ECE 343 Lab #1: Passive Filters

1 Introduction

Welcome to ECE343! This class provides an opportunity to get hands-on circuit experience as an accompaniment to ECE342. Each lab will follow a "*Prove, Simulate, Verify*" methodology:

1. First we mathematically prove concepts. This provides intuition for what comes next.
2. Next we introduce non-idealities by using `SPICE` models in simulation.
3. Finally, we validate all of the above by doing real measurements (using M2K Kits and lab equipment)

Lab 1 serves as a quick review of key concepts from ECE 210, as well as an introduction to the tools you'll be using for the rest of the lab. We will specifically be doing that in the context of a simple RC filter.

1.1 Learning Objectives

- Review Fourier/Laplace transforms and filter design
- Use LTspice to simulate circuit performance
- Use graphs to present simulation results
- Setup software/hardware needed in the experiments
- Take measurements using SCOPY and lab equipment
- Compare and analyze theoretical, simulated, and experimental results
- Explain the impact of R_{in} and R_{out} on the gain/frequency response of a system

2 Analysis – RC Filters

The work in this section involves only "paper work" and no bench setups. It requires you to use circuit analysis techniques learned in ECE 210 to compute the frequency response $\mathbf{H}(\omega)$ for circuit shown in Figs. 1 and 2. This exercise will also help set up the framework for Section 3.

2.1 First Order RC Filters

While we do not require you to memorize Fourier transform derivations for this lab, you should be able to do common time domain to/from frequency domain conversions. Consider the circuit shown below:

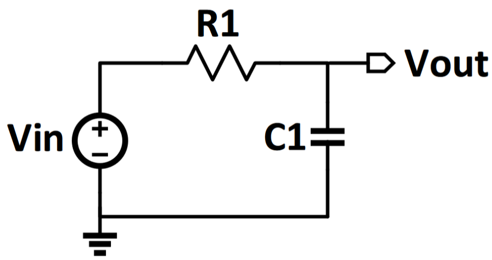

Figure 1: First Order RC Filter

1. Based on your intuition, comment on the type of filter. (i.e. Low-pass, High-pass, ...)(Hint: Think about what the impedance of a capacitor is at $\omega = 0$ and $\omega = \infty$.)

Low-pass

2. Compute the complex frequency response $\mathbf{H}(\omega)$.

$$\mathbf{H}(\omega) = \frac{\mathrm{V_{in}}}{\mathrm{V_C}} = \frac{\mathrm{Z_C}}{\mathrm{Z_C} + \mathrm{Z_R}} = \frac{1/j\omega \mathrm{C_1}}{\mathrm{R_1} + 1/j\omega \mathrm{C_1}} = \frac{1}{1 + j\omega \mathrm{R_1 C_1}}$$

3. Compute and sketch the magnitude response $|\mathbf{H}(\omega)|$. Compute the 3-dB cutoff frequency. Check the calculated result with your judgement in Question 1.

Therefore,

$$|\mathbf{H}(\omega)| = \frac{1}{\sqrt{1 + (\omega \mathrm{R_1 C_1})^2}}$$

Set $|\mathbf{H}(\omega)| = 1/\sqrt{2}$.

$$\begin{aligned} 1 + (\omega \mathrm{R_1 C_1})^2 &= 2 \\ \omega \mathrm{R_1 C_1} &= 1 \\ \omega &= \boxed{1/\mathrm{R_1 C_1}} \end{aligned}$$

It makes sense. As $\omega \to 0$, $|\mathbf{H}(\omega)| \to 1$. As $\omega \to \infty$, $|\mathbf{H}(\omega)| \to 0$. So, it's indeed low pass filter. Sketch the magnitude response:

