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# Assessing Game Balance with AlphaZero: Exploring Alternative Rule Sets in Chess

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Nenad Tomašev\*  
DeepMind

Ulrich Paquet\*  
DeepMind

Demis Hassabis  
DeepMind

Vladimir Kramnik  
World Chess Champion  
2000–2007<sup>§</sup>

## Abstract

It is non-trivial to design engaging and balanced sets of game rules. Modern chess has evolved over centuries, but without a similar recourse to history, the consequences of rule changes to game dynamics are difficult to predict. AlphaZero provides an alternative *in silico* means of game balance assessment. It is a system that can learn near-optimal strategies for any rule set from scratch, without any human supervision, by continually learning from its own experience. In this study we use AlphaZero to creatively explore and design new chess variants. There is growing interest in chess variants like Fischer Random Chess, because of classical chess’s voluminous opening theory, the high percentage of draws in professional play, and the non-negligible number of games that end while both players are still in their home preparation. We compare nine other variants that involve atomic changes to the rules of chess. The changes allow for novel strategic and tactical patterns to emerge, while keeping the games close to the original. By learning near-optimal strategies for each variant with AlphaZero, we determine what games between strong human players might look like if these variants were adopted. Qualitatively, several variants are very dynamic. An analytic comparison show that pieces are valued differently between variants, and that some variants are more decisive than classical chess. Our findings demonstrate the rich possibilities that lie beyond the rules of modern chess.

## 1. Introduction

Rule design is a critical part of game development, and small alterations to game rules can have a large effect on a game’s overall playability and the resulting game dynamics. Fine-tuning and balancing rule sets in games is often a laborious and time-consuming process. Automating the balancing process is an open area of research (Jaffe et al., 2012; de Mesentier Silva et al., 2017), and machine learning and evolutionary methods have recently been used to help game designers balance games more efficiently (Andrade et al., 2005; Leigh et al., 2008; Halim et al., 2014; Grau-Moya et al., 2018). Here we examine the potential of AlphaZero (Silver et al., 2018) to be used as an exploration tool for investigating game balance and game dynamics under different rule sets in board games, taking chess as an example use case.

Popular games often evolve over time and modern-day chess is no exception. The original game of chess is thought to have been conceived in India in the 6th century, from where it initially spread to Persia, then the Muslim world and later to Europe and globally. In medieval times, European chess was still largely based on Shatranj, an early variant originating from the Sasanian Empire that was based on the Indian Chaturaṅga (Murray, 1913). Notably, the queen and the bishop (alfin) moves were much more restricted, and the pieces were not as powerful as those in modern chess. Castling did not exist, but the king’s leap and the queen’s leap existed instead as special first king and queen moves. Apart from checkmate, it was also possible to win by baring the opposite king, leaving the piece isolated with the entirety of its army having been captured. In Shatranj, stalemate was considered a win, whereas these days it is considered a draw. The evolution of chess variants over the centuries can be viewed through the lens of changes in search space complexity and the expected final outcome uncertainty throughout the game, the latter being emphasized by modern rules and seen as important for the overall entertainment value (Cinotti et al., 2007). Modern chess was introduced in the 15th century, and is one of the most popular games to date,

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\*Equal contribution

<sup>§</sup>Classical (2000–2006); FIDE and Undisputed (2006–2007)

captivating the imagination of players around the world.

The interest in further development of chess has not subsided, especially considering a decreasing number of decisive games in professional chess and an increasing reliance on theory and home preparation with chess engines. This trend, coupled with curiosity and desire to tinker with such an inspiring game, has given rise to many variants of chess that have been proposed over the years (Gollon, 1968; Pritchard, 1994; Wikipedia, 2019). These variants involve alterations to the board, the piece placement, or the rules, to offer players “something subtle, sparkling, or amusing which cannot be done in ordinary chess” (Beasley, 1998). Probably the most well-known and popular chess variant is the so-called Chess960 or Fischer Random Chess, where pieces on the first rank are placed in one of 960 random permutations, making theoretical preparation infeasible.

Chess and artificial intelligence are inextricably linked. Turing (1953) asked, “*Could one make a machine to play chess, and to improve its play, game by game, profiting from its experience?*” While computer chess has progressed steadily since the 1950s, the second part of Alan Turing’s question was realised in full only recently. AlphaZero (Silver et al., 2018) demonstrated state-of-the-art results in playing Go, chess, and shogi. It achieved its skill without any human supervision by continuously improving its play by learning from self-play games. In doing so, it showed a unique playing style, later analysed in Game Changer (Sadler & Regan, 2019). This in turn gave rise to new projects like Leela Chess Zero (Lc0, 2018) and improvements in existing chess engines. CrazyAra (Czech et al., 2019) employs a related approach for playing the Crazyhouse chess variant, although it involved pre-training from existing human games. A model-based extension of the original AlphaZero system was shown to generalise to domains like Atari, while maintaining its performance on chess even without an exact environment simulator (Schrittwieser et al., 2019). AlphaZero has also shown promise beyond game environments, as a recent application of the model to global optimisation of quantum dynamics suggests (Dalgaard et al., 2020).

AlphaZero lends itself naturally to the problem of finding appealing and well-balanced rule sets, as no prior game knowledge is needed when training AlphaZero on any particular game. Therefore, we can rapidly explore different rule sets and characterise the arising style of play through quantitative and qualitative comparisons. Here we examine several hypothetical alterations to the rules of chess through the lens of AlphaZero, highlighting variants of the game that could be of potential interest for the chess community. One such variant that we have examined with AlphaZero, No-castling chess, has been publicly championed by Vladimir Kramnik (Kramnik, 2019), and has already had its moment in professional play on 19 December 2019, when Luke Mc-

Shane and Gawain Jones played the first-ever grandmaster No-castling match during the London Chess Classic. This was followed up by the very first No-castling chess tournament in Chennai in January 2020, which resulted in 89% decisive games (Shah, 2020).

## 2. Methods

In this section we motivate nine alterations to the modern chess rules, describe the key components of AlphaZero that are used in the analysis in Section 3, and outline how AlphaZero was trained for Classical chess and each of the nine variants.

### 2.1. Rule Alterations

There are many ways in which the rules of chess could be altered and in this work we limit ourselves to considering atomic changes that keep the game as close as possible to classical chess. In some cases, secondary changes needed to be made to the 50-move rule to avoid potentially infinite games. The idea was to try to preserve the symmetry and the aesthetic appeal of the original game, while hoping to uncover dynamic variants with new opening, middlegame or endgame patterns and a novel body of opening theory. With that in mind, we did not consider any alterations involving changes to the board itself, the number of pieces, or their arrangement. Such changes were outside of the scope of this initial exploration. Rule alterations that we examine are listed in Table 1. The variants in Table 1 are by no means new to this paper, and many are guised under other names: Self-capture is sometimes referred to as “Reform Chess” or “Free Capture Chess”, while Pawn-back is called “Wren’s Game” by Pritchard (1994). None have yet come under intense scrutiny, and the impact of counting stalemate as a win is a lingering open question in the chess community.

Each of the hypothetical rule alterations listed in Table 1 could potentially affect the game either in desired or undesired ways. As an example, consider No-castling chess. One possible outcome of disallowing castling is that it would result in an aggressive playing style and attacking games, given that the kings are more exposed during the game and it takes time to get them to safety. Yet, the inability to easily safeguard one’s own king might make attacking itself a poor choice, due to the counterattacking opportunities that open up for the defending side. In Classical chess, players usually castle prior to launching an attack. Therefore, such a change could alternatively be seen as leading to unenterprising play and a much more restrained approach to the game.

Historically, the only way to assess such ideas would have been for a large number of human players to play the game over a long period of time, until enough experience and understanding has been accumulated. Not only is this a long

Variant	Primary rule change	Secondary rule change
No-castling	Castling is disallowed throughout the game	-
No-castling (10)	Castling is disallowed for the first 10 moves (20 plies)	-
Pawn one square	Pawns can only move by one square	-
Stalemate=win	Forcing stalemate is a win rather than a draw	-
Torpedo	Pawns can move by 1 or 2 squares anywhere on the board. En passant can consequently happen anywhere on the board.	-
Semi-torpedo	Pawns can move by two square both from the 2nd and the 3rd rank	-
Pawn-back	Pawns can move backwards by one square, but only back to the 2nd/7th rank for White/Black	Pawn moves do not count towards the 50 move rule
Pawn-sideways	Pawns can also move laterally by one square. Captures are unchanged, diagonally upwards	Sideway pawn moves do not count towards the 50 move rule
Self-capture	It is possible to capture one's own pieces	-

Table 1. A list of considered alterations to the rules of chess.

process, but it also requires the support of a large number of players to begin with. With AlphaZero, we can automate this process and simulate the equivalent of decades of human play within a day, allowing us to test these hypotheses *in silico* and observe the emerging patterns and theory for each of the considered variations of the game.

Figure 1 illustrates each of the variants with an example position.

## 2.2. Key components of AlphaZero

AlphaZero is an adaptive learning system that improves through many rounds of self-play (Silver et al., 2018). It consists of a deep neural network  $f_\theta$  with weights  $\theta$  that compute

$$(\mathbf{p}, v) = f_\theta(s) \quad (1)$$

for a given position or state  $s$ . The network outputs a vector of move probabilities  $\mathbf{p}$  with elements  $p(s'|s)$  as prior probabilities for considering each move and hence each next state  $s'$ .<sup>1</sup> If we denote game outcome numerically by +1, for a win, 0 for a draw and -1 for a loss, the network addi-

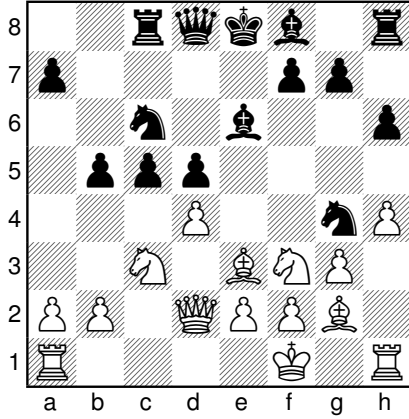
tionally outputs a scalar value  $v \in (-1, 1)$  which estimates the expected outcome of the game from position  $s$ .

The two predictions in (1) are used in Monte Carlo tree search (MCTS) to refine the assessment of a board position. The prior network  $\mathbf{p}$  assigns weights to candidate moves at a “first glance” of the board, yielding an order in which moves are searched with MCTS. The output  $v$  can be viewed as a neural network evaluation function for position  $s$ . The statistical estimates of the game outcomes after each move are refined through MCTS, which runs repeated simulations of how the game might unfold up to a certain ply depth. In each MCTS simulation,  $f_\theta$  is recursively applied to a sequence of positions (or nodes) up to a certain ply depth if they have not been processed in an earlier simulation. At maximum ply depth, the position is evaluated with (1), and that evaluation is “backed up” to the root, for each node adjusting its “action selection rule” to alter which moves will be selected and expanded in the next MCTS simulation. After a number of such MCTS simulations, the root move that was visited (or expanded) most is played.

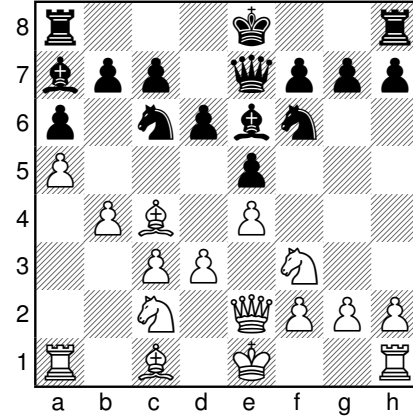
## 2.3. Training and evaluation

We trained AlphaZero from scratch for each of the rule alterations in Table 1, with the same set of model hyperpa-

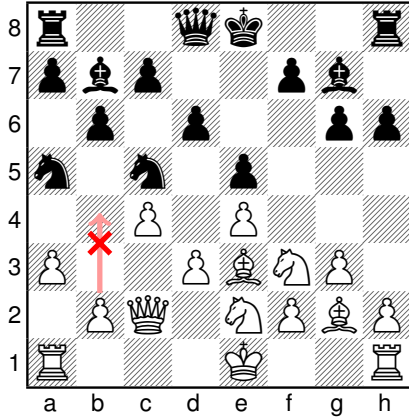
<sup>1</sup>We’ve suppressed notation somewhat; the probabilities are technically over actions or moves  $a$  in state  $s$ , but as each action  $a$  deterministically leads to a separate next position  $s'$ , we use the concise  $p(s'|s)$  in this paper.



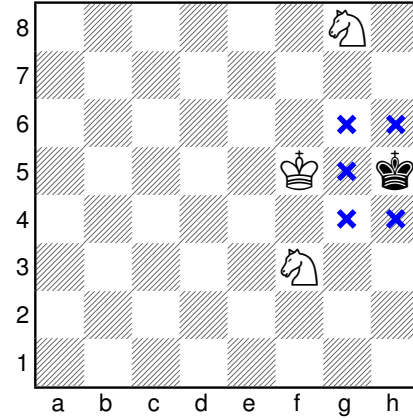
(a) An example from No-castling chess: This is a typical position where both kings haven't found immediate safety and remain exposed into the middlegame.



(b) An example from No-castling(10) chess: The play tends to be slower and more strategic, to allow for later castling. Here, on the 11th move, Black castles at the very first opportunity and White castles immediately after as well.

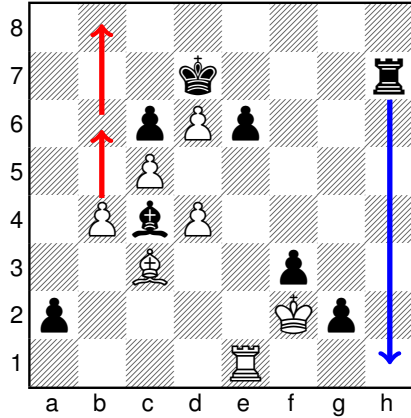


(c) An example from Pawn-one-square chess: Black just moved the knight to a5. In Classical chess this would seem counter-intuitive due to the potential of playing the pawn to b4, forking the knights. Here, however, the pawn cannot move to that square in a single move, justifying the manoeuvre.

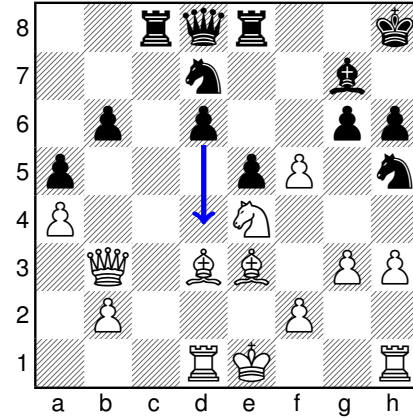


(d) An example from Stalemate=win chess: An endgame position that would have been a draw in Classical chess is now a win instead.

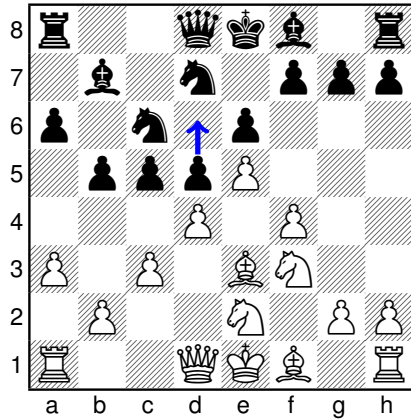
Figure 1. Examples of new strategic and tactical themes that arise in the explored chess variants. Figure 1e continues on the following page.



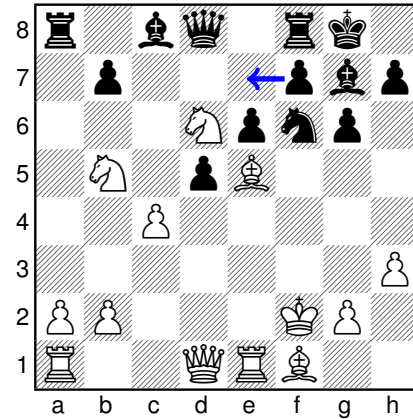
(e) An example from Torpedo chess: White needs to generate rapid counterplay, and does so with a torpedo move: b4-b6. Black responds with Rh1, to which White promotes to a queen with yet another torpedo move, b6-b8=Q.



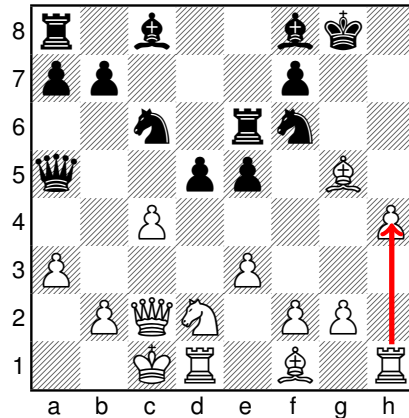
(f) An example from Semi-torpedo chess: The ability to rapidly advance pawns from the 3rd/6th rank enables Black the following energetic option: d6-d4, resulting in a forced tactical sequence. See Game AZ-19 in Appendix B.6 for details.



(g) An example from Pawn-back chess: Here, Black uses this possibility to challenge White's central pawns, while opening up the diagonal for the b7 bishop, by a pawn-back move d5-d6.



(h) An example from Pawn-sideways chess: After sacrificing the knight on f2 the previous move, Black utilises a sideways pawn move f7-e7 for tactical purposes, opening the f-file towards the White king, while attacking the knight on d6.



(i) An example from Self-capture chess: a self-capture move Rxh4 generates threats against the Black king.

Figure 1. (Continued from previous page.) Examples of new strategic and tactical themes that arise in the explored chess variants.



rameters. The models were trained for 1 million training steps, with a batch size of 4096 and allowing for an average 0.12 samples per position from self-play games. In order to encourage exploration during training, a small amount of noise was injected in the prior move probabilities (1) before search, sampled from a Dirichlet  $\text{Dir}(0.3)$  distribution, followed by a renormalization step (Silver et al., 2018). Further diversity was promoted by stochastic move selection in the first 30 plies of each of the training self-play games, by selecting the final moves proportionally to the softmax of the MCTS visit counts. The remaining game moves from ply 31 onwards were selected as top moves based on MCTS. Training self-play games were generated using 800 MCTS simulations per move.

