Problem 1

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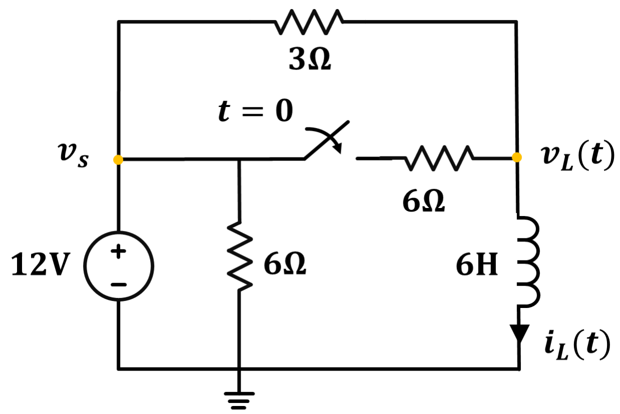
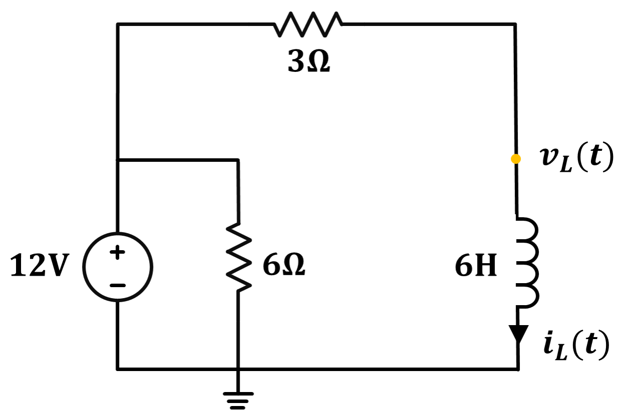
Sign: Yuqing Zhai

Problem 2



(a)

Before the switch close, it looks like



In the DC-steady state, the inductor will act like a short, therefore the voltage across it will be 0V. Using node-voltage method, we will see that

$$v_L(0^-) - 0\text{V} = 0\text{V}$$

which means that $\boxed{v_L(0^-) = 0\text{V}}$

Using node method, we see that

$$v_s - 0\text{V} = 12\text{V}$$

and therefore $v_s = 12\text{V}$. On node v_L, we see that

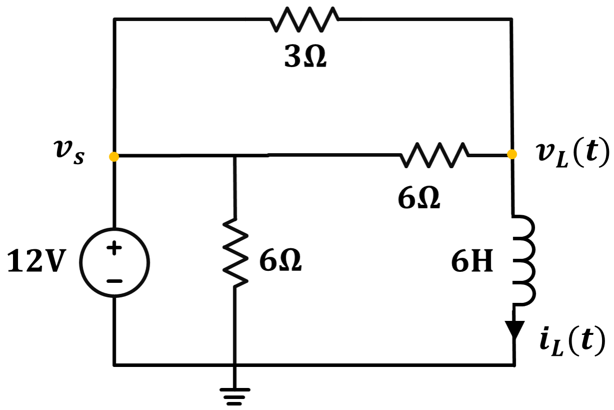
$$\frac{v_s - v_L(0^-)}{3\Omega} = i_L(0^-)$$

and therefore $\boxed{i_L(0^-) = 4\mathrm{A}}$

(b)y

See after (c)

(c)



Using node voltage method on node $v_L(t)$

$$\frac{v_s - v_L(t)}{3} + \frac{v_s - v_L(t)}{6} = i_L(t)$$
$$\frac{v_s - v_L(t)}{2} = i_L(t)$$

We know that the inductor follows

$$v_L(t) = L\frac{\mathrm{d}i_L(t)}{\mathrm{d}t}$$

Setting up the equation

$$\begin{aligned} v_s - v_L(t) &= 2 \cdot i_L(t) \\ v_s - L\frac{\mathrm{d}i_L(t)}{\mathrm{d}t} &= 2 \cdot i_L(t) \\ v_s - 6 \cdot \frac{\mathrm{d}i_L(t)}{\mathrm{d}t} &= 2 \cdot i_L(t) \\ i_L'(t) + \frac{1}{3} \cdot i_L(t) &= \frac{1}{3}(\frac{1}{2}v_s) \end{aligned}$$

