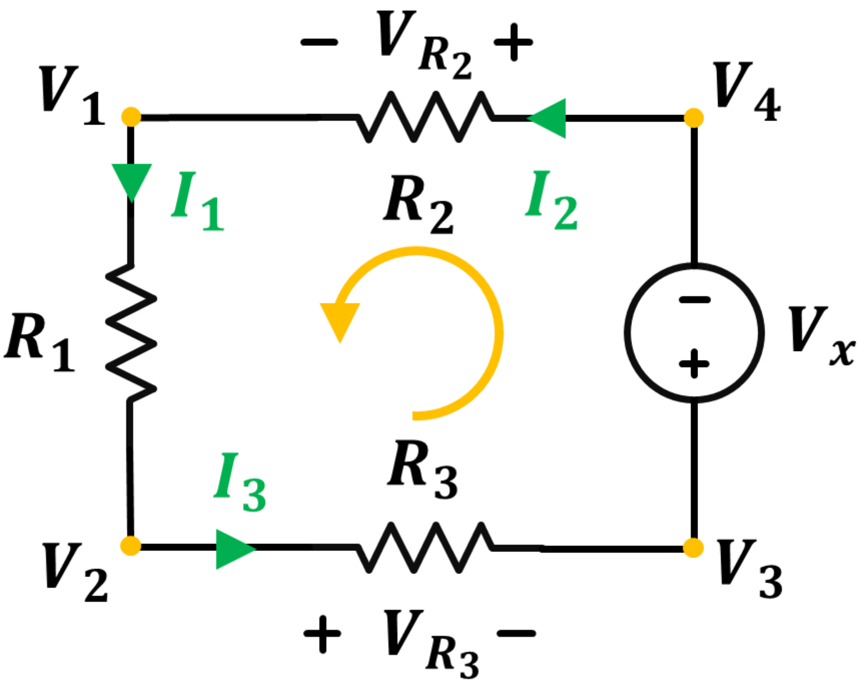
Problem 1

> Sign acknowledging you will abide by this course's and the University's Academic Integrity policies or face sanctions for not doing so. These policies include, among others, plagiarism: representing the words or ideas of others as your own; cheating: using unauthorized materials and/or information. The possible sanctions include, among others, reduced letter grades and an F in the course. If your solution upload does not include your signature, your homework will NOT be graded, resulting in a zero.

Sign: Yuqing Zhai

Problem 2

> Consider the circuit below, where $R_1 = 5\Omega$, $R_3 = 3\Omega$, $V_{R_2} = \frac{2}{3}\mathrm{V}$ and $V_{R_3} = 1\mathrm{V}$. Let the top-right node be the reference node. Determine the resistance R_2 and the voltage V_x in the following circuit, as well all the remaining node voltages.
>
> 
>

Since all components are in series, it's obvious that $I_1 = I_2 = I_3$. Using Ohm's Law on R_3, we see that

$$I_3 R_3 = V_{R_3}$$
$$I_3 = \frac{V_{R_3}}{R_3} = \frac{1\mathrm{V}}{3\Omega} = \frac{1}{3}\mathrm{A}$$

and we see $I_1 = I_2 = I_3 = \frac{1}{3}\mathrm{A}$. Applying Ohm's Law on R_2,

$$I_2 R_2 = V_{R_2}$$
$$\boxed{R_2 = \frac{V_{R_2}}{I_2} = \frac{\frac{2}{3}V}{\frac{1}{3}A} = 2\Omega}$$

and then apply KVL for the loop

$$I_1 R_1 + V_{R_2} + V_{R_3} + V_X = 0$$
$$\frac{1}{3}A \cdot 5\Omega + \frac{2}{3}\mathrm{V} + 1\mathrm{V} + V_X = 0$$
$$\boxed{V_X = -\frac{10}{3}\mathrm{V}}$$

Since we are taking V_4 as reference point, that means $V_4 = 0\mathrm{V}$, and therefore (node method)

$$I_2 R_2 = V_{R_2} = V_4 - V_1$$
$$\boxed{V_1 = V_4 - V_{R_2} = -\frac{2}{3}\mathrm{V}}$$