The absence of baselines makes it hard to formally assess the strength of each model, which is why it was important to couple the quantitative analysis and metrics observed at training and test time with a qualitative assessment in collaboration with Vladimir Kramnik, a renowned chess grandmaster and former world chess champion. As the rule changes that are considered in this study are mostly minor in practical terms, it is reasonable to assume that the trained models are of similar strength, although it is equally reasonable to expect that some of them could be further fine-tuned to account for the differences in game length and the average number of legal moves that need to be considered at each position. Given the nature of the study, the high level of observed play in trained models, and the number of rule alterations considered, we decided not to pursue such a potentially laborious process, as it would not alter any of the high-level conclusions that we present and discuss.

### 3. Quantitative assessment

There are marked differences between the styles of chess that arises from each of the rule alterations. Aesthetically, each variant has its own appeal, and we highlight them further in Section 4. Here we provide a quantitative comparison between variants, to complement the qualitative observations. Using a large quantity of self-play games, we infer the expected draw rate and first-move advantage for each variant, expressed as the expected score for White (Section 3.2). We then illustrate how the same opening can lead to vastly different outcomes under different chess variants in Section 3.3, and that these opening-specific differences can differ from the aggregate differences across all openings. An analysis of the utilisation of the newly introduced options made possible by the new rule alterations in Section 3.4 shows that the non-classical moves are used in a large percentage of games, often multiple times per game, in each of the variants. This suggests that the new options are indeed useful, and contribute to the game. We estimate the diversity of opening play by looking at the opening trees

which we construct from AlphaZero’s network priors (1) for the first couple of moves and show that the breadth of opening possibilities in each of these chess variants seems to be inversely related to their relative decisiveness (Section 3.5). Sections 3.6 and 3.7 highlight the difference in opening play according to the prior distributions of the variants. Rule adjustments, especially those affecting piece mobility, are also expected to affect the relative material value of the pieces. Finally, Section 3.8 provides approximations for piece values in each of the variants, computed from a sample of 10,000 fast-play AlphaZero games.

#### 3.1. Self-play games

For each chess variant, we generated a diverse set of  $N = 10,000$  AlphaZero self-play games at 1 second per move, and  $N = 1,000$  games at 1 minute per move. The outcomes of the fast self-play games are presented in Figure 2a; the longer games follow in Figure 2b. As AlphaZero is approximately deterministic given the same MCTS depth and number of rollouts, we promote diversity in games by sampling the first 20 plies in each game proportional to the softmax of the MCTS visit counts, followed by playing the top moves for the rest of the game.

In addition to that, we generated a set of  $N = 1,000$  fast-play games from fixed starting positions arising from the Dutch Defence, Chigorin Defence, Alekhine Defence and King’s Gambit for each of the variants, as further discussed in Section 3.3.

The two sets of diverse self-play games are used in Section 3.2 to compare the decisiveness of each variant, in Section 3.4 to analyse how many special moves are used, and in Section 3.8 to estimate piece values across variants.

A selection of these games is presented in Appendix B.

#### 3.2. Expected scores and draw rates

It is widely hypothesised that classical chess is theoretically drawn; that the odds  $\pi = (\pi_{\text{win}}, \pi_{\text{draw}}, \pi_{\text{lose}})$  of white winning, drawing and losing are  $(0, 1, 0)$  at optimal play. We determine how favourable for white or how “drawish” different variants are by estimating the expected scores and draw rates at non-optimal play under the *same* conditions. We keep the conditions that chess variants are played against themselves with AlphaZero fixed, like the move selection criteria or Monte Carlo Tree Search (MCTS) evaluation time.

The overall decisiveness in the generated game sets depends on the time controls involved. We see in Figures 2a and 2b that across all variations the percentage of drawn games increases with longer thinking times, and longer thinking times also affect the expected score for White, as shown in Table 2. This suggests that the starting position might be

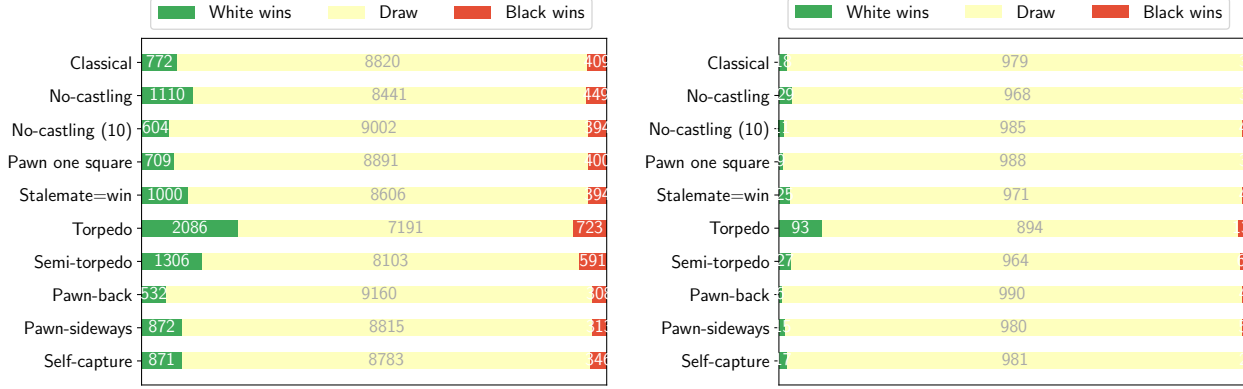


Figure 2. AlphaZero self-play game outcomes under different time controls. As moves are determined in a deterministic fashion given the same conditions, diversity was enforced by sampling the first 20 plies in each game proportional to their MCTS visit counts. Across all variations the percentage of drawn games increases with longer thinking times. This seems to suggest that the starting position might be theoretically drawn in these chess variants, like in Classical chess, and that some of the variants are simply harder to play, involving more calculation and richer patterns.

Variant	Training	1sec	1min
Classical	54.1%	51.8%	50.8%
No castling	55.7%	53.3%	51.3%
No castling (10)	52.5%	51.0%	50.4%
Pawn one square	53.5%	51.6%	50.3%
Stalemate=win	54.9%	53.0%	51.1%
Torpedo	57.0%	56.8%	54.0%
Semi-torpedo	54.7%	53.6%	50.9%
Pawn-back	53.0%	51.1%	50.1%
Pawn-sideways	54.8%	52.8%	50.5%
Self-capture	54.2%	52.6%	50.8%

Table 2. Empirical score for White under different game conditions, for each chess variant: self-play games at the end of model training, 1 second per move games, and 1 minute per move games. Diversity in 1 second per move games and 1 minute per move games was enforced by sampling the first 20 plies in each game proportional to their MCTS visit counts.

theoretically drawn in these chess variants, like in Classical chess, and that some of the variants are simply harder to play, involving more calculation and richer patterns. We hypothesise that the relative differences in AlphaZero’s win rates might translate to differences in human play, although this hypothesis would need to be practically validated in the future. Yet, in absence of any existing human games, we can use these results as a preliminary guess of what those results might be, assuming that what is difficult to calculate for AlphaZero may be difficult for human players as well.

### 3.2.1. INFERENCE FOR GAME ODDS

To compare variants, we first infer the odds of their outcomes under set playing conditions. For a given variant, let the game outcomes  $\mathcal{G}$  be  $n_{\text{win}}$  wins and  $n_{\text{lose}}$  losses for white, and  $n_{\text{draw}} = N - n_{\text{win}} - n_{\text{lose}}$  draws. If we assume a uniform Dirichlet prior on  $\pi$  and multinomial likelihood for winning, drawing or losing, the posterior distribution is Dirichlet,

$$p(\pi|\mathcal{G}) = \text{Dir}(n_{\text{win}} + 1, n_{\text{draw}} + 1, n_{\text{lose}} + 1). \quad (2)$$

### 3.2.2. DRAW RATES

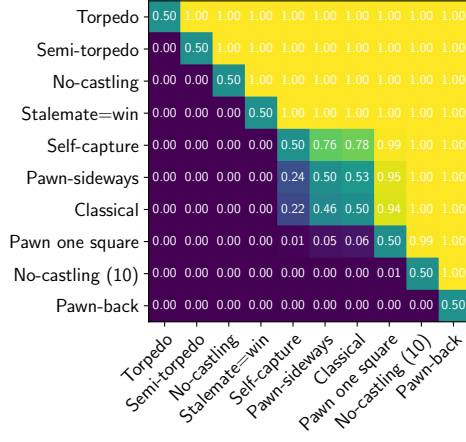
To compare the decisiveness of chess variants, we infer the probability that variant A has a lower draw rate than variant B, given the games played  $\mathcal{G}^A$  and  $\mathcal{G}^B$  under the same conditions:<sup>2</sup>

$$p(\pi_{\text{draw}}^A < \pi_{\text{draw}}^B) = \iint \mathbb{I}[\pi_{\text{draw}}^A < \pi_{\text{draw}}^B] p(\pi^A|\mathcal{G}^A) p(\pi^B|\mathcal{G}^B) d\pi^A d\pi^B. \quad (3)$$

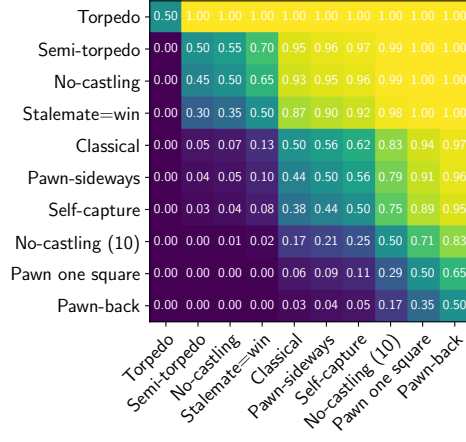
The integral is not available in closed form; we evaluate it with a Monte Carlo estimate by drawing pairs of samples from  $p(\pi^A|\mathcal{G}^A)$  and  $p(\pi^B|\mathcal{G}^B)$  – using (2) – and computing the fraction of times that samples satisfy  $\pi_{\text{draw}}^A < \pi_{\text{draw}}^B$ .

Figure 3a provides an indication of the relative *decisiveness* of variants, when played by AlphaZero at approximately 1 second per move, and Figure 3b provides the comparison at

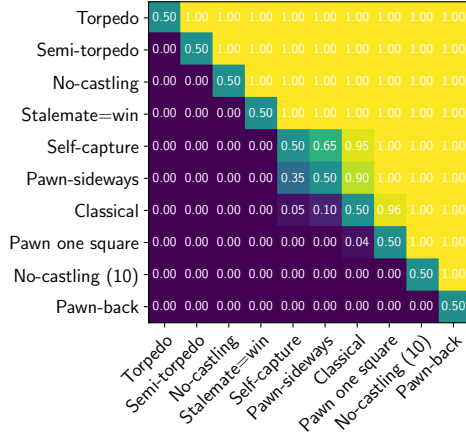
<sup>2</sup>This approach follows MacKay (2003, Chapter 37.1).



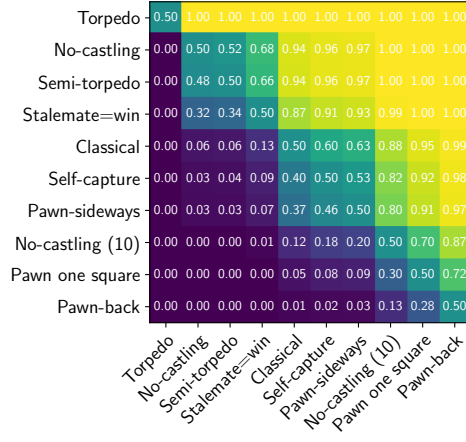
(a) A draw rate comparison  $p(\pi_{\text{draw}}^{\text{row}} < \pi_{\text{draw}}^{\text{column}})$  at approximately 1 seconds per move, on 10,000 AlphaZero games per variation.



(b) A draw rate comparison  $p(\pi_{\text{draw}}^{\text{row}} < \pi_{\text{draw}}^{\text{column}})$  at approximately 1 minute per move, on 1,000 AlphaZero games per variation.



(c) A comparison of expected scores  $p(e^{\text{row}} > e^{\text{column}})$  at 1 second per move, on 10,000 games per variation.



(d) A comparison of expected scores  $p(e^{\text{row}} > e^{\text{column}})$  at 1 minute per move, on 1,000 games per variation.

Figure 3. A comparison of draw rates. The most decisive chess variants under both time controls are Torpedo, Semi-torpedo, No-castling and Stalemate=win. These four variants also give White the largest first-move advantage.

1 minute per move. Under both time controls, the most decisive chess variants we explored are Torpedo, Semi-torpedo, No-castling and Stalemate=win. Torpedo and Semi-torpedo have increased pawn mobility, allowing for faster, more dynamic play, leading to more decisive outcomes. There are also more moves to consider at each juncture. No-castling chess makes it harder to evacuate the king to safety, similarly affecting the draw rate. Finally, Stalemate=win removes one important drawing resource for the weaker side, converting a number of important endgame positions from being drawn to being winning for the stronger side. Under the same conditions of play, the slower Pawn one square chess variant and Pawn-back chess variant are the most drawish. Pawn-back chess incorporates additional defensive resources, and the ability to go back to protect the weak squares seems to

be more important for defending worse positions than it is for attacking – given that attacking tends to involve moving forward on the board.

### 3.2.3. EXPECTED SCORES

The decisiveness of a chess variant under imperfect play does not necessarily have to correspond to the first-move advantage. In classical chess, White scores higher on average. Top-level chess players tend to press for an advantage with the White pieces and defend with the Black pieces, looking for opportunities to counter-attack. The reason is the first-move advantage; it is an initiative that, with good play, persists throughout the opening phase of the game. This not a universal property that would hold in any game ,



as playing the first move might also disadvantage a player in some types of games. It is therefore important to estimate the effect of the rule changes on the first-move advantage in each chess variant, expressed as the expected score for White.

The expected score for White is defined as:

$$e = \pi_{\text{win}} + \frac{1}{2}\pi_{\text{draw}} \quad (4)$$

for a particular set of conditions like time controls, the move selection criteria and the AlphaZero model playing the game. Given the game outcomes  $\mathcal{G}^A$  and  $\mathcal{G}^B$  of variants A and B, the probability of white having a higher first-move advantage in variant A is

$$p(e^A > e^B) = \iint \mathbb{I} \left[ \pi_{\text{win}}^A + \frac{1}{2}\pi_{\text{draw}}^A > \pi_{\text{win}}^B + \frac{1}{2}\pi_{\text{draw}}^B \right] p(\pi^A | \mathcal{G}^A) p(\pi^B | \mathcal{G}^B) d\pi^A d\pi^B, \quad (5)$$

which we again evaluate with a Monte Carlo estimate.

White’s first-move advantage with approximately 1 second and 1 minute per move in AlphaZero games is compared in Figures 3c and 3d respectively. The relative ordering of variations follows the ranking in general decisiveness, suggesting that the new chess variants that are more decisive in AlphaZero games are also more advantageous for White, possibly due to an increase in dynamic attacking options.

### 3.3. Differences in specific openings

To further illustrate how different alterations of the rule set would require players to adjust their opening repertoires, we provide a comparison of how favourable specific opening positions are for the first player, for each of the variants previously introduced in Table 1. Figure 4 shows the win, draw, and loss percentages for White under 1 second per move, for the Dutch Defence, Chigorin Defence, Alekhine Defence and King’s Gambit, on a sample of 1000 self-play games. The only variant we did not include in these comparisons is Pawn one square, as the lines used in the comparisons involve the double-pawn-moves which are not legal in that variant.

These four opening systems are not considered to be the most principled ways of playing Classical chess. They are therefore particularly interesting for establishing if a certain rule change pushes the evaluation of each of these openings from “slightly inferior” to “unsound” or “unplayable”.

In case of Dutch Defence in Figure 4a, we see that it is more favourable for White in Torpedo and Stalemate=win chess than in Classical chess. This is in line with the overall increase in decisiveness in those variations, but is not more favourable in case of No-castling chess, despite No-castling chess otherwise being more decisive than Classical

chess. We can already see in this one example that the overall differences in decisiveness between variants are not equally distributed across all possible opening lines, and that the evaluation of the difference in the expected score will depend on the style of opening play.

In case of Chigorin Defence in Figure 4b, Pawn-sideways chess seems to be refuting the variation, based on our initial findings. In a smaller sample of games played at 1 minute per move, we have seen a 100% score being achieved by AlphaZero in this line of Pawn-sideways chess, though these are still preliminary conclusions. To the human eye the line does not appear to be very forcing; it is not a short tactical refutation, but results in a fairly long-term strategic advantage, which AlphaZero converts into a win. This line also seems to be harder to defend in No-castling chess and Torpedo, but not in Stalemate=win chess, unlike the Dutch Defence.

The Alekhine Defence in Figure 4c seems to be less sound in all of the variations considered, compared to Classical chess, with a major increase in decisiveness in Pawn-sideways chess, No-castling chess and Torpedo chess.

Finally, King’s Gambit in Figure 4d seems to give a substantial advantage to Black across all chess variants considered, although in No-castling chess and Torpedo chess, White has somewhat better winning chances than in Classical chess. Pawn-sideways chess, again, seems to be the worst of the variants to consider playing this line in. Still, in our preliminary experiments with games at longer thinking times, most games would still ultimately end in a draw. This suggests that it is still likely a playable opening, when played at a very high level with deep calculation.

### 3.4. Utilisation of special moves

Several of the variants that are explored in this study involve additional move options that are not permitted under the rules of Classical chess, like additional pawn moves and self-captures. It is not clear from the outset how often these newly introduced moves would be utilised in each of the variants. Will they make a difference? We use the set of 10,000 games at 1 second per move from Section 3.1 to quantify how often the additional moves are played.

#### 3.4.1. TORPEDO MOVES

In Semi-torpedo chess, 88% of all games have at least one torpedo move, and 1.20% of all moves played in the game are torpedo moves. In Torpedo chess, these percentages are even higher: 94% of games utilise torpedo moves and these represent 2.40% of all moves played in the game. Furthermore, 28.7% of games featured pawn promotions with a torpedo move, highlighting the speed at which a passed pawn can be promoted to a queen.

