(a)

Let $s \in L_1$.

Assume $x \in L_{ee}$ for every $x \in L_1$ and $|x| < |s|$.

There are three cases to consider:

- If $s = \varepsilon$, then $\#(0, s) = 0$ and $\#(1, s) = 0$ and thus $s \in L_{ee}$
- If $s = x0101$ or $s = x1010$, for $x \in L_{ee}$

$$\begin{aligned}
\#(0, x0101) &= \#(0, x) + \#(0, 0101) = \#(0, x) + 2 \\
\#(1, x0101) &= \#(1, x) + \#(1, 0101) = \#(1, x) + 2 \\
\#(0, x1010) &= \#(0, x) + \#(0, 1010) = \#(0, x) + 2 \\
\#(1, x1010) &= \#(1, x) + \#(1, 1010) = \#(1, x) + 2
\end{aligned}$$

  Since $|x| < |s|$, according to the assumption, $\#(0, x)$ and $\#(1, x)$ is even number, and thus $\#(0, s)$ and $\#(1, s)$ is too. $s \in L_{ee}$

- If $s = x00y$ or $s = x11y$, for $xy \in L_{ee}$. Then

$$\begin{aligned}
\#(0, x00y) &= \#(0, x) + \#(0, 00) + \#(0, y) = \#(0, x) + \#(0, y) + 2 \\
\#(1, x00y) &= \#(1, x) + \#(1, 00) + \#(1, y) = \#(1, x) + \#(1, y) + 0 \\
\#(0, x11y) &= \#(0, x) + \#(0, 11) + \#(0, y) = \#(0, x) + \#(0, y) + 0 \\
\#(1, x11y) &= \#(1, x) + \#(1, 11) + \#(1, y) = \#(1, x) + \#(1, y) + 2
\end{aligned}$$

  since $|x| < |s|$ and $|y| < s$, according to the assumption $\#(0, x)$ and $\#(0, y)$ and $\#(1, x)$ and $\#(1, y)$ is even number, and thus $\#(0, s)$ and $\#(1, s)$ too. $s \in L_{ee}$

Thus $L_1 \subseteq L_{ee}$

(b)

Let $s \in L_{ee}$,

Assume $x \in L_1$ for every $x \in L_{ee}$ and $|x| < |s|$.

There are three cases to consider:

- $s = \varepsilon$, according to definition, $s \in L_1$
- $s = x0101$ or $x = x1010$, $x = L_{ee}$, according to the definition, $|x| < |s|$ and thus $x \in L_1$. s has the form of $x0101$ and $x1010$, and according to the definition of the L_1, $s \in L_1$.
- $s = x00y$ or $s = x11y$, $x, y = L_{ee}$, according to the definition, $|x| < |s|$ and $|y| < |s|$ and thus $x \in L_1$, $y \in L_1$. Thus if $xy \in L_1$, then $s \in L_1$. We could recursively apply this case to x and y until write s as a product of string in L_1 with no subpattern 00 and 11. In case, it will become previous two cases.

Then $L_{ee} \subseteq L_1$