RECENT ADVANCES IN RETINEX THEORY

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It is a cultural commonplace deriving from Newton that the color of an object in the world around us depends on the relative amounts of red, green, and blue light coming from the object to our eyes. In contradiction, it has long been known that the color of an object when it is part of a general scene will not change markedly with those considerable changes in the relative amounts of red, green, and blue light which characterize illumination from sunlight versus blue skylight versus grey-day light versus tungsten light versus fluorescent light. The visual phenomena associated with this contradiction were named "color constancy". We will not examine here the interesting explanations of color constancy by Helmholtz and those who have followed him during the last century because, from the point of view of Retinex Theory, as the following experiments show, the paradox does not really exist: the color of an object is not determined by the composition of the light coming from the object.

The first group of experiments is carried out with an arrangement of real fruits and vegetables. A photograph of the scene is shown in Fig. 1.

In a dark room with black walls, three illuminating projectors with clear slides in the slide-holders are directed on the scene. The brightness of the projectors is individually controllable. An interference filter passing 450 nm is placed in front of one projector, 550 nm in front of the second, and 610 nm in front of the third. The measurements in this experiment are all carried out with a telescopic photometer that reads the flux towards the eye from a circular area about 8 mm in diameter on the surface of an object. The readings are in watts per steradian per square meter. All readings are made with light from *only one* projector at a time. Three synchronized camera shutters on the illuminating projectors make possible the comparison of the colors in the fruit and vegetables as seen with continuous illumination and as seen with illumination for only a fraction of a second.

The meter is directed at an orange, the fluxes towards the eye on the three wavebands are set equal to each other, and the orange is observed when the whole scene is illuminated with the combined light from the three illuminators. The orange is orange-colored.

The process is repeated for a green pepper so that the identical radiation comes to the eye — the same three equal wattages as came from the orange. The green pepper is green.

Similarly, when the identical radiation reaches the eye from a yellow banana, the banana is yellow.

When the identical radiaton reaches the eye from a dark red pepper, the pepper is dark and the pepper is red.

Observations in a pulse of light

One of the most important experiments in this group is to compare the scene as viewed in continuous illumination with the scene viewed in a fractional-second pulse. In view of the historic tendency to involve adaptation and eye-motion as causal factors in "color constancy", it is impressive to see that for every new setting, as we turn our attention from object to object, the colors seen in a pulse are correct: whether the scene is viewed in continuous illumination or in a fractional-second pulse, straw is straw-color, rye bread is rye-bread-color, limes are lime-color, green apples are green and red apples are red with the illuminators of the scene so set in each case that the very same ratio of wattages of long, middle and short wave radiation comes to the eye from each of the objects in this list. That is, observed through a spectrophotometer, they would all look alike. Since this experiment establishes that the color of an object is not a function of the composition of the light coming from the object, there is nothing surprising about the failure of the objects to change color when the

composition of the illumination changes: within the framework of Retinex Theory, "color constancy" is the name of a non-existent paradox.

In Fig. 1, a pile of pigment is located in front of the peppers. The pigment is red lead oxide, or minium, referred to in Newton's *Opticks* (1704) in Proposition X, Problem V

> "... Minium reflects the least refrangible or red-making rays most copiously, and thence appears red. ... Every Body reflects the Rays of its own Colour more copiously than the rest, *and from their excess and predominance in the reflected Light has its Colour.*"

The demonstrations in this group of experiments prove that the part of Newton's proposition which we have italicized is incorrect. When the scene is so illuminated that the minium sends to the eye fluxes with the same three equal wattages, the minium continues to look its own brilliant orange-red color—even though there is not at all an "excess and predominance in the reflected light" of "red-making rays."

This group of experiments leads to the first statement in Retinex Theory:

I. The composition of the light from an area in an image does not specify the color of that area.

When the fruit and vegetable scene is illuminated by light of one waveband, we observe that the very light objects stay very light and the very dark objects stay very dark as we alter the brightness of the illumination over nearly the whole range between extinction and maximum illumination. For example, with middle-wave illumination the red pepper will be always almost black and the green pepper always a light grey-green. This situation will be reversed when we change to long wave illumination; that is, the red pepper will always be light and the green pepper always dark.

Based on these observations of the peppers, it is reasonable to venture the hypothesis that an object which always looks light with middle-wave illumination on the scene and always looks dark with long-wave illumination on the scene will look green when the scene is illuminated with both illuminators and *will continue to look green* as we change the relative brightness of the long- and short-wave illuminators and hence the relative fluxes from the object to the eye. Similarly, an object which is dark in middle-wave illumination and light in long-wave illumination

will look red when the scene is illuminated with both illuminators and *will continue to look red* as we change the relative brightness of the two illuminators. Similar relationships can be established for the short-wave illumination.

Since these experiments indicate that color can indeed be predicted on the basis of the three lightnesses at a point, we are led to the question of how to predict *for each waveband separately*, each of the three lightnesses of a point in an image. We would like to know how to compute the number on which lightness is based with the expectation that we will find that a given trio of numbers will always be a single color, a color uniquely corresponding to the given trio.

These expectations lead to the second statement in Retinex Theory:

II. The color of a unit area is determined by a trio of numbers each computed on a single waveband to give the relationship for that waveband between the unit area and the rest of the unit areas in the scene.

