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sign: Yuqing Zhai

Question 2

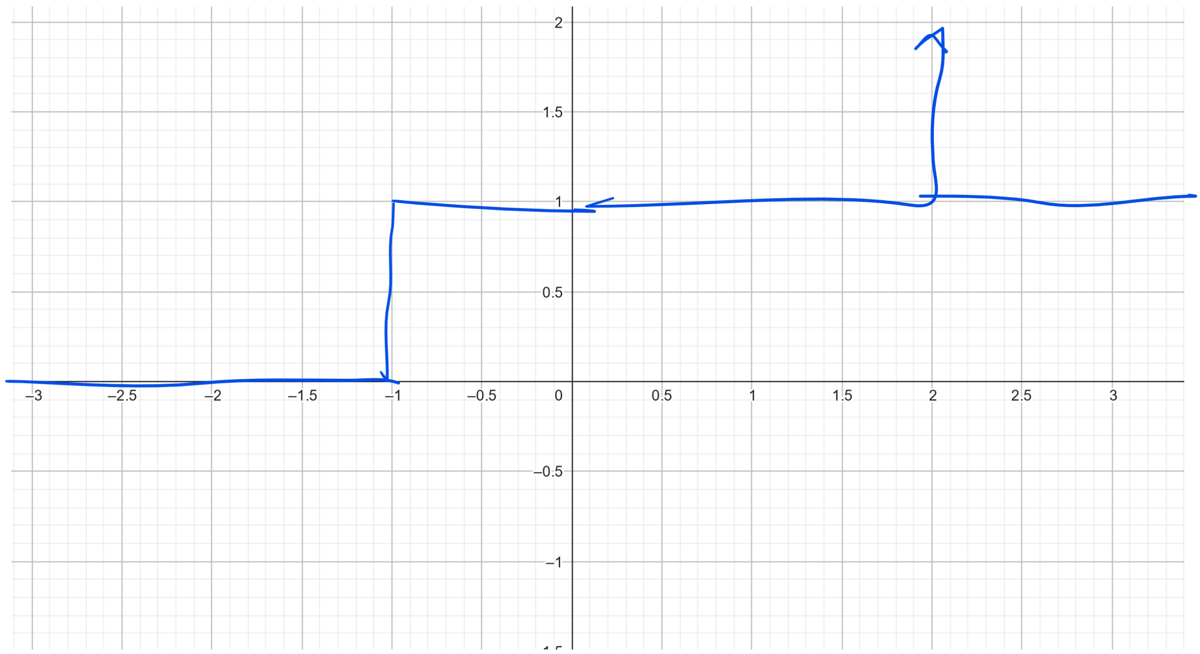
Simplify the following expressions involving the impulse, unit step function, rectangular pulse, and/or triangular pulse and sketch the results.

(a)

$$\begin{aligned} g(t) &= \cos(2\pi t)\left(\frac{\mathrm{d}u(t)}{\mathrm{d}t} + \delta(t+0.5)\right) \\ &= \cos(2\pi t)(\delta(t) + \delta(t+0.5)) \\ &= \cos(2\pi t)\delta(t) + \cos(2\pi t)\delta(t+0.5) \\ &= \cos(2\pi \cdot 0)\delta(t) + \cos(2\pi \cdot (-0.5))\delta(t+0.5) \\ &= \delta(t) - \delta(t+0.5) \end{aligned}$$

