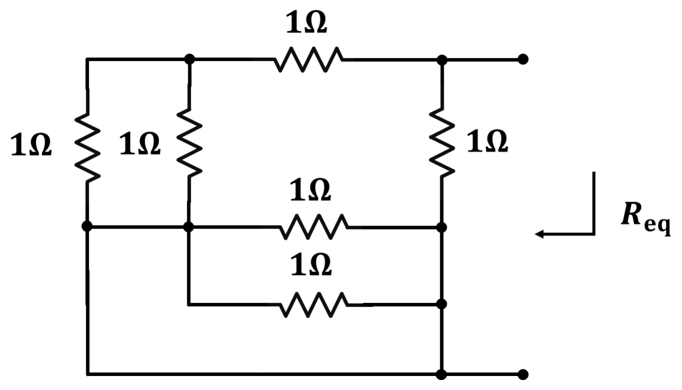
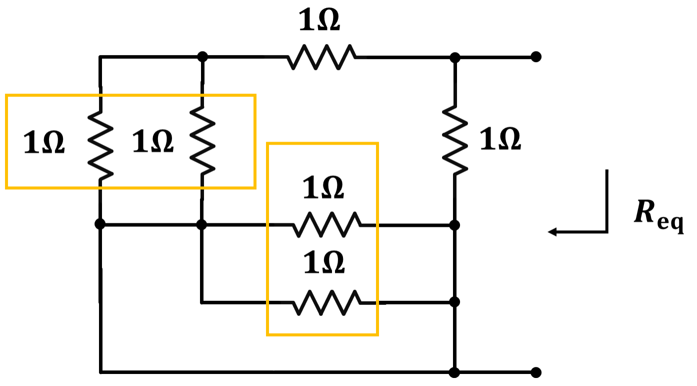
Problem 2

> For each one of the following two circuits, obtain R_{eq}

(a)

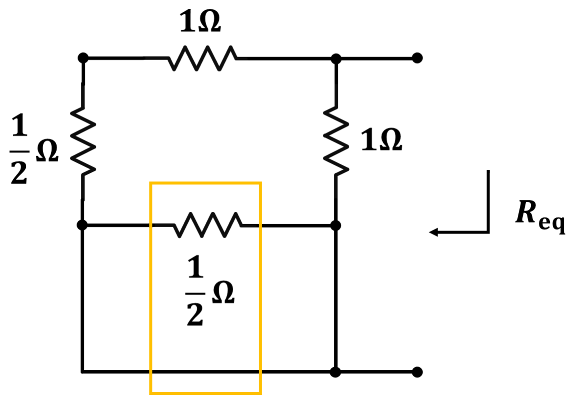




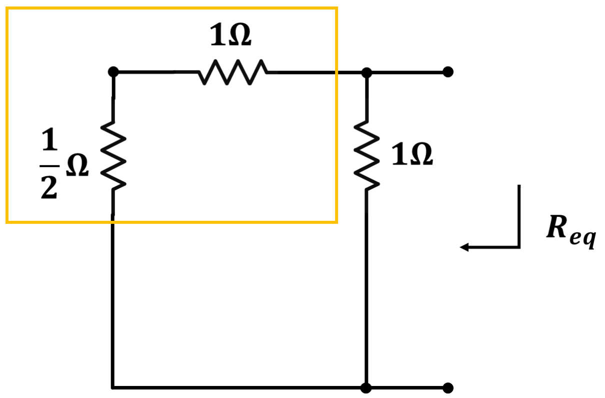
Apply the rule of **resistor in parallel**, the equivalent resistor for the section A and B, R_A and R_B respectively, is therefore

$$R_{\text{A}} = \frac{1\Omega \cdot 1\Omega}{1\Omega + 1\Omega} = \frac{1}{2}\Omega$$
$$R_{\text{B}} = \frac{1\Omega \cdot 1\Omega}{1\Omega + 1\Omega} = \frac{1}{2}\Omega$$

So we simplify the circuit:



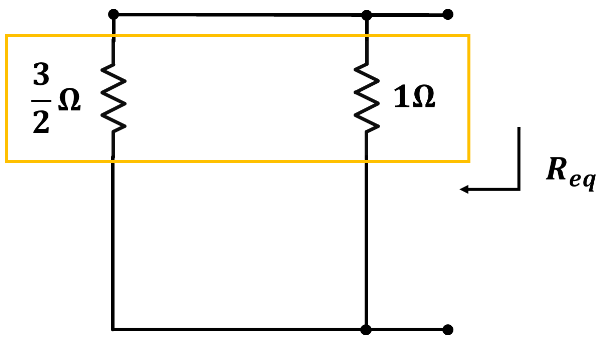
The resistor circled is in parallel with a wire, so it get short-circuited, and we could further simplify the circuit as:



Apply the rule of **resistor in series**, the equivalent resistor for that part is

$$R_{\text{eqiv}} = 1\Omega + \frac{1}{2}\Omega = \frac{3}{2}\Omega$$

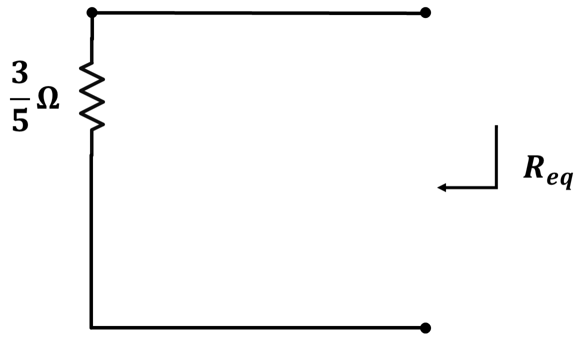
So we simplify the circuit:



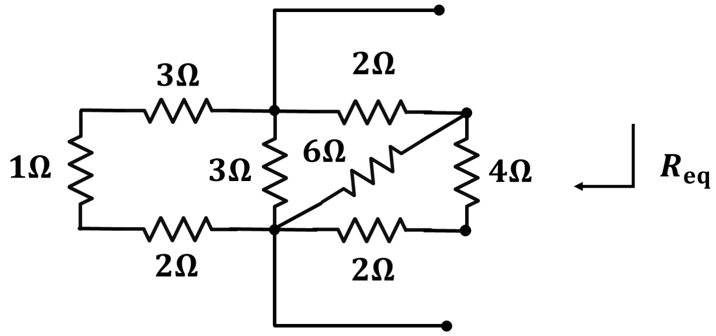
Apply the rule of **resistor in parallel** again, the equivalent resistor for that part is

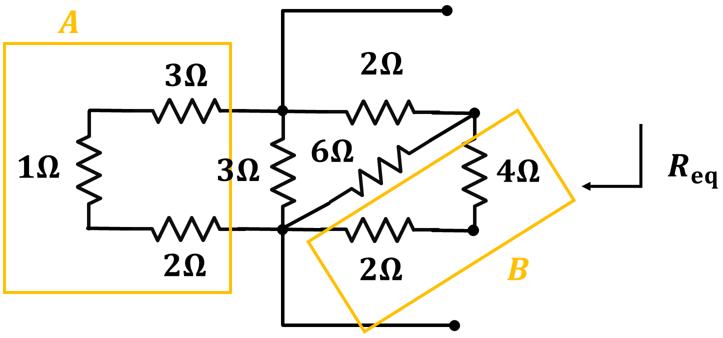
$$\boxed{R_{\text{eq}} = \frac{\frac{3}{2}\Omega \cdot 1\Omega}{\frac{3}{2}\Omega + 1\Omega} = \frac{3}{5}\Omega}$$

So we simplify the circuit:



(b)