This suggests testing the simple hypothesis that (for each of the three wavebands) the lightness of an object and of a unit area on it might be determined by the ratio of the flux from that unit area to the average of the fluxes from all the unit areas in the display. As we contemplate the real still-life arrangement, still illuminating with one waveband, we note that the lightness of any given object does not change no matter where we put it in the whole arrangement, no matter what the background. If we then put a neutral wedge into the slideholder of the illuminating projector so that the illumination transversely across the display is changing, for example, by a factor of ten from one side to the other, we find that the lightness of an object still does not change as we move it from one side of the display to the other. The dark pepper is dark on either side; the light pepper is light on either side. Clearly the simple hypothesis is threatened: the ratio of flux per unit area from the pepper to the average flux per unit area from the whole field must change by a factor of ten as the pepper is moved across the field; yet the *lightness* of the pepper is almost unchanged. Nevertheless, in spite of this significant difficulty with the simple hypothesis, there is great appeal to this method for finding one of the three numbers for three wavebands, numbers which we hope will be our new designators for the color of that unit area.
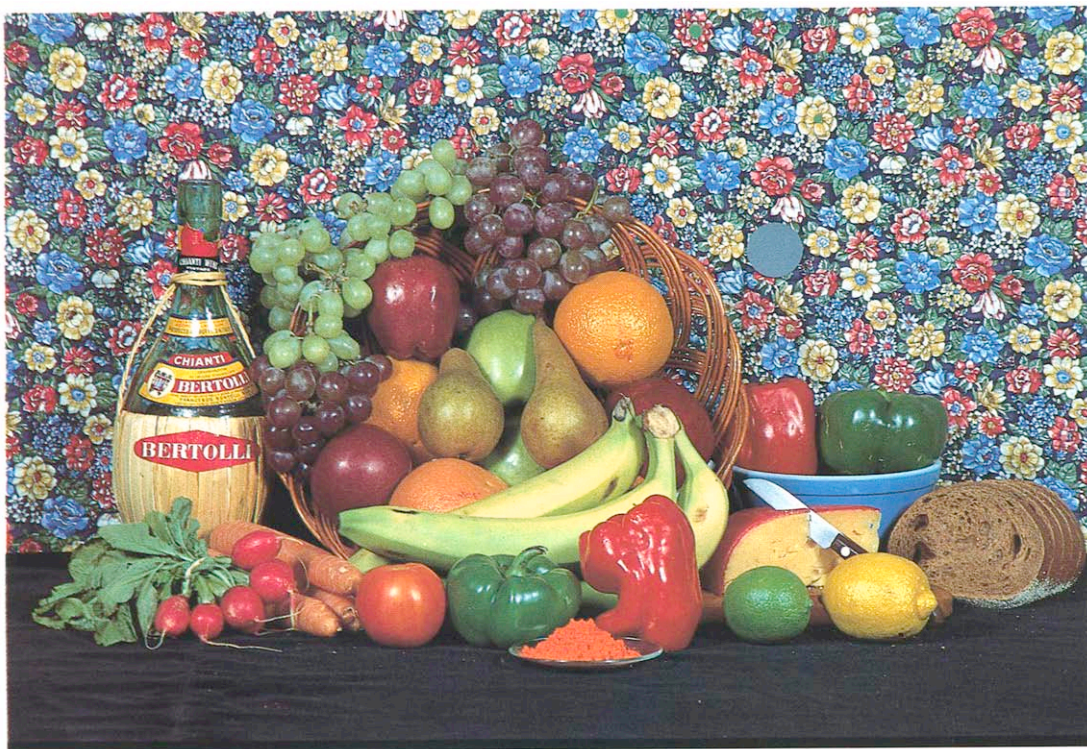
Since this simplest of approaches for the

Fig. 1. Still life with minium.

computation on a single waveband involves the ratio of the unit area flux to the average of all the unit area fluxes in the field of view, let us note that there are two initially equivalent ways of stating the averaging process. The first is to take the ratio of the flux from the one unit area to the average of the flux from all the other unit areas. The second, which may seem somewhat esoteric, is to take the ratio of the unit area flux to each of the other unit area fluxes in the field, separately, and then to average all of these ratios. We might say that in the first case we have established the relationship between the unit area and the average of *all* the other unit areas, whereas in the second case, we have found the average of all the *separate* relationships between the unit area and each of all the other unit areas. These two procedures yield identical results. The second one, however, lends itself to modification that will cause the computation to give the same value for the unit area when the overall illumination is altered to be mottled or uniform or oblique, whereas the first procedure will give variable results with such variation in the regularity of illumination.*

The useful modification of the second procedure comprises noting any chain of unit areas from a distant one to the one we are characterizing and using the chain of ratios from the distant one to give the relationship to the distant one. It is required that when the individual ratios in the chain approximate unity, they act as if they *are* unity. With this procedure, in the presence of irregular illumination, the relationship of the unit area being evaluated to the distant unit area will yield approximately the same number as the ratio of their fluxes in uniform illumination. If we now average a large number of relationships of our terminal unit area to many distant unit areas, we arrive at the number which characterizes that unit area for that waveband.

This second type of relationship is represented in the algorithm below which contains inherently all of the procedures necessary to satisfy the conditions to generate one number which will be a member of the trio of numbers that will be the color.

Thus the third statement in Retinex Theory is the algorithm itself.
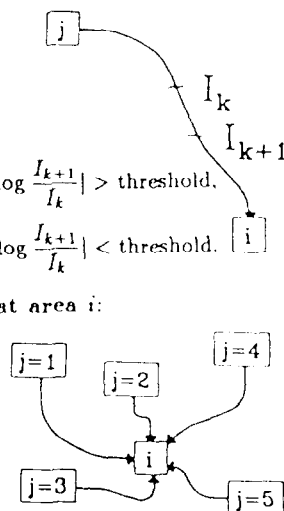
III. The Retinex algorithm:

The relationship of i to j:

$$R^{\Lambda}(i,j) = \sum_k \delta \log \frac{I_{k+1}}{I_k}$$

$$\delta \log \frac{I_{k+1}}{I_k} = \begin{cases} \log \frac{I_{k+1}}{I_k} & \text{if } |\log \frac{I_{k+1}}{I_k}| > \text{threshold,} \\ 0 & \text{if } |\log \frac{I_{k+1}}{I_k}| < \text{threshold.} \end{cases}$$

Average of relationships at area i:

$$\bar{R}^{\Lambda}(i) = \frac{\sum_{j=1}^{N} R^{\Lambda}(i,j)}{N}$$

The relationship of the flux at i to the flux at j, at one of the three principle wavebands, Λ, is given by the first two equations. The third equation gives the number which is one of the trio of numbers we have been looking for. Its value is the average of the relationships between unit area i and each of several hundred unit areas, j's, randomly distributed over the whole field of view. This number, $\bar{R}^{\Lambda}$, is the *designator* for one waveband. There will be three designators, one for each waveband. Together they can be represented as a point in a Retinex three-space (Fig. 2).