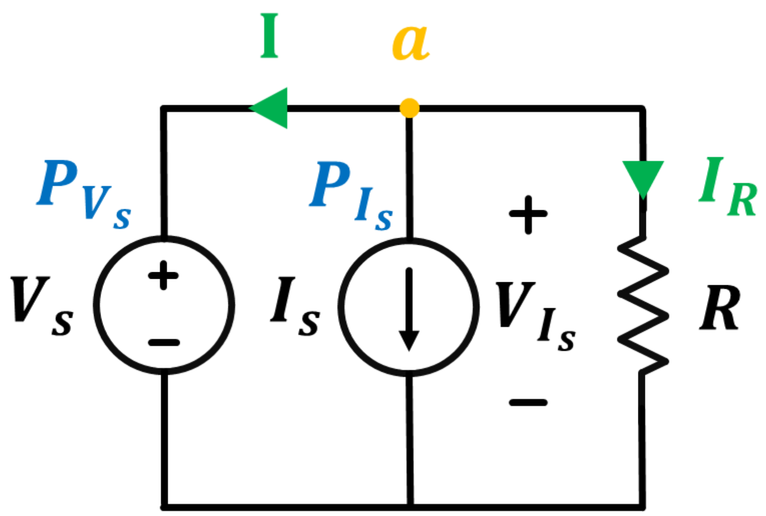
Similarly

$$I_1 R_1 = V_1 - V_2$$
$$\boxed{V_2 = V_1 - I_1 R_1 = -\frac{2}{3}\mathrm{V} - \frac{5}{3}\mathrm{V} = -\frac{7}{3}\mathrm{V}}$$

and

$$I_3 R_3 = V_{R_3} = V_3 - V_2$$
$$\boxed{V_3 = V_2 - V_{R_3} = -\frac{7}{3}\mathrm{V} - 1\mathrm{V} = -\frac{10}{3}\mathrm{V}}$$

Problem 3

> Consider the circuit below, where $V_s = 8\mathrm{V}$, $I_s = 2\mathrm{A}$ and $R = 4\Omega$. Determine the current I and calculate the absorbed power at each circuit element. Clearly indicate if the power is absorbed or injected.
>
> 
>

Using Ohm's Law on R, we could get

$$I_R \cdot R = V_s$$
$$I_R = \frac{V_s}{R} = \frac{8\mathrm{V}}{4\Omega} = 2\mathrm{A}$$

Applying KCL at a, we see that

$$I_R + I_s + I = 0$$
$$\boxed{I = -I_R - I_s = -2\mathrm{A} - 2\mathrm{A} = -4\mathrm{A}}$$

Power used by R

$$\boxed{P_R = I_R^2 R = (2A)^2 \cdot 4\Omega = 16\mathrm{W}}$$

Since V_s and I_s are parallel, we see that $V_{I_s} = V_s = 8\mathrm{V}$. So

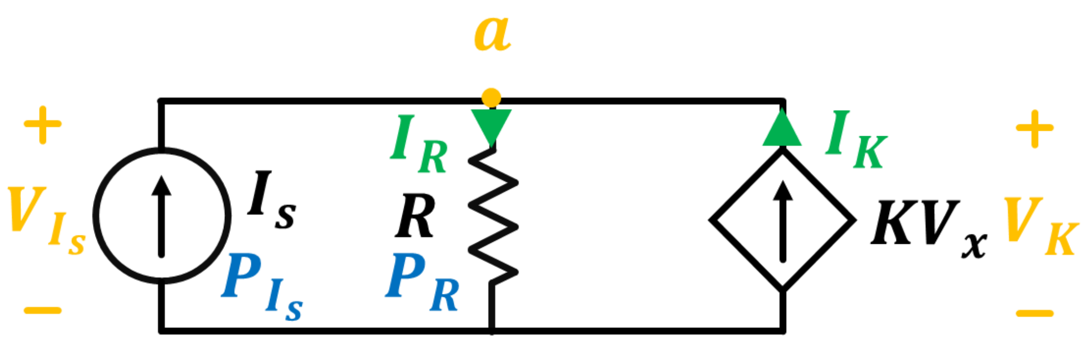
$$\boxed{P_{I_s} = V_{I_s} I_s = 8\mathrm{V} \cdot 2\mathrm{A} = 16\mathrm{W}}$$