From the textbook 3.4.2, we know this form of equation has solution of

$$i_L(t) = (i_L(0^-) - \frac{1}{2}v_s)e^{-t/3} + \frac{1}{2}v_s$$

which is

$$i_L(t) = (4-6)e^{-t/3} + 6$$
$$\boxed{i_L(t) = (-2e^{-t/3} + 6)\mathrm{A}}$$

and thus

$$\boxed{v_L(t) = L\frac{\mathrm{d}i_L(t)}{\mathrm{d}t} = 6 \cdot -2 \cdot (-\frac{1}{3})e^{-t/3} = (4e^{-t/3})\mathrm{V}}$$

It's obvious that their $\tau = 3\mathrm{s}$

(b)

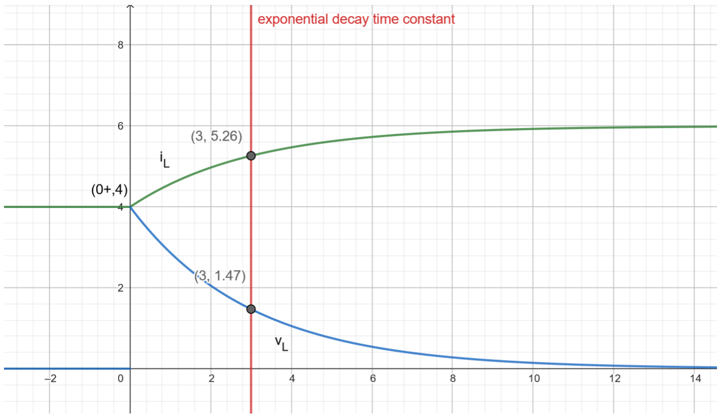
From (c), we see that

$$i_L(t) = (i_L(0^-) - \frac{1}{2}v_s)e^{-t/3} + \frac{1}{2}v_s$$
$$i_L(t) = \frac{1}{2}v_s(1 - e^{-t/3}) + i_L(0^-)e^{-t/3}$$

and we could see that

- Let $i_L(0^-) = 0$, the **zero-state response** is $\boxed{i_{L,\text{ZS}}(t) = \frac{1}{2}v_s(1 - e^{-t/3}) = 6(1 - e^{-t/3})}$
- Let $v_s = 0$, the **zero-input response** is $\boxed{i_{L,\text{ZI}} = i_L(0^-)e^{-t/3}} = 4e^{-t/3}\text{A}$

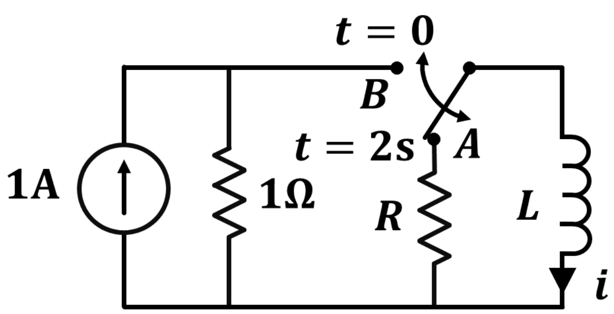
(d)



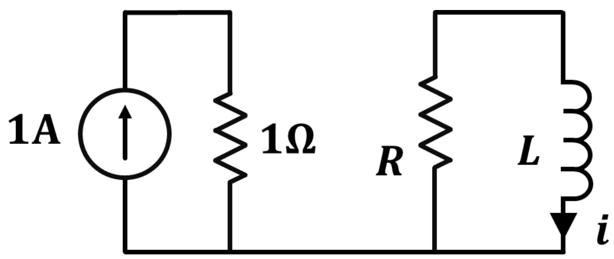
$\tau = 3\text{s}$

- The blue line is $v_L(t)$. At $t = 0^+, v_L = 4\text{V}$. As $t \to \infty$, $v_L \to 0\text{V}$. At $t = \tau$, $v_L(t) = 4e^{-1}\text{V}$
- The green line is $i_L(t)$. At $t = 0$, $i_L = 4\text{A}$. As $t \to \infty$, $v_L = 6\text{V}$. At $t = \tau$, $i_L(t) = (-2e^{-1} + 6)\text{A}$

Problem 3



Before the $t = 0$:



Since the switch has been in position A for a long time, the circuit will be in a steady state, and the inductor will act like a short.

