1.

(a)

$$(0+1)^*111(0+1)^*10+(0+1)^*1110$$

The string requires a suffix 10 and a substring 111. There two subcases

1. the substring 111 in string is separated from the suffix 10 of the string. The corresponding regular expression is $(0+1)^*111(0+1)^*10$. $(0+1)^*$ is the expression for any string.
2. the last letter of substring 111 in string is the first letter in the suffix 10 of the string. The corresponding regular expression is $(0+1)^*1110$

(b)

$$1^*0^*1^*$$

If the string does not contain subsequence 010, it should not have any 0 appears after the string already have a subsequence of 01.

(c)

$$(0^*10^*10^*10^*)^*1(0^*10^*10^*10^*)^*+(0^*10^*10^*10^*)^*1(0^*10^*10^*10^*)^*1(0^*10^*10^*10^*)^*$$

The regular expression $(0^*10^*10^*10^*)^*$ means any string where `1` appears $3n(n\in\mathbb{N}^+)$ times. To make number of `1`s not be divisible by 3. The `1` should appear either $3n+1$ or $3n+2$ times $(n\in\mathbb{N}^+)$. We insert extra `1`s into the regular expression, $(0^*10^*10^*10^*)^*$. This corresponds to $(0^*10^*10^*10^*)^*1(0^*10^*10^*10^*)^*$ and $(0^*10^*10^*10^*)^*1(0^*10^*10^*10^*)^*1(0^*10^*10^*10^*)^*$, respectively.

(d)

$$w_1+w_2+w_3+\cdots+w_k$$

Treat each string w_i in L as a regular expression. The regular expression that define the L is the one that accept any string in L. The regular expression is thus the union of all the string in L.

(e)

Define $h=\max_{1\le i\le k}|w_i|$ as the maximum length of all the string in language L. Therefore, there are several cases to consider:

1. For $l\le h$, we could pick a subset $S_l\subseteq L$ that $S_l=\{s:|s|=l\}$. We could also have Σ^l (that is the set of all string with the length l). Define $K_l=\Sigma^l\backslash S_l$. Then K_l is all the string in the $\bar{L}$ with length l, and the regular expression $x_l=\sum_{s\in K_l}s$ (the $\sum$ means regular expression union). For $0\le i\le h$, the regular expression for all string in $\bar{L}$ with length less than or equal to h is expressed by (call this regular expression x)

$$x=x_0+x_i+\cdots+x_h$$

2. For any string s that $|s|>h$, It's in $\bar{L}$. It could be expressed as $(0+1)^{h+1}(0+1)^*$. $(0+1)^{h+1}$ means $(0+1)(0+1)\cdots(0+1)$ for $h+1$ times)

The resulting regular expression is just $x+(0+1)^{h+1}(0+1)^*$.