

| | <code>vector<T></code> | <code>sorted_vector<T></code> | <code>linked_list<T></code> (single, head) | <code>linked_list<T></code> (single, head, tail) | <code>linked_list<T></code> (double, head) | <code>linked_list<T></code> (double, head, tail) |
|------------|------------------------------|-------------------------------------|---|---|---|---|
| push_back | $O(1)$ (amortized) | $O(n)$ | $O(n)$ | $O(1)$ | $O(n)$ | $O(1)$ |
| pop_back | $O(1)$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(1)$ |
| push_front | $O(n)$ | $O(n)$ | $O(1)$ | $O(1)$ | $O(1)$ | $O(1)$ |
| pop_front | $O(n)$ | $O(n)$ | $O(1)$ | $O(1)$ | $O(1)$ | $O(1)$ |
| insert | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ |
| remove | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ |
| find | $O(n)$ | $O(\log n)$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ |

Note: `sorted_vector<T>` trade input time for find time, and never use a `doubly_linked_list<T>` that only has head.

| | <code>stack<T></code> | <code>queue<T></code> |
|----------|-----------------------------|-----------------------------|
| push | $O(1)$ | $O(1)$ |
| pop | $O(1)$ | $O(1)$ |
| is_empty | $O(1)$ | $O(1)$ |

| | <code>binary_search_tree<T></code> | <code>avl_tree<T></code> | <code>b_tree<T, M></code> |
|--------|--|--------------------------------|---------------------------------|
| push | avg: $O(h = \log n)$ worst: $O(h = n)$ | $O(\log n)$ | $O(m \log_m n = \log n)$ |
| remove | avg: $O(h = \log n)$ worst: $O(h = n)$ | $O(\log n)$ | $O(m \log_m n = \log n)$ |
| find | avg: $O(h = \log n)$ worst: $O(h = n)$ | $O(\log n)$ | $O(\log_m n = \log n)$ |

| | <code>hash_map<T></code> |
|--------|--------------------------------|
| set | $O(1)$ |
| get | $O(1)$ |
| remove | $O(1)$ |

| | <code>heap<T></code> |
|--------------|----------------------------|
| create | $O(n)$ |
| heapify up | $O(\log n)$ |
| heapify down | $O(\log n)$ |
| insert | $O(\log n)$ |
| remove | $O(\log n)$ |

| | <code>disjoint_set<T></code> |
|--|------------------------------------|
|--|------------------------------------|

| | |
|-------|------------------------------------|
| | <code>disjoint_set<T></code> |
| union | $O(\log^* n = 1)$ |
| find | $O(\log^* n = 1)$ |