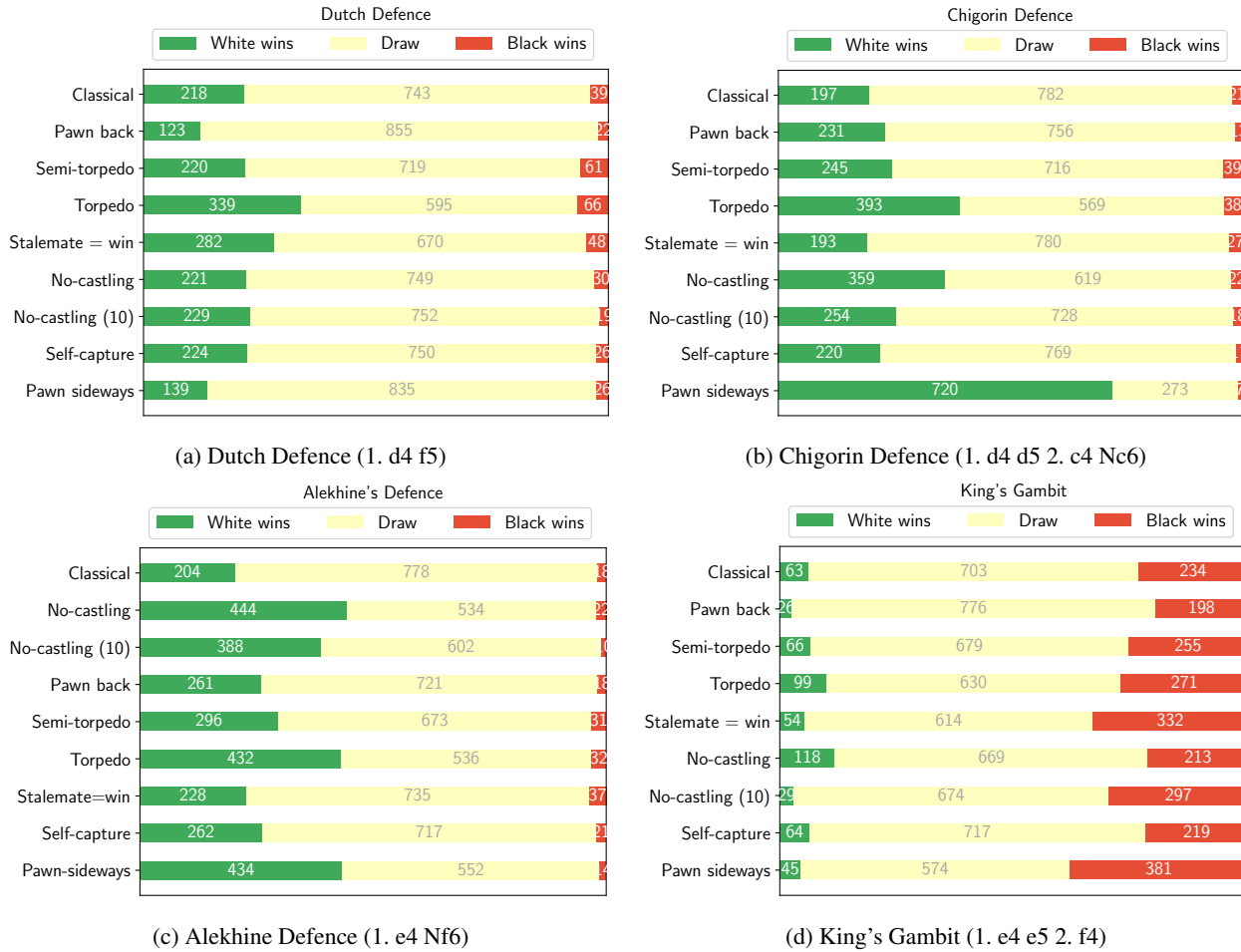


Figure 4. The same opening position can give vastly different degrees of advantage to either play, depending on the variant under consideration, as shown here by the number of games won, drawn and lost for AlphaZero as White when playing at approximately 1 second per move, for a sample of 1000 games, while always playing the best move without any additional noise being added for play diversity. The stochasticity captured in the results stems from the asynchronous execution of MCTS threads during search. Therefore, these results indicate how favorable the 'main line' continuation is, for each of the following openings: the Dutch Defence, the Chigorin Defence, Alekhine Defence and the King's Gambit.

### 3.4.2. BACKWARDS AND LATERAL PAWN MOVES

In Pawn-back chess, 96.3% of the games involved a backwards pawn move. In Pawn-sideways chess, 99.6% of games features lateral pawn moves, and a total of 11.4% of all moves in the game were lateral pawn moves, as the reconfiguring of pawn formations was common in AlphaZero's playing style in this chess variant.

### 3.4.3. SELF-CAPTURES

In Self-capture chess, 52.5% of games featured self-capture moves, which represented 0.7% of all moves played. The most common self-captures involved sacrificing a pawn (86.9%), although sacrificing a bishop (5.3%) or a knight (4.5%) was not uncommon. Rook self-capture sacrifices were rare (2.3%) and occasionally AlphaZero would self-

capture a queen (1%), though these were mostly unnecessary captures in winning positions, given that AlphaZero was not incentivised to win in the fastest possible way.

### 3.4.4. WINNING THROUGH STALEMATE

In Stalemate=win chess the percentage of all decisive games that were won by stalemate rather than mate in AlphaZero games was 37.2%, though this number is inflated due to the fact that AlphaZero would often stylistically stalemate rather than mate the opponent in positions where both are possible.

The percentages listed above suggest that the rule changes featured in these chess variants did indeed leave a trace on how the game is being played, and that they are useful additional options that can potentially change the game dy-

namics. Yet, it is important to note that the resulting games are still of approximately similar length, as shown in Figure 8 in Appendix A, with some changes in the empirical duration of decisive games. This means that playing a game in one of these chess variants is unlikely to prolong or shorten the game by a large amount, meaning that classical time controls should still be appropriate. Note that the numbers in Figure 8 that correspond to the number of plies in AlphaZero games are an upper bound on game length, since AlphaZero was trained without discounting, and would therefore not play the fastest winning sequence in its decisive games.

### 3.5. Diversity

For a game to be appealing, it has to be rich enough in options that these options do not get quickly exhausted, as play would then become repetitive. We use the average information content (entropy) of the first  $T = 20$  plies of play from each variant’s prior as a surrogate diversity measure. The trained AlphaZero policy priors model the move probabilities of the positions in self-play training data, and reflects the statistics at which opening lines appear there. An entropy of *zero* corresponds to there being one and only one forcing sequence of moves to be playable for White and Black, all other moves leading to substantially worse positions for each side. A higher entropy implies a wider and more balanced opening tree of variations, leading to a more diverse set of middlegame positions. The intuition that there would be many more plausible opening lines in slower variants like Pawn one square, holds true experimentally. In simulation, more decisive variants like Torpedo chess typically have fewer plausibly playable opening lines.

The decomposition of the entropy as a statistical expectation can help identify whether there exist defensive lines that equalise the game in an almost forcing way. In Classical chess, one such defensive resource is the Berlin Defence in the Ruy Lopez, taking the sting out of 1. e4. We show in Section 3.5.2 that AlphaZero, when trained on Classical chess, expresses a strong preference for the Berlin Defence, similarly to the human consensus on the solidity of the Berlin endgame. Without the option to castle, this particular line disappears in No-castling chess.

#### 3.5.1. AVERAGE INFORMATION CONTENT

The prior network from (1) defines the probability of *a priori* considering move  $a_t$  in state  $s_t$ , but as move  $a_t$  leads to state  $s_{t+1}$  deterministically, we shall abbreviate the prior with  $p(s_{t+1}|s_t)$ .

The prior is a weighted list of *possible moves* for state  $s_t$  that are utilised in AlphaZero’s MCTS search. The weights specify how *plausible* each move is before MCTS calculation; they specify candidates for consideration. In information-

Variant	Entropy	Equivalent 20-ply games
No-castling	27.65	$1.02 \times 10^{12}$
Torpedo	27.89	$1.30 \times 10^{12}$
Self-capture	27.94	$1.36 \times 10^{12}$
No-castling (10)	27.97	$1.40 \times 10^{12}$
Classical	28.58	$2.58 \times 10^{12}$
Stalemate=win	29.01	$3.97 \times 10^{12}$
Semi-torpedo	31.63	$5.45 \times 10^{13}$
Pawn-back	32.30	$1.07 \times 10^{14}$
Pawn-sideways	34.16	$6.85 \times 10^{14}$
Pawn one square	38.95	$8.24 \times 10^{16}$
Uniform random	64.96	$1.63 \times 10^{28}$

Table 3. The average information content in nats in the first 20 plies of the AlphaZero prior for each chess variant. The uniform random baseline assumes an equal probability for each move in Classical chess, and provides rough indication of the ratio between “plausible” and “possible” games according to the AlphaZero prior. The uniform random baseline depends on the number of legal moves per position, and is marginally different but of the same magnitude for other variations.

theoretic terms, the entropy

$$H(s_t) = - \sum_{s_{t+1}} p(s_{t+1}|s_t) \log p(s_{t+1}|s_t) \quad (6)$$

is a function of state  $s_t$  and represents the number of nats (or bits, if  $\log_2$  is used) that are needed to encode the weighted moves in position  $s_t$ .

If there are  $M(s_t)$  legal moves in state  $s_t$ , then the number of candidate moves  $m(s_t)$  – the number that a top player would realistically consider – is much smaller than  $M(s_t)$ . In de Groot (1946)’s original framing,  $M(s_t)$  is a player’s legal freedom of choice, while  $m(s_t)$  is their objective freedom of choice. Iida et al. (2003) hypothesise that  $m(s_t) \approx \sqrt{M(s_t)}$  on average. Because  $p(s_{t+1}|s_t)$  is a distribution on all legal moves, we define the number of candidate moves  $m(s_t)$  by

$$m(s_t) = \exp(H(s_t)) ; \quad (7)$$

it is the number of *uniformly weighted* moves that could be encoded in the *same* number of nats as  $p(s_{t+1}|s_t)$ .<sup>3</sup>

We provide insight into the diversity of the prior opening tree through two quantities, the move sequence entropy  $\mathcal{H}(t)$  at depth  $t$  from the opening position, and the average number of candidate moves at ply  $t$ ,  $\mathcal{M}(t)$ .

<sup>3</sup>As an illustrative example, if the number of candidate moves is  $m(s_t) = 3$  for some  $p(s_{t+1}|s_t)$  that might put non-zero mass on all of its moves, then  $m(s_t)$  is also equal to the number of candidate moves of a probability vector  $\mathbf{p} = [\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, \dots, 0]$  that puts equal non-zero mass on only three moves.

**Move sequence entropy** Let  $\mathbf{s} = \mathbf{s}_{1:t} = [s_1, s_2, \dots, s_t]$  be the sequence of states after  $t$  plies, starting at  $s_0$ , the initial position. The prior probability – without search – of move sequence  $\mathbf{s}_{1:t}$  is  $p(\mathbf{s}_{1:t}|s_0) = \prod_{\tau=1}^t p(s_\tau|s_{\tau-1})$ . The entropy of the move sequence is

$$\begin{aligned} \mathcal{H}(t) &= - \sum_{\mathbf{s}_{1:t}} p(\mathbf{s}_{1:t}) \log p(\mathbf{s}_{1:t}) \\ &= \mathbb{E}_{\mathbf{s}_{1:t} \sim p(\mathbf{s}_{1:t})} \left[ - \log p(\mathbf{s}_{1:t}) \right], \end{aligned} \quad (8)$$

where the starting position  $s_0$  is dropped from notation for brevity. An entropy  $\mathcal{H}(t) = 0$  implies that, according to the prior, one and only one reasonable opening line could be considered by White and Black up to depth  $t$ , with all deviations form that line leading to substantially worse positions for the deviating side. A higher  $\mathcal{H}(t)$  implies that we would *a priori* expect a wider opening tree of variations, and consequently a more diverse set of middlegame positions.

**Average number of candidate moves** The entropy of a chess variant’s prior opening tree is an unwieldy number that doesn’t immediately inform us how many move options we have in each chess variant. A more naturally interpretable number is the expected number of (good) candidate moves at each ply as the game unfolds. The average number of candidate moves at ply  $t$  is

$$\mathcal{M}(t) = \sum_{\mathbf{s}_{1:t}} p(\mathbf{s}_{1:t}) m(s_t) = \mathbb{E}_{\mathbf{s}_{1:t} \sim p(\mathbf{s}_{1:t})} [m(s_t)]. \quad (9)$$

Both the sums in (8) and (9) are over an exponential number of move sequences. We compute Monte Carlo estimates of  $\mathcal{H}(t)$  and  $\mathcal{M}(t)$  by sampling  $10^4$  sequences from  $p(\mathbf{s})$  and averaging the negative log probabilities of those sequences to obtain  $\mathcal{H}(t)$ , or averaging  $m(s_t)$  over all samples at depth  $t$  to obtain  $\mathcal{M}(t)$ . We defer a presentation of the breakdown of the average number of candidate moves per variant to Figure 11 in Appendix A, and will encounter  $\mathcal{M}(t)$  next in Figure 6 when Classical and No-castling chess are compared side by side.

The entropy of the AlphaZero prior opening tree is given in Table 3 for each variation. Similar to the calculation in (7) we give an estimate of the equivalent number of 20-ply sequences as  $\exp(\mathcal{H}(t))$ . As a baseline comparison, we take a prior distribution for Classical chess where all legal moves are *equally* playable, and estimate the entropy of the “Uniform random” move selection criteria. It affords us a crude estimate of the number of possible classical openings, as opposed to the number of plausibly playable or candidate openings. The estimates in Table 3 for Classical chess and “Uniform random Classical chess” corroborate the claim that the number of playable opening lines – a player’s objective freedom of choice – is roughly the square root of the number of legal opening lines (Iida et al., 2003).

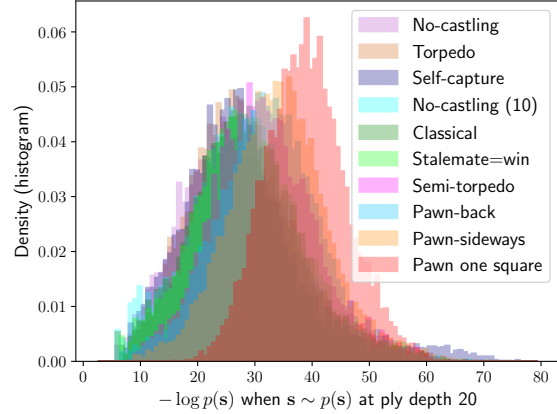


Figure 5. Histograms of  $-\log p(\mathbf{s})$  when  $\mathbf{s} \sim p(\mathbf{s})$  for each variant. Following (8), the means of these distributions give the entropies in Table 3. The individual histograms are separately presented in Figure 9 in Appendix A.

The two variants that have the largest entropy and hence largest opening tree in Table 3, Pawn-sideways and Pawn one square, also happen to be among the most drawish, according to Figures 3a and 3b. The two variants that have the smallest opening trees under our analysis, No-castling and Torpedo, are also the most decisive and give White some of the largest advantages, according to Figures 3a to 3d. Importantly, we estimate the size of the opening trees of these more decisive versions to still be of the same order of magnitude as that of Classical chess.

Figure 5 (a separate figure for each variant appears in Figure 9 in Appendix A) visualises the density of  $-\log p(\mathbf{s})$  when state sequences  $\mathbf{s}$  are drawn from  $p(\mathbf{s})$ . The mean of each density is the entropy of (8), and an overlap in the histograms of two variants implies that their opening trees contain a similar number of lines that are considered as candidates with similar odds. In Figure 5, a histogram that is shifted to the left means that fewer move sequences are considered *a priori*, and each has higher probability. A histogram that is shifted to the right implies that a larger variety of move sequences are *a priori* considered, and each has to be considered with a smaller probability. “Uniform random” is shown in Figure 9j, and would appear as a tall narrow spike centred around 64 in this figure. In the following section, we shall use log probability histograms as a tool to highlight the differences between Classical and No-castling chess.

### 3.5.2. CLASSICAL VS. NO-CASTLING CHESS

In Classical chess AlphaZero has a strong preference for playing the Berlin Defence 1... e5 2. Nf3 Nc6 3. Bb5 Nf6 in response to 1. e4, and here 4. O-O is White’s main reply,



Variant	Entropy	Equiv. 21-ply games
Classical (e4)	23.72	$2.00 \times 10^{10}$
Classical (Nf3)	29.54	$6.75 \times 10^{12}$
No-castling (e4)	27.42	$8.10 \times 10^{11}$
No-castling (Nf3)	28.40	$2.16 \times 10^{12}$

Table 4. The average information content in nats of the AlphaZero prior for Classical and No-castling chess, estimated on the 20 plies following 1. e4 and 1. Nf3.

which is not an option in no-castling chess. Yet, castling is also an integral part of most other lines in the Ruy Lopez, affecting each move when considering relative preferences. In the absence of castling, AlphaZero does not have as strong a preference for a particular line for Black after 1. e4, suggesting either that it is not as easy to fully neutralise White’s initiative, or alternatively that there is a larger number of promising defensive options.

To indicate the difference between Classical and No-castling chess, we compare the prior’s opening trees after 1. e4 and 1. Nf3 in Figure 6. If we examine the density of  $-\log p(s_{2:21}|s_1)$  under  $p(s_{2:21}|s_1)$ , where  $s_1$  is the board position after either 1. e4 or 1. Nf3, we see a marked shift in the characteristics of the AlphaZero prior opening trees (see Figures 6a and 6b). Statistically, the AlphaZero prior after 1. e4 is much more forcing than after 1. Nf3 in Classical chess. This is also evident from the average information content of the 20 plies after 1. e4 and 1. Nf3 in Table 4. In No-castling chess, 1. e4 seems as flexible as 1. Nf3, with a much wider variety of emerging preferential lines of play in the AlphaZero model.