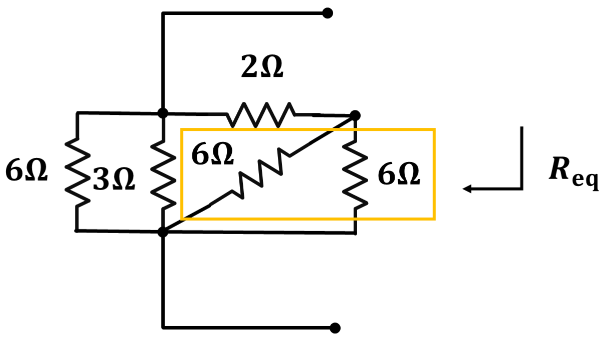




Apply the rule of **resistor in series**, the equivalent resistor for the section A and B, R_A and R_B respectively, is therefore

$$R_A = 1\Omega + 2\Omega + 3\Omega = 6\Omega$$
$$R_B = 2\Omega + 4\Omega = 6\Omega$$

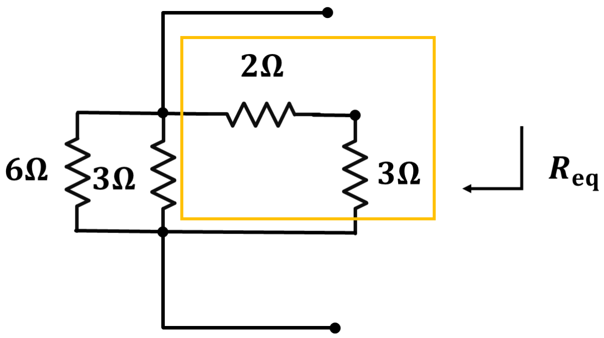
So we simplify the circuit:



Apply the rule of **resistor in parallel**, the equivalent resistor for that part is

$$R_{\text{eqiv}} = \frac{6\Omega \cdot 6\Omega}{6\Omega + 6\Omega} = 3\Omega$$

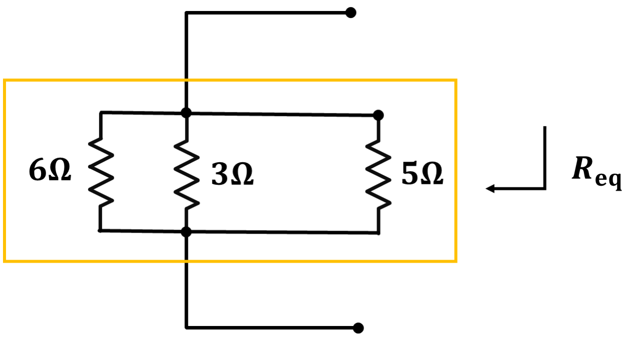
So we simplify the circuit:



Apply the rule of **resistor in series**. the equivalent resistor for that part is

$$R_{\text{eqiv}} = 2\Omega + 3\Omega = 5\Omega$$

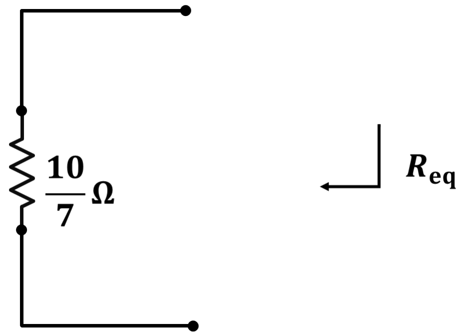
So we simplify the circuit:



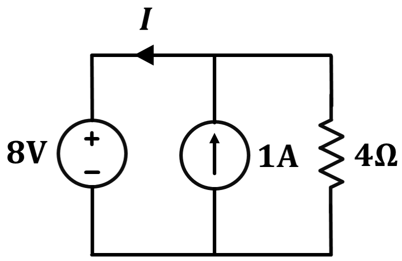
Apply the rule of **resistor in parallel**, the equivalent resistor for that part is

$$\boxed{R_{\text{eq}} = \frac{1}{\frac{1}{6\Omega} + \frac{1}{3\Omega} + \frac{1}{5\Omega}} = \frac{10}{7}\Omega}$$

So we simplify the circuit:



Problem 3

> Determine the current I in the following circuit using source transformation
>
> 
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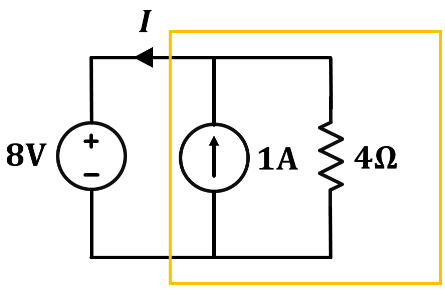
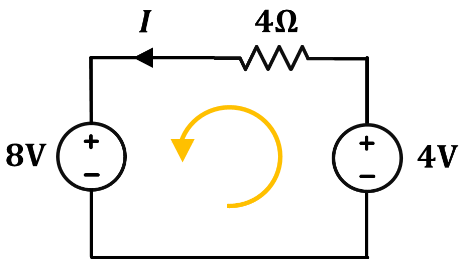


Transform the selected region. We know that for a current source in parallel with a resistor, we could transform that into a voltage source in series with a resistor, where the I flows from $-$ to $+$ in the voltage source, and the voltage for that source is

$$V = 1\text{A} \cdot 4\Omega = 4\text{V}$$

Therefore we get:



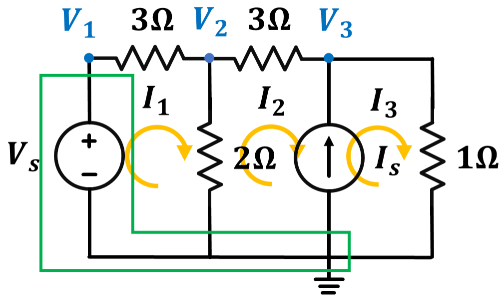
we could now apply KVL on the circuit and get

$$4\Omega \cdot I + 8\text{V} - 4\text{V} = 0$$

$$\boxed{I = -\frac{4\text{V}}{4\Omega} = -1\text{A}}$$

Problem 4

> Consider the circuit below



(a)

> Use the loop-current method to obtain a set of three linearly independent equations, in terms of the loop currents I_1, I_2 and I_3, and the sources V_s and I_s, but no other variables, that can be used to determine the loop currents. Simplify your equations and write them with integer-valued coefficients.

On the left loop, we obtain (using KVL)

$$V_s = I_1 \cdot 3\Omega + (I_1 - I_2) \cdot 2\Omega$$

We also find

$$I_s = I_3 - I_2$$

On the outer loop, we obtain (using KVL)

$$V_s = I_1 \cdot 3\Omega + I_2 \cdot 3\Omega + I_3 \cdot 1\Omega$$

We could simply these three equations.

$$\begin{gathered} V_s = I_1 \cdot 5\Omega - I_2 \cdot 2\Omega \\ I_s = I_3 - I_2 \\ V_s = I_1 \cdot 3\Omega + I_2 \cdot 3\Omega + I_3 \cdot 1\Omega \end{gathered}$$

and we get

$$\boxed{\begin{aligned} I_1 &= \frac{3}{13\Omega}V_s - \frac{1}{13}I_s \\ I_2 &= \frac{1}{13\Omega}V_s - \frac{5}{26}I_s \\ I_3 &= \frac{1}{13\Omega}V_s + \frac{21}{26}I_s \end{aligned}}$$

(b)

> Use the node-voltage method to obtain a set of three

we see that on the node labeled V_2, it has (using KCL)

$$\frac{V_1 - V_2}{3\Omega} = \frac{V_2 - 0}{2\Omega} + \frac{V_2 - V_3}{3\Omega}$$

we see that on the node labeled V_3, it has (using KCL)

$$\frac{V_2 - V_3}{3\Omega} + I_s = \frac{V_3 - 0}{1\Omega}$$

We see that on the node labeled V_1, we get

$$V_1 - 0 = V_s$$

and therefore, simply the equation

$$\begin{gathered} 2(V_1 - V_2) = 3V_2 + 2(V_2 - V_3) \\ (V_2 - V_3) + 3\Omega \cdot I_s = 3V_3 \\ V_1 = V_s \end{gathered}$$

