Problem 1

(a)

We know that the capacitance is

$$C = \varepsilon_0 \frac{A}{d} = \varepsilon_0 \frac{\pi R^2}{d}$$

(b)

We know that

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = I = C\frac{\mathrm{d}V}{\mathrm{d}t}$$

and it's true that

$$\mathrm{d}V = \frac{1}{C} I \mathrm{d}t$$

therefore

$$V = \frac{1}{C}\int I \mathrm{d}t = \frac{I_0}{\varepsilon_0 \pi R^2} d \int e^{i\omega t} \mathrm{d}t = -\frac{1}{\varepsilon_0 \pi R^2}\frac{i}{\omega} I_0 e^{i\omega t} d$$

(c)

Since there is no current $\vec{J}$ between the plates, we have

$$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 \varepsilon_0 \iint \frac{\partial \vec{E}}{\partial t} \cdot \mathrm{d}\vec{A}$$

and therefore we see that

$$E = \frac{V}{d} = \frac{I_0}{\varepsilon_0 \pi R^2}\frac{e^{i\omega t}}{i\omega}$$

and that

$$\frac{\partial E}{\partial t} = \frac{I_0}{\varepsilon_0 \pi R^2} e^{i\omega t}$$

and use ampere loop between the plates:

$$B \cdot 2\pi s = \mu_0 \varepsilon_0 \cdot \frac{I_0}{\varepsilon_0 \pi R^2} e^{i\omega t} \cdot \pi s^2$$

and therefore

$$B = \frac{\mu_0}{2\pi} I_0 e^{i\omega t} \cdot \frac{s}{R^2}$$

(d)

we see that total magnetic flux is

$$\begin{aligned}\Phi = \int_0^R B(s)\cdot \mathrm{d}A = \int_0^R \frac{\mu_0}{2\pi} I_0 e^{i\omega t}\cdot \frac{s}{R^2}\cdot d\mathrm{d}s \\ = \frac{\mu_0}{2\pi} I_0 e^{i\omega t}\cdot \frac{d}{R^2}\int_0^R s\mathrm{d}s \\ = \frac{\mu_0}{2\pi} I_0 e^{i\omega t}\cdot \frac{d}{R^2}\frac{R^2}{2} \\ = \frac{\mu_0}{4\pi} I_0 e^{i\omega t} d\end{aligned}$$

and therefore

$$\mathcal{E} = -\frac{\partial \Phi}{\partial t} = -\frac{\mu_0}{4\pi}\frac{I_0 e^{i\omega t}}{i\omega} d = \frac{\mu_0}{4\pi}\frac{i}{\omega}\cdot I_0 e^{i\omega t} d$$

we see that the factor for $\mathcal{E}$ is $\frac{\mu_0}{4\pi}\frac{i}{\omega}$, and the factor for V is $-\frac{1}{\varepsilon_0 \pi R^2}\frac{i}{\omega}$, and we see that they have a π phase difference.