Figure 6 additionally shows the average number of candidate moves at each ply. In Classical chess, White has more options than Black in both lines, the difference slowly diminishing over time as the first-move advantage decreases. 1. Nf3 offers more options, as it is less forcing. In No-castling chess, there seems to be a higher number of effective available moves for both sides after 1. e4 in the first couple of plies, based on the AlphaZero model.

The Berlin Defence is a contributing factor to the narrower opening tree footprint we see in Figure 6a. As defensive tool for Black, Vladimir Kramnik successfully used the Berlin Defence in his World Championship Match with Garry Kasparov in 2000. He describes his choice as follows:

“Back in the 90s, the engines of the time seemed to think that White had the advantage in the Berlin endgame, giving evaluations around +1 in White’s favour. I thought that things weren’t as simple, given that Black’s only real problem was the loss of castling rights, and the difficulty of connecting rooks. The first time that I had a

deeper look at it was when I was preparing for the match with Kasparov, and I thought that the opening was a good choice against Kasparov’s playing style. Pursuing it required a belief in instinct and the human assessment of the position. Nowadays, it is considered to be a very solid opening, and modern engines assess most arising positions as being equal.”

### 3.6. Differences between opening trees

We compare how similar opening trees are by considering how likely a given sequence of moves is under two variants. To compare, we define one variant  $p$  as the reference variant, and generate a move sequence  $s$  according to its prior. The Kullback-Leibler divergence is a measure of how likely such sequences of moves are under the opening book of variant  $q$  compared to that of  $p$ . Given two distributions  $p(s)$  and  $q(s)$ , the Kullback-Leibler divergence from  $q$  to  $p$  is the relative entropy of variant  $p$  with respect to  $q$ ,

$$D_{KL}[p||q] = \sum_s p(s) \log \frac{p(s)}{q(s)} = \mathbb{E}_{s \sim p(s)} [\log p(s) - \log q(s)]. \quad (10)$$

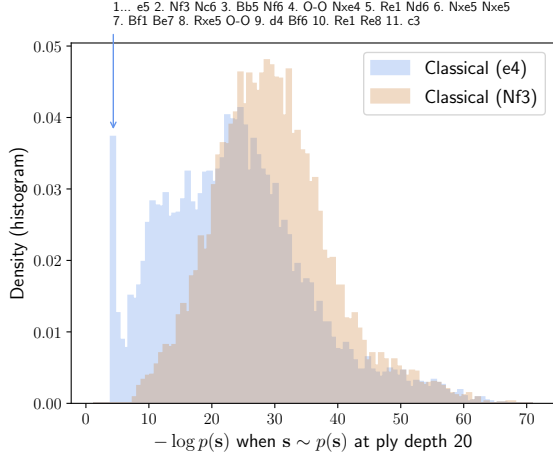
It is the expected number of extra nats (or bits if  $\log_2$  is used) that is required to compress move sequences from variant  $p$  using variant  $q$ ’s opening book distribution. The calculation in (10) involves a sum that is exponential in the length of  $s$ , and we estimate it with a Monte Carlo average of  $\log p(s)/q(s)$  over  $10^4$  sampled sequences from  $p(s)$ .

A legal move in variant  $p$  may be illegal in variant  $q$ , in which case there is no way in which sequences in  $p$  can be encoded in  $q$ . The Kullback-Leibler divergence in (10) is then infinite. More formally, this happens when  $q(s_{t+1}|s_t)$  puts zero mass on state transitions which are possible in  $p$ . We therefore need to ensure that the reference variant  $p$  is chosen so that its legal moves are a *subset* of those of  $q$ . In Table 5 we show all divergences with respect to Classical chess, and distinguish between two kinds of variants:

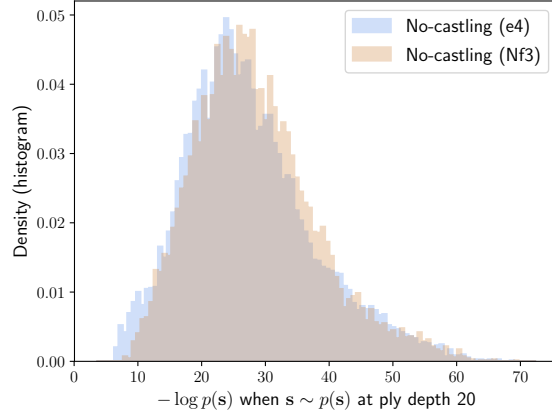
1. variants that add moves to Classical chess, and whose legal moves are supersets of Classical chess;
2. variants that remove legal moves from Classical chess, and whose moves are subsets of Classical chess.

The legal moves of Stalemate=win correspond to that of Classical chess, and it is included as both a superset and a subset in Table 5. The density of samples from (10) is given in Figure 10 in Appendix A. The divergence is largest for variants that introduce the largest number of additional pawn moves or the most restrictions. Self-capture chess, despite

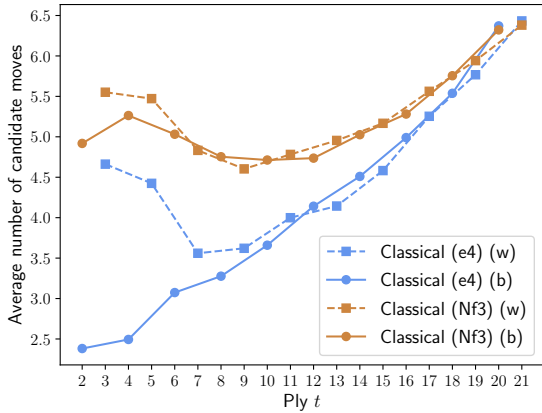




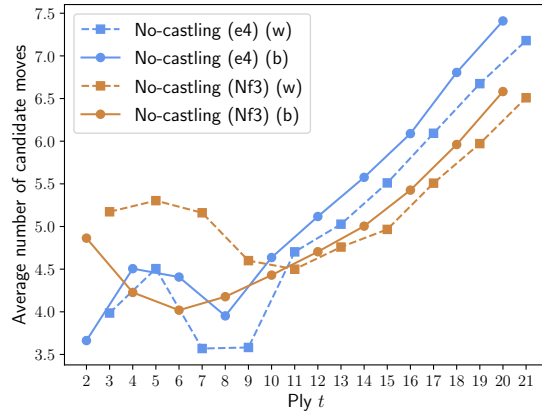
(a) The density of (negative) log likelihoods for opening lines in Classical chess after 1. e4 and 1. Nf3 when move sequences are sampled from the AlphaZero prior. There is a marked difference in overlap between the histograms, suggesting that AlphaZero *a priori* considers “narrower” opening lines after 1. e4 than after 1. Nf3. We identify the samples  $s$  at the high likelihood spike with a particular line in the Berlin Defence.



(b) The density of (negative) log likelihoods for opening lines in No-castling chess after 1. e4 and 1. Nf3 when move sequences are sampled from the AlphaZero prior. Without the option of castling a king to safety, the prior opening trees after 1. e4 and 1. Nf3 have more similar “distributional footprints” compared to Classical chess in Figure 6a.



(c) The average number of candidate moves  $\mathcal{M}(t)$ , as computed with (9), for Classical chess.



(d) The average number of candidate moves  $\mathcal{M}(t)$ , as computed with (9), for No-castling chess.

Figure 6. The diversity of responses to 1. e4 and 1. Nf3 in Classical and No-castling chess, as well as the average number of candidate moves available for White and Black at each ply. The spike is in the classical chess 1. e4 response distribution is at 1... e5 2. Nf3 Nc6 3. Bb5 Nf6 4. O-O Nxe4 5. Re1 Nd6 6. Nxe5 Nxe5 7. Bf1 Be7 8. Rxe5 O-O 9. d4 Bf6 10. Re1 Re8 11. c3, a known equalising line in the Berlin Defence, leading to drawish positions.

the plethora of additional opportunities for self-capture, is statistically closer to Classical chess because of the low frequency at which the extra moves are played.

### 3.7. How much opening theory should be relearned?

Although the relative entropy expresses how many more nats are required to encode prior moves of one variant given another, it does not tell us whether one variant’s player is considering the right candidate moves when playing another

variant. How many *more* candidate moves should a player Q, who was trained on one variant of chess, take into consideration when wanting to play at player P’s level in another variation? Let  $q(s)$  be the candidate prior for the variation that player Q was trained on, and  $p(s)$  the prior for variant P, variant that Q wants to play. We define the combination

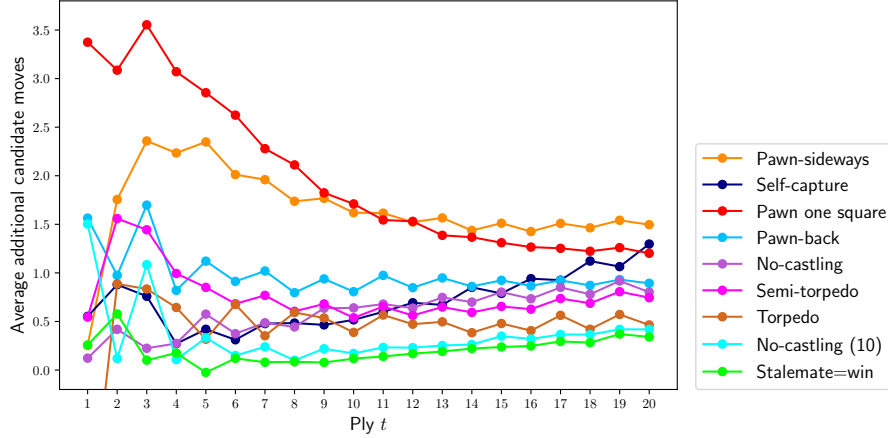


Figure 7. The average number of additional candidate moves  $\mathcal{A}_q(t)$  that a Classical player Q with prior  $q(s_{t+1}|s_t)$  should consider in order to match player P’s candidate moves from prior  $p(s)$  for each of the evaluated variants; see (15). (The order of the variants in the legend matches their ordering at ply  $t = 20$ .)

	Variant $p$	Variant $q$	$D_{\text{KL}}[p  q]$
Supersets	Classical	Stalemate=win	2.59
	Classical	Self-capture	5.24
	Classical	Semi-torpedo	10.35
	Classical	Pawn-back	11.70
	Classical	Torpedo	11.89
	Classical	Pawn-sideways	24.23
Subsets	Stalemate=win	Classical	2.50
	No-castling (10)	Classical	7.17
	No-castling	Classical	13.19
	Pawn one square	Classical	20.28

Table 5. Differences in the opening tree of the new chess variants and Classical chess. These are expressed as Kullback-Leibler (KL) divergences, the direction depending on whether a particular variant is a superset or a subset of Classical chess, based on the rule change. In all cases but Stalemate=win the reverse KL divergences are infinite as when there are legal opening lines  $s$  in variant  $p$  that don’t exist in  $q$ , and hence for which  $q(s) = 0$  when  $p(s)$  is not (contributing  $-\log 0$  to the divergence).

of the two priors as the normalized supremum

$$r(s_{t+1}|s_t) = \frac{\max \{p(s_{t+1}|s_t), q(s_{t+1}|s_t)\}}{\sum_{s'_{t+1}} \max \{p(s'_{t+1}|s_t), q(s'_{t+1}|s_t)\}} \quad (11)$$

There is a particular reason behind our choice of definition for the combined prior in (11): The number of candidate moves that the combination of players P and Q would consider, is always smaller than the sum of candidate moves that P and Q would consider individually.

Put more formally, define the number of candidate moves for the combined player as the number of uniformly weighed moves that could be encoded in the same number of nats as  $r(s_{t+1}|s_t)$ ,<sup>4</sup>

$$m_r(s_t) = \exp \left( - \sum_{s_{t+1}} r(s_{t+1}|s_t) \log r(s_{t+1}|s_t) \right) \quad (12)$$

For any choice of priors  $p$  and  $q$  the number of candidate moves that are considered by the combined player in state  $s_t$  is lower bounded by

$$m_r(s_t) \leq m_p(s_t) + m_q(s_t), \quad (13)$$

which we prove in Appendix A.1.

We now define the difference

$$\text{additional}(s_t) = m_r(s_t) - m_q(s_t) \quad (14)$$

to represent the number of additional candidate moves that player Q should consider, to play at the level of P in position

<sup>4</sup>The perceptive reader would recognise equation (12) as equation (7). We restate it here with a subscript to indicate the explicit dependence on the distribution.

$s_t$ . The additional number of candidates  $\text{additional}(s_t)$  is zero when the priors match,  $q = p$ , and intuitively Q doesn't need to consider any further candidate moves. The number of additional moves may be negative; intuitively, Q puts enough weight on all candidates that P deems important, and doesn't need to consider any further candidate moves. The number of additional candidate moves and is upper bounded by  $\text{additional}(s_t) \leq m_p(s_t)$  according to (13); at the very worst, Q would additionally have to consider all of P's candidates.

We consider positions up to ply  $t$  plies sampled from prior for P, and at ply  $t$  evaluate how many additional candidate moves Q should consider on average:

$$\mathcal{A}_q(t) = \mathbb{E}_{\mathbf{s}_{1:t} \sim p(\mathbf{s}_{1:t})} [\text{additional}(s_t)] . \quad (15)$$

The expectation is estimated with a Monte Carlo average over  $10^4$  samples from  $p(\mathbf{s}_{1:t})$ .

Figure 7 shows the average additional number of candidate moves if Q is taken as the Classical chess prior, with P iterating over all other variants. From the outset, Pawn one square places 60% of its prior mass on 1. d3, 1. e3, 1. c3 and 1. h3, which together only account for 13% of Classical's prior mass. As pawns are moved from the starting rank and pieces are developed,  $\mathcal{A}_q(t)$  slowly decreases for Pawn one square. As the opening progresses, Stalemate=win slowly drifts from zero, presumably because some board configurations that would lead to drawn endgames under Classical rules might have a different outcome. Torpedo puts 66% of its prior mass on one move, 1. d4, whereas the Classical prior is broader (its top move, 1. d4, occupies 38% of its prior mass). The truncated plot value for Torpedo is  $\mathcal{A}_q(1) = -1.8$ , signifying that the first Classical candidate moves effectively already include those of Torpedo chess. There is a slow upward drift in the average number of additional candidates that a Classical player has to consider under Self-capture chess as a game progresses. We hypothesise that it can, in part, be ascribed to the number of reasonable self-capturing options increasing toward the middle game.

### 3.8. Material

Material plays an important role in chess, and is often used to assess whether a particular sequence of piece exchanges and captures is favourable. Material sacrifices in chess are made either for concrete tactical reasons, e.g. mating attacks, or to be traded off for long-term positional strengthening of the position. Understanding the material value of pieces in chess helps players master the game and is one of the very first pieces of chess knowledge taught to beginners. Changes to the rules of chess affect piece mobility, and hence also the relative value of pieces. Without a basic estimate of what the relative piece values in each variant are, it would be harder for human players to start playing these chess variants. As a

guide, we provide an experimental approximation to piece values based on outcomes of AlphaZero games under 1 second per move.

We approximate piece values from the weights of a linear model that predicts the game outcome from the difference in numbers of each piece only. As background, the real AlphaZero evaluation  $v$  in  $(\mathbf{p}, v) = f_\theta(s)$  is the output of a deep neural network with weights  $\theta$ . The expected game outcome  $v$  is the result of a final tanh activation to ensure an output in  $(-1, 1)$ . If  $z \in \{-1, 0, 1\}$  indicates the playing side's game outcome, AlphaZero's loss function includes the mean squared error  $(z - v)^2$  (Silver et al., 2018). We create a simplified evaluation function  $g_w(s)$  that only takes piece counts on the board into consideration. For a position  $s$  we construct a feature vector  $d \stackrel{\text{def}}{=} [1, d_\Delta, d_\square, d_\blacktriangle, d_\blacksquare, d_\clubsuit]$  that contains the integer differences between the playing side and their opponent's number of pawns, knights, bishops, rooks and queens. We define  $g_w$  with weights  $w \in \mathbb{R}^6$  as

$$g_w(s) = \tanh(w^T d) . \quad (16)$$

When trained on the 10,000 AlphaZero self-play board positions from Section 3.1 for each variant, the piece weights  $w$  provide an indication of their relative importance. Let  $(s, z) \sim \text{games}$  represent a sample of a position and final game outcome from a variant's self-play games. We minimise

$$\ell(w) = \mathbb{E}_{(s,z) \sim \text{games}} \left[ (z - g_w(s))^2 \right] \quad (17)$$

empirically over  $w$ , and normalise weights  $w$  by  $w_\Delta$  to yield the relative piece values. The recovered piece values for each of the chess variants are given in Table 6.






Variant					
Classical	1	3.05	3.33	5.63	9.5
No castling	1	2.97	3.13	5.02	9.49
No castling (10)	1	3.14	3.40	5.37	9.85
Pawn one square	1	2.95	3.14	5.36	9.62
Stalemate=win	1	2.95	3.13	4.76	8.96
Self-capture	1	3.10	3.22	5.34	9.42
Pawn-back	1	2.65	2.85	4.67	9.39
Semi-torpedo	1	2.72	2.95	4.69	8.3
Torpedo	1	2.25	2.46	3.58	7.12
Pawn-sideways	1	1.8	1.98	2.99	5.92

Table 6. Estimated piece values from AlphaZero self-play games for each variant.

In Classical chess, piece values vary based on positional considerations and game stage. The piece values in Table 6 should not be taken as a gold standard, as the sample of AlphaZero games that they were estimated on does not fully capture the diversity of human play, and the game lengths do not correspond to that of human games, which tend to be

shorter. For comparison, we have included the piece value estimates that we obtain by applying the same method to Classical chess, showing that the estimates do not deviate much from the known material values. Over the years, many material systems have been proposed in chess. The most commonly used one (Capablanca & de Firmian, 2006) gives 3–3–5–9 for values of knights, bishops, rooks and queens. Another system (Kaufman, 1999) gives 3.25–3.25–5–9.75. Yet, bishops are typically considered to be more valuable than the knights, and there is usually an additive adjustment while in possession of a bishop pair. The rook value varies between 4.5 and 5.5 depending on the system and the queen values span from 8.5 to 10. The relative piece values estimated on the AlphaZero game sample for Classical chess, 3.05–3.33–5.63–9.5, do not deviate much from the existing systems. This suggests that the estimates for the new chess variants are likely to be approximately correct as well.