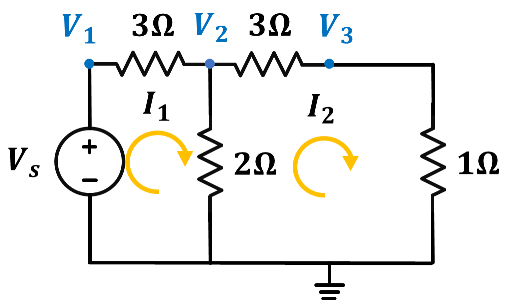
and

$$\boxed{\begin{aligned} &V_1 = V_s \\ &V_2 = \frac{4}{13}V_s + \frac{3}{13}\Omega \cdot I_s \\ &V_3 = \frac{1}{13}V_s + \frac{21}{26}\Omega \cdot I_s \end{aligned}}$$

(c)

> It is known that $V_3 = k_1 V_s + k_2 I_s$. Use superposition to determine the values of k_1 and k_2.

First remove the I_s.



Use node voltage method, we see that on the node labeled V_2, it has (using KCL)

$$\frac{V_1 - V_2}{3\Omega} = \frac{V_2 - 0}{2\Omega} + \frac{V_2 - V_3}{3\Omega}$$

and one the node labeled V_3,

$$\frac{V_2 - V_3}{3\Omega} = \frac{V_3 - 0}{1\Omega}$$

and we see that on the node labeled V_1, we get

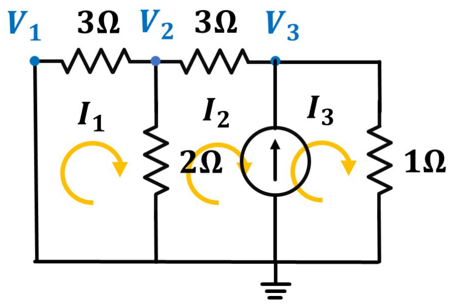
$$V_1 - 0 = V_s$$

So

$$\begin{gathered} 2(V_1 - V_2) = 3V_2 + 2(V_2 - V_3) \\ (V_2 - V_3) = 3V_3 \\ V_1 = V_s \end{gathered}$$

We get $\boxed{V_3 = \frac{1}{13}V_s}$ which matches our value in (b).

Then, remove the V_s:



Use node-method, it basically follow the same procedure:

we see that on the node labeled V_2, it has (using KCL)

$$\frac{V_1 - V_2}{3\Omega} = \frac{V_2 - 0}{2\Omega} + \frac{V_2 - V_3}{3\Omega}$$

we see that on the node labeled V_3, it has (using KCL)

$$\frac{V_2 - V_3}{3\Omega} + I_s = \frac{V_3 - 0}{1\Omega}$$

We see that on the node labeled V_1, we get

$$V_1 = 0$$

and therefore, simply the equation

$$\begin{gathered} -2V_2 = 3V_2 + 2(V_2 - V_3) \\ (V_2 - V_3) + 3\Omega \cdot I_s = 3V_3 \\ V_1 = 0 \end{gathered}$$

and therefore we see that $\boxed{V_3 = \frac{21}{26} I_s}$, which matches our value in (b).

We therefore see that $\boxed{k_1 = \frac{1}{13}}$ and $\boxed{k_2 = \frac{21}{26}}$

(d)