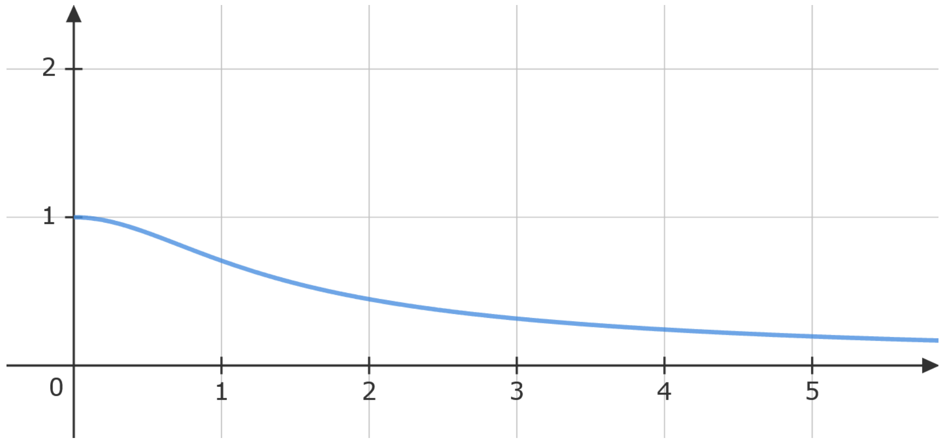

Figure Solution 1: Magnitude of $\mathbf{H}(\omega)$ (V) / Frequency ω (Hz) given ($R_1C_1 = 1$)

2.2 RC filter with load resistor

In this section we expand the analysis done in section 2.1 by attaching a load resistor at the output node.

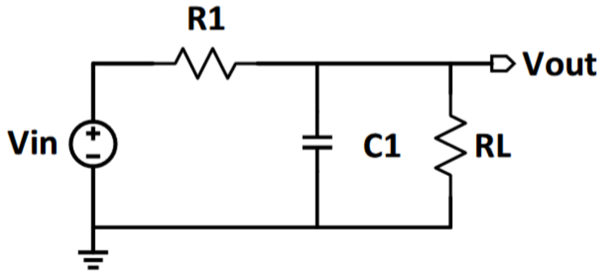

Figure 2: First Order RC Filter with load resistor

For the circuit shown in Fig. 2 above:

1. Find the complex frequency response $\mathbf{H}(\omega)$

$$Z_C \parallel Z_L = \frac{R_L/j\omega C_1}{R_L + 1/j\omega C_1} = \frac{R_L}{1 + j\omega R_L C_1}$$

$$\begin{aligned}\mathbf{H}(\omega) &= \frac{Z_C \parallel Z_L}{Z_R + (Z_C \parallel Z_L)} \\ &= \frac{R_L/(1 + j\omega R_L C_1)}{R_1 + R_L/(1 + j\omega R_L C_1)} \\ &= \frac{R_L}{R_L + R_1 + j\omega R_1 R_L C_1}\end{aligned}$$

2. Compute and sketch the magnitude response. Compute the $\mathbf{3-dB}$ cut-off frequency.

$$|\mathbf{H}(\omega)| = \frac{R_L}{\sqrt{(R_L + R_1)^2 + (\omega R_1 R_L C_1)^2}}$$

$$|\mathbf{H}(0)| = \frac{R_L}{R_L + R_1}$$

Set $|\mathbf{H}(\omega)\ /\ |\mathbf{H}(0)| = 1/\sqrt{2}$,

$$\frac{R_L}{\sqrt{(R_L + R_1)^2 + (\omega R_1 R_L)^2}} = \frac{1}{\sqrt{2}} \frac{R_L}{R_L + R_1}$$
$$\frac{R_L + R_1}{\sqrt{(R_L + R_1)^2 + (\omega R_1 R_L C_1)^2}} = \frac{1}{\sqrt{2}}$$
$$\frac{(R_L + R_1)^2 + (\omega R_1 R_L C_1)^2}{(R_L + R_1)^2} = 2$$
$$\frac{\omega R_1 R_L C_1}{R_L + R_1} = 1$$

and we get

$$\boxed{\omega = \frac{R_L + R_1}{R_1 R_L C_1} = \frac{1}{(R_1 \parallel R_L)\, C_1}}$$

Sketch the magnitude response:

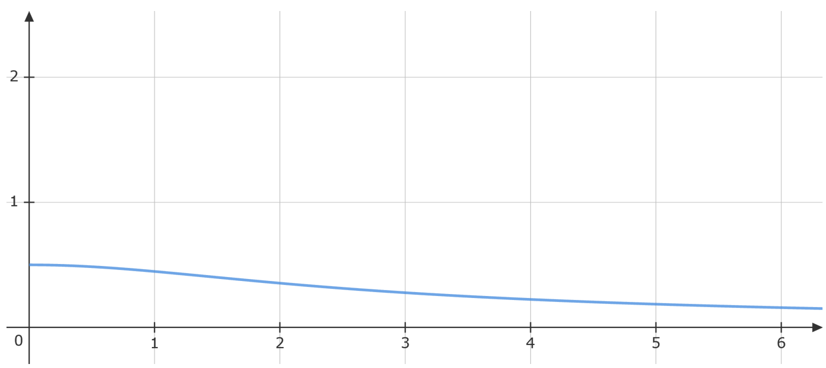

Figure Solution 2: Magnitude of $\mathbf{H}(\omega)$ (V) / Frequency ω (Hz) given $(R_1, R_L, C_1 = 1)$

3. If we want the **3 − dB** cut-off frequency of the circuit in Fig. 2 be the same as that of Fig. 1, how will you change the value of the load resistor in Fig. 2

Set $R_L = \infty$

2.3 RC filter with Op-Amp

In this section we expand the analysis done in section 2.2 by inserting an Op-Amp between the filter and load resistor.

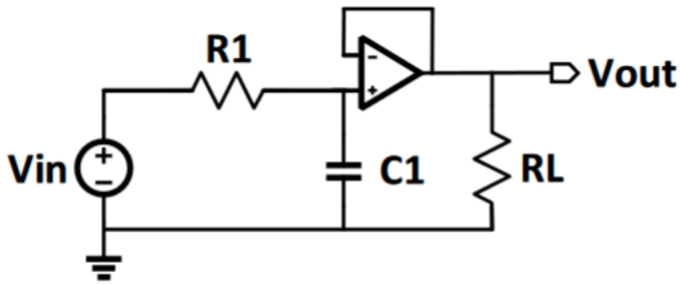

Figure 3: First Order RC Filter with Op-Amp

For the circuit shown in Fig. 3 above:

1. What is the input resistance looking into the (ideal) Op-Amp?

Infinity

2. Find the complex frequency response $\mathbf{H}(\omega)$ and compute the $\mathbf{3-dB}$ cut-off frequency.

Assume ideal Op-Amp, then $\mathrm{I}_+ = \mathrm{I}_- = 0\mathrm{A}$, which means that $\mathrm{I}_{\mathrm{R}_1} = \mathrm{I}_{\mathrm{C}_1}$ and $\mathrm{I}_{\mathrm{R}_1}(\mathrm{Z}_1 + \mathrm{Z}_\mathrm{C}) = \mathrm{V}_{\mathrm{in}}$. We recover the same equation as in section 2.1. So

$$\mathbf{H}(\omega) = \frac{1}{1 + j\omega \mathrm{R}_1\mathrm{C}_1}$$
$$|\mathbf{H}(\omega)| = \frac{1}{\sqrt{1 + (\omega \mathrm{R}_1\mathrm{C}_1)^2}}$$

So, the result is

$$\boxed{\omega = 1/\mathrm{R}_1\mathrm{C}_1}$$

3. Compare the result of section 2.2 and 2.3, what is the advantage of the filter with Op-Amp.

We see that section 2.3 has a same response function as section 2.1. (Equivalent to section 2.2 if we set $\mathrm{R}_\mathrm{L} \to \infty$, but that is practically impossible).

The advantage of the filter with Op-Amp is that we could get the ideal response function as in section 2.1, (as it separates the RC circuit from the load resistor) and the response $\mathbf{H}(\omega)$ is independent from the $\mathrm{R_L}$ that we choose. Therefore we could freely choose the $\mathrm{R_L}$ we want to achieve desired effect.

3 Simulation

In this section we will verify the analysis done in section 2 using a circuit simulation software called LTspice. Please refer to the tutorial on the course website to get started with LTspice.