We can see similar piece values estimated for No-castling, No-castling(10), Pawn-one-square chess, Self-capture and Stalemate=win. This is not surprising, given that these variants do not involve a major change in piece mobility. Estimated piece values look quite different in the remaining variations, where pawn mobility has been increased: Pawn-back, Semi-torpedo, Torpedo and Pawn-sideways. In Pawn-sideways chess, minor pieces seem to be worth approximately two pawns, which is in line with our anecdotal observations when analysing AlphaZero games, as such exchanges are frequently made. Like Torpedo chess, pawns become much stronger and more valuable than before. Changes in Pawn-back and Semi-torpedo are not as pronounced.

#### 4. Qualitative assessment

To evaluate the differences in play between the set of chess variations considered in this study, we couple the quantitative assessment of the variations with expert analysis based on a large set of representative games. While the overall decisiveness and opening diversity add to the appeal of any chess variation, the subjective questions of aesthetic value and the types of positions, moves and patterns that arise are not possible to fully capture quantitatively. For providing a deep qualitative assessment of the appeal of these chess variations, we rely on the experience of chess grandmaster Vladimir Kramnik, an ex-world chess champion and an authority on the game. By characterising typical patterns, we hope to provide players with insights to help them judge for themselves if they would find some of these chess variants interesting enough to try out in practice. What we provide here are preliminary findings.

The detailed qualitative assessment of the chess variants presented in this article, along with typical motifs and illus-

trative games, is provided in the Appendix (Section B). For this analysis, we use the 1,000 1-minute per move games of Section 3.1 as well as 200 1-minute per move games from a diverse set of early opening positions that all of the major opening systems. By looking at the former, we were able to assess AlphaZero’s preferred style of play in each chess variant, and by looking at the latter, we could assess how the treatment of different opening lines changes and which of those become more or less promising under each of the rule changes. Figure 1 shows an illustrative example position for each of the considered chess variants.

What follows is a short summary of the main takeaways from the qualitative analysis for each of the variants, provided by GM Vladimir Kramnik.

**No-castling chess** is a potentially exciting variant, given that king safety is often compromised for both players, allowing for simultaneous attacking and counter-attacking and the equality, when reached, tends to be dynamic in nature rather than “dry”. The multitude of approaches to evacuate the king, and their timing, adds complexity to the opening play. **No-castling (10)**, where castling is not permitted for the first 10 moves (20 plies) is a partial restriction, rather than an absolute one – which does not change the game to the same extent. Due to castling being such a powerful option, the lines preferred by AlphaZero all tend to involve castling, only delayed – resulting in a preference for slower, closed positions, and a less attractive style of play. Such partial castling restrictions can be considered if the desire is to sidestep opening theory and preparation, but this may not be of interest for the wider chess audience.

**Pawn one square chess** variant may appeal to players who enjoy slower, strategic play – as well as a training tool for understanding pawn structures, due to the transpositional possibilities when setting up the pawns. The reduced pawn mobility makes it harder to launch fast attacks, making the game overall less decisive.

**Stalemate=win chess** has little effect on the opening and middlegame play, mostly affecting the evaluation of certain endgames. As such, it does not increase decisiveness of the game by much, as it seems to almost always be possible to defend without relying on stalemate as a drawing resource. Therefore, this chess variant is not likely to be useful for sidestepping known theory or for making the game substantially more decisive at the high level. The overall effect of the change seems to be minor.

**Torpedo and Semi-torpedo chess** both make the game more dynamic and more decisive, and Torpedo chess in particular leads to new motifs and changes in all stages of the game. Creating passed pawns becomes very important, as they are hard to stop. The attacking possibilities make Torpedo chess quite appealing, and it is likely to be of

interest for players that enjoy tactical play.

**Pawn-back chess** makes it possible to regain control of the weakened squares in the position and remove some square weaknesses. It also introduces additional possibilities for opening up diagonals and making squares available for the pieces. Counter-intuitively, even though moving the pieces backwards is usually a defensive manoeuvre, this can make more aggressive options possible, given that pawns can now be pushed further earlier on, as there is always an option of moving them back to cover the weakened squares. AlphaZero has a strong preference for playing the French defence with Black, which is particularly interesting.

**Pawn-sideways chess** is incredibly complex, resulting in patterns that are at times quite “alien” when one is used to classical chess. The pawn structures become very fluid and it is impossible to create permanent pawn weaknesses. Given how important this concept is in classical chess, this chess variant requires us to rethink how we approach any given position, making it very concrete and relying on deep calculation. Restructuring the pawn formation takes time, and players need to use that time for creating other types of advantages. Many of AlphaZero games in this variant have been quite tactical, some involving novel tactics that are not possible under classical rules.

**Self-capture chess** is quite entertaining, as it introduces additional options for sacrificing material – and material sacrifices have a certain aesthetic appeal. Self-capture moves can feature in all stages of the game. Not every game involves self-captures, as giving away material is not always required, but they do feature in a substantial percentage of the games, and in some games they occur multiple times. Self-capture moves can be used to open files and squares for the pieces in the attack; opening up a blockade by sacrificing a pawn in the pawn chain; or in defence, while escaping the mating net.

## 5. Conclusions

We have demonstrated how AlphaZero can be used for prototyping board games and assessing the consequences of rule changes in the game design process, as demonstrated on chess, where we have trained AlphaZero models to evaluate 9 different chess variants, representing atomic changes to the rules of classical chess. Training an AlphaZero model under these rule changes helped us effectively simulate decades of human play in a matter of hours, and answer the “what if” question: what the play would potentially look like under developed theory in each chess variant. We believe that a similar approach could be used for auto-balancing game mechanics in other types of games, including computer games, in cases when a sufficiently performant reinforcement learning system is available.

To assess the consequences of the rule changes, we coupled the quantitative analysis of the trained model and self-play games with a deep qualitative analysis where we identified many new patterns and ideas that are not possible under the rules of classical chess. We showed that there several chess variants among those considered in this study that are even more decisive than classical chess: Torpedo chess, Semi-torpedo chess, No-castling chess and Stalemate=win chess.

We additionally quantified the arising diversity of opening play and the intersection of opening trees between chess variations, showing how different the opening theory is for each of the rule changes. There is a negative correlation between the overall opening diversity and decisiveness, as the decisive variants likely require more precise play, with fewer plausible choices per move. For each of the chess variants, we estimated the material value of each of the pieces based on the results of 10,000 AlphaZero games, to provide insight into favourable exchange sequences and make it easier for human players to understand the game.

No-castling chess, being the first variant that we analysed (chronologically), has already been tried in an experimental blitz grandmaster tournament in Chennai, as well as a couple of longer grandmaster games. Our assessment suggests that several of the assessed chess variants might be quite appealing to interested players, and we hope that this study will prove to be a valuable resource for the wider chess community.

## Acknowledgements

We would like to thank chess grandmasters Peter Heine Nielsen, and Matthew Sadler for their valuable feedback on our preliminary findings and the early version of the manuscript. Oliver Smith and Kareem Ayoub have been of great help in managing the project. We would also like to thank the team of Chess.com for providing us with a platform to announce and discuss No-castling chess and present annotated games.

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## A. Quantitative Appendix

### A.1. Proof of equation (13)

Let  $\mathbf{p}$  and  $\mathbf{q}$  be two vectors with non-negative entries that sum to one. Define  $\mathbf{r}$  as a vector with elements

$$r_i = \frac{\max(p_i, q_i)}{\sum_{i'} \max(p_{i'}, q_{i'})}. \quad (18)$$

We show below that

$$e^{-\sum_i r_i \log r_i} \leq e^{-\sum_i p_i \log p_i} + e^{-\sum_i q_i \log q_i}. \quad (19)$$

Let  $R = \sum_i \max(p_i, q_i)$  be the normalizing constant in (18). It is bounded by  $1 \leq R \leq 2$ . We write the entropy as

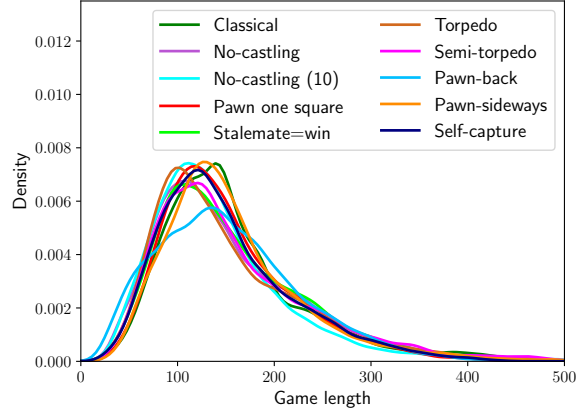
$$\begin{aligned} & -\sum_i r_i \log r_i \\ &= -\frac{1}{R} \sum_i \max(p_i, q_i) \log \max(p_i, q_i) + \log R \\ &= -\frac{1}{R} \sum_i \max(p_i \log p_i, q_i \log q_i) + \log R \\ &\leq -\sum_i \max(p_i \log p_i, q_i \log q_i) + \log R \\ &\leq -\frac{1}{2} \sum_i p_i \log p_i - \frac{1}{2} \sum_i q_i \log q_i + \log R \end{aligned} \quad (20)$$

where the last inequality in (20) follows from  $\max(a, b) \geq \frac{a+b}{2}$ . Exponentiating (20) and applying Jensen's inequality yields

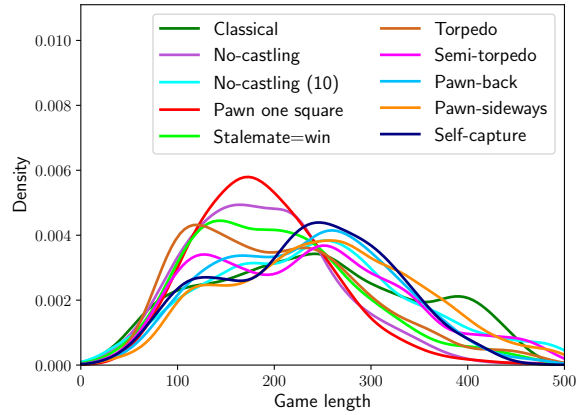
$$\begin{aligned} & e^{-\sum_i r_i \log r_i} \\ &\leq R e^{\frac{1}{2}(-\sum_i p_i \log p_i) + \frac{1}{2}(-\sum_i q_i \log q_i)} \\ &\leq R \left( \frac{1}{2} e^{-\sum_i p_i \log p_i} + \frac{1}{2} e^{-\sum_i q_i \log q_i} \right) \\ &\leq e^{-\sum_i p_i \log p_i} + e^{-\sum_i q_i \log q_i}. \end{aligned} \quad (21)$$

The final line follows from  $R/2 \leq 1$  as  $1 \leq R \leq 2$ . The bound is tight at  $R = 1$  when  $\mathbf{p}$  and  $\mathbf{q}$  both put probability mass uniformly on two non-intersecting same-sized subsets of elements.<sup>5</sup>

### A.2. Additional figures



(a) The game length distributions of the total number of plies for all self-play games for each variant.



(b) The game length distributions of the total number of plies for the subset of *decisive* (not drawn) self-games for each variant.

Figure 8. The game length distributions of the total number of plies of AlphaZero games in each chess variant, based on a sample of 10,000 games played at 1 second per move. The experimental setup is described in Section 3.1.

<sup>5</sup>An example of two vectors giving a tight bound in (19) is  $\mathbf{p} = [\frac{1}{2}, \frac{1}{2}, 0, 0, 0]$  and  $\mathbf{q} = [0, 0, \frac{1}{2}, \frac{1}{2}, 0]$ .

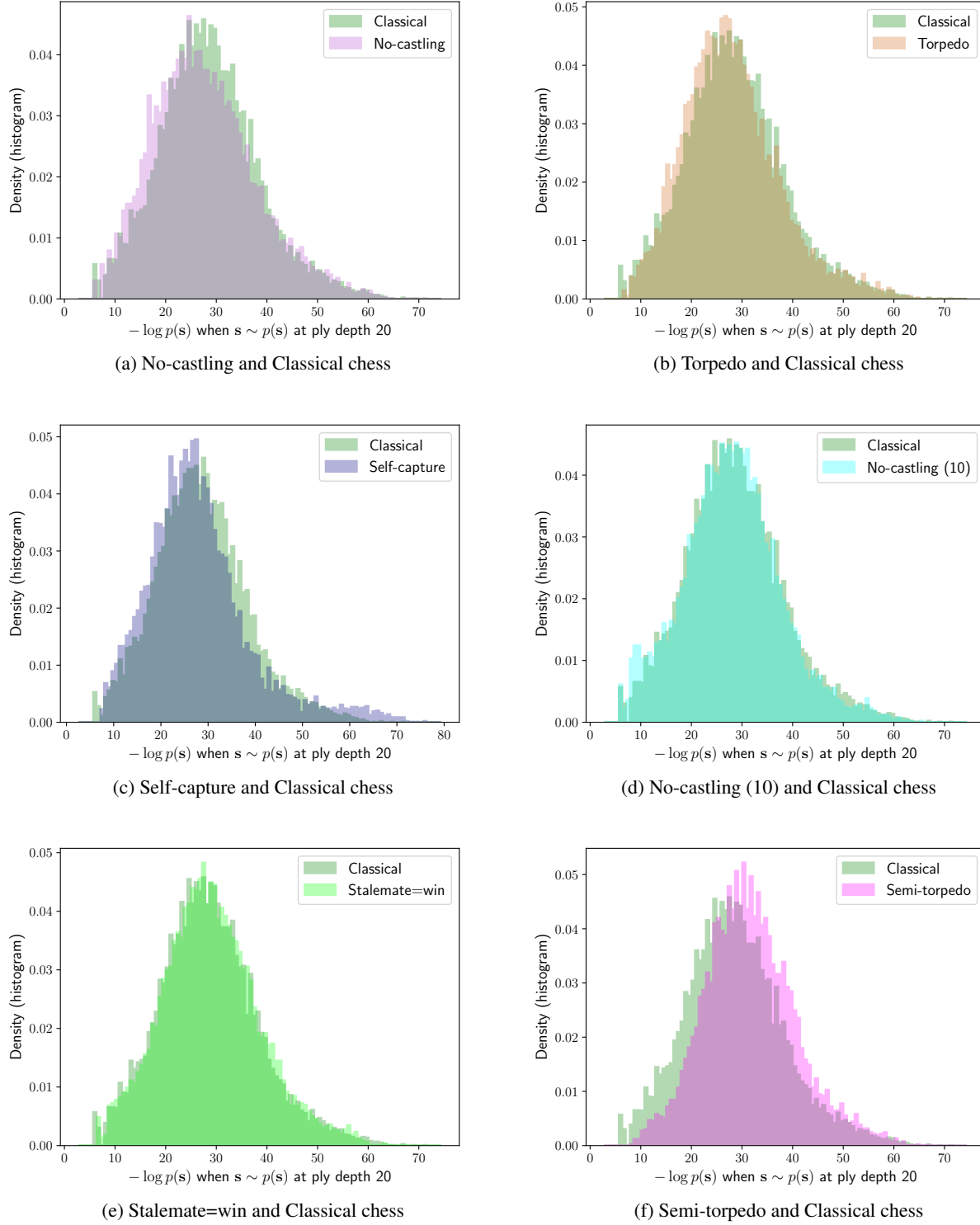
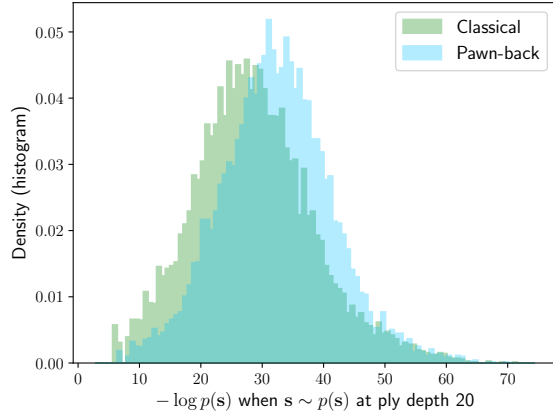
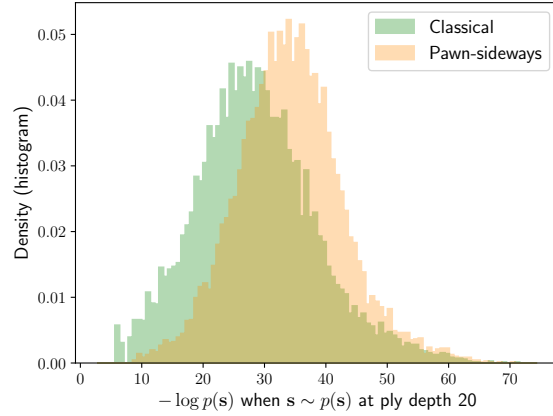


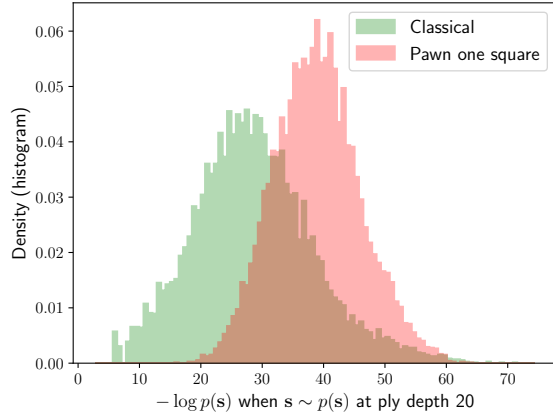
Figure 9. The density of (negative) log likelihoods for the prior opening lines for Classical chess and each of the variants. The mean of each histogram gives the entropy or average information content for each variant’s prior  $p(s)$ , as given in (8). The subfigures are ordered by entropy, following Table 3. Figure 9g continues on the next page.