When an image of a collage of colored papers (Fig. 3) or of a colored Mondrian (Fig. 4) is formed and the computational technique proposed is pursued, it is found that the three-space is populated in an orderly way. The points on one of the internal diagonals turn out to look black at one end, run through gray, and are white at the other end of the diagonal. There is a domain in which the greens reside, another for the reds, still another for the blues, and yet another for the yellows. It is a triumph of this computational technique that the overall variation in the composition of the illumination in terms of flux at a given wavelength or in terms of relative flux between wavelengths does not disturb the reliability with which a paper which looks red, no matter where it resides in the Mondrian, will have the same three designators as the other papers which look red. It will therefore be part of a family of reds which

appears in one domain of the three-space. Similarly. all the blues or greens or yellows, *wherever resident in the Mondrian* and however haplessly illuminated, will appear in their appropriate domains in the three-space (Fig. 5). It is the computation that leads each paper to have its position in the three-space; the proof of the pudding is that all things that appear in the same region of the three-space are the same color as one another, whatever their history in terms of geography and illumination on the Mondrian may have been.

Measurements and computations

The relative simplicity of the algorithm as a formula for determining the color of a unit area in any situation, natural or artificial, demanded the design of an instrument that could be swept across the surface of a display, whether projected, or planar like the real collages, to compute first the relationship between a unit area and any distant unit area (all on one waveband) and then, with the aid of a compact associated computer, the average of the relationships between the unit area and a large number of randomly selected unit areas throughout the field of view.

An "optical mouse" (Fig. 6) and the associated circuits have been designed by William Wray to carry out the algorithmic computation for a unit area in the Mondrian or the fruit scene. An observer is allowed to see the display only in one waveband while the optical mouse system is swept along the random pathways to give a number for that waveband for the unit area. The observer has not been allowed to see the display in full color. Then the waveband is changed and the process is repeated for the same unit area. Finally, the number for the third waveband is computed. We now have the three designators. The three-space, populated by a large number of previously computed colors such as those listed in Table 1, is projected in three dimensions onto a neighboring screen. The designators are located in the three-space with the aid of a three-dimensional pointer, the design of which takes advantage of the fact that the spectators are already wearing orthogonally polarized lenses to see the three-space stereoscopically. The color where the three-dimensional pointer locates itself, as dictated by the three designators, is noted. Then and only then all three illuminators or projectors are turned on together to see if the color predicted by the designators is indeed the color of the unit area in the display. The predictions for the Mondrian have been strikingly good. The predictions for the fruit scene are good with uniform illumination but for irregular illumination seem to require that the optical mouse treat the ratio of adjacent readings as unity when the ratio is farther from unity than is necessary for the obliquely illuminated collages. This suggests that, for the extreme intricacy of some parts of
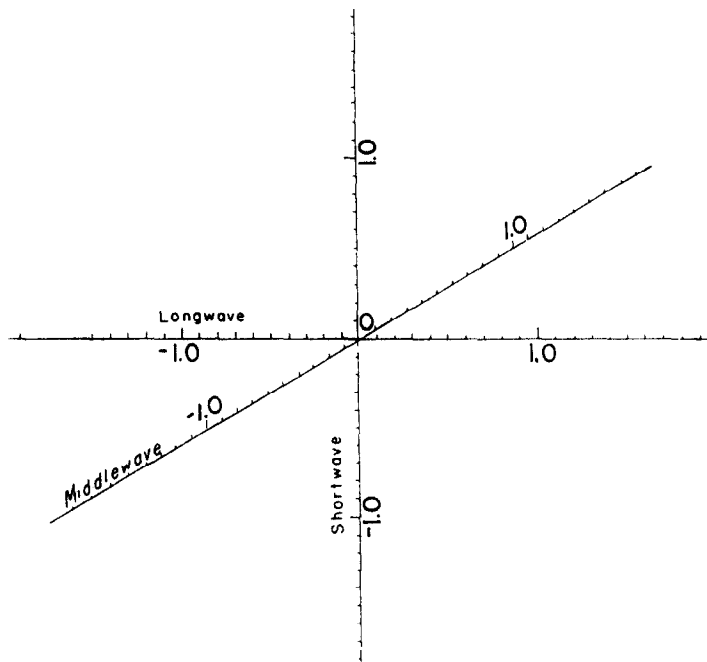

Fig. 2. Axes of the Retinex three-space.