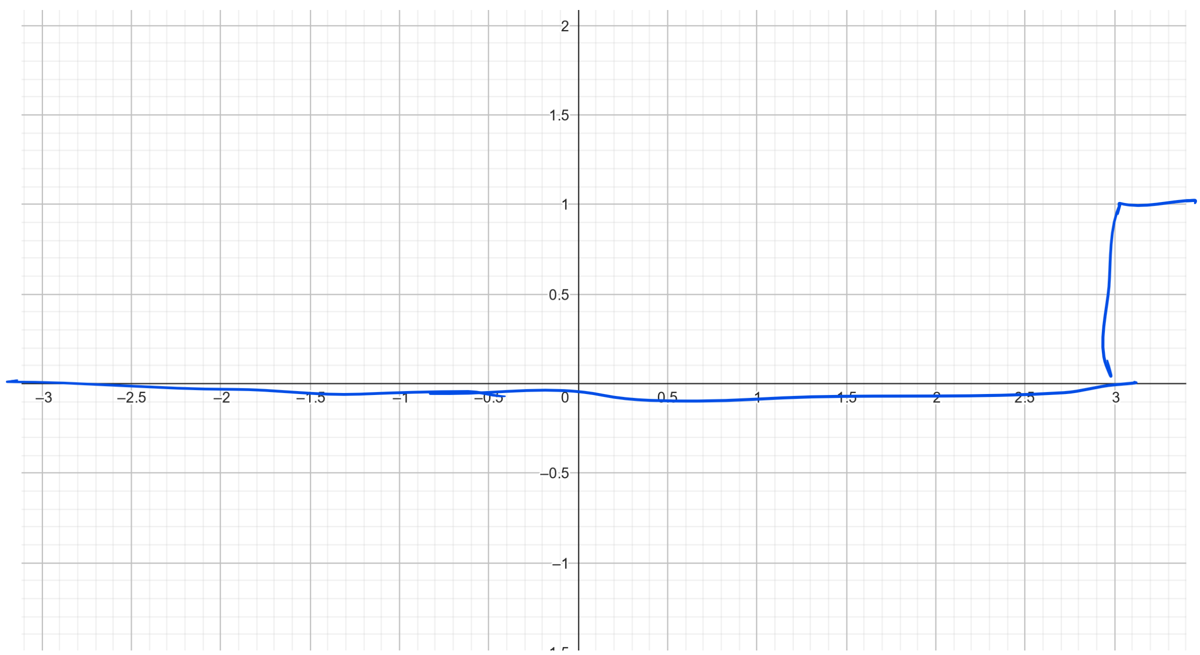
(b)

$$\begin{aligned} a(t) &= \int_{-\infty}^{t} \delta(\tau+1)\mathrm{d}\tau + \mathrm{rect}(\frac{t}{6})\delta(t-2) \\ &= u(t+1) + \mathrm{rect}(\frac{2}{6})\delta(t-2) \\ &= u(t+1) + \delta(t-2) \end{aligned}$$



(c)