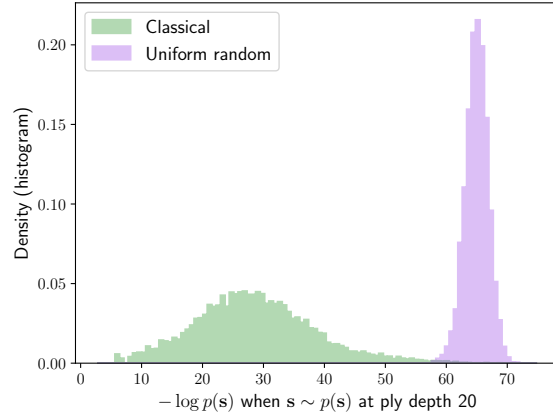
(g) Pawn-back and Classical chess



(h) Pawn-sideways and Classical chess

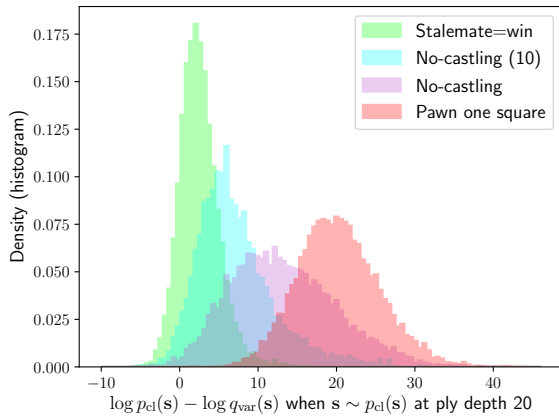


(i) Pawn one square and Classical chess

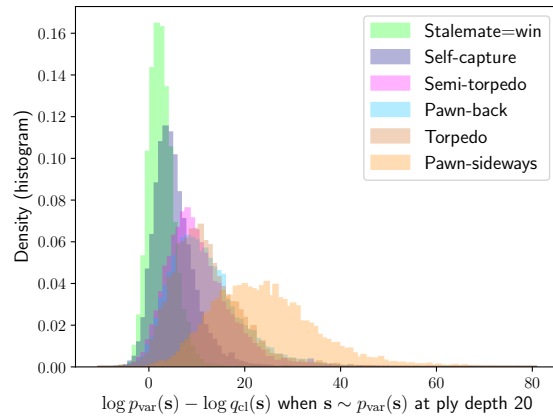


(j) Uniform random classical moves and Classical chess

Figure 9. (Continued from previous page.) The density of (negative) log likelihoods for the prior opening lines for Classical chess and each of the variants. The mean of each histogram gives the entropy or average information content for each variant's prior  $p(s)$ , as given in (8). The subfigures are ordered by entropy, following Table 3.

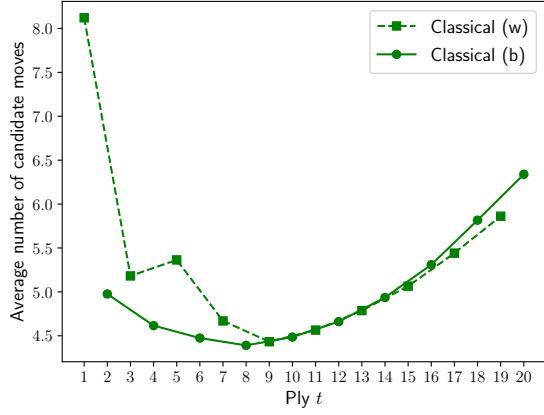


(a) A decomposition of the entropy of subset variants of Classical chess relative to Classical chess.

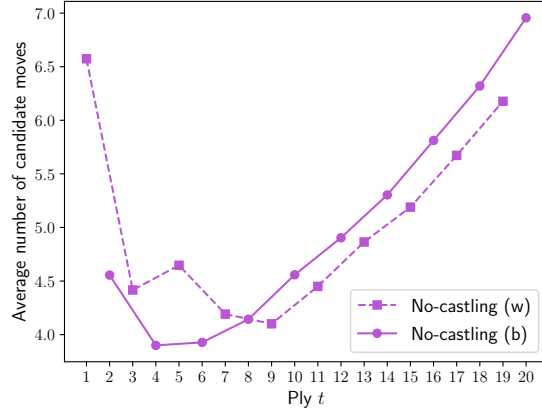


(b) A decomposition of the entropy of Classical chess relative to its superset variants.

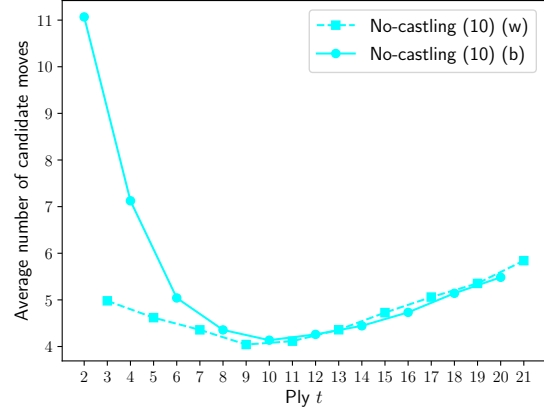
Figure 10. Histograms of the density of terms  $\log p(s) - \log q(s)$  whose mean under  $p(s)$  is the Kullback-Leibler divergence in (10).



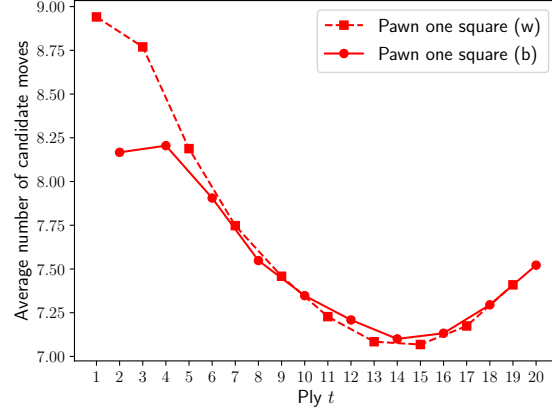
(a) Classical chess



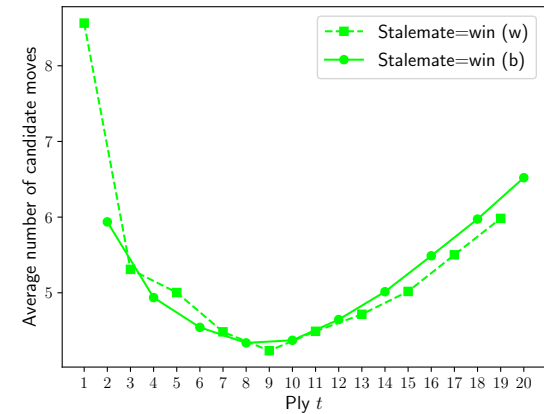
(b) No-castling chess



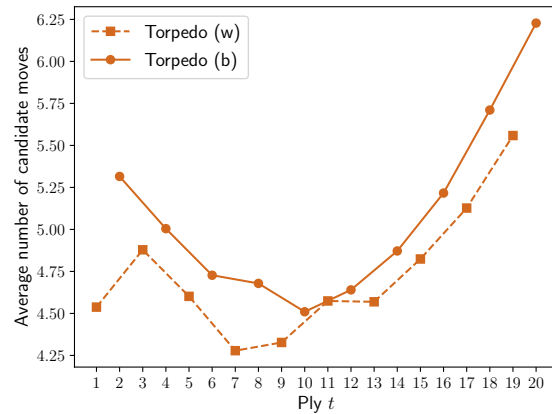
(c) No-castling (10) chess



(d) Pawn one square chess



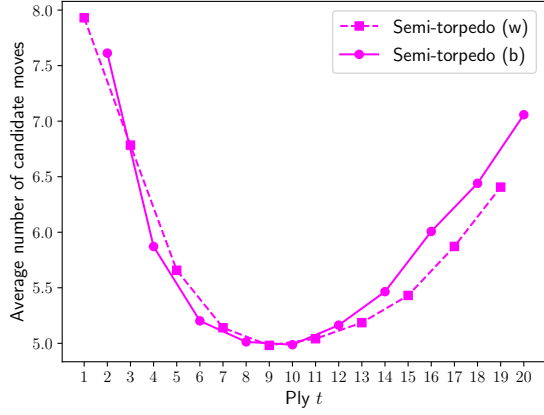
(e) Stalemate=win chess



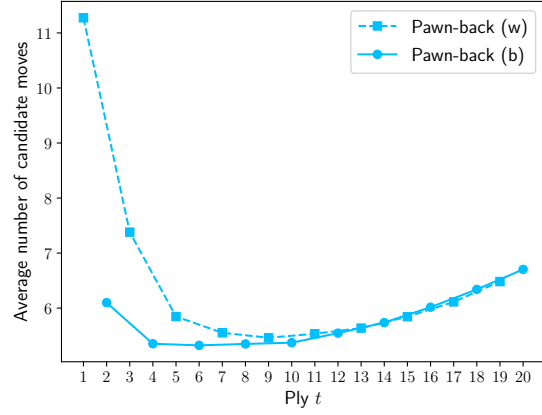
(f) Torpedo chess

Figure 11. The average number of candidate moves  $\mathcal{M}(t)$  from (9) for each of the variants, as computed from their prior distributions  $p(s)$ . Figure 11g continues on the next page.

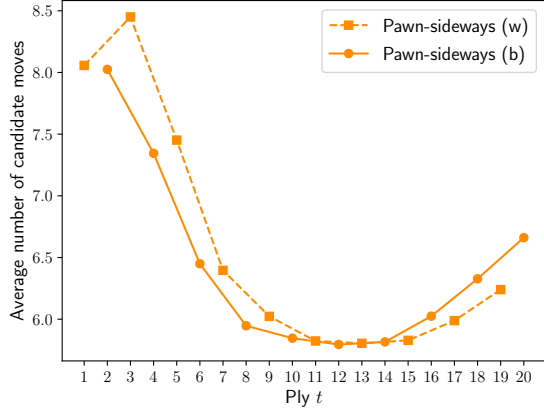




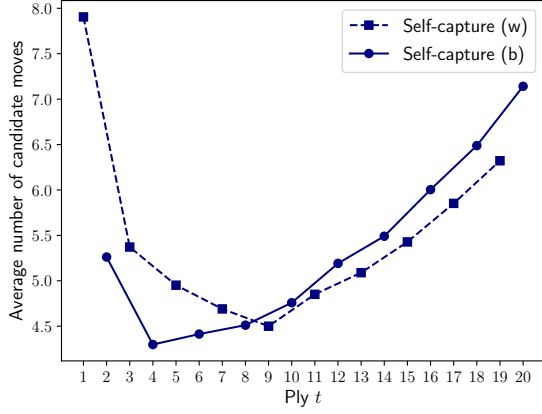
(g) Semi-torpedo chess



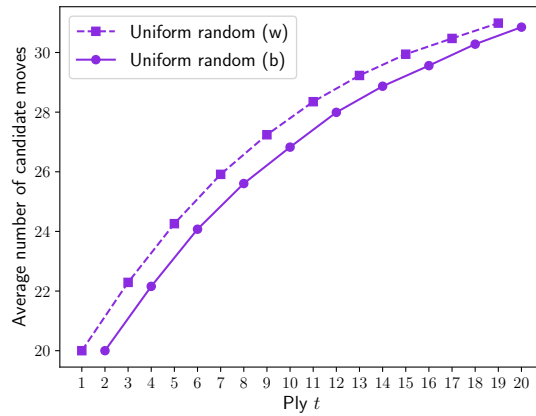
(h) Pawn-back chess



(i) Pawn-sideways chess



(j) Self-capture chess



(k) Uniform random moves under classical chess rules

 Figure 11. (Continued from previous page.) The average number of candidate moves  $\mathcal{M}(t)$  from (9) for each of the variants, as computed from their prior distributions  $p(s)$ .

## B. Appendix

Here we present a selection of instructive games for each of the chess variations considered in the study, along with a detailed assessment of the variations by Vladimir Kramnik.

Given that different rule changes that we examined had led to a different degree of departure from existing chess theory and patterns, we do not present an equal amount of instructive positions and games for each chess variation, and rather focus on those that have either been assessed to be of greater immediate interest or simply employ patterns that are unfamiliar and novel and require more time to introduce and understand.

The Appendix is organised into sections corresponding to each of the chess variations and rule alterations examined in this study, in the following order: No-castling chess (Page 25), No-castling (10) chess (Page 31), Pawn one square chess (Page 34), Stalemate=win chess (Page 37), Torpedo (Section 40), Semi-torpedo (Page 54), Pawn-back chess (Page 61), Pawn-sideways chess (Page 70) and Self-capture chess (Page 85).

Each of the variants-specific sections first introduces the rule change, sets out the motivation for why it seemed of interest to be tried out, gives a qualitative assessment and a high-level conceptual overview of the dynamics of arising play by Vladimir Kramnik and then concludes with several instructive games and positions, selected to illustrate the typical motifs that arise in AlphaZero play in these variations.

### B.1. No-castling

In No-castling chess, the adjustment to the original rules involved a full removal of castling as an option.

#### B.1.1. MOTIVATION

The motivation for the No-castling chess variant, as provided by Vladimir Kramnik:

“Adjustments to castling rules were chronologically the first type of changes implemented and assessed in this study. Firstly, excluding a single existing rule makes it comparatively easy for human players to adjust, as there is no need to learn an additional rule. Secondly, the right to castle is relatively new in the long history of the game of chess. Arguably, it stands out amongst the rules of chess, by providing the only legal opportunity for a player to move two of their own pieces at the same time.”

#### B.1.2. ASSESSMENT

The assessment of the no-castling chess variant, as provided by Vladimir Kramnik:

“I was expecting that abandoning the castling rule would make the game somewhat more favorable for White, increasing the existing opening advantage. Statistics of AlphaZero games confirmed this intuition, though the observed difference was not substantial to the point of unbalancing the game. Nevertheless, when considering human practice, and considering that players would find themselves in unknown territory at the very early stage of the game, I would expect White to have a higher expected score in practice than under regular circumstances.

One of the main advantages of no-castling chess is that it eliminates the nowadays overwhelming importance of the opening preparation in professional chess, for years to come, and makes players think creatively from the very beginning of each game. This would inevitably lead to a considerably higher amount of decisive games in chess tournaments until the new theory develops, and more creativity would be required in order to win. These factors could also increase the following of professional chess tournaments among chess enthusiasts.

With late middlegame and endgame patterns staying the same as in regular chess, there is a major difference in the opening phase of a no-castling chess game. The main conceptual rules of piece development and king safety are still valid, but most concrete opening variations of regular chess no longer apply, as castling is usually an essential part of existing chess opening variations.

For example, possibly opening a game with 1. f4, which is not a great idea in classical chess, might be one of the better options already, since it might make it easier to evacuate the king after Nf3, g3, Bg2, Kf2, Rf1, Kg1. Some completely new patterns of playing the openings start to make sense, like pushing the side pawns in order to develop the rooks via the “h” file or “a” file, as well as “artificial castling” by means of Ke2, Re1, Kf1 and others. Many new conceptual questions arise in this chess variation.

For instance, one has to think about what ought to be preferable: evacuating the king out of the center of the board as soon as possible or aiming to first develop all the pieces and claim space and central squares. Years of practice are likely

required to give a clear answer on the guiding principles of early play and best opening strategies. Even with the help of chess engines, it would likely take decades to develop the opening theory to the same level and to the same depth as we have in regular chess today. The engines can be helpful with providing initial recommendations of plausible opening lines of play, but the right understanding and timing of the implementation of new patterns is crucial in practical play.

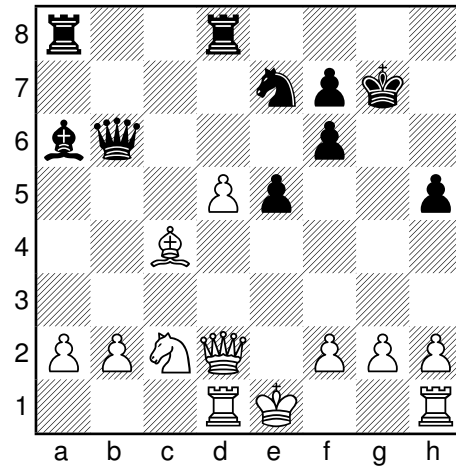
Studying the numerous no-castling games played by AlphaZero, I have noticed one major conceptual change. Since both kings have a harder time finding a safe place, the dynamic positional factors (e.g. initiative, piece activity, attack), seem to have more importance than in regular chess. In other words, a game becomes sharper, with both sides attacking the opponent king at the same time.

I am convinced that because of the aforementioned reasons we would see many interesting games, and many more decisive games at the top level chess tournaments in case the organisers decide to give it a try. Due to the simplicity of the adjustment compared to regular chess, it is also easy to implement this variation at any other level, including the online chess playing platforms, as it merely requires an agreement between the two players not to play castling in their game.

”

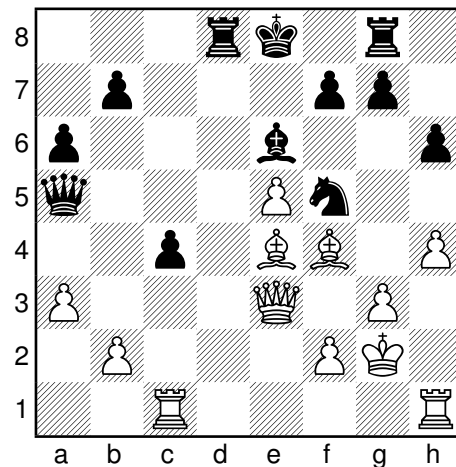
### B.1.3. MAIN LINES

Here we discuss “main lines” of AlphaZero under No-castling chess, when playing with roughly one minute per move from a particular fixed first move. Note that these are not purely deterministic, and each of the given lines is merely one of several highly promising and likely options. Here we give the first 20 moves in each of the main lines, regardless of the position.