3.1 Simulation – First Order RC Filter with Op-Amp

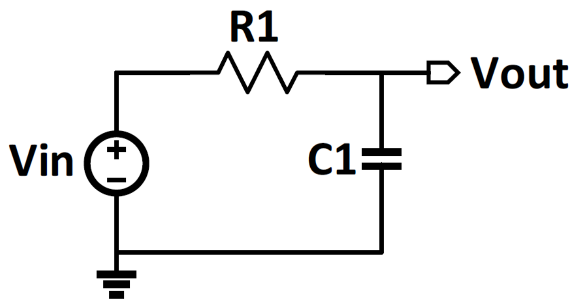

Figure 4: First Order RC Filter

Component Values: $\mathrm{V_{in}} = 1\sin(\omega t)\mathrm{V}, \mathrm{R_1} = 1\mathrm{k}\Omega$

1. Consider the first order RC circuit shown in Fig. 4. Compute the value of **C** so that the **3 − dB** cut-off frequency is at **160Hz**.

$$\mathrm{C_1} = \frac{1}{\mathrm{R_1}\cdot(2\pi f)} = \frac{1}{1000\Omega\cdot 2\pi\cdot 160\mathrm{Hz}} \approx 0.995\mu\mathrm{C} \approx 1\mu\mathrm{C}$$

2. Draw the circuit shown in Fig. 4 in LTspice. Use the capacitor value that you calculate above.

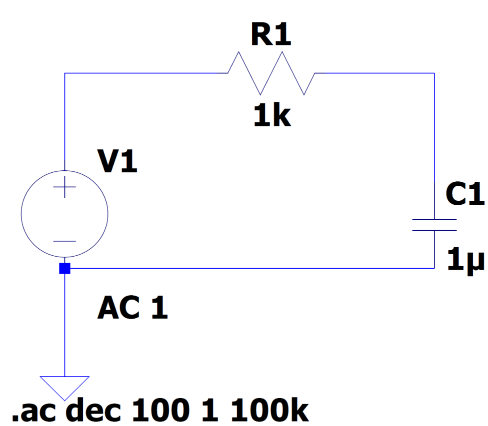

Figure Solution 3: Section 2.1 circuit in LTspice

3. Plot and **save** the magnitude and phase response of the RC filter in LTspice using the following parameters.
   - Type of Sweep: Decade
   - Number of points per decade: 100
   - Start Frequency: 1 Hz
   - Stop Frequency: 100 kHz

   NOTE: If you are using the MATLAB smaple code from course website, please make sure the exported data is saved in **Cartesian** format under *"Select Traces to Export"*