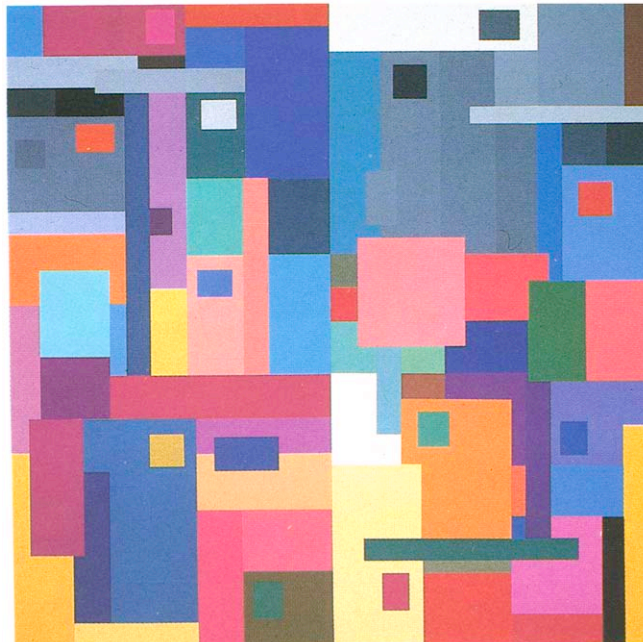Fig. 3. Collage of colored paper objects.
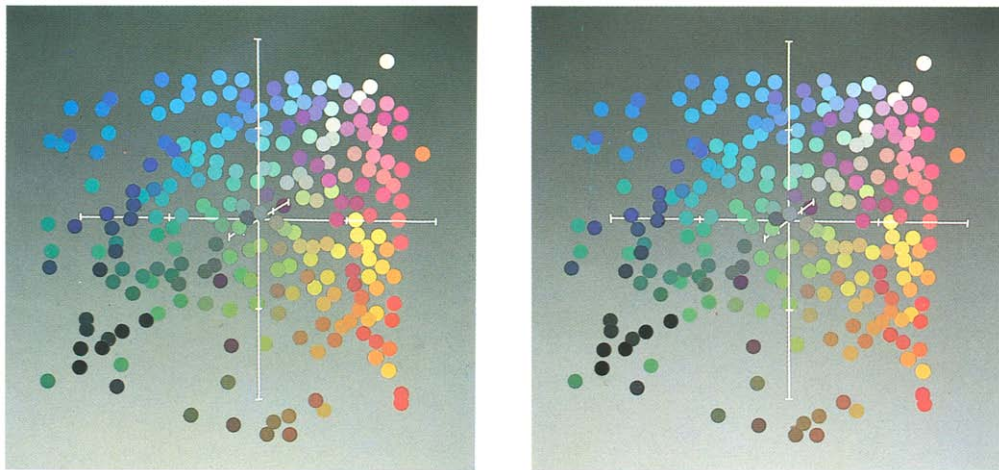

Fig. 4. Mondrian collage.

Fig. 5. (a) Stereoscopic view of the Retinex color three-space. Left eye view on the left. Three designators were determined for each paper according to the algorithm described here in order to locate the colored papers in the three-space. Note that black, greys, and white fall on a diagonal running from the lower left to the upper right. Other colors form colonies generating color domains. (The accuracy of this representation as opposed to the actual papers is limited by the sequence of photo-engraving procedures.)
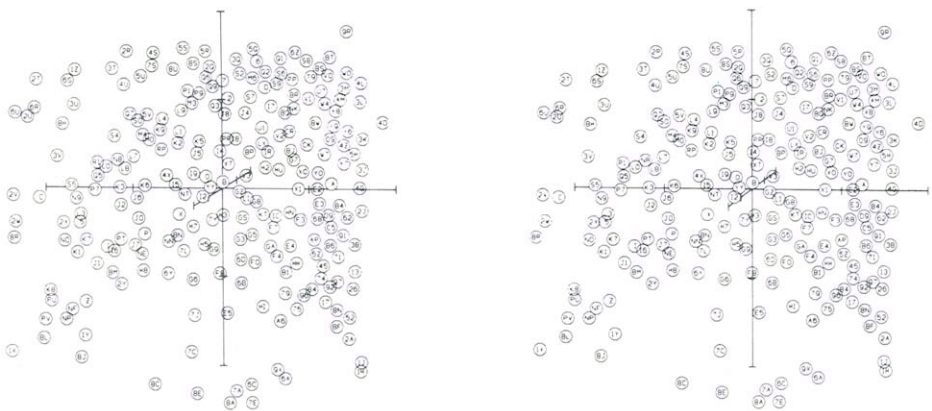

Fig. 5. (b) Stereoscopic key to the color three-space. The colored papers used are listed in Table 1.
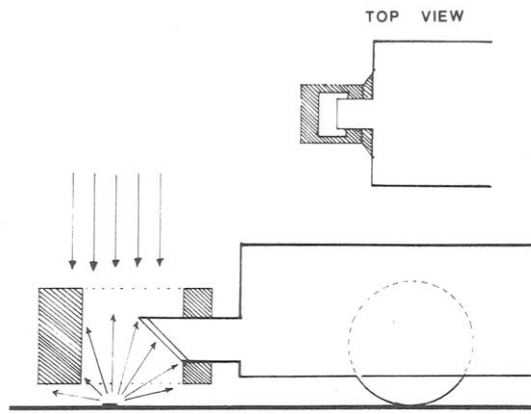
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TOP VIEW

Fig. 6. Diagram of optical mouse. The rectangular aperture in top view (right) rests on the surface of the collage or screen. The projected radiation passes through the aperture to the surface where it is scattered back toward the (sloping) photosensitive element. A large number of rapid readings are integrated and read into the minicomputer every third of an inch of travel.

Table 1. These numbered Coloraid (C) and Munsell (M) papers are those referred to in the Retinex three-space in Fig. 5.