**Main line after d4** The main line of AlphaZero after 1. d4 in No-castling chess is:

1. d4 (book) d5 2. c4 e6 3. Nc3 c5 4. cxd5 exd5 5. Nf3 Nf6 6. g3 Nc6 7. Bg2 h6 8. Kf1 Be6 9. Bf4 Rc8 10. h4 a6 11. Rc1 Rg8 12. a3 c4 13. Ne5 Bd6 14. e4 dxe4 15. Nxe4 Bxe5 16. dxe5 Nxe4 17. Bxe4 Qa5 18. Kg2 Rd8 19. Qe2 Nd4 20. Qe3 Nf5

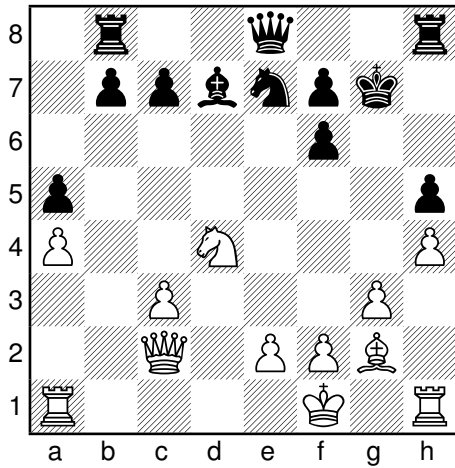


**Main line after e4** The main line of AlphaZero after 1. e4 in No-castling chess is:

1. e4 (book) c5 2. Nf3 Nc6 3. Nc3 Nf6 4. d4 cxd4 5. Nxd4 e6 6. Ndb5 d6 7. Bf4 e5 8. Bg5 a6 9. Na3 b5 10. Nd5 Be7 11. Bxf6 Bxf6 12. c4 Ne7 13. Nxf6+ gxf6 14. cxb5 h5 15. Qd2 Kf8 16. Bc4 Kg7 17. Rd1 d5 18. exd5 Qb6 19. bxa6 Rd8 20. Nc2 Bxa6

**Main line after c4** The main line of AlphaZero after 1. c4 in No-castling chess is:

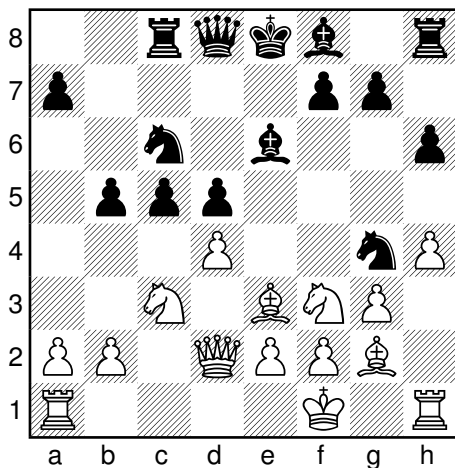
1. c4 (book) e5 2. Nc3 Nf6 3. Nf3 Nc6 4. d4 exd4 5. Nxd4 Bb4 6. Bf4 Bxc3 7. bxc3 d6 8. g3 Ne5 9. Bg2 Kf8 10. c5 Ng6 11. Be3 dxc5 12. Nb3 Qe8 13. h4 h5 14. Bxc5+ Kg8 15. Qc2 a5 16. Bd4 Ne7 17. Bxf6 gxf6 18. a4 Kg7 19. Nd4 Rb8 20. Kf1 Bd7



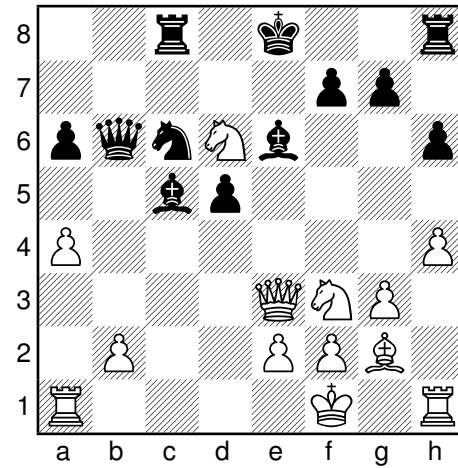
#### B.1.4. INSTRUCTIVE GAMES

**Game AZ-1: AlphaZero No-castling vs AlphaZero No-castling** The first ten moves for White and Black have been sampled randomly from AlphaZero’s opening “book”, with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

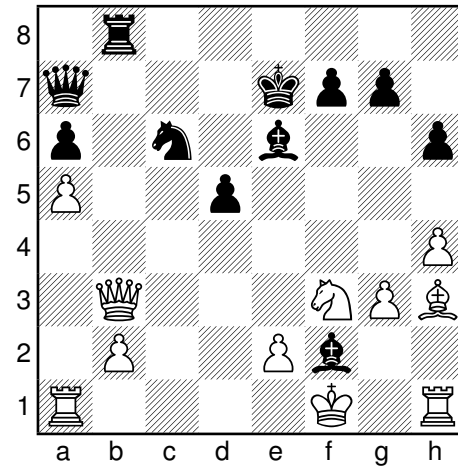
1. d4 d5 2. c4 e6 3. Nc3 c5 4. cxd5 exd5 5. Nf3 Nf6 6. g3 Nc6 7. Bg2 h6 8. h4 Be6 9. Kf1 Rc8 10. Be3 Ng4 11. Qd2 b5



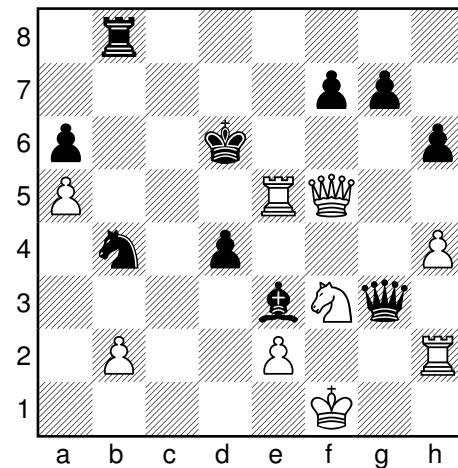
12. Nxb5 Qb6 13. a4 a6 14. dxc5 Nxe3+ 15. Qxe3 Bxc5 16. Nd6+



16... Ke7 17. Nxc8+ Rxc8 18. a5 Qa7 19. Qb3 Bxf2 20. Bh3 Rb8

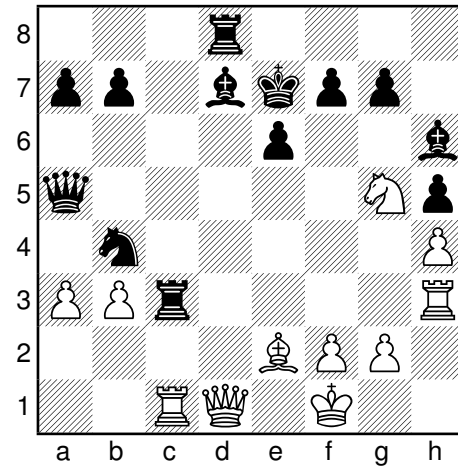


21. Qa3+ Bc5 22. Qd3 Nb4 23. Qh7 Qd7 24. Bxe6 Qxe6 25. Rc1 Be3 26. Rc3 d4 27. Rc5 Kd6 28. Re5 Qg4 29. Qf5 Qxg3 30. Rh2



30... Qg6 31. Rg2 Qxf5 32. Rxf5 Ke6 33. Rc5 Kd6 34. Rf5 Ke6 35. Re5+ Kf6 36. h5 Rc8 37. Rg4 Rc1+ 38. Kg2 Nc6

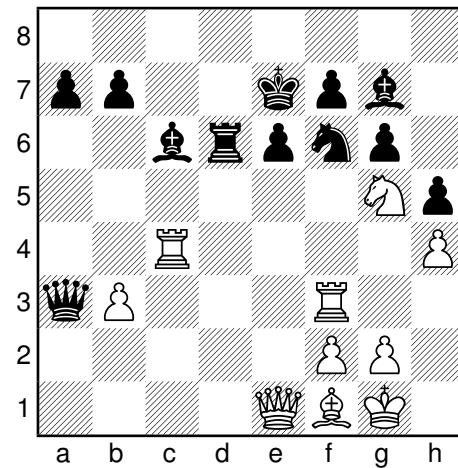
39. Re8 Rc2 40. Kh3 Rc5 41. Kh4 Bf2+ 42. Kh3 Be3  
 43. Rh4 Rxa5 44. Kg3 Ra1 45. Rhe4 Kf5 46. Nh4+ Kf6  
 47. Rc8 Ne7 48. Re8 Nc6 49. Nf3 Kf5 50. b3 Rb1 51. Nh4+  
 Kf6 52. Ra8 Ra1 53. Kh3 Ne5 54. Re8 Rh1+ 55. Kg3  
 Nc6 56. Ra8 Ra1 57. b4 Nxb4 58. Rd8 Rg1+ 59. Kh3  
 Rh1+ 60. Kg2 Rg1+ 61. Kh3 Rh1+ 62. Kg3 Rg1+ 63. Ng2  
 Nc2 64. Kh2 Rf1 65. Rc8 Kf5 66. Nxe3+ Nxe3 67. Rxd4  
 Kg5 68. Rc5+ f5 69. Kg3 Kxh5 70. Re4 Ng4 71. Kg2  
 Rf2+ 72. Kg1 Nf6 73. Re7 Rf4 74. Rxg7 Ng4 75. Rc3 Kh4  
 76. Re7 Kg5 77. Ra3 h5 78. Rxa6 Rb4 79. Ra5 h4 80. Ra3  
 Nf6 81. Rg7+ Kh5 82. Rf7 Kg5 83. Rg7+ Kh5 84. Kh1  
 Rb2 85. Ra5 Kh6 86. Rg2 Rb1+ 87. Rg1 Rxg1+ 88. Kxg1  
 Kg5 89. Ra8 Ne4 90. Kg2 Kg4 91. Ra4 Kg5 92. Rb4 Kg4  
 93. Rd4 Kh5 94. Kh3 Ng5+ 95. Kh2 Ne4 96. Kg2 Kg4  
 97. Rb4 Kg5 98. Kf3 Nd2+ 99. Ke3 Ne4 100. Rb7 Kg4  
 101. Rg7+ Ng5 102. Rg8 h3 103. Kf2 f4 **1/2-1/2**



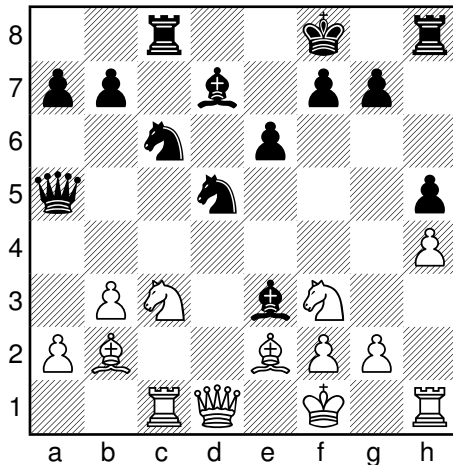
20. Rhxc3 Bc6 21. Qe1 Qxa3 22. Kg1 g6 23. Bf1 Bg7  
 24. Re3 Rd6 25. Rc4 Nd5 26. Rf3 Nf6

**Game AZ-2: AlphaZero No-castling vs AlphaZero No-castling** The first ten moves for White and Black have been sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

1. Nf3 e6 2. d4 d5 3. e3 c5 4. b3 h5 5. dxc5 Bxc5 6. Bb2  
 Kf8 7. c4 Nf6 8. h4 Bd7 9. Nc3 Nc6 10. Be2 Rc8 11. Rc1  
 Qa5 12. cxd5 Nxd5 13. Kf1 Bxe3



27. Rff4 Qxb3 28. Be2 Rd7 29. Rc1 Qb2 30. Bf3 Bxf3  
 31. Rxf3 Qd2 32. Qf1 Qd5 33. Qe1 Qd2 34. Qf1 Qd5  
 35. Qe2 Bh6 36. Qb2 Bxg5 37. hxg5 Ng4 38. Re1 Qd2  
 39. Qa3+ Rd6 40. Rb1 Kf8 41. g3 Ne5 42. Rf4 Qd3 43. Qxd3  
 Nxd3 44. Ra4 Rd5 45. Rxb7 Rxg5 46. Ra3 Rd5 47. Rbxa7  
 Ne5 48. R7a5 Rd1+ 49. Kg2 Ng4 50. Ra1 Rd4 51. R5a4  
 Rd3 52. R4a3 Rd4 53. Ra4 Rd3 54. R4a3 Rd2 55. R3a2  
 Rd7 56. Ra7 Rd6 57. R7a6 Rd7 58. Ra7 Rd6 59. R7a6 Rd5  
 60. R6a5 Rd2 61. R5a2 Rd5 62. Ra5 Rd2 63. R5a2 **1/2-1/2**

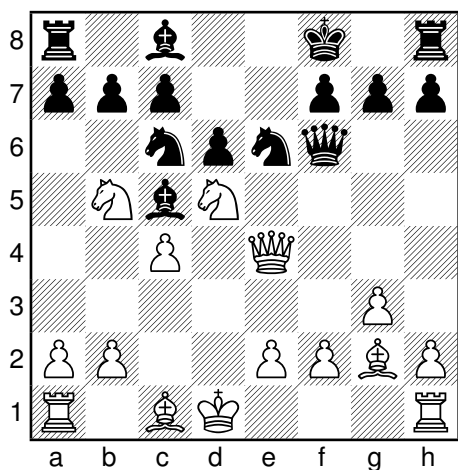


14. Rc2 Bh6 15. Ng5 Ncb4 16. Rc1 Ke7 17. Rh3 Rhd8  
 18. a3 Nxc3 19. Bxc3 Rxc3

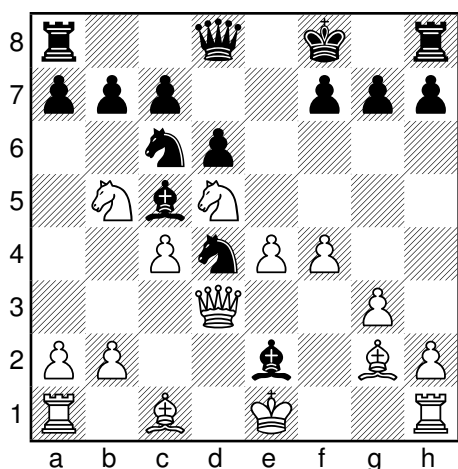
**Game AZ-3: AlphaZero No-castling vs AlphaZero No-castling** The first ten moves for White and Black have been sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

1. c4 e5 2. Nc3 Nf6 3. Nf3 Nc6 4. d4 exd4 5. Nxd4 Bb4  
 6. g3 Ne4 7. Qd3 Nc5 8. Qe3+ Kf8 9. Bg2 Qf6 10. Ndb5  
 Ne6 11. Kd1 Bc5 12. Qe4 d6 13. Nd5

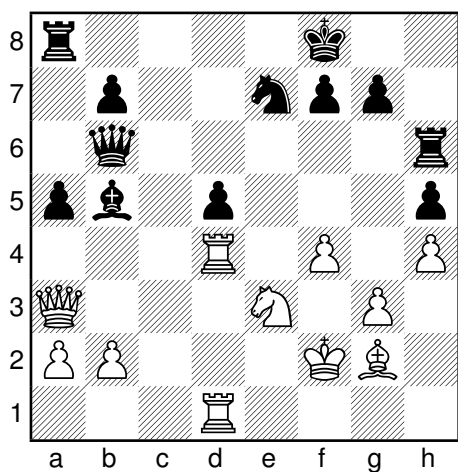




13... Qd8 14. f4 Ned4 15. Qd3 Bf5 16. e4 Bg4+ 17. Ke1 Be2



18. Qc3 Nxb5 19. cxb5 Bxb5 20. Be3 Bxe3 21. Nxe3 Ne7 22. Kf2 h5 23. h4 Rh6 24. Rac1 c6 25. Rhd1 Qb6 26. Rd4 a5 27. Rcd1 d5 28. exd5 cxd5 29. Qa3

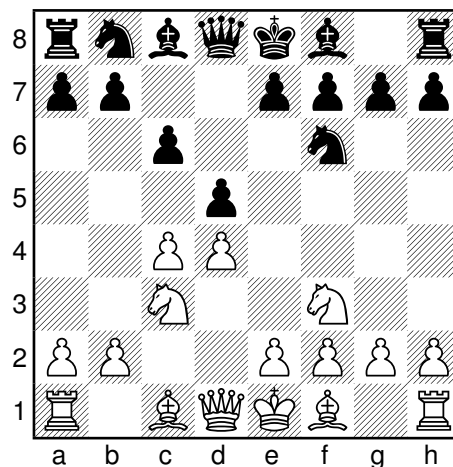


The game soon ended in a draw.

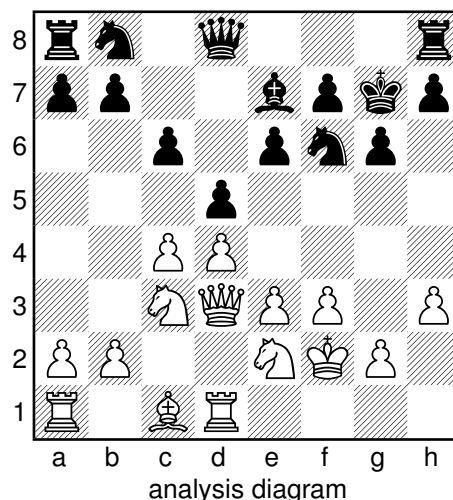
### B.1.5. HUMAN GAMES

Here we take a brief look at a couple of recently played blitz games between professional chess players from the tournament that took place in Chennai in January 2020 (Shah, 2020). We focus on new motifs in the opening stage of the game, and show how these might be counter-intuitive compared to similar patterns in classical chess.