$$b(t) = \delta(t-3) * u(t) = u(t-3)$$

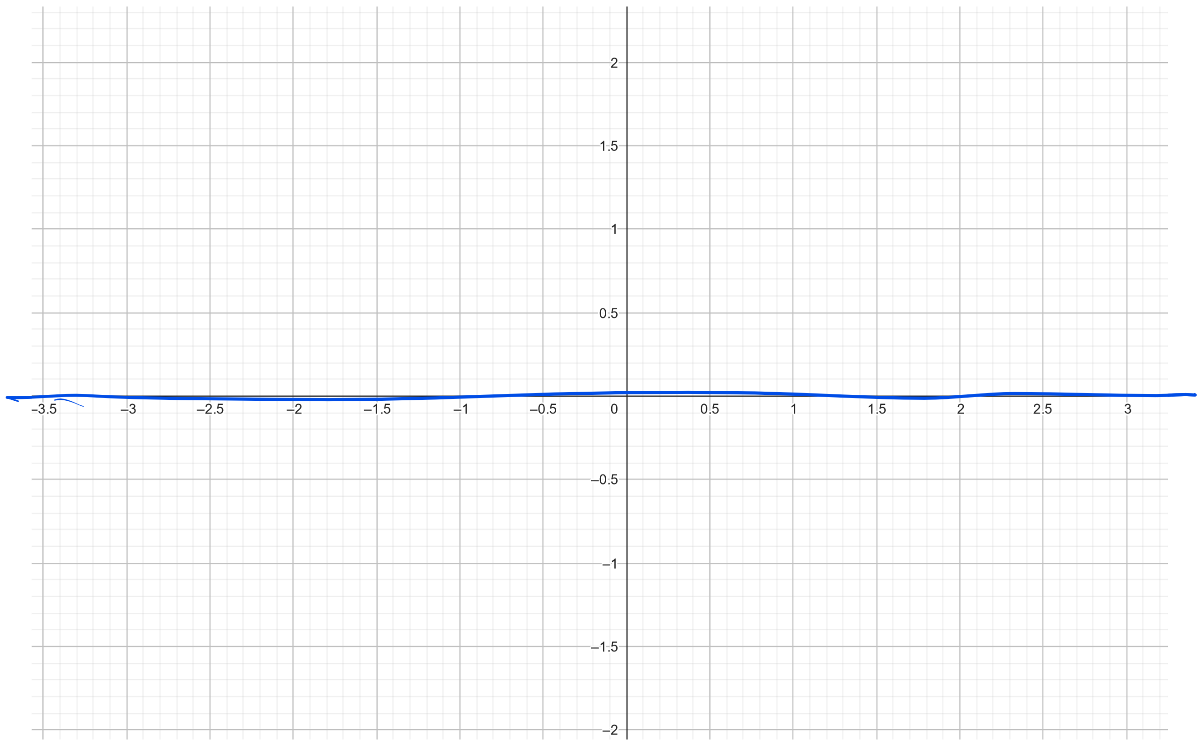


(d)

$$\begin{aligned} f(t) &= (1+t^2)(\delta(t) - \delta(t+1)) \\ &= (1+t^2)\delta(t) - (1+t^2)\delta(t+1) \\ &= (1+0^2)\delta(t) - (1+(-1)^2)\delta(t+1) \\ &= \delta(t) - 2\delta(t+1) \end{aligned}$$

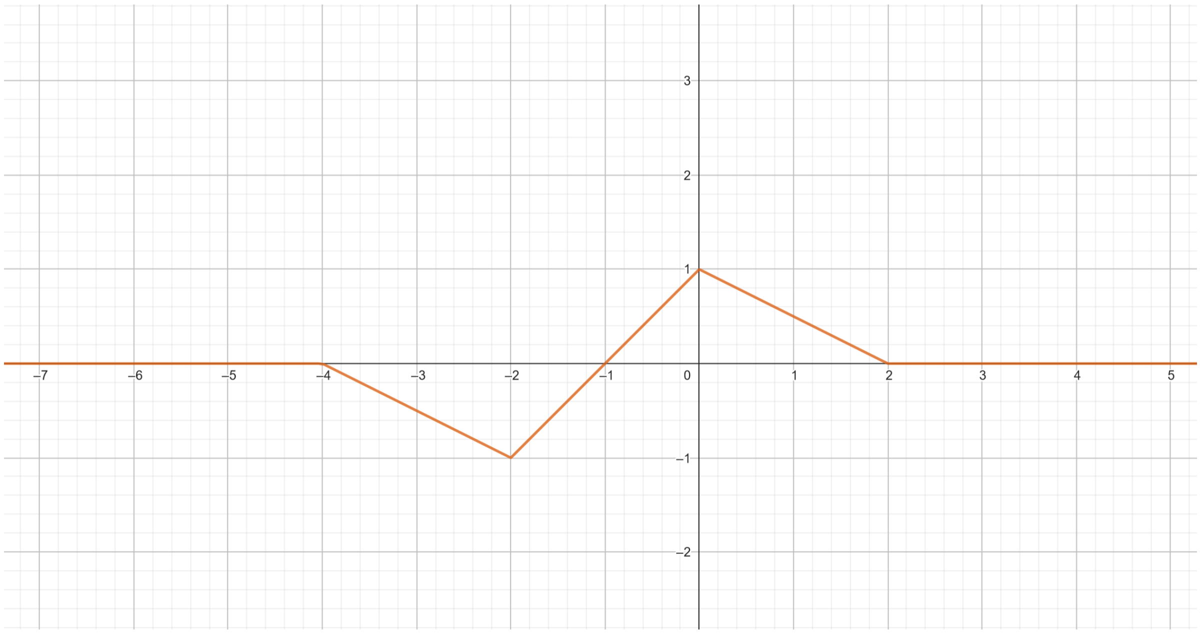
(e)

$$y(t) = \int_{-1}^{\infty} (\tau^2 + 1)\delta(\tau + 2)\mathrm{d}\tau = 0$$



(f)

$$\begin{aligned} c(t) &= \triangle(\frac{t}{4}) * (\delta(t) - \delta(t+2)) \\ &= \triangle(\frac{t}{4}) * \delta(t) - \triangle(\frac{t}{4}) * \delta(t+2) \\ &= \triangle(\frac{t}{4}) - \triangle(\frac{t+2}{4}) \end{aligned}$$