Similarly,

$$\boxed{P_{V_s} = V_s I = 8\mathrm{V} \cdot (-4\mathrm{A}) = -32\mathrm{W}}$$

$\boxed{R \text{ and } I_s \text{ absorbs}}$ power (indicated by $P_R > 0\mathrm{W}$ and $P_{I_s} > 0\mathrm{W}$), but $\boxed{V_s \text{ injects}}$ power (indicated by $P_{V_s} < 0\mathrm{W}$)

Problem 4

> Consider the circuit below, where $I_s = 5\mathrm{A}$, $R = 2\Omega$ and $K = 2\mathrm{A/V}$. Determine the voltage V_x and calculate the absorbed power at each circuit element. Clearly indicate if the power is absorbed or injected.
>
> 
>

Applying KCL on node a, we see that

$$I_R = I_S + I_K$$

and applying Ohm's Law on R, we see that

$$I_R \cdot R = -V_x$$
$$(I_s + I_K) \cdot R = -V_x$$
$$(I_s + KV_x) \cdot R = -V_x$$
$$\boxed{V_x = -\frac{R \cdot I_s}{RK + 1} = -\frac{2\Omega \cdot 5\mathrm{A}}{2\Omega \cdot 2\mathrm{A}/V + 1} = -2\mathrm{V}}$$

Therefore

$$I_R = -\frac{V_x}{R} = -\frac{-2\mathrm{V}}{2\Omega} = 1\mathrm{A}$$

and

$$\boxed{P_R = (-V_x) I_R = -(-2\mathrm{V}) \cdot 1\mathrm{A} = 2\mathrm{W}}$$

We see that I_s, R and V_K are in parallel, so $V_{I_s} = V_K = -V_x = 2\mathrm{V}$, and therefore

$$\boxed{P_{I_s} = (-V_{I_s}) I_s = 5\mathrm{A} \cdot (-2\mathrm{V}) = -10\mathrm{W}}$$

We know that

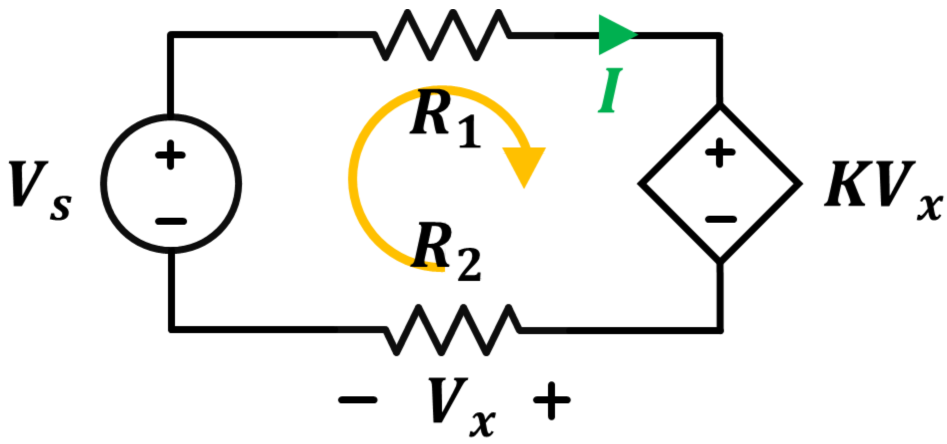
$$I_R = I_s + I_K$$
$$I_K = I_R - I_s = 1\text{A} - 5\text{A} = -4\text{A}$$

and therefore

$$\boxed{P_{I_K} = (-V_K) \cdot I_K = -2\text{V} \cdot (-4\text{A}) = 8\text{W}}$$

$\boxed{R \text{ and } I_K \text{ absorbs}}$ power (indicated by $P_R > 0\text{W}$ and $P_{I_K} > 0\text{W}$), but $\boxed{I_s \text{ injects}}$ power (indicated by $P_{I_s} < 0\text{W}$)