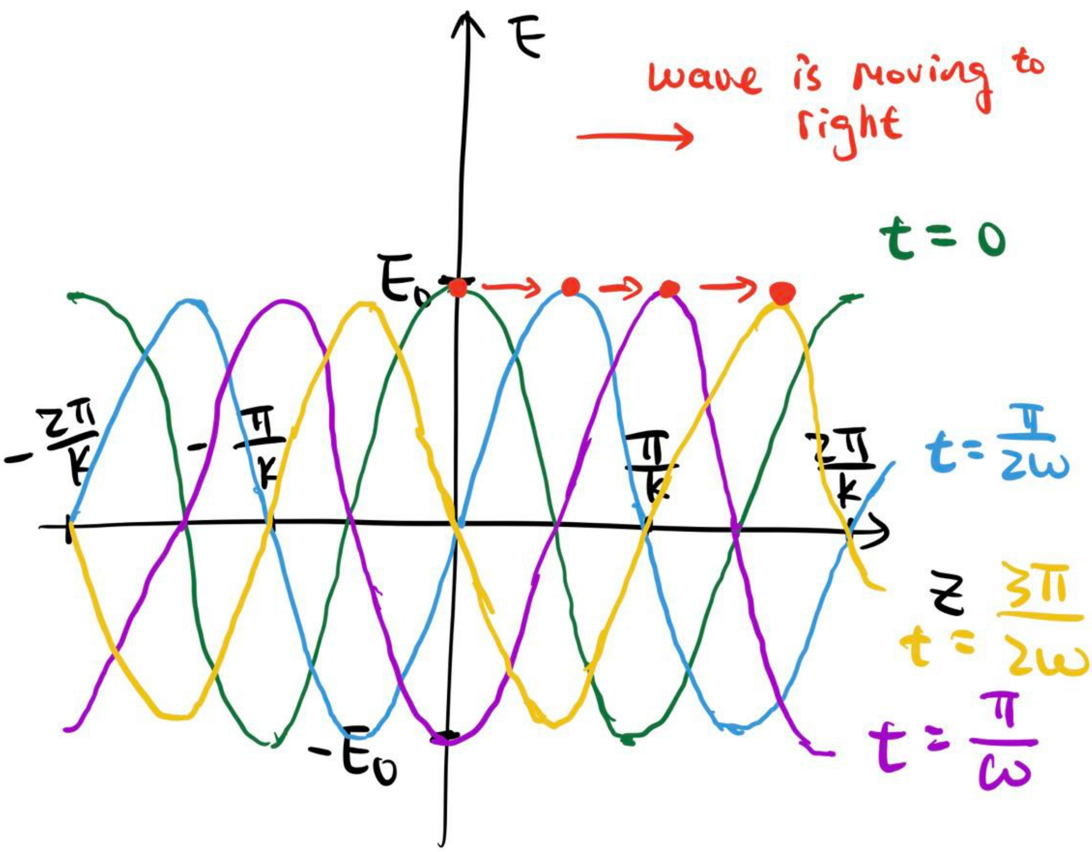
Problem 2

(a)

From lecture, we know that $k = \frac{\omega}{c}$, where ω is the angular frequency of the wave and c is the speed of light. The explicit formula for such plane wave is:

$$\vec{E} = \hat{x} E_0 \cos(kz - \omega t)$$

(b)



See that red dot moves in four different timestamp.

(c)

we see that

$$\vec{B} = \frac{1}{c}\hat{k} \times \vec{E} = \frac{1}{c}(\hat{z} \times \hat{x})E_0 \cos(kz - \omega t) = \frac{E_0}{c}\hat{y}\cos(kz - \omega t)$$

(d)

we see that

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} = \frac{E_0^2}{c\mu_0}\cos^2(kz - \omega t)\hat{z}$$

and we find that

$$\langle \vec{S} \rangle = \frac{E_0^2}{c\mu_0}\langle \cos^2(kz - \omega t)\rangle \hat{z} = \frac{E_0^2}{2c\mu_0}\hat{z}$$

(e)

we see that

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 E_0^2 \cos^2(kz - \omega t)$$

and peak value is

$$u_E = \frac{1}{2}\varepsilon_0 E_0^2$$

and therefore

$$\langle u_E \rangle = \frac{1}{2}\varepsilon_0 E_0^2 \langle \cos^2(kz-\omega t)\rangle = \frac{1}{4}\varepsilon_0 E_0^2$$

It's half of the peak value.

(f)

Notice that $\mu_0 c^2 = \mu_0 \cdot \frac{1}{\varepsilon_0\mu_0} = 1/\varepsilon_0$

we see that

$$u_B = \frac{1}{2\mu_0}B^2 = \frac{1}{2}\frac{E_0^2}{\mu_0 c^2}\cos^2(kz-\omega t) = \frac{1}{2}\varepsilon_0 E_0^2 \cos^2(kz-\omega t)$$

and then

$$\langle u_B \rangle = \frac{1}{2}\frac{E_0^2}{\mu_0 c^2}\langle \cos^2(kz-\omega t)\rangle = \frac{E_0^2}{4\mu_0 c^2} = \frac{1}{4}\varepsilon_0 E_0^2$$

we see that the $u_B = u_E$ at all time and space.

(g)

we see that

$$\langle u \rangle = \langle u_B \rangle + \langle u_E \rangle = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{E_0^2}{2c^2\mu_0}$$

and that

$$|\langle \vec{S} \rangle| = \frac{E_0^2}{2c\mu_0}$$

we see that they are not equal. However, they looks similar, only by a $1/c$ factor off from each other. ($|\langle \vec{S} \rangle| = c\langle u \rangle$)