 119 |
| 73 | 132 |
| 68 | 118 |
| 72 | 135 |
| 70 | 117 |

1. ***Identify the null hypothesis.***

   The null hypothesis is that there is no significant difference between the mean heart rate at rest and the mean heart rate after skipping for 3 minutes.

2. ***Work out the number of pairs (n).***

   Each row in the table represents one pair of data (from one individual).

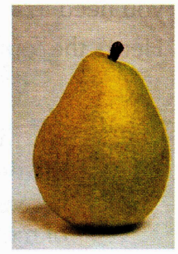
   There are 8 rows in the table, so $n = \mathbf{8}$

# Student's *t*-Test

**3** ***Calculate t.***

Just pop the values into the formula: $\boxed{t = \frac{\bar{d}\sqrt{n}}{s_d}}$

$$t = \frac{\bar{d}\sqrt{n}}{s_d} = (52.5 \times \sqrt{8}) \div 8.2 = \mathbf{18.11\ (2\ d.p.)}$$



Darcy was alarmed to hear that her data should be recorded in pears.

**4** ***Use n to work out the degrees of freedom.***

The formula for the number of degrees of freedom you should use is: $\boxed{df = n - 1}$

8 − 1 = **7**

**5** ***Use a critical value table to find out if the result is significant.***

The **probability level** you're interested in is 0.01.

This is the column for a P value of 0.01.

This is the row for $df = 7$.

| *df* | Probability level (P) | | | |
|---|---|---|---|---|
| | **0.10** | **0.05** | **0.02** | **0.01** |
| 4 | 2.132 | 2.776 | 3.747 | 4.604 |
| 5 | 2.015 | 2.571 | 3.365 | 4.032 |
| 6 | 1.943 | 2.447 | 3.143 | 3.707 |
| 7 | 1.895 | 2.365 | 2.998 | 3.499 |
| 8 | 1.860 | 2.306 | 2.896 | 3.355 |
| 9 | 1.833 | 2.262 | 2.821 | 3.250 |
| 10 | 1.812 | 2.228 | 2.764 | 3.169 |
| 11 | 1.796 | 2.201 | 2.718 | 3.106 |
| 12 | 1.782 | 2.179 | 2.681 | 3.055 |



The critical value of $t$ is **3.499**

18.11 is **more than** 3.499 so the difference **is significant** at the 1% level.

This means that the **null hypothesis can be rejected**, so you can conclude that:

**There is a significant difference between the mean heart rate at rest and the mean heart rate after skipping for 3 minutes.**

## Practice Questions

Q1 Use the critical value table above to determine whether these values of $t$ are significant.

a) $t = 3.645$ at probability level 0.01 when $df = 6$.

b) $t = 1.965$, where $p \leq 0.05$ when $df = 10$.

c) $t = 3.451$, where $p \leq 0.01$ when $df = 8$.

d) $t = 2.179$ at the 5% level when $df = 12$.

# Student's *t*-Test

Q2 An experiment was carried out to investigate the effect of exercise on breathing rate. The breathing rates of a group of ten participants were measured at rest and then again after running for five minutes on a treadmill. The results are in the table below.

| Participant | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Breathing rate at rest / breaths per minute | 10 | 12 | 15 | 19 | 16 | 16 | 11 | 20 | 17 | 14 |
| Breathing rate after exercise / breaths per minute | 41 | 50 | 51 | 42 | 43 | 44 | 49 | 45 | 40 | 49 |

Mean difference = 30.4
Standard deviation of difference = 6.0

The breathing rates before and after exercise were compared using the paired Student's *t*-test.

a) Explain why the paired Student's *t*-test was the statistical test used to analyse this data, instead of the unpaired Student's *t*-test.

b) Write down the null hypothesis for this investigation.

c) Calculate the *t*-value for these results.

d) Use the table on the previous page to find the critical value for this experiment. Are these results significant at the 1% level?

Q3 Marty is investigating whether light intensity affects the size of ivy leaves. He selects at random twenty ivy leaves growing on the north-facing (shaded) side of a tree stump and twenty ivy leaves growing on the south-facing (unshaded) side. His results are summarised in the table below.

| | Mean width / cm | Standard deviation ($s$) / cm |
|---|---|---|
| Shaded | 7.6 | 1.4 |
| Unshaded | 6.3 | 1.5 |

a) Write down the null hypothesis for this study.

b) Calculate the unpaired Student's *t*-test value for these results.

c) Calculate the degrees of freedom for this investigation.

d) The critical value for this experiment at a probability level of 0.05 is 2.024. Explain what Marty can conclude from this investigation.

Q4 As part of a study investigating the effect of living conditions on health, a researcher surveyed 15 people in the same city about their BMI and the area of the city they lived in. The results are shown in the table below.

| | BMI | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| City centre | 19.9 | 22.1 | 25.3 | 26.0 | 24.5 | 27.2 | 28.7 | 24.0 |
| Suburbs | 21.8 | 19.5 | 24.4 | 22.2 | 26.0 | 26.5 | 25.2 | – |

The mean BMI of the people living in the city centre was found to be 24.71, with a standard deviation of 2.79. The mean BMI of the people living in the suburbs was 23.66, with a standard deviation of 2.56.

a) Calculate the observed *t*-value for this data using the formula for the unpaired Student's *t*-test.

b) Use the table on the previous page to determine whether these results are significant at $p \leq 0.05$.

**Assam, Darjeeling, Earl Grey, Lapsang — this t-test is right up my street...**

*Bother. Wrong sort of tea. But these pages are still important. Make sure you remember when to use a Student's t-test and what all the little bits of those formulas mean. And when you're talking about the significance of the result don't forget that you need a t-value that's equal to or bigger than the critical value before you can reject the null hypothesis.*