Problem 5

> Consider the circuit below, where $V_s = 3\text{V}$, $R_1 = 1\Omega$, $R_2 = 2\Omega$, and $K = 2$. Determine the current I and the voltage V_x.
>
> 
>

Since all components are in series, they have same current I passing through them indicated in the graph. Using KVL indicated in the graph, we have

$$IR_1 + KV_x + V_x - V_s = 0 \tag{1}$$

also, we could apply the Ohm's Law on R_2, and we get

$$I \cdot R_2 = V_x$$

substitute this to (1), we get

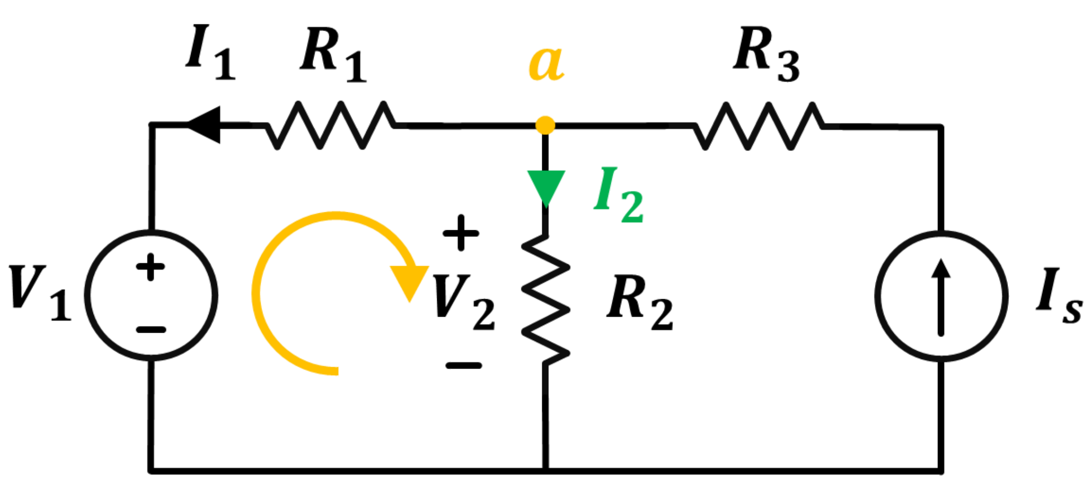
$$V_s = (K+1)IR_2 + IR_1$$
$$I((K+1)R_2 + R_1) = V_s$$
$$\boxed{I = \frac{V_s}{(K+1)R_2 + R_1} = \frac{3\text{V}}{(2+1) \cdot 2\Omega + 1\Omega} = \frac{3\text{V}}{7\Omega} = \frac{3}{7}\text{A}}$$

and therefore

$$\boxed{V_x = I \cdot R_2 = \frac{3}{7}\text{A} \cdot 2\Omega = \frac{6}{7}\text{V}}$$

Problem 6

> Consider the circuit below, where $R_1 = 6\Omega$, $R_2 = 6\Omega$, $R_3 = 2\Omega$, $V_2 = 18\text{V}$, and $I_s = 5\text{A}$. Determine the current I_1 and the voltage V_1.