Table of Colors					
Loc.	Color	Loc.	Color	Loc.	Color
0	(M)5G 5/2	84	(M)10YR 6/8	J7	(M)5G 4/6
2	(M)5B 6/4	8A	(C)Y S-3	J8	(M)7.5G 7/6
5	(M)5B 3/4	8C	(C)YO S-3	K1	(M)7.5G 4/6
6	(M)7.5B 7/6	8E	(C)O S-3	K2	(M)10G 6/6
7	(M)7.5B 6/6	8J	(C)R S-3	K3	(M)10G 5/6
A	(M)10RP 5/8	8N	(C)V S-3	K5	(M)10G 6/4
B	(M)N 4.75	8R	(C)B S-3	K6	(M)10G 5/4
C	(C)VBV HUE	8S	(C)BGB S-3	K7	(M)10G 4/4
D	(C)VBV T-1	8U	(C)GBG S-3	K8	(M)10G 3/4
I	(M)5G 4/4	91	(M)2.5Y 8/10	K9	(M)2.5BG 6/8
P	(M)5G 4/2	92	(M)2.5Y 7/10	KT	(M)2.5GY 6/4
X	(M)5G 4/1	96	(M)2.5Y 6/8	L0	(M)2.5BG 5/8
Z	(M)5G 2.5/1	97	(M)5Y 8.5/12	L1	(M)2.5BG 6/6
09	(M)2.5R 6/6	9R	(C)WHITE	L3	(M)2.5BG 4/6
13	(M)5R 5/12	9X	(C)SIENNA	L4	(M)5BG 6/8
17	(M)5R 4/10	A6	(M)5Y 6/8	L7	(M)5BG 5/6
1H	(C)ROR HUE	AM	(M)10RP 7/8	L8	(M)2.5BG 5/6
1J	(C)R HUE	AP	(M)10RP 4/8	L9	(M)7.5BG 6/6
1X	(C)YG HUE	B1	(M)5Y 6/6	LI	(M)5GY 6/4
1Y	(C)YGY HUE	B4	(M)7.5Y 9/10	LX	(M)5GY 6/1
1Z	(C)COBALT	B6	(M)7.5Y 8/10	M0	(M)7.5BG 5/6
26	(M)7.5R 5/12	BF	(M)7.5YR 7/16	M3	(M)10BG 6/6
2A	(C)Y T-1	BH	(M)5B 4/10	M4	(M)10BG 5/6
2J	(C)R T-1	BJ	(M)5P 4/12	M6	(M)10BG 7/4
2Q	(C)BVB T-1	BL	(M)N 1.75	N0	(M)10BG 3/4
2R	(C)B T-1	BM	(M)N 2.75	N8	(M)5B 4/6
2T	(C)BG T-2	BN	(M)N 3.75	N9	(M)5B 3/6
2U	(C)GBG T-1	BP	(M)N 5.75	NE	(M)10GY 4/2
2V	(C)G T-1	BQ	(M)N 6.75	NF	(M)10GY 3/2
2W	(C)GYG T-1	BR	(M)N 7.75	NK	(M)10GY 7/1
2X	(C)YG T-1	BS	(M)N 8.75	NN	(M)10GY 4/1
2Y	(C)YGY T-1	BT	(M)N 9.5	NP	(M)10GY 2.5/1
3B	(C)YOY T-2	BW	(M)2.5R 6/4	NT	(M)2.5G 5/4
3J	(C)R T-2	C5	(M)10Y 8.5/10	P1	(M)10B 5/8
3K	(C)RVR T-2	CK	(M)5R 6/4	P7	(M)10B 3/6
3L	(C)RV T-2	CR	(M)5R 6/2	P9	(M)2.5PB 5/8
3M	(C)VRV T-2	D9	(M)2.5GY 8.5/10	PP	(M)10G 8/2
3Q	(C)BVB T-2	E2	(M)2.5GY 8.5/8	PR	(M)10G 6/2
3T	(C)BG T-3	E3	(M)2.5GY 8/8	PT	(M)10G 4/2
3U	(C)GBG T-2	E4	(M)2.5GY 7/8	PU	(M)10G 3/2
3V	(C)G T-2	E5	(M)2.5GY 6/8	PV	(M)10G 2.5/2
45	(M)10R 5/8	F0	(M)2.5GY 6/6	Q1	(M)2.5PB 7/6
4D	(C)OYO T-3	F3	(M)5GY 8/10	Q2	(M)2.5PB 6/6
4G	(C)RO T-3	F4	(M)5GY 7/10	Q3	(M)2.5PB 5/6
4L	(C)RV T-3	F7	(M)5GY 7/8	Q5	(M)2.5PB 3/6
4S	(C)BGB T-3	F8	(M)5GY 6/8	Q6	(M)5PB 5/10
4U	(C)GBG T-3	G3	(M)5GY 6/6	Q7	(M)5PB 4/10
4X	(C)YG T-3	G5	(M)7.5GY 7/10	Q9	(M)5PB 5/8
4Z	(M)2.5R 6/8	G6	(M)7.5GY 6/10	R1	(M)5PB 5/8
52	(M)2.5YR 6/14	G7	(M)7.5GY 8/8	RP	(M)7.5BG 5/4
5H	(C)ROR T-4	G8	(M)7.5GY 7/8	S0	(M)7.5PB 3/12
5Q	(C)BVB T-4	G9	(M)7.5GY 6/8	S2	(M)7.5PB 5/10
5R	(C)B T-4	GA	(M)7.5YR 5/4	S4	(M)7.5PB 3/10
5S	(C)BGB T-4	GS	(M)10YR 5/2	S5	(M)7.5PB 2/10
5U	(C)GBG T-4	GZ	(M)10YR 5/1	S6	(M)10PB 5/10
5V	(C)G T-4	H2	(M)7.5GY 7/6	S7	(M)10PB 4/10
5Z	(M)2.5R 4/8	H3	(M)7.5GY 6/6	S8	(M)10PB 6/8
62	(M)5YR 7/10	H5	(M)10GY 6/10	S9	(M)10PB 5/8
68	(M)5YR 6/6	H7	(M)10GY 6/8	T9	(M)2.5P 6/8
6A	(C)Y S-1	H8	(M)10GY 5/8	U1	(M)2.5P 4/8
6B	(C)YOY S-1	HH	(M)2.5Y 6/6	U7	(M)5P 5/10
6C	(C)YO S-1	HI	(M)2.5Y 5/6	V0	(M)5P 6/8
6D	(C)OYO S-1	HN	(M)2.5Y 6/4	V1	(M)5P 5/8
6R	(C)B S-1	HU	(M)2.5Y 6/2	V2	(M)5P 4/8
6S	(C)BGB S-1	I2	(M)10GY 6/6	V3	(M)5P 3/8
6U	(C)GBG S-1	I4	(M)2.5G 7/8	W0	(M)7.5P 6/10
6Y	(C)YGY S-1	I5	(M)2.5G 6/8	W4	(M)7.5P 5/8
6Z	(M)5B 8/4	I6	(M)2.5G 5/8	W7	(M)10P 4/10
71	(M)7.5YR 7/10	I7	(M)2.5G 8/6	X0	(M)10P 4/8
74	(M)7.5YR 6/8	I8	(M)2.5G 7/6	X1	(M)10P 3/10
75	(M)7.5YR 5/8	I9	(M)2.5G 6/6	X8	(M)2.5RP 6/8
79	(M)7.5YR 5/6	IC	(M)5Y 6/4	XT	(M)2.5P 4/4
7A	(C)Y S-2	IR	(M)5Y 6/1	Y0	(M)2.5RP 4/8
7C	(C)YO S-2	J0	(M)2.5G 5/6	Y6	(M)5RP 6/10
7E	(C)O S-2	J1	(M)2.5G 4/6	Y7	(M)5RP 5/10
7J	(C)R S-2	J2	(M)5G 5/8	Y9	(M)5RP 6/8
7L	(C)RV S-2	J4	(M)5G 7/6	YY	(M)5P 4/1
7S	(C)BGB S-2	J5	(M)5G 6/6		
7Y	(C)YGY S-2	J6	(M)5G 5/6		