Question 3

(a)

We know that $f(t) * \delta(t - t_0) = f(t - t_0)$. So we have

$$h(t) * \mathrm{rect}(\frac{t}{3}) = \mathrm{rect}(\frac{t-3}{3})$$
$$\delta(t-3) * \mathrm{rect}(\frac{t}{3}) = \mathrm{rect}(\frac{t-3}{3})$$

so $h(t) = \delta(t - 3)$

(b)

Notice that $y(t) = \frac{1}{2}(f(t - \frac{3}{2}) - f(t - \frac{9}{2}))$, then it's obvious that

$$\frac{1}{2}(\delta(t - \frac{3}{2}) - \delta(t - \frac{9}{2})) * f(t) = y(t)$$
$$h(t) = \frac{1}{2}(\delta(t - \frac{3}{2}) - \delta(t - \frac{9}{2}))$$

(c)

Notice that $y(t) = f(t-2) + f(t-3) + f(t-4)$, then it's obvious that

$$(\delta(t-2) + \delta(t-3) + \delta(t-4)) * f(t) = y(t)$$
$$h(t) = \delta(t-2) + \delta(t-3) + \delta(t-4)$$

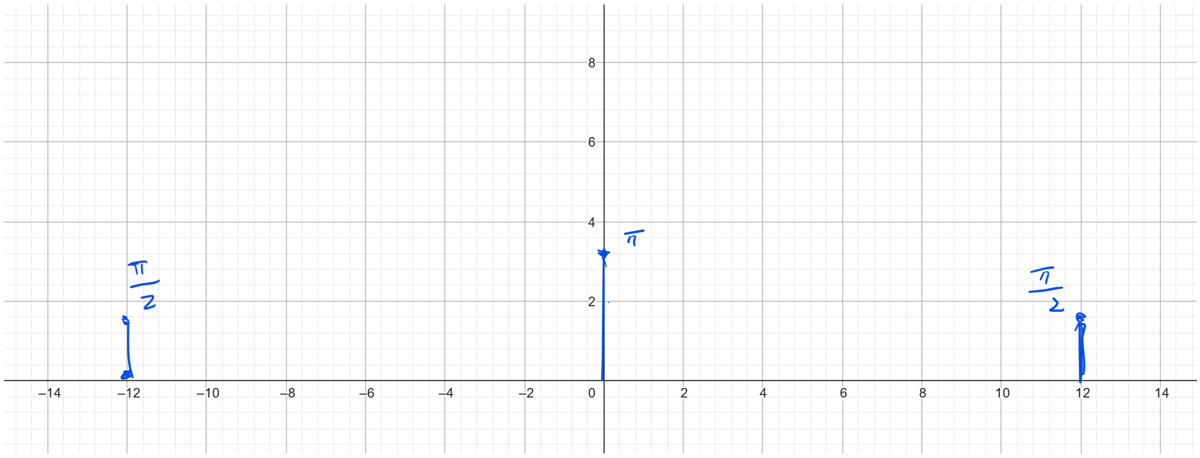
Problem 4

(a)

$$\mathcal{F}(f(t)) = 5\mathcal{F}(\cos(5t)) + 3\mathcal{F}(\sin(15t))$$
$$= 5\pi(\delta(\omega-5) + \delta(\omega+5)) + j \cdot 3\pi(\delta(\omega+15) - \delta(\omega-15))$$
$$= 5\pi\delta(\omega-5) + 5\pi\delta(\omega+5) + 3j\pi\delta(\omega+15) - 3j\pi\delta(\omega-15))$$