Applying Ohm's Law on R_2, we see that

$$I_2 = \frac{V_2}{R_2} = \frac{18\mathrm{V}}{6\Omega} = 3\mathrm{A}$$

Applying KCL on node a, we see that

$$I_s = I_1 + I_2$$

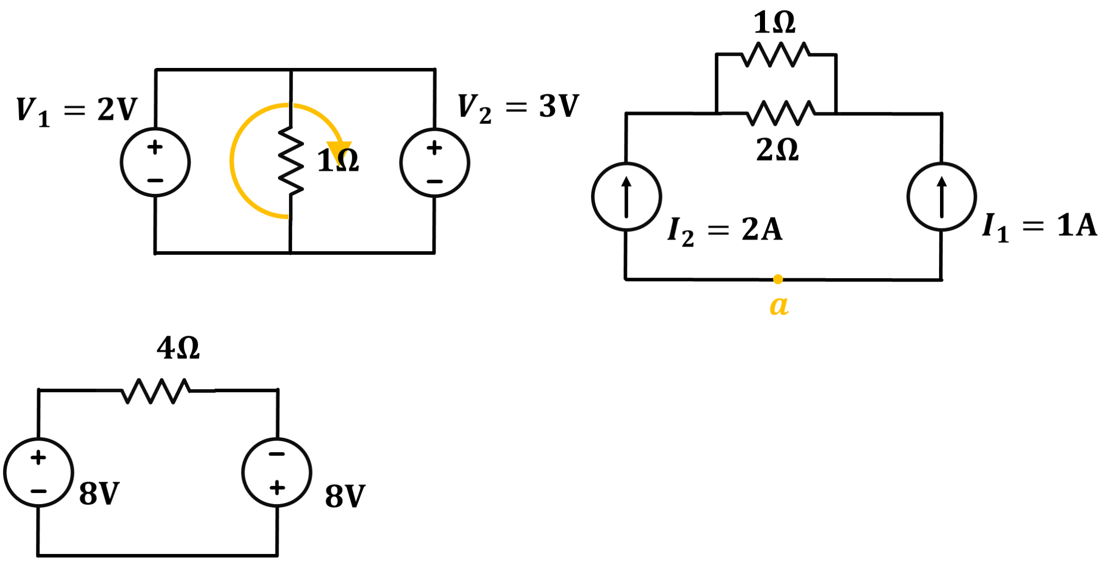
$$\boxed{I_1 = I_s - I_2 = 5\mathrm{A} - 3\mathrm{A} = 2\mathrm{A}}$$

Applying KVL indicated in the graph, we see that

$$-I_1R_1 + V_2 - V_1 = 0$$

$$\boxed{V_1 = V_2 - I_1R_1 = 18\mathrm{V} - 2\mathrm{A}\cdot 6\Omega = 6\mathrm{V}}$$

Problem 7

> Some of the following circuits violate $\mathrm{KVL}/\mathrm{KCL}$ and / or basic definitions of two-terminal elements given in Section 1.3. For each one of the circuits, determine if it is correct or ill-specified. If it is ill-specified, explain the problem and indicate what will happen if the incorrect circuit has been built up in your life?
>
> 
>

$a)$ and $b)$ is ill-formed because:

If we apply KVL indicated in the graph (a), we will find out

$$V_2 - V_1 = 3\text{V} - 2\text{V} = 1\text{V} \neq 0\text{V}$$

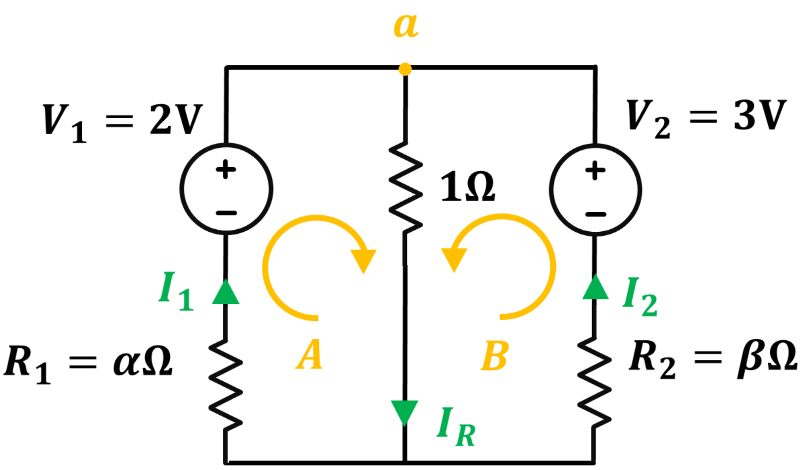
which violates KVL, so the circuit is ill-formed in this case.

If we apply KCL on node a in the graph (b), we will find out that

$$I_1 + I_2 = 3\text{A} \neq I_{\text{flow in}} = 0\text{A}$$

which violates KCL, so the circuit is ill-formed in this case.