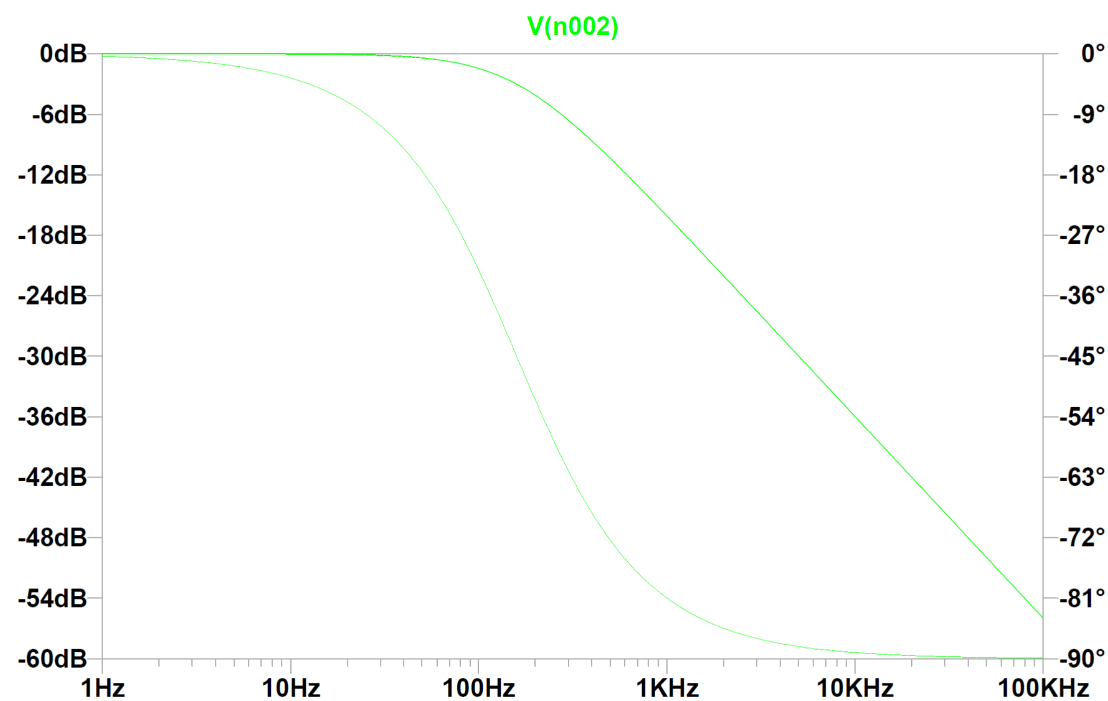

Figure Solution 4: V_{out} Section 2.1 circuit in LTspice

4. Export the data from LTspice to MATLAB using the instruction given in LTspice tutorial on the website. Then plot and **save** each of the graphs specified below (in MATLAB):
   - Magnitude (dB) vs log Frequency
   - Magnitude (dB) vs linear Frequency
   - Phase (deg) vs log Frequency
   - Magnitude (V/V) vs log Frequency

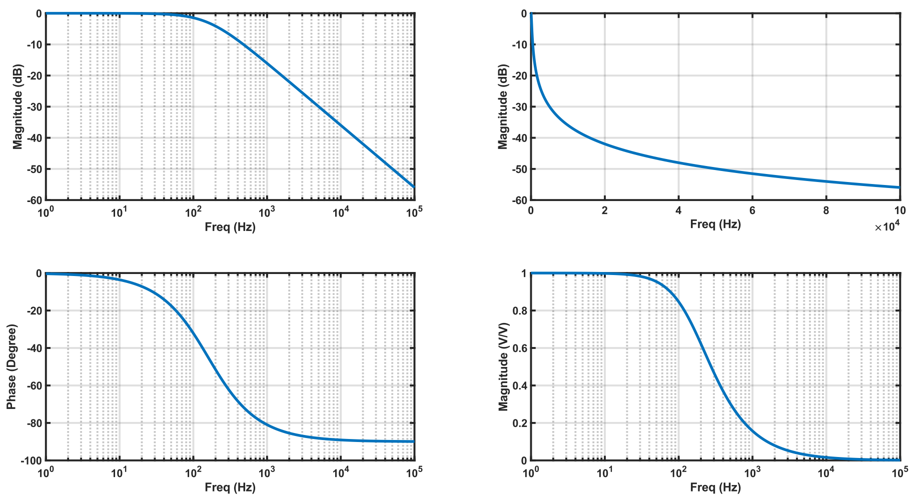

Figure Solution 5: Magnitude (V/V or dB) and Phase (degree)/ Frequency ω (Hz or log Hz)

5. Fill out Table 1. Note: you can either do this in LTspice (by changing the axes from linear to logarithm) or do it in MATLAB.

Frequency	$\lvert\mathbf{H}(\omega)\rvert$ [V/V]	$\lvert\mathbf{H}(\omega)\rvert$ [dB]	$\angle\mathbf{H}(\omega)$ [deg]
40 Hz	0.9701	−0.2636	−14.0437
60 Hz	0.9352	−0.5817	−20.7366
80 Hz	0.8948	−0.9659	−26.5234
100 Hz	0.8467	−1.4451	−32.1419
140 Hz	0.7979	−1.9608	−37.0676
160 Hz	0.7479	−2.5229	−41.5898
180 Hz	0.7086	−2.9921	−44.8799
200 Hz	0.6583	−3.6310	−48.8264
220 Hz	0.6236	−4.1021	−51.4219

Table 1: Simulation Results -First Order RC filter

6. After looking at your plots, why do we plot magnitude vs log frequency most of the time?

Plotting magnitude vs frequency directly results in a graph where magnitude quickly decreases to 0, making it harder to analysis the characteristic of the response function. Plotting magnitude vs log frequency results in a graph where it's easier to see the relationship between magnitude and frequency. Therefore, we plot magnitude vs log frequency most of the time.