Since in this case the 1Ω resistor will have $1\mathrm{A}$ current (provided by the current source), using KCL on the node a, we see that

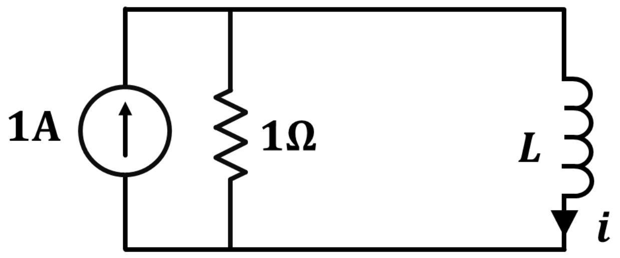
$$I + 1 = 1$$

that means the $I = 0\mathrm{A}$, and therefore, do the KCL on the node b, we see the current on R is just i . Therefore, do the KVL as indicated, we see

$$Ri = v_L$$

since the inductor acts like a switch, $v_L = 0\mathrm{V}$, and therefore $i(0) = 0\mathrm{A}$

At $t = 0$, when the switch moves to B, we see



From the textbook 3.4.2, this kind of circuit has the solution of

$$i(t) = (i(0^-) - I_s)e^{-\frac{t}{L/R}} + I_s$$

given $I_s = 1\mathrm{A}$, $R = 1\Omega$ in this case, it is

$$i(t) = 1 - e^{-t/L}$$

at $t = 2\mathrm{s}$, we want $i(2) = (1 - e^{-1})\mathrm{A}$

$$\begin{gathered} i(2) = 1\mathrm{A}(1 - e^{-2/L}) = (1 - e^{-1})\mathrm{A} \\ -\frac{2}{L} = -1 \\ \boxed{L = 2\mathrm{H}} \end{gathered}$$

Then after $t = 2\mathrm{s}$, it moves back to A again, as mentioned earlier, that means

$$\begin{gathered} Ri(t) = v_L(t) \\ Ri(t) = L\frac{\mathrm{d}i(t)}{\mathrm{d}t} \\ \frac{R}{L}\mathrm{d}t = \frac{1}{i(t)}\mathrm{d}i(t) \\ \int_{2\mathrm{s}}^{t} \frac{R}{L}\mathrm{d}s = \int_{i(2\mathrm{s})}^{i(t)} \frac{1}{i}\mathrm{d}i \\ \ln i(t) = \ln i(2) - \frac{R}{L}(t-2) \\ i(t) = i(2)e^{-\frac{R}{L}(t-2)} \end{gathered}$$

we know that $i(2) = (1 - e^{-1})\mathrm{A}$. Therefore

$$i(t) = (1 - e^{-1})\mathrm{A} \cdot e^{-\frac{R}{L}(t-2)}$$

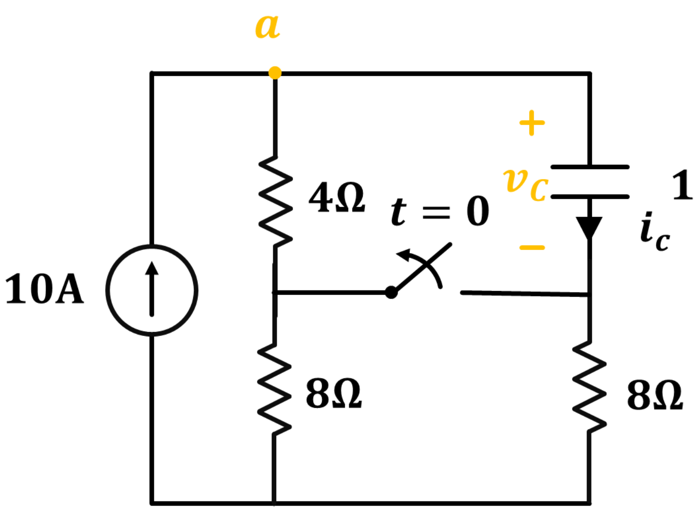
we want $i(8) = (1 - e^{-1})e^{-2}\mathrm{A}$, that is

$$i(8) = (1 - e^{-1})\mathrm{A} \cdot e^{-\frac{R}{L}(8-2)} = (1 - e^{-1})e^{-2}\mathrm{A}$$

$$-\frac{R}{L}(8-2) = -2$$

$$\boxed{R = \frac{L}{3} = \frac{2}{3}\Omega}$$

Problem 4



(a)

Since the switch has been closed for a long time, the capacitor will act like a open-circuit. Therefore $\boxed{i_C = 0\mathrm{A}}$ We see that in this case, the current flowing through the 4Ω is $10\mathrm{A}$. Therefore

$$\boxed{v_R(0^-) = 4\Omega \cdot 10\mathrm{A} = 40\mathrm{V}}$$

doing the KVL as indicated, we see that

$$\boxed{v_C(0^-) = v_R(0^-) = 40\mathrm{V}}$$

(b)

See after (d)