The independent voltage sources and independent current sources seen in graph are ideal sources. In reality, the non-ideal sources could be roughly modeled by



> TL;DR. They voltage source will likely get burnt in real life.

Each non-ideal voltage source could be represent roughly by an ideal voltage source and a internal resistance (the R_1 and R_2, it's usually small) in series. Applying KCL on node a, we see that

$$I_1 + I_2 = I_R$$

and applying KVL on Loop A and B, we see

$$\begin{aligned} I_R \cdot 1\Omega + I_1 R_1 = V_1 \\ I_R \cdot 1\Omega + I_2 R_2 = V_2 \end{aligned}$$

and we could solve the them

$$\begin{aligned} (I_1 + I_2) \cdot 1\Omega + I_1 \cdot \alpha\Omega = 2\text{V} \\ (I_1 + I_2) \cdot 1\Omega + I_2 \cdot \beta\Omega = 3\text{V} \end{aligned}$$

and

$$\begin{aligned} I_1(1+\alpha)\Omega + I_2(1)\Omega = 2\text{V} \\ I_1(1)\Omega + I_2(1+\beta)\Omega = 3\text{V} \end{aligned}$$

we find that

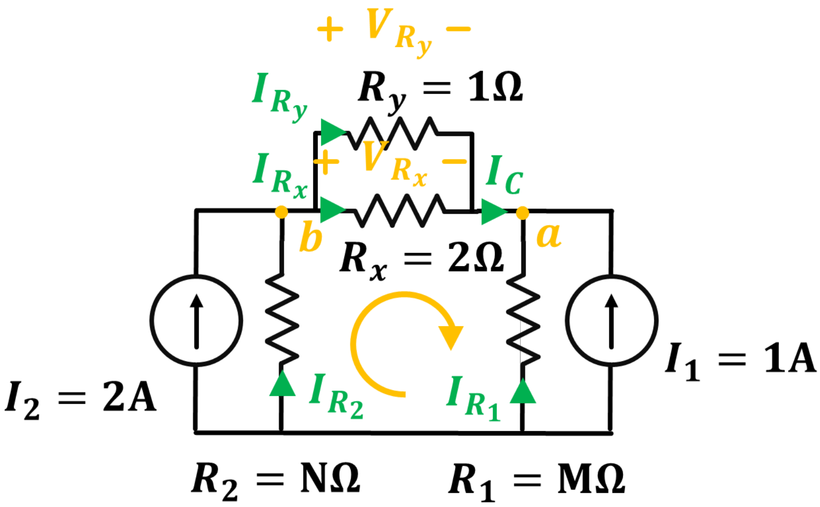
$$\begin{aligned} I_1 &= \frac{2(1+\beta) - 3}{(1+\alpha)(1+\beta) - 1}\text{A} \\ I_2 &= \frac{3(1+\alpha) - 2}{(1+\alpha)(1+\beta) - 1}\text{A} \end{aligned}$$

Since α and β are usually are far less than 1 (the R_1 and R_2 are usually few mili-ohms), we could check what is the power dissipation on the internal resistor of the power source as it becomes more ideal

$$\lim_{\alpha \to 0} P_{I_1} = \lim_{\alpha \to 0} I_1^2 R_1 = \lim_{\alpha \to 0} \left(\frac{2(1+\beta)-3}{(1+\alpha)(1+\beta)-1} \right)^2 \alpha \mathrm{W} = \infty \mathrm{W}$$

$$\lim_{\beta \to 0} P_{I_2} = \lim_{\beta \to 0} I_2^2 R_2 = \lim_{\beta \to 0} \left(\frac{3(1+\alpha)-2}{(1+\alpha)(1+\beta)-1} \right)^2 \beta \mathrm{W} = \infty \mathrm{W}$$

The power (heat) dissipation on the internal resistor in each voltage source will grow to infinity as the itself becomes more ideal. Usually, the voltage source will have a relatively small internal resistance (few milliohms). That means it's likely to have high heat dissipation which could be dangerous and damage the power source.