7. In the plot of magnitude (dB) vs log Freq, mark the gains at **1kHz** and **10kHz** with the 'Data Cursors' in MATLAB. **Save** this plot with both markers included. Finally, compute the slope of the line segment between the two frequencies (in units of **dB/decade**)

Frequency	$\lvert\mathbf{H}(\omega)\rvert$ [V/V]	$\lvert\mathbf{H}(\omega)\rvert$ [dB]
1 kHz	0.1572	-16.0722
10 kHz	0.0159	-35.9647

Table Solution 1: Magnitude at 1kHz and 10kHz

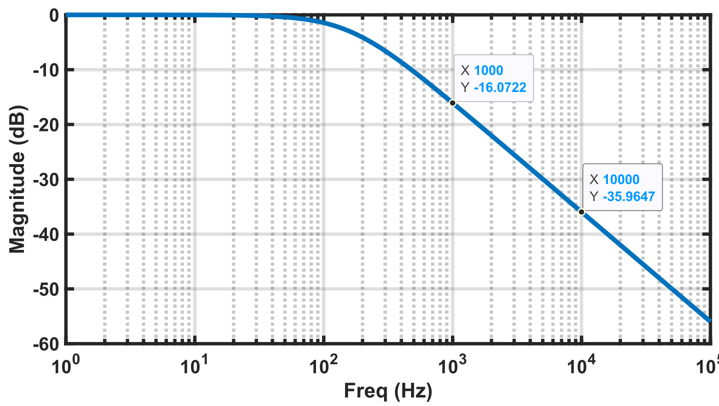

Figure Solution 6: Magnitude (dB) / Frequency ω (log Hz) with marked 1kHz and 10kHz datapoint

$$\text{Slope [dB/decade]} = -35.9647 - (-16.0722) = \boxed{-19.8925}$$

3.2 RC filter with load resistor

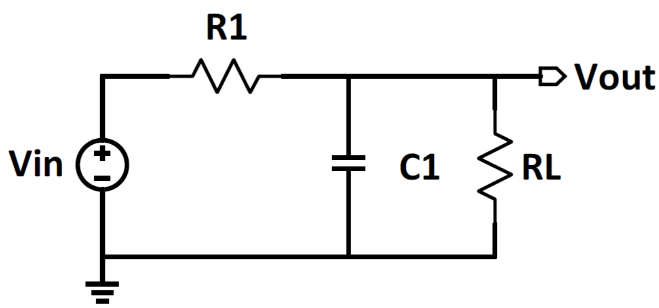

Figure 5: RC filter with load resistor

Component Values: $V_{in} = 1\sin(\omega)V$, $R_1 = 1k\Omega$, $R_L = 330\Omega, 1k\Omega, 5k\Omega$, $C_1 = 1\mu F$

1. Draw the circuit shown in Fig. 6 in LTspice. (Note: It's always a good idea to draw different circuits in a new LTspice schematic in case the TA asks you to see your circuit. You can always copy and paste from an existing schematic so you don't have to draw from scratch.)

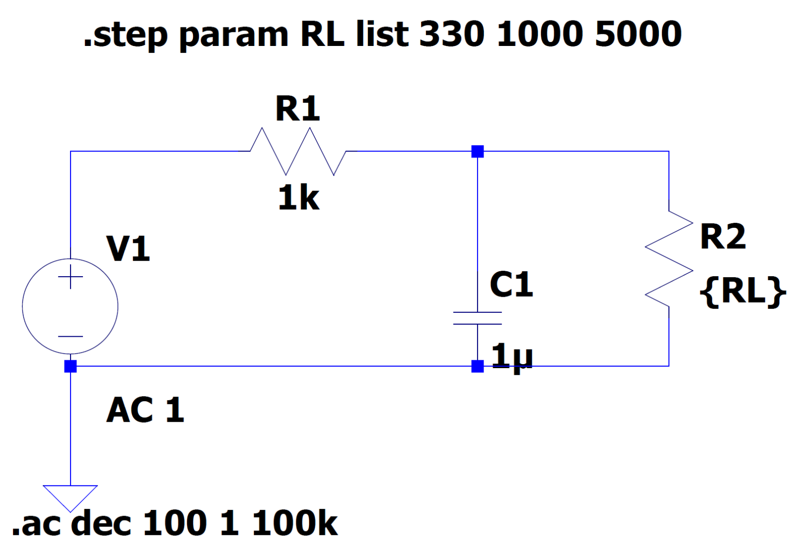

Figure Solution 7: Section 2.2 circuit in LTspice

2. Fill out Table 2.

RL	**3 − dB** cut-off frequency
330Ω	645.6542Hz
1000Ω	316.2278Hz
5000Ω	190.5461Hz

Table 2: Simulation Results - RC filter with load resistor

3. Based on the simulation result (or your intuition), how will the **3 − dB** cut-off frequency change if the load resistance is very large. (Give an approximate frequency).