| | Edge List | Adjacency Matrix | Adjacency List |
|---------------------------------|------------|------------------|---------------------------|
| <code>space</code> | $O(V + E)$ | $O(V^2)$ | $O(V + E)$ |
| <code>insert_vertex(v)</code> | $O(1)$ | $O(V)$ | $O(1)$ |
| <code>remove_vertex(v)</code> | $O(E)$ | $O(V)$ | $O(\deg V)$ |
| <code>insert_edge</code> | $O(1)$ | $O(1)$ | $O(1)$ |
| <code>remove_edge</code> | $O(1)$ | $O(1)$ | $O(1)$ |
| <code>incident_edge(v)</code> | $O(E)$ | $O(V)$ | $O(\deg V)$ |
| <code>are_adjacent(v, w)</code> | $O(E)$ | $O(1)$ | $O(\min(\deg V, \deg W))$ |

```
function prim(graph, s) {
  let arr = graph.vertices.map((vertex, i) => vertex == s
    ? [vertex, 0]
    : [vertex, Infinity])
  let pq = container(arr); // O(V)
  let distance_map = map(arr);
  let processor_map = map();

  let new_graph = Graph.new();

  while (pq.is_not_empty()) { // O(V) * Inner
    let [least, _] = pq.smallest() // heap: O(log V), array: O(V);
    new_graph.add_vertex(least); // adjacency_matrix : O(V), adjacency_list :
O(1);
    new_graph.add_edge(least, processor_map[least]) // O(1)

    for (let [neigh, neigh_edge] in least.neighbors_with_edge()) { // O(deg
V)
      if (new_graph.has(neigh)) continue;
      if (neigh_edge.weight < distance_map[neigh]) {
        distance_map[neigh] = neigh_edge.weight; // O(1)
        pq[neigh] = neigh_edge.weight; // heap : O(log V), array: O(1)
      }
    }
  }

  return new_graph
}

// total time:
// adjacency_matrix + array : O(V) * (O(V) + O(V) + O(deg V) * O(1)) = O(V^2 + E)
// adjacency_matrix+heap: O(V) * (O(log V) + O(V) + O(deg V) * O(log V)) = O(V^2
+ E log V)
```

```
// adjacency_list + array :  $O(V) * (O(V) + O(\log V) + O(\deg V) * O(1)) = O(V^2 + E)$ 
// adjacency_list + heap :  $O(V) * (O(\log V) + O(\deg V) * O(\log V)) = O(V \log V + E \log V)$ 
// adjacency_list + fibonacci_heap :  $O(V) * (O(\log V) + O(\deg V) * O(1)) = O(V \log V + E)$ 
```

| Prim | adjacency_matrix | adjacency_list |
|-------|---------------------|---------------------|
| array | $O(V^2 + E)$ | $O(V^2 + E)$ |
| heap | $O(V^2 + E \log V)$ | $O((V + E) \log V)$ |

```
function kruskal(graph) {
  let edges = graph.edges.sort((a, b) => a.weight < b.weight); //  $O(E \log E)$ 
  let dset = disjoint_set(graph.vertices);
  let new_graph = Graph.new();
  for (let edge in edges) { //  $O(V)$ 
    if (new_graph.vertices.length >= n - 1) break;
    new_graph.add_vertex(edge.vertexA);
    new_graph.add_vertex(edge.vertexB);
    new_graph.add_edge(edge.vertexA, edge.vertexB);
    if (dset.find(edge.vertexA) != dset.find(edge.vertexB)) {
      dset.union(edge.vertexA, edge.vertexB);
    }
  }
  return new_graph;
}
```

| Kruskal | |
|---------|---------------|
| heap | $O(E \log E)$ |
| sorting | $O(E \log E)$ |

```
function dijkstra(graph, start, destin) {
  let arr = graph.vertices.map((vertex, i) => vertex == s
    ? [vertex, 0]
    : [vertex, Infinity])
  let prev_map = {};
  let distance_map = map(arr);
  let pq = priority_queue(arr);
  let visited = {};

  distance_map[start] = 0;
  let current = start;

  while (pq.top != destin) { //  $O(V)$ 
    let current = pq.pop(); //  $O(\log V)$ 
    for (let [neigh, edge] in current.neighbor_with_edge()) { //  $O(\deg V)$ 
      if (visited[neigh] != undefined) return;
      if (distance_map[neigh] < distance_map[current] + edge.weight) {
```

```

        distance_map[neigh] = distance_map[current] + edge.weight; //
fibonacci_heap : O(1), heap : O(log V)
        prev_map[neigh] = current;
    }
}
visited[current] = true;
}
let path = []
while (prev_map[current] !== undefined) {
    path.push(current);
    current = prev_map[current];
}
return [path, distance_map[destin]];
}
// time complexity:
// heap : O(V) * (O(log V) + O(deg V) * O(log V)) = O(Vlog V) + O(Elog V) = O((V
+ E)log V)
// fibo_heap : O(V) * (O(log V) + O(deg V) * O(1)) = O(Vlog V) + O(E) = O(E +
Vlog V)

```

time complexity is $O(E + V \log V)$ (fibonacci heap)