(b)

$$x(t) = \cos^2(6t) = \frac{1}{2}(\cos(12t) + 1)$$
$$\mathcal{F}(x(t)) = \frac{1}{2}\mathcal{F}(\cos(12t)) + \frac{1}{2}\mathcal{F}(1)$$
$$= \frac{\pi}{2}(\delta(\omega-12) + \delta(\omega+12)) + \pi\delta(\omega)$$



(c)

$$\begin{aligned}\mathcal{F}(y(t)) &= \mathcal{F}(e^{-2t}u(t) * \cos(2t)) = \mathcal{F}(e^{-2t}u(t))\mathcal{F}(\cos(2t)) \\ &= (\frac{1}{2+j\omega}) \cdot \pi(\delta(\omega-2)+\delta(\omega+2)) \\ &= \frac{\pi}{2+j\omega}(\delta(\omega-2)+\delta(\omega+2)) \\ &= \frac{\pi}{2+2j}\delta(\omega-2) + \frac{\pi}{2-2j}\delta(\omega+2)\end{aligned}$$

(d)

$$\begin{aligned}\mathcal{F}(z(t)) &= \mathcal{F}(e^{-t}u(t)) + \mathcal{F}(e^{-t}u(t)\cos(3t)) \\ &= (\frac{1}{1+j\omega}) + (\frac{1+j\omega}{(1+j\omega)^2+9})\end{aligned}$$

Problem 5

(a)

$f_0 = 2F = 40 \cdot 2 = 80\text{KHz}$

(b)

$$\mathcal{F}(f(t)) = \frac{1}{40}\text{rect}(\frac{\omega}{80\pi})$$

$\Omega = 40\pi \cdot \text{rad Hz}$, $F = \frac{\Omega}{2\pi} = 20\text{Hz}$,

$f_0 = 2F = 20 \cdot 2 = 40\text{Hz}$

(c)

$$\begin{aligned}\mathcal{F}(g(t)) &= \frac{1}{100}\text{rect}(\frac{\omega}{200\pi}) + \frac{1}{2\pi}\frac{1}{40}\text{rect}(\frac{\omega}{80\pi}) * \pi(\delta(\omega-200\pi)+\delta(\omega+200\pi)) \\ &= \frac{1}{100}\text{rect}(\frac{\omega}{200\pi}) + \frac{1}{2}\frac{1}{40}(\text{rect}(\frac{\omega-200\pi}{80\pi}) + \text{rect}(\frac{\omega+200\pi}{80\pi}))\end{aligned}$$

non-zero frequency: $-50\text{Hz} \sim 50\text{Hz}$, $80\text{Hz} \sim 120\text{Hz}$, $-120\text{Hz} \sim -80\text{Hz}$

So, $F = 120\text{Hz}$ and $f_0 = 2F = 120 \cdot 2 = 240\text{Hz}$

Problem 6

(a)

$$\begin{aligned}\mathcal{F}(f_1(t)) &= F(\omega) * j\pi(\delta(\omega + 4000\pi) - \delta(\omega - 4000\pi)) \\ &= j\pi(F(\omega + 4000\pi) - F(\omega - 4000\pi))\end{aligned}$$

non-zero frequency: $-4000\pi - \Omega \sim -4000\pi + \Omega$, $4000\pi - \Omega \sim 4000\pi + \Omega$.

So, $\omega_s = 8000\pi + 2\Omega$ rad/s

(b)

$$\begin{aligned}\mathcal{F}(f_2(t)) &= \frac{1}{2\pi} F(\omega) \cdot j\pi(\delta(\omega + 4000\pi) - \delta(\omega - 4000\pi)) \\ &= \frac{1}{2} j(F(-4000\pi)\delta(\omega + 4000\pi) - F(4000\pi)\delta(\omega - 4000\pi))\end{aligned}$$

If $\Omega < 4000\pi$, then the entire Fourier transformed function will be evaluated to 0, and $w_s = 0$

Else, the $w_s = 2 \cdot 4000\pi = 8000\pi$ rad/s

(c)

$$\mathcal{F}(f_3(t)) = f(t) * f(t) = \frac{1}{2\pi} F(\omega) F(\omega)$$

This still has the same bandwidth as the $F(\omega)$, so $\omega_s = 2\Omega$ rad/s