(c)

Apply KCL at node a,

$$i_s = i_R + i_C$$

and applying node-method at node a

$$v_a = i_R \cdot (4+8) = v_C + 8 \cdot i_C$$

At the capacitor

$$i_C = C\frac{\mathrm{d}v_C}{\mathrm{d}t} = 1\frac{\mathrm{d}v_C}{\mathrm{d}t}$$

and therefore

$$
\begin{aligned}
12(i_s - i_C) &= v_C + 8i_C \\
12i_s &= 20i_C + v_C \\
20v_C' + v_C &= 12i_s \\
v_C' + \frac{1}{20}v_C &= \frac{1}{20}(12i_s)
\end{aligned}
$$

From textbook 3.4.1, we know this kind of differential equation has solution

$$v_C(t) = (v_C(0^-) - 12i_s)e^{-t/20} + 12i_s$$

we know that $I_s = 10\mathrm{A}$ and $v_C(0^-) = 40\mathrm{V}$, therefore

$$\boxed{v_C(t) = (-80e^{-t/20} + 120)\mathrm{V}}$$

therefore we know that

$$\boxed{i_C = \frac{\mathrm{d}v_C}{\mathrm{d}t} = 4e^{-t/20}\mathrm{A}}$$

(d)

It follows that

$$i_R = i_s - i_c = (10 - 4e^{-t/20})\mathrm{A}$$

and therefore

$$\boxed{v_R = 4\Omega \cdot i_R = (40 - 16e^{-t/20})\mathrm{V}}$$

(b)

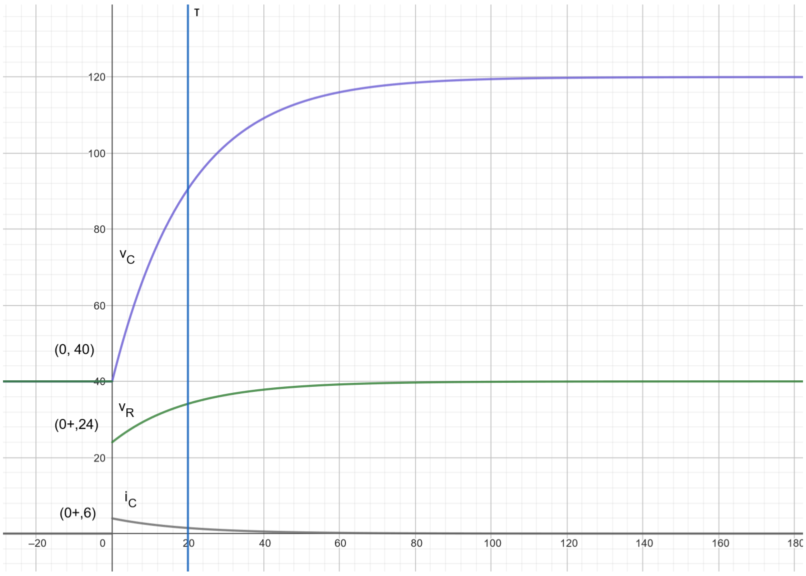
From (c), we see that

$$
\begin{aligned}
v_C(t) &= (v_C(0^-) - 12i_s)e^{-t/20} + 12i_s \\
v_C(t) &= 12i_s(1 - e^{-t/20}) + v_C(0^-)e^{-t/20}
\end{aligned}
$$

and we could see that

- Let $v_C(0^-) = 0$, the **zero-state response** is $\boxed{i_{L,\mathrm{ZS}}(t) = 12i_s(1 - e^{-t/20})}$
- Let $i_s = 0$, the **zero-input response** is $\boxed{i_{L,\mathrm{ZI}} = v_C(0^-)e^{-t/20} = 40e^{-t/20}\mathrm{V}}$