Set $V_s = 10\text{V}$, and $I_s = 1\text{A}$. Therefore,

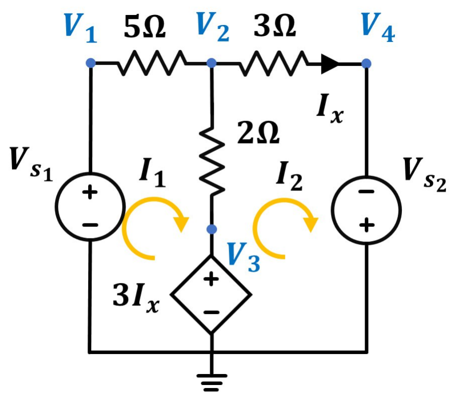
$$\boxed{\begin{aligned} I_1 &= \frac{3}{13\Omega}10\text{V} - \frac{1}{13}1\text{A} = \frac{30}{13}\text{A} - \frac{1}{13}\text{A} = \frac{29}{13}\text{A} \\ I_2 &= \frac{1}{13\Omega}10\text{V} - \frac{5}{26}1\text{A} = \frac{10}{13}\text{A} - \frac{5}{26}\text{A} = \frac{15}{26}\text{A} \\ I_3 &= \frac{1}{13\Omega}10\text{V} + \frac{21}{26}1\text{A} = \frac{10}{13}\text{A} + \frac{21}{26}\text{A} = \frac{41}{26}\text{A} \end{aligned}}$$

(e)

Set $V_s = 10\text{V}$, and $I_s = 1\text{A}$. Therefore,

$$\boxed{\begin{aligned} V_1 &= V_s = 10\text{V} \\ V_2 &= \frac{4}{13}10\text{V} + \frac{3}{13}\Omega \cdot 1\text{A} = \frac{40}{13}\text{V} + \frac{3}{13}\text{V} = \frac{43}{13}\text{V} \\ V_3 &= \frac{1}{13}10\text{V} + \frac{21}{26}\Omega \cdot 1\text{A} = \frac{10}{13}\text{V} + \frac{21}{26}\text{V} = \frac{41}{26}\text{V} \end{aligned}}$$

Problem 5

> Consider the circuit below:
>
> 
>

(a)

> Use the loop-current method to obtain a set of linearly independent equations, in terms of the loop currents I_1 and I_2, and the sources V_{s_1} and V_{s_2}, but no other variables, that can be used to determine the loop currents. Simplify your equations and write them with integer-valued coefficients.

On the left loop, we find (using KVL)

$$I_1 \cdot 5\Omega + (I_1 - I_2) \cdot 2\Omega + 3\frac{\mathrm{V}}{A} I_x = V_{s_1}$$

On the right loop, we find

$$I_2 \cdot 3\Omega + (I_2 - I_1) \cdot 2\Omega = V_{s_2} + 3\frac{\mathrm{V}}{A} I_x$$

we also know

$$I_2 = I_x$$

and therefore we find that

$$\begin{aligned} I_1 &= \frac{1}{8\Omega} \cdot V_{s_1} - \frac{1}{16\Omega} \cdot V_{s_2} \\ I_2 &= \frac{1}{8\Omega} \cdot V_{s_1} + \frac{7}{16\Omega} \cdot V_{s_2} \end{aligned}$$

(b)

On the V_1, we find

$$V_1 = V_{s_1}$$

and on the V_2 (using KCL), we find

$$\frac{V_1 - V_2}{5\Omega} = \frac{V_2 - V_3}{2\Omega} + I_x$$

and on the V_3, we find that

$$V_3 = 3I_x$$

and on the V_4, we find that

$$V_4 = -V_{s_2}$$

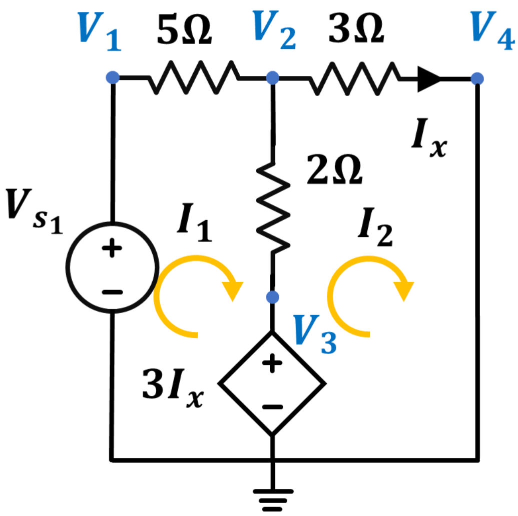
and we also know that

$$\frac{V_2 - V_4}{3\Omega} = I_x$$

therefore

$$\boxed{\begin{aligned} &V_1 = V_{s_1} \\ &V_2 = \frac{3}{8}V_{s_1} + \frac{5}{16}V_{s_2} \\ &V_3 = \frac{3}{8}V_{s_1} + \frac{21}{16}V_{s_2} \\ &V_4 = -V_{s_2} \end{aligned}}$$

