Question 1

> Find $X \otimes Y \left|\psi\right\rangle$, where
>
> $$\left|\psi\right\rangle = \frac{|0\rangle|1\rangle - |1\rangle|0\rangle}{\sqrt{2}}$$

We know that

$$X = |0\rangle\langle 1| + |1\rangle\langle 0|$$
$$Y = -i|0\rangle\langle 1| + i|1\rangle\langle 0|$$

thus

$$X \otimes Y \left|\psi\right\rangle = \frac{X|0\rangle Y|1\rangle - X|1\rangle Y|0\rangle}{\sqrt{2}} = \frac{|1\rangle(-i\left|0\right\rangle) - |0\rangle i\left|1\right\rangle}{\sqrt{2}} = \frac{-i(|10\rangle + |01\rangle)}{\sqrt{2}}$$

Question 2

> If
>
> $$\left|\psi\right\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$
>
> find $I \otimes Y \left|\psi\right\rangle$

$$I \otimes Y \left|\psi\right\rangle = \frac{I|0\rangle Y|0\rangle + I|1\rangle Y|1\rangle}{\sqrt{2}} = \frac{|0\rangle i|1\rangle + |1\rangle(-i|0\rangle)}{\sqrt{2}} = \frac{i(|01\rangle - |10\rangle)}{\sqrt{2}}$$

Question 3

> Calculate the matrix representation of $X \otimes Y$

we know that

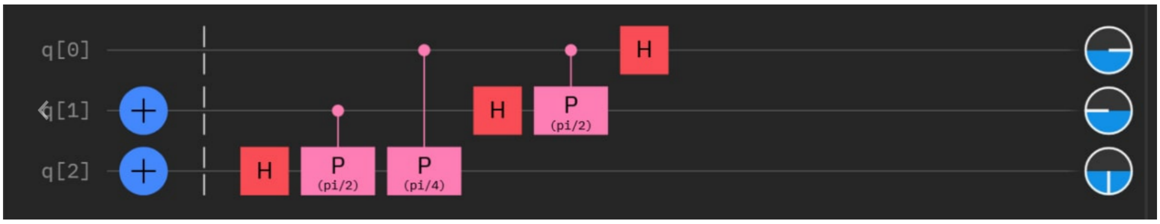
$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

and thus

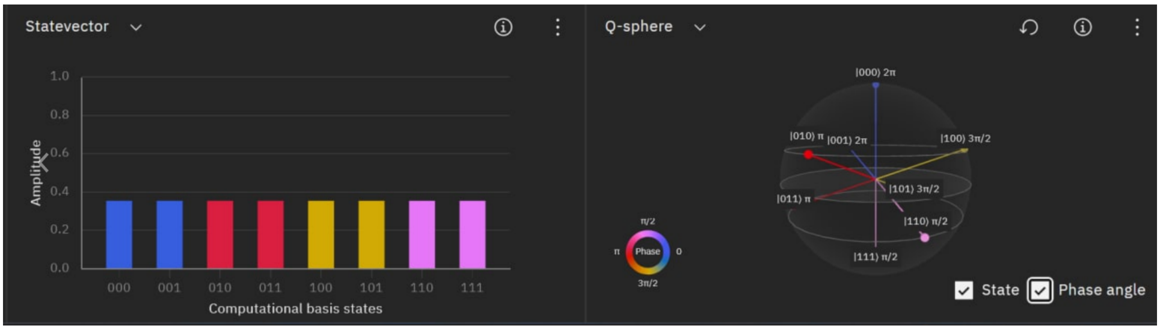
$$X \otimes Y = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}$$

Question 4

Notice, the IBM compose use the bottom bit as the most significant bit (so the entire circuit is flipped upside down). We first prepare $|110\rangle$ using the X gate. Then we apply the Fourier Transform.



The state vector diagram is



which phase angle could be seen in the Q-sphere.

The result state is

$$\frac{1}{2\sqrt{2}}(|000\rangle + |001\rangle + e^{2\pi i\cdot\frac{4}{8}}|010\rangle + e^{2\pi i\cdot\frac{4}{8}}|011\rangle + e^{2\pi i\cdot\frac{6}{8}}|100\rangle + e^{2\pi i\cdot\frac{6}{8}}|101\rangle + e^{2\pi i\cdot\frac{2}{8}}|110\rangle + e^{2\pi i\cdot\frac{2}{8}}|1$$

and it could be simplified as

$$\frac{1}{\sqrt{2}}(|0\rangle + e^{2\pi i\frac{3}{4}}|1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle + e^{2\pi i\frac{1}{2}}|1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle + e^{2\pi i\cdot 0}|1\rangle)$$

According to the textbook, the Fourier transform would have the result of form

$$\frac{1}{\sqrt{2}}(|0\rangle + e^{2\pi i 0.x_2x_1x_0}|1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle + e^{2\pi i 0.x_1x_0}|1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle + e^{2\pi i 0.x_0}|1\rangle)$$

In this case, $x_2 = 1, x_1 = 1, x_0 = 0$, and $0.x_2x_1x_0 = 0.110_2 = 0.75_{10}$, $0.x_1x_0 = 0.10_2 = 0.5_{10}$, and $0.x_0 = 0.0_2 = 0.0_{10}$, which is consistent with the result in the composer.