> TL;DR. It is also likely going to get burnt.

Now we see the what happened in b), each non-ideal current source could be considered as an ideal current source and an internal resistance (the R_1 and R_2, it's usually large) in parallel. (Take this approximation with grain of salt, it might not *exactly* reflect the real situation) First applying KCL on both node a and b, we see that

$$I_{R_2} = I_C - I_2$$
$$I_{R_1} = -(I_C + I_1)$$

and since R_x and R_y are in parallel, $V_{R_x} = V_{R_y}$ and $I_{R_y} + I_{R_x} = I_C$ and

$$I_{R_x} R_x = I_{R_y} R_y$$

and

$$I_{R_x} + \frac{R_x}{R_y} I_{R_x} = I_C$$

$$I_C = (1 + \frac{R_x}{R_y}) I_{R_x} = (1 + \frac{2\Omega}{1\Omega}) I_{R_x} = 3 I_{R_x}$$

Applying KVL on the loop indicated in the graph

$$I_{R_2} R_2 + R_x I_{R_x} - I_{R_1} R_1 = 0\mathrm{V}$$
$$(I_C - I_2) R_2 + R_x I_{R_x} + (I_C + I_1) R_1 = 0\mathrm{V}$$
$$(3 I_{R_x} - 2\mathrm{A}) \cdot N\Omega + I_{R_x} \cdot 2\Omega + (3 I_{R_x} + 1\mathrm{A}) M\Omega = 0\mathrm{V}$$

Solving the equation

$$I_{R_x} = \frac{2N - M}{3N + 3M + 2} \mathrm{A}$$

and therefore

$$I_C = 3I_{R_x} = \frac{6N - 3M}{3N + 3M + 2}\mathrm{A}$$

and

$$\begin{aligned} I_{R_2} &= I_C - I_2 = \frac{6N - 3M}{3N + 3M + 2} - 2 = -\frac{9M + 4}{3N + 3M + 2} \\ I_{R_1} &= -(I_C - I_1) = -\left(\frac{6N - 3M}{3N + 3M + 2} + 1\right) = -\frac{9N + 2}{3N + 3M + 2} \end{aligned}$$

this is hard to analysis. Typically, in real life, the internal resistance is rather big (like few kilo or mega ohms), and if we assume $N = \alpha M$. We could see (given M is large enough)

$$\begin{aligned} I_{R_2} &= -\frac{9M + 4}{3\alpha M + 3M + 2} \approx \frac{9}{3\alpha + 3} = \frac{3}{\alpha + 1}\mathrm{A} \\ I_{R_1} &= -\frac{9\alpha M + 2}{3\alpha M + 3M + 2} \approx \frac{9\alpha}{3\alpha + 3} = \frac{3\alpha}{\alpha + 1}\mathrm{A} \end{aligned}$$

and we have

$$P_{R_2} = (I_{R_2})^2 \cdot R_2 = \left(\frac{3}{\alpha + 1}\right)^2 \cdot \alpha M\mathrm{W}$$

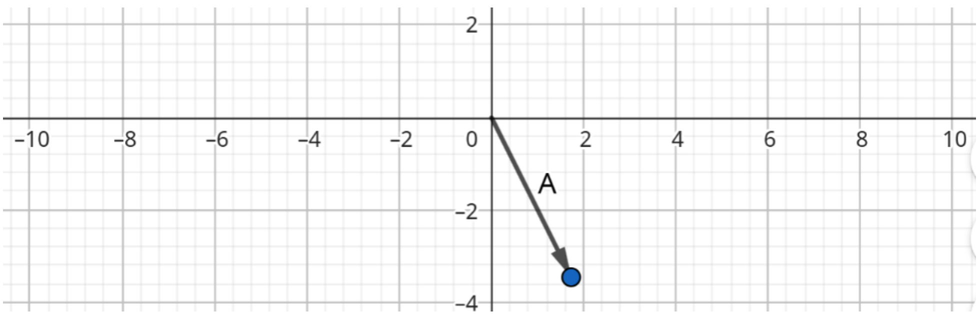
$$P_{R_1} = (I_{R_1})^2 \cdot R_1 = \left(\frac{3\alpha}{\alpha + 1}\right)^2 \cdot M\mathrm{W}$$

Then, if we *assume* α is small relative to M (that is $a \ll M$), and M is large enough. That means the resulting P_{R_2} and P_{R_1} will likely be big. (Take $\alpha = 1$ and $M = 1000$ as example, this will produce a few kW power, which is not small). This high power (the heat dissipation in this case) will burn the circuit.

