1

Say we have two vectors written in the computational basis:

$$|a\rangle = 3|0\rangle + 2i|1\rangle, |b\rangle = -|0\rangle + 2|1\rangle$$

a)

$$\langle a| = 3\langle 0| - 2i\langle 1|$$
$$\langle b| = -\langle 0| + 2\langle 1|$$

b)

$$\langle a|b\rangle = -3 - 4i$$
$$\langle b|a\rangle = -3 + 4i$$

c)

$$|c\rangle = |a\rangle + 2|b\rangle = |0\rangle + (4+2i)|1\rangle$$
$$\langle c|a\rangle = 3 + (4-2i)2i = 3 + 8i + 4 = 7 + 8i$$

d)

$$||a\rangle| = \sqrt{9+4} = \sqrt{13}$$
$$||b\rangle| = \sqrt{1+4} = \sqrt{5}$$
$$|\tilde{a}\rangle = \frac{3}{\sqrt{13}}|0\rangle + \frac{2i}{\sqrt{13}}|1\rangle$$
$$|\tilde{b}\rangle = -\frac{1}{\sqrt{5}}|0\rangle + \frac{2}{\sqrt{5}}|1\rangle$$

e)

No, since $\langle a|b\rangle$ is not zero.

2

Photon horizontal and vertical polarization states are written as $|h\rangle$ and $|v\rangle$. Suppose

$$|\psi_1\rangle = \frac{1}{2}|h\rangle + \frac{\sqrt{3}}{2}|v\rangle$$
$$|\psi_2\rangle = \frac{1}{2}|h\rangle - \frac{\sqrt{3}}{2}|v\rangle$$
$$|\psi_3\rangle = |h\rangle$$

Find

$$|\langle\psi_1|\psi_2\rangle|^2, |\langle\psi_1|\psi_3\rangle|^2, |\langle\psi_3|\psi_2\rangle|^2$$

$$|\langle\psi_1|\psi_2\rangle|^2 = \left(\frac{1}{4} - \frac{3}{4}\right)^2 = \frac{1}{4}$$

$$|\langle\psi_1|\psi_3\rangle|^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$|\langle\psi_3|\psi_2\rangle|^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

3

$$|q_1\rangle = |\tilde{u}_1\rangle = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$|u_2'\rangle = |u_2\rangle - \langle q_1|u_2\rangle|q_1\rangle = \begin{pmatrix} 1 \\ 2 \\ 4 \\ 5 \end{pmatrix} - 6\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \\ 1 \\ 2 \end{pmatrix}$$

$$|q_2\rangle = |\tilde{u}_2\rangle = \begin{pmatrix} -\frac{2}{\sqrt{10}} \\ -\frac{1}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} \\ \frac{2}{\sqrt{10}} \end{pmatrix}$$

$$|u_3'\rangle = |u_3\rangle - \langle q_1|u_3\rangle|q_1\rangle - \langle q_2|u_3\rangle|q_2\rangle$$

$$= \begin{pmatrix} 1 \\ -3 \\ -4 \\ -2 \end{pmatrix} - (-4)\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} - (-\frac{7}{\sqrt{10}})\begin{pmatrix} -\frac{2}{\sqrt{10}} \\ -\frac{1}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} \\ \frac{2}{\sqrt{10}} \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ -3 \\ -4 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \\ 2 \\ 2 \end{pmatrix} + \begin{pmatrix} -\frac{7}{5} \\ -\frac{7}{10} \\ \frac{7}{10} \\ \frac{7}{5} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{8}{5} \\ -\frac{17}{10} \\ -\frac{13}{10} \\ \frac{7}{5} \end{pmatrix}$$

$$|q_3\rangle = |\tilde{u}_3\rangle = \begin{pmatrix} \frac{8\sqrt{910}}{455} \\ -\frac{17\sqrt{910}}{910} \\ -\frac{\sqrt{910}}{70} \\ \frac{\sqrt{910}}{65} \end{pmatrix}$$