As $\mathrm{R_L} \to \infty$, $\omega_{\mathrm{3dB}} \to 1/\mathrm{R_1C_1} \approx 160\mathrm{Hz}$

4. In the plot of magnitude (dB) vs log Freq, mark the gains at **1kHz** and **10kHz**. **Save** this plot with both markers included. Finally, compute the slope of the line segment between the two frequencies (in units of **dB/decade**)(You only need to do this with one load resistance.)

Choose $\mathrm{R_L} = 5000\Omega$,

Frequency	$\lvert\mathbf{H}(\omega)\rvert$ [V/V]	$\lvert\mathbf{H}(\omega)\rvert$ [dB]
1 kHz	0.1563	−16.1192
10 kHz	0.0159	−35.9652

Table Solution 2: Magnitude at 1kHz and 10kHz

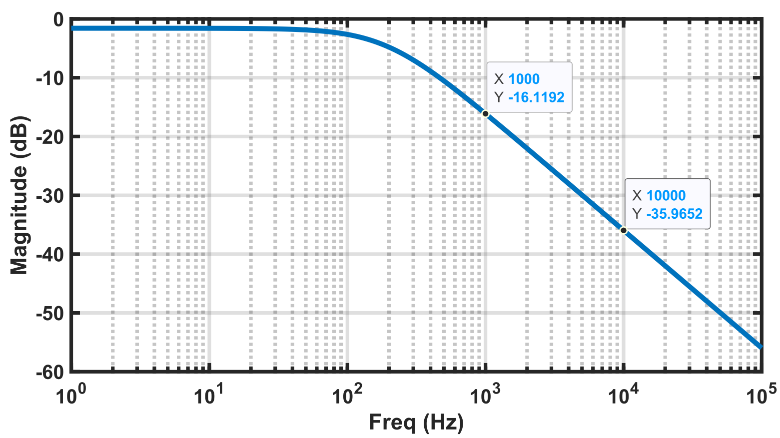

Figure Solution 8: Magnitude (dB) / Frequency ω (log Hz) with marked 1kHz and 10kHz datapoint

$$\text{Slope [dB/decade]} = -35.9652 - (-16.1192) = \boxed{-19.8460}$$

3.3 RC filter with Op-Amp

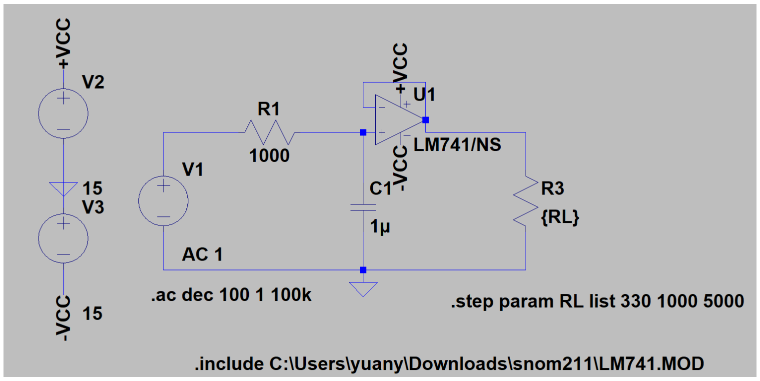

Figure 6: RC filter with Op-Amp

Component Values: $V_{in} = 1\sin(\omega)V$, $R_1 = 1k\Omega$, $R_L = 330\Omega, 1k\Omega, 5k\Omega$, $C_1 = 1\mu F$

1. Draw the circuit shown in Fig. 6 in LTspice. (Note: Use the opamp2 LTspice symbol and include the .MOD file of the Op-Amp. Change the value of component to “LM741/NS”)