the pictures of real objects, the optical mouse may need a smaller aperture and a closer spacing of adjacent readings than employed in these tests.

Applications of Retinex Theory and the Retinex algorithm

The predictive power of the algorithm is shown not merely by the examples we have discussed but also by many other experiments. Of fundamental importance is the ability of the algorithm to predict darks, lights, shadows, highlights, light sources, and extended areas in surrounds with extremely low reflectivity. As a quite different example, the color of an area located on the Mondrian can be predicted and determined, while the flux to the eye is held constant, by modifying the designators within the area by computed changes in the rest of the field, for each waveband separately. Thus, the area can be changed from a specified white to a specified dark purple—without changing the flux to the eye from the area—by changes in the whole Mondrian computed for each waveband. In an important corollary experiment the area which is being changed from white to purple is surrounded by a very wide black border (produced by blocking the light at the slide-holder of the illuminating projector). The computed changes in the Mondrian, which had altered the designators for the area to make it white or purple, continue to be effective in the presence of the wide black border. While the experiment with this black annulus shows that the com-

putation for the lightness of a point is carried out over extensive areas of the field of view, there is nothing in the experiment to tell us to what extent the computation is carried on in the retina as opposed to the cortex.

The following experiment (Fig. 7), carried out in collaboration with David Hubel and Margaret Livingstone of Harvard Medical School and Hollis Perry and Michael Burns of The Rowland Institute, shows that the cortex is essential to the computation (Land *et al.*, 1983). A Mondrian was modified by covering half of it with velvet. In the middle of and just to the velvet side of the vertical line where the velvet met the rest of the Mondrian, we placed a target area having a fixation point within it. The flux from the target area to the observer's eye was kept constant while the illumination of the Mondrian was altered to create a new set of designators for the target area. For the normal observer, this procedure altered the color of the target area from purple to white. An observer who was normal, *except that for medical purposes his corpus callosum had been cut through*, sat in a chair several feet in front of the target area. For everyone, the two lefthand sides of the retinas are connected to the left sides of our brains, and the two righthand sides of our retinas are connected to the right sides of our brains. Consequently, one half of the field of view is served by one half of the brain and the other half of the field of view by the other half

of the brain. If the computation for the Mondrian were carred out entirely in the retina, the results of the computation for any point in the field of view would be independent of which half of the brain was used to read the results. The subject with the split brain would report "white" when the rest of us saw white and "purple" when the rest of us saw purple. This was not the case.

