Question 1

> Using $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, calculate the expectation value $\langle X \rangle$ for the states $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$ (related to McMahon 3.3)

The expectation value $\langle X \rangle$ for $|+\rangle$ and $|-\rangle$

$$\begin{aligned} \langle + | X | + \rangle &= \langle + | + \rangle = 1 \\ \langle - | X | - \rangle &= -\langle - | - \rangle = -1 \end{aligned}$$

It's expected since $|+\rangle$ and $|-\rangle$ is the eigenstate of the X.

Question 2

> Determine the unitary operator needed to transform from the $|0\rangle$, $|1\rangle$ "computational" basis to the $|+\rangle$, $|-\rangle$ basis. Using this unitary matrix, transform X, above, from the computational to the $|\pm\rangle$ basis and compare with the result in McMahon 3.3

This is the Hadamard Gate H, since

$$\begin{aligned} H|0\rangle &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |+\rangle \\ H|1\rangle &= \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = |-\rangle \end{aligned}$$

$$X' = HXH = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

which has the same result in McMahon 3.3. This indicates the unitary matrix (the change-basis matrix) is indeed correct.

Question 3

> A three-state system is in the state
>
> $$|\psi\rangle = \frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle + \frac{i}{\sqrt{2}}|2\rangle$$
>
> Write down the necessary projection operators and calculate the probabilities $\Pr(0)$, $\Pr(1)$ and $\Pr(2)$

So,

$$\begin{aligned} P_0 &= |0\rangle\langle 0| \\ P_1 &= |1\rangle\langle 1| \\ P_2 &= |2\rangle\langle 2| \end{aligned}$$

$$\Pr(0) = \langle\psi| P_0 |\psi\rangle = \langle\psi| \frac{1}{2} |0\rangle = \frac{1}{4}$$

$$\Pr(1) = \langle\psi| P_1 |\psi\rangle = \langle\psi| \frac{1}{2} |1\rangle = \frac{1}{4}$$

$$\Pr(2) = \langle\psi| P_2 |\psi\rangle = \langle\psi| \frac{i}{\sqrt{2}} |2\rangle = -\frac{i}{\sqrt{2}} \cdot \frac{i}{\sqrt{2}} = \frac{1}{2}$$