(c)



First remove V_{s_2}. Using node-voltage method, we see that

On the V_1, we find

$$V_1 = V_{s_1}$$

and on the V_2 (using KCL), we find

$$\frac{V_1 - V_2}{5\Omega} = \frac{V_2 - V_3}{2\Omega} + I_x$$

and on the V_3, we find that

$$V_3 = 3\frac{\mathrm{V}}{\mathrm{A}}I_x$$

and on the V_4, we find that

$$V_4 = 0\mathrm{V}$$

and we also know that

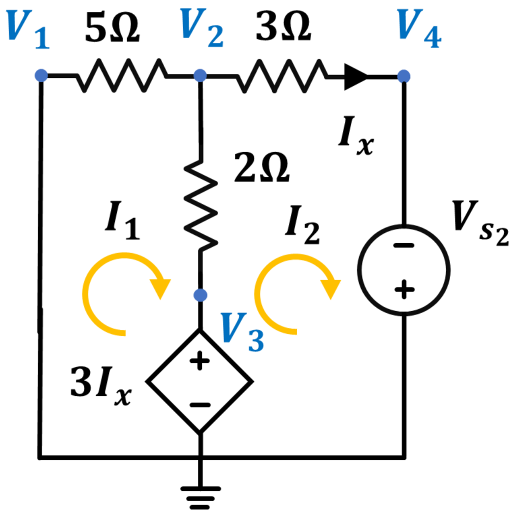
$$\frac{V_2 - V_4}{3\Omega} = \frac{V_2 - 0\mathrm{V}}{3\Omega} = I_x$$

therefore

$$V_1 = V_{s_1}$$
$$V_2 = \frac{3}{8}V_{s_1}$$
$$V_3 = \frac{3}{8}V_{s_1}$$
$$V_4 = 0\text{V}$$

we find $\boxed{k_1 = \frac{3}{8}}$

Similarly, remove V_{s_1}. Using node-method again,



we find that

On the V_1, we find

$$V_1 = 0$$

and on the V_2 (using KCL), we find

$$\frac{V_1 - V_2}{5\Omega} = \frac{V_2 - V_3}{2\Omega} + I_x$$

and on the V_3, we find that

$$V_3 = 3I_x$$

and on the V_4, we find that

$$V_4 = -V_{s_2}$$

and we also know that

$$\frac{V_2 - V_4}{3\Omega} = I_x$$

therefore

$$V_1 = 0\text{V}$$
$$V_2 = \frac{5}{16}V_{s_2}$$
$$V_3 = \frac{21}{16}V_{s_2}$$
$$V_4 = -V_{s_2}$$

we see that $\boxed{k_2 = \frac{21}{16}}$.

(d)

Set $V_{s_1} = 10\text{V}$ and $V_{s_2} = 4\text{V}$.

$$I_1 = \frac{1}{8\Omega} \cdot 10\text{V} - \frac{1}{16\Omega} \cdot 4\text{V} = 1\text{A}$$
$$I_2 = \frac{1}{8\Omega} \cdot 10\text{V} + \frac{7}{16\Omega} \cdot 4\text{V} = 3\text{A}$$

(e)

Set $V_{s_1} = 10\text{V}$ and $V_{s_2} = 4\text{V}$.

$$\boxed{\begin{array}{l} V_1 = 10\text{V} \\ V_2 = \frac{3}{8} \cdot 10\text{V} + \frac{5}{16} 4\text{V} = 5\text{V} \\ V_3 = \frac{3}{8} \cdot 10\text{V} + \frac{21}{16} 4\text{V} = 9\text{V} \\ V_4 = -4\text{V} \end{array}}$$