If the computations are carried out *not* in the retina, but in the cortex, there should be a striking difference between the results for a normal observer and results for a subject with a severed corpus callosum. If we assume that normally the computation extends over the whole field of view, it follows that the computation would extend across both halves of the brain by way of the corpus callosum. When the normal observer regards a Mondrian, half of which is velvet and half of which is the standard Mondrian, the color he sees in the target area depends on the whole computation across the whole field of view. When our subject with the severed corpus callosum regards the same field of view, one half of his brain will be computing for Mondrian plus target and the other half of his brain, for black velvet plus target. When the verbal half of his brain is dealing with *Mondrian* plus target, he reports "purple" or "white", as the case may be, at the same time that we do. When the verbal half of his brain is dealing with *velvet* plus target, then he reports only "white"
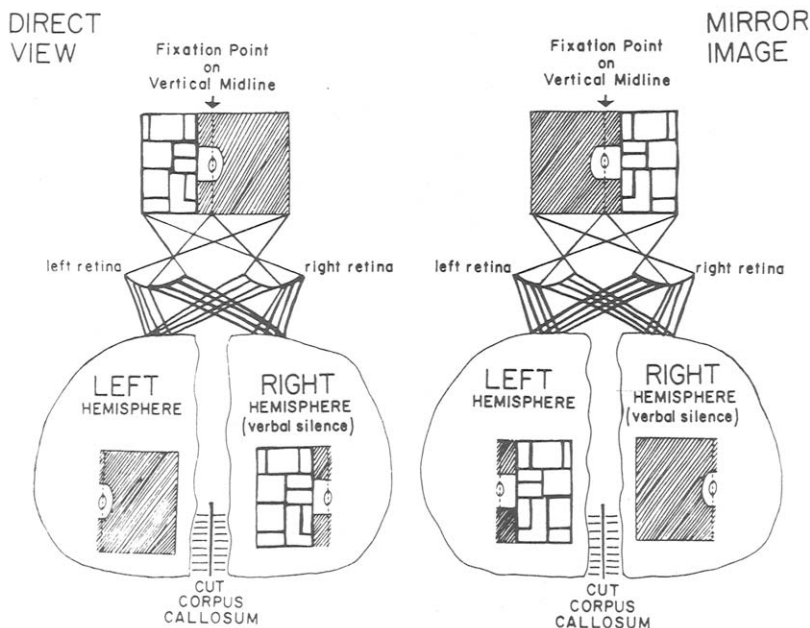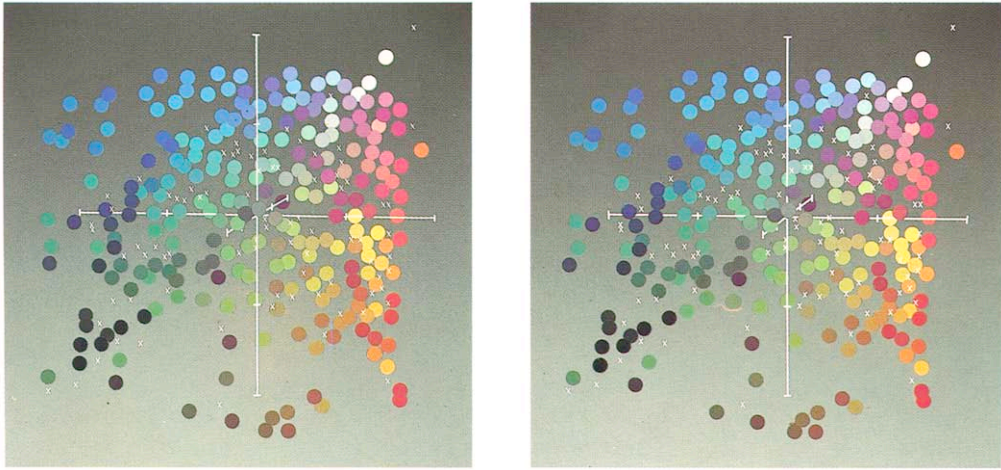

Fig. 7. Diagram of split brain experiment.

Fig. 8. Stereoscopic view of three-space. Left-eye view on the left. X's showing location of colors seen in red and white projection

while we are seeing the changes from white to purple. (We determined this choice of presentations by having him look at the display directly or with a mirror.) Thus, the experiment establishes that the computations (i) are long-range and (ii) require the cortex.

A most dramatic application of Retinex Theory is the quantitative prediction of the colors available in the classical experiment of projecting with red and white light. As you will remember from our two-color projection experiments, we project two records, a middle-wave record and a long-wave record. Let us consider the red-and-white pictures in terms of designators. The middle-wave and short-wave designators are equal to each other because when we project the middle-wave record with white light, we are, in effect, placing on the screen three identical images in superposition

(i) the image carried by the portion of the white light to which the short-wave photosensitive cells respond,

(ii) the image carried by the portion of the white light to which the middle-wave photosensitive cells respond, and

(iii) the image carried by the portion of the white light to which the long-wave photosensitive cells respond.

When we add a second projector equipped with a red filter and project the long-wave record, the energies transmitted by this red projector are combined geographically with the energy from the long-wave portion of the white light that previously had been placed on the screen. From this summation of long-wave energies, a final image, in terms of lightness, is then computed for the long-wave retinex.

Thus, the short-wave and middle-wave retinexes will have equal designators for every place in a projected red-and-white image, and the long-wave retinex will have designators computed from the combined long-wave energies. When we photograph a variegated colored scene and compute the trio of designators for the colors that will appear in a red-and-white picture, all the points in three-space that represent the original scene fall on a plane that cuts through the whole array of colors in the three-space. The x's in the plane are points representing colors in the original scene (Fig. 8). It is obvious that, although all the colors are confined to a plane, that plane passes through enough color domains in the color space to yield a very colorful picture. There are many other

interesting experiments that can be carried out to study this plane. For example, when we move the red filter from the long-wave record to the middle-wave record, all the colors change to quite different new ones falling on the same plane in predictable new positions.

If the two projections of the red and white light are made in combination through a large long-wave transmitting filter and then through a large middle-wave transmitting filter and then through a large short-wave transmitting filter, the optical mouse, reading the image on each of the three wavebands separately, will correctly predict the three designators for locating the correct colors in the three-space.

Ingle (1985) undertook the investigation of the question. "Is the goldfish a Retinex animal?" The answer is dramatically in the affirmative: a goldfish selects a color he has been trained for when he is removed from the training tank and placed in a tank at one end of which is a Mondrian. In a series of tests, the Mondrian is so illuminated as to send to the fish's eye from an area of whatever color he is trained for the same number of photons of the same frequency as came from each of many other colors in the Mondrian under some standard illumination (Fig. 9). Nevertheless, the fish continues to choose the correct color. In recent tests, we used a whole series of Mondrians in each of which the color for which the fish was trained was randomly located amongst all the other colors of the Mondrian. In these tests, the fish swam to the color he had been trained for even though the display was completely new to him and the flux to his eye on the three wave-bands was what would come in some
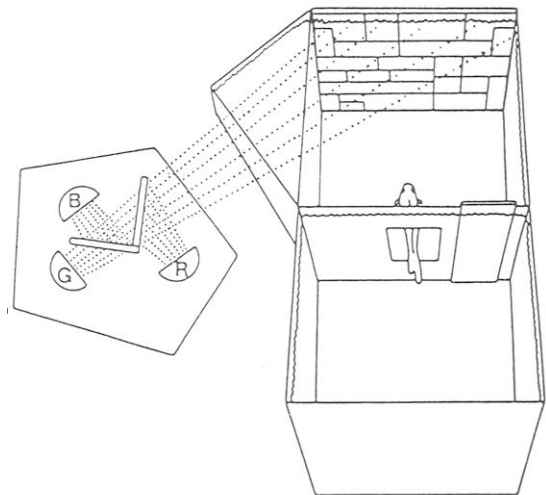

Fig. 9. Goldfish entering Mondrian chamber.

standard white light from each of a series of differently colored papers.