Problem 8

Let $A = \sqrt{3} - j \cdot 2\sqrt{3}$ and Let $B = -3 - j \cdot \sqrt{3}$.

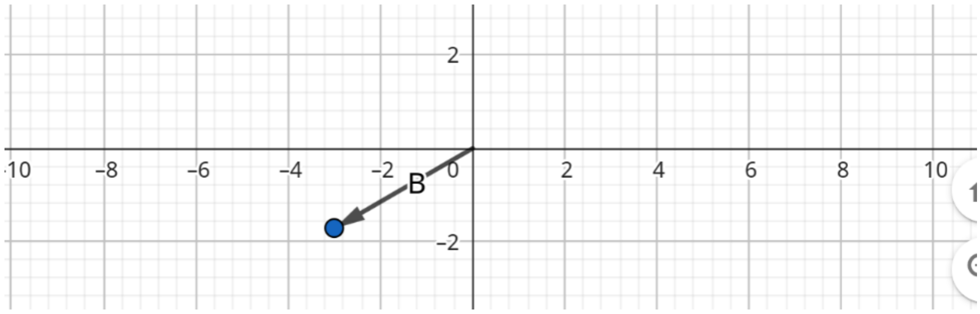
(a)



we see the magnitude is $r = \sqrt{(\sqrt{3})^2 + (2\sqrt{3})^2} = 4$ and the angle/phase is $\arctan\left(-\frac{2\sqrt{3}}{\sqrt{3}}\right) = \arctan(-2)$ (it's in fourth quadrant). Therefore

$$A = 4e^{i\arctan(-2)}$$

(b)



we see that the magnitude is $r = \sqrt{(-3)^2 + (-\sqrt{3})^2} = 2\sqrt{3}$ and the angle/phase is $\arctan\left(\frac{-\sqrt{3}}{-3}\right) + \pi = \frac{7}{6}\pi$

$$B = 2\sqrt{3}e^{i\frac{7}{6}\pi}$$

(c)

$$A + B = (\sqrt{3} - 3) - j \cdot (3\sqrt{3})$$
$$A - B = (\sqrt{3} + 3) - j \cdot \sqrt{3}$$

$$|A + B| = \sqrt{(\sqrt{3} - 3)^2 + (-3\sqrt{3})^2} = \sqrt{9 + 3 - 6\sqrt{3} + 27} = \sqrt{39 - 6\sqrt{3}}$$

$$|A - B| = \sqrt{(\sqrt{3} + 3)^2 + (-\sqrt{3})^2} = \sqrt{9 + 3 + 6\sqrt{3} + 3} = \sqrt{15 + 6\sqrt{3}}$$

(d)

$$\begin{aligned} AB &= -(\sqrt{3} - j \cdot 2\sqrt{3})(3 + j \cdot \sqrt{3}) \\ &= -(3\sqrt{3} + j \cdot 3 - j \cdot 6\sqrt{3} + 6) \\ &= -(3\sqrt{3} + 6) + j \cdot (6\sqrt{3} - 3) \end{aligned}$$

$$\begin{aligned} A/B &= \frac{AB^*}{BB^*} = \frac{AB^*}{|B|^2} \\ &= \frac{(\sqrt{3} - j \cdot 2\sqrt{3})(-3 + j\sqrt{3})}{12} \\ &= \frac{1}{12}(-3\sqrt{3} + j \cdot 3 + j \cdot 6\sqrt{3} + 6) \\ &= \frac{1}{4}(2 - \sqrt{3}) + \frac{1}{4}j \cdot (2\sqrt{3} + 1) \end{aligned}$$