Subproblems

Define $C(i,j,m)$ for indices $1 \le i \le m \le j \le n$ such that $C(i,j,m)$ represents the cost of constructing a proper binary tree using the subsequence $\langle a_i, a_{i+1}, \ldots, a_j \rangle$ with a_m as the root of the tree. Obviously, the final result is $\sum_{m=1}^{n} C(0,n,m)$.

Recursive Formula

For base case, if the subsequence has a single element then it is a leaf, and there are no edges so that cost $C(i,i,i) = 0$. If the subsequence has even number of elements, then it is invalid, because you could not construct a proper binary tree from it, so $C(i, i+2k, *) = \infty$ for $k \in \mathbb{Z}$.

For subsequence has odd number of elements (and is more than one element), then we need to pick a element a_m to be the root, then we need to total cost is the cost to construct left subtree (plus an edge) and right subtree (plus an edge):

$$C(i,j,m) = L(i,m) + R(m,j)$$

where the cost for left part and right part is:

$$\begin{aligned} L(i,m) &= \min_{k \in [1,m-1]} C(i, m-1, k) + |a_m - a_k| \\ R(m,j) &= \min_{\ell \in [m+1,j]} C(m+1, j, \ell) + |a_m - a_\ell| \end{aligned}$$

That is, the minimum for constructing the subtree plus the edge cost.

Evaluation Order

We notice that for $C(i,j,m)$, the case depends on case $C(i',j',m')$ where the interval length $L = j - i + 1$ is bigger than $L' = j' - i' + 1$. So, we evaluate all intervals $L \in [1,n]$ in increasing order (we only evaluate odd length as even length is already a base case). For m, we evaluate them in $m \in (i,j)$. (We could just evaluate m in any order, but let's evaluate it in increasing order).

Complexity

There are $O(n^2)$ intervals for $(i \le i \le j \le m)$, for each interval, we evaluate $O(n)$ choices of m. For each choice, we iterate over k and ℓ so it will take $O(n)$ time. So, in total it will take $O(n^4)$ time. For space complexity, since we are storing for each interval with each choice of m, it will take $O(n^3)$ space.