When the goldfish makes his choice from some 50 papers on the basis of the color of the piece of paper for which he was trained, he is ignoring completely the luminance of the piece of paper. Not only is the luminance of the paper to which the fish directs himself arbitrary, so also are the luminances from other areas in the Mondrian. These Retinex experiments show that it is not the luminance on each waveband or the aggregate luminance to which he is conditioned, rather it is the three lightnesses. Thus, if having measured the fluxes in a normal white light, from a piece of dark blue paper, we then cause those same three fluxes to be reflected from a bright yellow paper in the Mondrian, the bright yellow paper, as we have learned, continues to look bright yellow to us and the fish will direct himself to the bright yellow paper for which he was trained. In short, the experiment shows that the permanence in terms of which he was trained did not involve the value of three fluxes or three luminances on three wavebands but rather the value of three lightnesses.

It has been our practice to check all experiments with humans by carrying them out from time to time with a pulse of light—as short as a microsecond, and to reject results which are not apparent in a single pulse. We could not apply this particular check to the experiments with goldfish but we have made every effort to minimize accidental adaptation and adaptation time. In the 1923 experiments by Burkamp [the serious limitations of which are discussed in Ingle's (1985) paper], the adaptation time was chosen in response to one long-held theory that adaptation is a basic constituent of a color constancy mechanism and was extended to several hours.

Daw's early discovery of double-opponent cells in the goldfish opened the way for correlation between the opponent ideas of Hering and physiological investigations. Although most naturally implemented with simple color cells, the Retinex algorithm is compatible with double-opponent cells. A simple linear combination of the long, middle and short inputs to the Retinex procedure can give three "wavebands" which correspond to double-opponent terminology [Fig. 10(a)]. Using these new "wavebands" as input to the Retinex algorithm, the same color space is generated except that the coordinate axes are rotated [Fig. 10(b)]. Thus

$$RG = \frac{1}{\sqrt{2}}L - \frac{1}{\sqrt{2}}M$$

$$YB = \frac{1}{\sqrt{6}}L + \frac{1}{\sqrt{6}}M - \sqrt{\frac{2}{3}}S$$

$$BW = \frac{1}{\sqrt{3}}L + \frac{1}{\sqrt{3}}M + \frac{1}{\sqrt{3}}S$$

$$L = \frac{1}{\sqrt{2}}RG + \frac{1}{\sqrt{6}}YB + \frac{1}{\sqrt{3}}BW$$

$$M = -\frac{1}{\sqrt{2}}RG + \frac{1}{\sqrt{6}}YB + \frac{1}{\sqrt{3}}BW$$

$$S = -\sqrt{\frac{2}{3}}YB + \frac{1}{\sqrt{3}}BW$$

Fig. 10(a). An example of a linear transformation between simple and double-opponent wavebands. The specific numbers are meant to be taken suggestively and not literally.
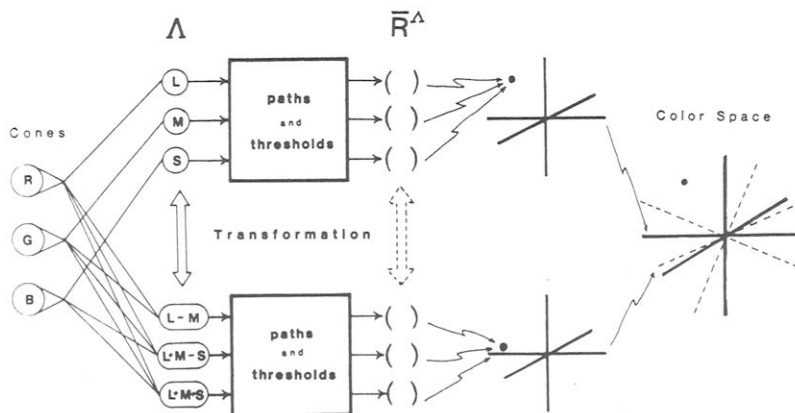

Fig. 10(b). The color space with a coordinate system corresponding to the above transformation.

predictions of Retinex Theory in the two implementations are very similar. They are not identical, however, because the threshold operation in the algorithm is nonlinear. Because of Daw's (1984) important comprehensive paper we will not go further into discussion of the neurophysiological role of Retinex Theory.

Livingstone and Hubel (1983), in their extensive report on, "The anatomy and physiology of a color system in the primate visual cortex", say that their results "suggest that a system involved in the processing of color information, especially color-spatial interactions, runs parallel to and separate from the orientation specific system. Color encoded in three coordinates by the major blob cell types, red-green, yellow-blue, and black-white, can be transformed into the three coordinates, red, green, and blue, of the Retinex algorithm ... "

Zeki (1980) has discovered color-reading cells in the V4 region of the prestriate visual cortex of the rhesus monkey. The image of a Mondrian is formed on the retina of the anesthetized monkey, and Mondrian experiments analogous to those carried out on humans are carried out with the animal. The results for the monkey, as reported by his cortical cells, are strikingly similar to the experiential results as reported by humans.

In summary the three propositions of Retinex Theory are:

I. The composition of the light from an area in an image does not specify the color of that area.

II. The color of a unit area is determined by a trio of numbers each computed on a single waveband to give for that waveband the relationship between the unit area and the rest of the unit areas in the scene.

III. The trio of numbers, the three $\bar{R}^\wedge$'s, as computed by the Retinex algorithm, are the designators for the point in Retinex three-space which is the color of the unit area.

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