(d)

$$\mathcal{F}(f_4(t)) = \frac{1}{2\pi} F(\omega) * F(\omega)$$

$\Delta\omega_F = 2\Omega$, $\Delta\omega_{F_4} = \Delta\omega_F + \Delta\omega_F = 2\Delta\omega_F = 4\Omega$. (Width property)

$\omega_s = \Delta\omega_{F_4} = 4\Omega$ rad/s

(e)

$$\mathcal{F}(f_5(t)) = F(\omega) e^{-2j\omega}$$

This still has the same bandwidth as the $F(\omega)$, so $\omega_s = 2\Omega$ rad/s

Problem 7

(a)

$$h(t) = \frac{\mathrm{d}g(t)}{\mathrm{d}t} = 5\delta(2t - 5) \cdot 2 = 10\delta(2t - 5)$$

(b)

$$h(t) = \frac{\mathrm{d}g(t)}{\mathrm{d}t} = 3t^2 u(t) + t^3 \delta(t) = 3t^2 u(t)$$

(c)

$$h(t) = \frac{\mathrm{d}g(t)}{\mathrm{d}t} = e^{-t} u(t - 5) + (2 - e^{-t})\delta(t - 5) = e^{-t} u(t - 5) + (2 - e^{-5})\delta(t - 5)$$

Problem 8

(a)

Convolution is linear operation, so it's linear.

It's not time-invariant, $y'(t) = y(t-u) = 5f(t-u) * u(t-u) \neq 5f(t-u) * u(t)$.

It's $\int_{-\infty}^{\infty} 5f(\tau) \cdot u(t-\tau) \mathrm{d}\tau$, this considered all future input of $f(t)$, so it's not causal.

It's not BIBO, consider simplest function $f(t) = 1$, then $\int_{-\infty}^{\infty} 5u(\tau)f(t-\tau)\mathrm{d}\tau = \int_{-\infty}^{\infty} 5u(\tau)d\tau$ is infinity and certainly not bounded.

(b)

Taking the square of input signal f is not linear operation. Consider $f' = 2f$, then $\delta(t-4) * (2f)^2 = 4\delta(t-4) * f^2 \neq 2y(t)$

It's causal, since it only uses present and past value.

It's BIBO stable since $y(t) = f^2(t-4)$, if f is bounded, then y is bounded.

It's not time-invariant, as it depends on t other than the input function.

(c)

It's linear since integral is a linear operation.

It's causal, we see it only depends on the input from $t-2$ to past infinity.

It's time-invariant since $\int_{-\infty}^{t-2} f(\tau - u)d\tau = \int_{-\infty}^{t-u-2} f(x)\mathrm{d}x = y(t-u)$.

It's not BIBO, even like $f(t) = 1$, this function will not be bounded, (it will increase to infinity as t increase)

(d)

It's linear since addition is linear operation.

It's not causal since it uses the future input $f(t+1)$

It's time invariant, $y'(t) = y(t-u) = f(t-u-1) + f(t-u+1) = f'(t-1) + f'(t+1)$

It's BIBO, as both $f(t-1)$ and $f(t+1)$ is bounded.

(e)

It's causal since it only uses present input.

It's not time-invariant, say $f' = f(t-u)$, then $y(t-u) = f(2(t-u) = f(2t-2u) \neq f(t-u)$

It's linear suppose $y(t) = f(2t), x(t) = g(2t)$, then $af(2t) + bg(2t) = ay(t) + bg(t)$

It's BIBO, as $y(t) = f(2t)$ is bounded.

(f)

It's causal since it only uses present input.

It's not time-invariant, $y(t-u) = (t+u-1)f(t-u) \neq (t+1)f(t-u)$

It's linear, suppose $y(t) = (t+1)f(t), x(t) = (t+1)g(t)$, then $(t+1)(af(t) + bg(t)) = a(t+1)f(t) + b(t+1)g(t) = ay(t) + bx(t)$

It's not BIBO, as (t+1) could be arbitrarily large.