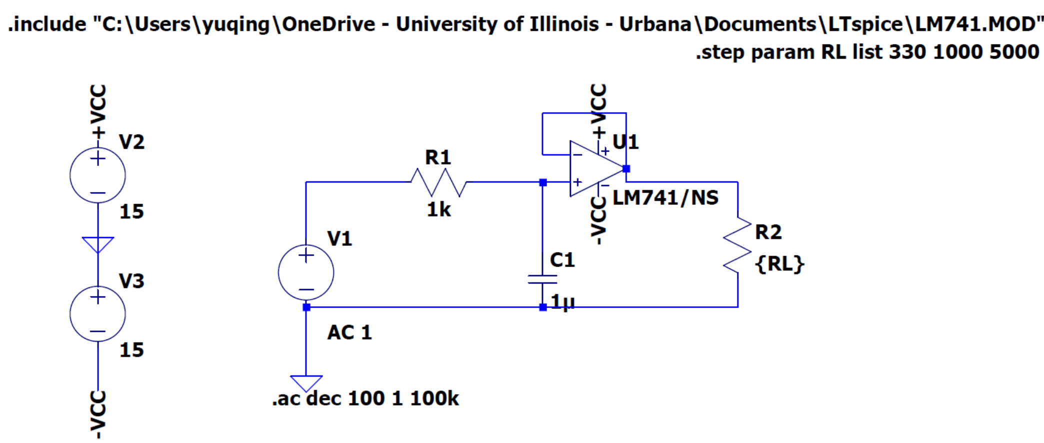

Figure Solution 9: Section 2.3 circuit in LTspice

2. Fill out Table 3.

RL	**3 − dB** cut-off frequency
330Ω	158.4893Hz
1000Ω	158.4893Hz
5000Ω	158.4893Hz

Table 3: Simulation Results - RC filter with load resistor

3. Check the voltage at the two input nodes of the Op-Amp. Why can we assume that the voltage at two input pins of the Op-Amp are equal to each other?

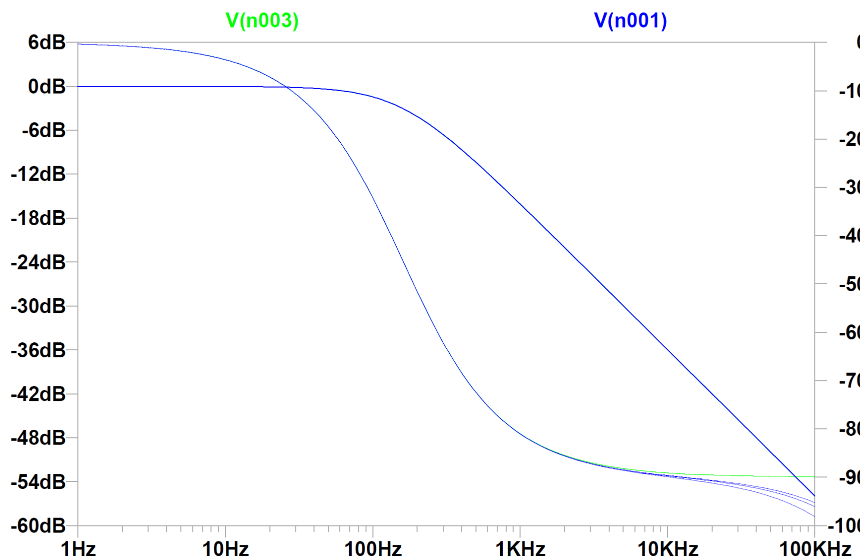

Figure Solution 10: Section 2.3 circuit in LTspice

From the empirical data, we see that under three different load resistor, the positive input node `V(n003)` and negative input node `V(n001)` are almost same across different frequency (except at high frequency when they diverge in phase). Soi we could safely assume that they are equal.

Also, from Ideal Op-Amp assumption, we could assume they have same voltage, since the internal resistance connects them are assumed to be infinity.

4. What is the functionality of the Op-Amp in the circuit?

Op-Amp separates the load resistor from the RC circuit (no current will flow from R_1 to R_L), so that the voltage of positive input side (i.e. the V_{out}) only depends on R_1 and C_1, which makes equivalent to the circuit in section 2.1. Therefore, the V_{out} only depends on RC circuit and we could pick whatever R_L we wish without worrying changing the behavior of response function.