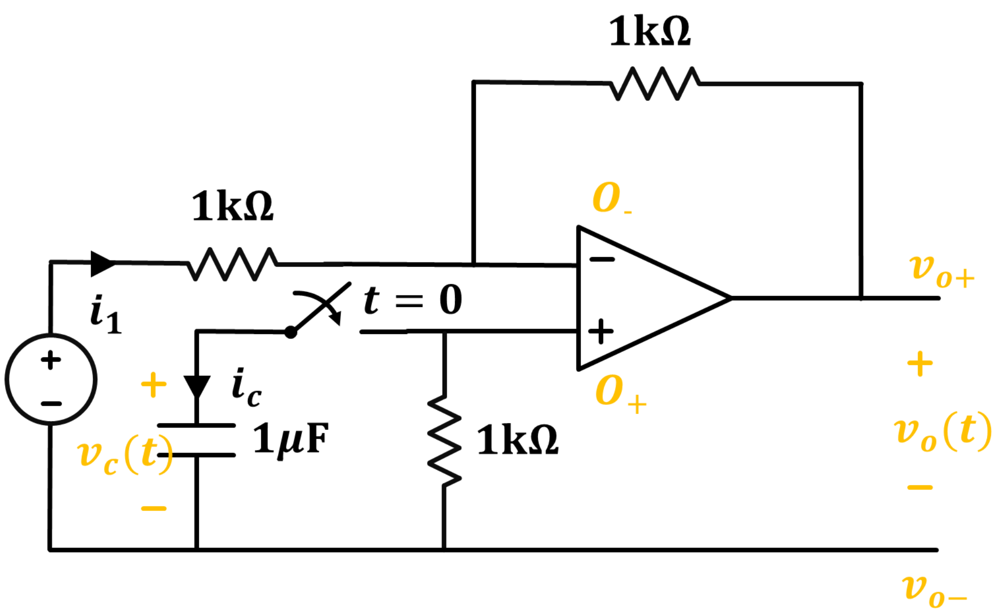
(e)



$\tau = 20\text{s}$

- The purple line is the $v_C(t)$. At $t = 0$, $v_C(t) = 40\text{V}$. As $t \to \infty$, $v_C(t) \to 120\text{V}$. At $t = \tau$, $v_C(t) = (-80e^{-1} + 120)\text{V}$
- The green line is the $v_R(t)$. At $t = 0^+$, $v_R(t) = 24\text{V}$. As $t \to \infty$, $v_R(t) = 40\text{V}$. At $t = \tau$, $v_R(t) = (40 - 16e^{-1})\text{V}$
- The gray line is the $i_C(t)$. At $t = 0^+$, $i_C(t) = 4\text{A}$. As $t \to \infty$, $i_C(t) = 0\text{A}$. At $t = \tau$, $i_C(t) = 4e^{-1}\text{A}$

Problem 5



Using node-voltage method, we see that

$$v_{O+} - 0\text{V} = v_c(t)$$

therefore $v_{O+} = v_c(t)$, and therefore $v_{O-} = v_{O+} = v_c(t)$ under the ideal op-amp approximation. Therefore, the current i_1 is

$$i_1 = \frac{1\text{V} - v_c(t)}{1\text{k}\Omega}$$

since under ideal op-amp approximation

$$i_{O-} = i_{O+} = 0\mathrm{A}$$

therefore at node O_- (using KCL)

$$\frac{v_c(t) - v_{o+}(t)}{1\mathrm{k}\Omega} = \frac{1V - v_c(t)}{1\mathrm{k}\Omega}$$

> Notice the difference between v_{O+} and v_{o+}

and therefore we get

$$v_{o+}(t) = (2v_c(t) - 1)\mathrm{V}$$

we see that at node O_+

$$\frac{v_c(t) - 0\mathrm{V}}{1\mathrm{k}\Omega} = -i_c(t)$$

and at the same time

$$i_c(t) = C\frac{\mathrm{d}v_c(t)}{\mathrm{d}t} = 1\mu\mathrm{F} \cdot \frac{\mathrm{d}v_c(t)}{\mathrm{d}t}$$

therefore

$$v_c(t) = -1\mathrm{k}\Omega \cdot 1\mu\mathrm{F} \cdot \frac{\mathrm{d}v_c(t)}{\mathrm{d}t} = -1m\mathrm{V} \cdot \mathrm{s}\frac{\mathrm{d}v_c}{\mathrm{d}t}$$

and therefore (the derivation is similar to problem 3, so the process is skipped)

$$v_c(t) = v_c(0)e^{-1000t}\mathrm{V}$$

therefore

$$v_{o+}(t) = (2v_c(0)e^{-1000t} - 1)\mathrm{V}$$

Let $t = 2ms$, we get

$$v_{o+}(t) = (2 \cdot (-2\mathrm{V}) \cdot e^{-1000 \cdot 2\mathrm{m}s} - 1)\mathrm{V}$$
$$v_{o+}(t) = (-4e^{-2} - 1)\mathrm{V}$$

we see that therefore

$$\boxed{v_o(t) = v_{o+}(t) - v_{o-}(t) = (-4e^{-2} - 1)\mathrm{V} - 0\mathrm{V} = (-4e^{-2} - 1)\mathrm{V}}$$