**Game H-1: Arjun, Kalyan (2477) vs D. Gukesh (2522) (blitz)** 1. d4 d5 2. c4 c6 3. Nc3 Nf6 4. Nf3



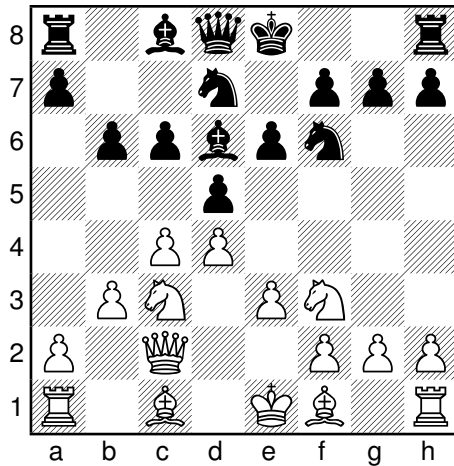
Interestingly, even at an early stage we can see an example of a difference in patterns that originate in Classical chess and those that arise in No-castling chess. The positioning of the knight on f3 is very natural, but is in fact an imprecision. AlphaZero prefers keeping the option open of playing the pawn to f3 instead, in order to tuck the king away to safety. It gives the following line as its favored continuation: 4. e3 Bf5 5. Bd3 g6 6. h3 e6 7. Nge2 Be7 8. f3 Bxd3 9. Qxd3 Kf8 10. Kf2 Bg7 11. Rd1.



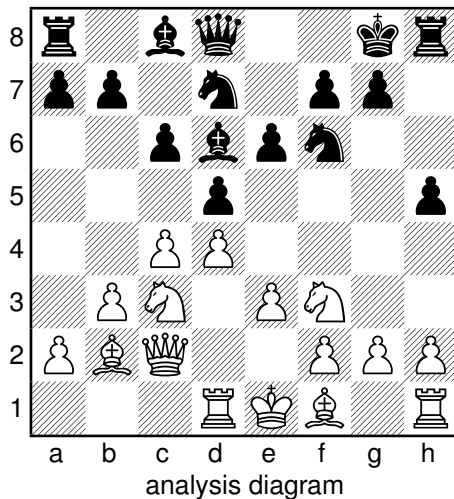
analysis diagram

1/2–1/2 Yet, 4. Nf3 was played in the game, which continued:

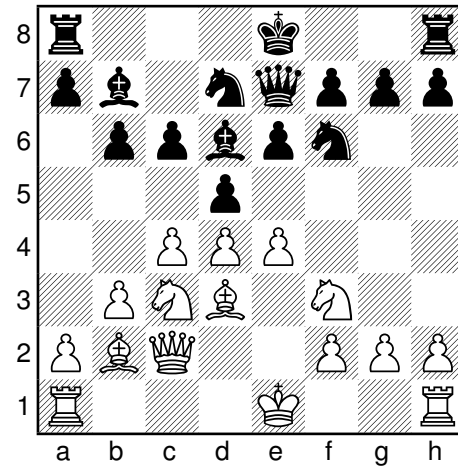
4... e6 5. e3 Nbd7 6. Qc2 Bd6 7. b3 b6



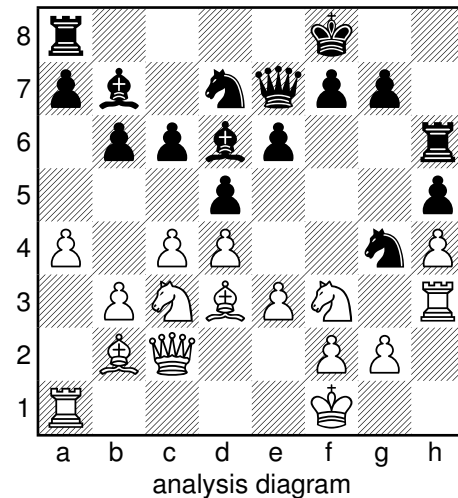
Here AlphaZero suggests that it was instead time to move the king to safety. Deciding on when exactly to initiate the evacuation of the king from the centre and choosing the best way of achieving it is one of the key motifs of No-castling chess. This decision is less clear than the decision to castle in Classical chess, due to a larger number of options and the fact that the sequence takes more moves that all need to be staged accordingly. Instead of moving the pawn to b6, AlphaZero suggests the following instead: 7... h5 8. Bb2 Kf8 9. Rd1 Kg8.



Going back to the game continuation, after 7... b6 White has the upper hand. The game continued: 8. Bb2 Bb7 9. Bd3 Qe7 10. e4

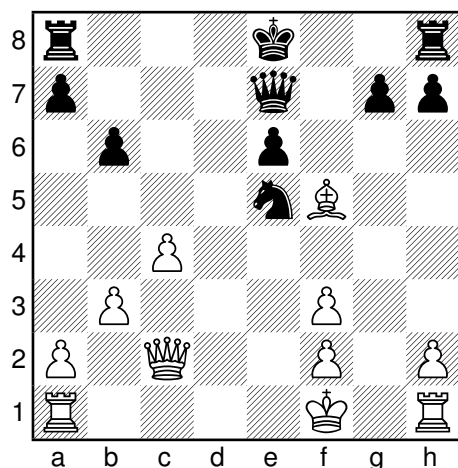


This is another example of mistiming the evacuation of the king. Instead of playing 10. e4, it was the right time to move the king to safety instead, retaining a large plus for White after: 10. Kf1 Kf8 11. h4 h5 12. a4 Ng4 13. Rh3 Rh6



Going back to the position after 10. e4, the game continuation goes as follows:

10... dxe4 11. Nxe4 (Giving away the advantage. Recapturing with the bishop was correct, even though it might seem as otherwise counter-intuitive.) 11... Nxe4 12. Bxe4 f5. (This is looking bad for Black; 12... Nf6 is the preferred move.) 13. Bd3 c5 (At this point, AlphaZero assesses the position as winning for White.) 14. Kf1 (The advantage could have been kept with 14. d5.) 14... Bxf3 15. gxf3 cxd4 (15... Rf8 may have been equalizing) 16. Bxd4 (Gives the advantage to Black. White ought to have captured on f5 instead. The right way to respond to the game move would have been 16... Qh4.) 16... Be5 17. Bxe5 Nxe5 18. Bxf5

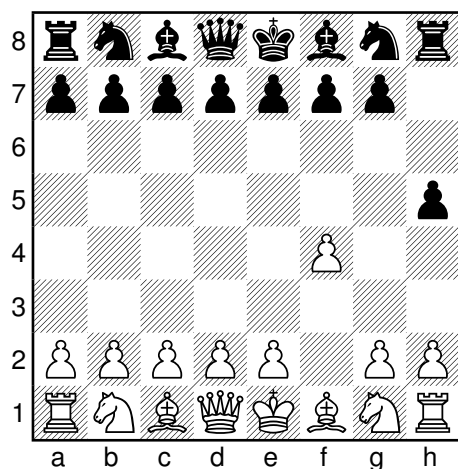


A brilliant piece sacrifice.

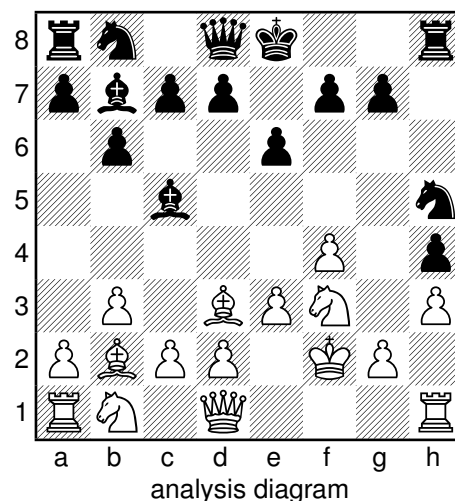
18... exf5 19. Re1 Kd8 20. Qxf5 (20. Qd2+ may have been stronger) 20... Re8 21. f4 Qb7 22. Rg1 Ng6 (The final mistake, it appears that 22. Nf7 might hold) 23. Rd1+ Ke7 24. Rg3 Qh1+ 25. Ke2 Qe4+ 26. Re3 Qxe3+ 27. fxe3 Rad8 28. Rxd8 Rxd8 29. Qe4+ Kf8 30. Qb7 **1-0**

#### Game H-2: Gelfand, Boris vs Kramnik, Vladimir (blitz)

1. f4 h5 Already Kramnik demonstrates a motif that is quite strong in no-castling chess, pushing one of the side pawns early.

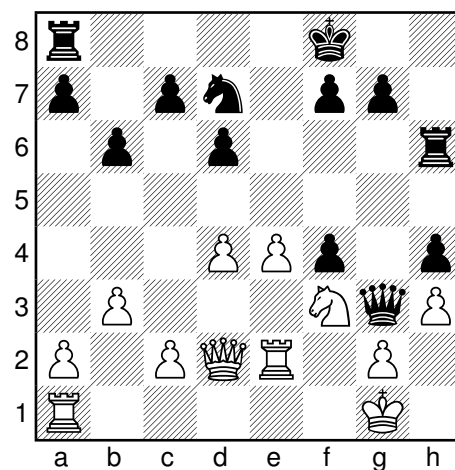


2. Nf3 e6 3. e3 Nf6 4. b3 (Interestingly, AlphaZero doesn't like this very normal-looking move, giving Black a slight plus after 4... c5 5. Bb2 Be7 6. Be2 d5 7. Rf1 Kf8 8. Kf2 Nc6 9. Kg1 Kg8 10. a4 Bd7.) 4... b6 5. Bb2 Bb7 6. Bd3 (5. Be2 might have been better.) 6... h4 (Not the most precise, according to AlphaZero, suggesting that 6... c5 7. Rf1 Be7 8. Kf2 h4 9. Ng5 Kf8 10. Kg1 Rh6 11. Be2 Nc6 was still slightly better for Black.) 7. h3 (This turns out to be the wrong reaction, giving the advantage back to Black again.) 7... Nh5 8. Kf2 Be7 (Here, there was an opportunity to play 8... Bc5 instead:



which would have kept a big plus for Black.)

9. Re1 Bf6 10. Bxf6 (10. Nc3) 10... Qxf6 (10... gxf6 was the better recapture) 11. Nc3 Ng3 12. Kg1 d6 (12... Ke7 was the correct plan) 13. Ng5 Nd7 14. Nce4 Nxe4 15. Bxe4 Bxe4 16. Nxe4 Qg6 17. Ng5 Ke7 18. e4 (18. Qe2) 18... e5 19. d4 exf4 20. Nf3 Kf8 21. Qd2 Qg3 22. Re2 Rh6



23. Rf1 (Black gains the upper hand.) 23... Re6 24. Nh2 (A mistake, 24. e5 was required.) 24... Rae8 25. Rxf4 Nf6 26. e5 dxe5 27. Rf3 (Another mistake, 27. Rxe5 was correct.) 27... Qg6 28. d5 (Taking on e5 was still a better continuation.) 28... R6e7 29. c4 e4 30. Rc3 Nh5 31. Nf1 Kg8 32. Qe1 Nf4 33. Rd2 e3 34. Rxe3 Rxe3 35. Nxe3 Qe4 **0-1**

#### B.2. No-castling (10)

In the No-castling (10) variant of chess, castling is only allowed from move 11 onwards, both for the first and the second player.

## B.2.1. MOTIVATION

When it comes to limit the impact of castling on the game, it is possible to consider different types of partial limitations, the easiest of which is disallowing it for a fixed number of opening moves. In this variation, we have explored the impact of disallowing castling for the first 10 moves, but any other number could have been used instead. Each choice leads to a slightly different body of opening theory, as particular lines either become viable or stop being viable under different circumstances.

## B.2.2. ASSESSMENT

The assessment of the No-castling (10) chess variant, as provided by Vladimir Kramnik:

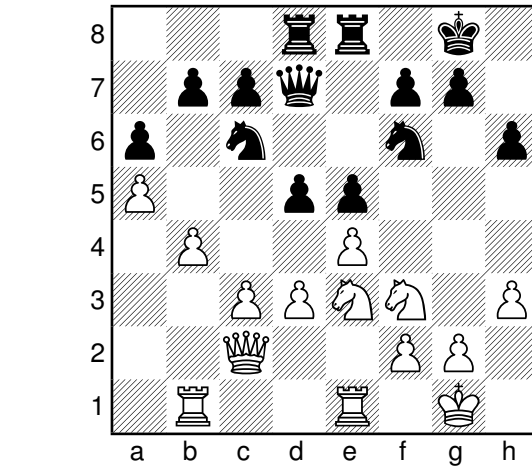
“The main purpose of the partial restriction to castling, as a hypothetical adjustment to the rules of chess, would be to sidestep opening theory. As such, it is aimed at professional chess as an option to potentially consider. The game itself does not change in other meaningful ways, and AlphaZero usually aims at playing slower lines where castling does indeed take place after the first 10 moves. This makes sense, given that castling is a fast and powerful move, so aiming to take advantage of it if available makes for a good approach. Yet, the slowing down of the game could as a side-effect lead to an increased number of draws. Another disadvantage is the need to count and keep track of the move number when considering variations.”

## B.2.3. MAIN LINES

Here we discuss “main lines” of AlphaZero under No-castling (10) chess, when playing with roughly one minute per move from a particular fixed first move. Note that these are not purely deterministic, and each of the given lines is merely one of several highly promising and likely options. Here we give the first 20 moves in each of the main lines, regardless of the position.

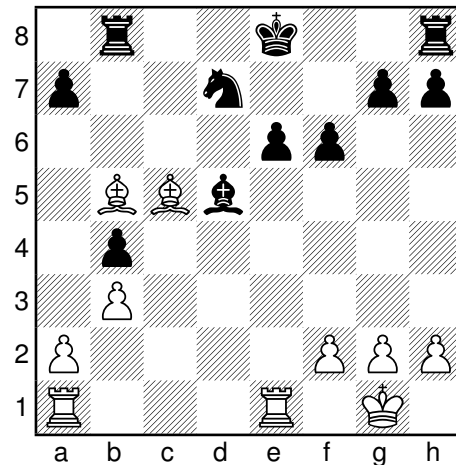
**Main line after e4** The main line of AlphaZero after 1. e4 in No-castling (10) chess is:

1. e4 (book) e5 2. Bc4 Nc6 3. Nf3 Nf6 4. Qe2 Bc5 5. c3 Qe7 6. b4 Bb6 7. a4 a6 8. a5 Ba7 9. d3 d6 10. Na3 Be6 11. Nc2 O-O 12. O-O h6 13. Be3 Qd7 14. Bxa7 Nxa7 15. Rfe1 Nc6 16. h3 Rfe8 17. Bxe6 Qxe6 18. Ne3 d5 19. Qc2 Rad8 20. Rab1 Qd7



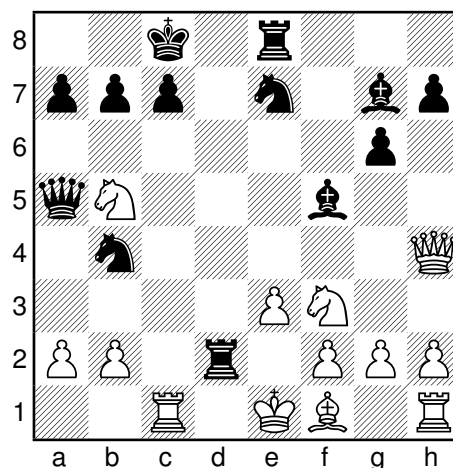
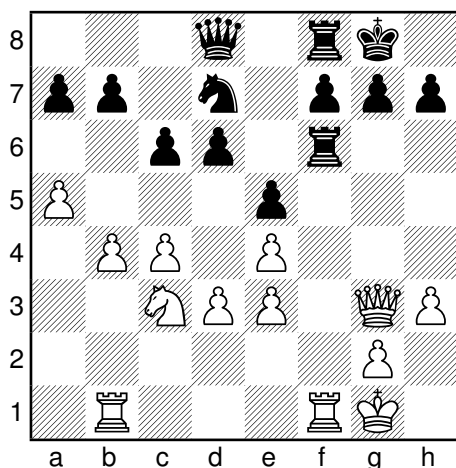
**Main line after d4** The main line of AlphaZero after 1. d4 in No-castling (10) chess is:

1. d4 (book) d5 2. Nf3 Nf6 3. c4 dxc4 4. Nc3 e6 5. Qa4+ c6 6. Qxc4 b5 7. Qd3 Bb7 8. e4 b4 9. Na4 Qa5 10. b3 c5 11. Ne5 cxd4 12. Qb5+ Qxb5 13. Bxb5+ Nfd7 14. Bb2 f6 15. Nxd7 Nxd7 16. Bxd4 Bxe4 17. O-O Bd6 18. Rfe1 Bd5 19. Nc5 Bxc5 20. Bxc5 Rb8



**Main line after c4** The main line of AlphaZero after 1. c4 in No-castling (10) chess is:

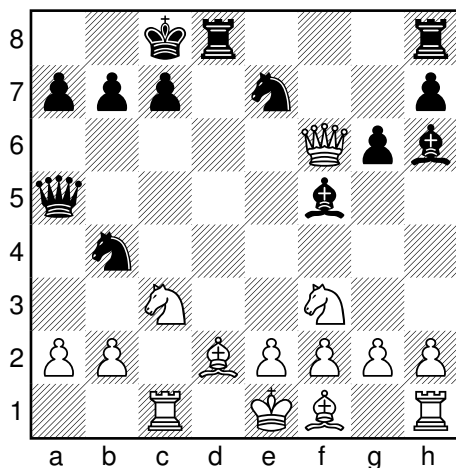
1. c4 (book) e5 2. Nc3 Nf6 3. Nf3 Nc6 4. e4 Bb4 5. d3 d6 6. a3 Bc5 7. b4 Bb6 8. Be3 Bg4 9. Be2 Bxf3 10. Bxf3 Nd4 11. Na4 Nxf3+ 12. Qxf3 Bxe3 13. fxe3 Nd7 14. O-O O-O 15. Nc3 c6 16. h3 Qb6 17. Rab1 Rae8 18. a4 Re6 19. a5 Qd8 20. Qg3 Rf6



## B.2.4. INSTRUCTIVE GAMES

**Game AZ-4: AlphaZero No-castling (10) vs AlphaZero No-castling (10)** The first ten moves for White and Black have been sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

1. c4 e5 2. d4 exd4 3. Qxd4 Nc6 4. Qe3+ Nge7 5. Nf3 d5 6. exd5 Qxd5 7. Nc3 Qa5 8. Qg5 Bf5 9. Bd2 f6 10. Qh5+ g6 11. Qh4 Nb4 12. Rc1 O-O-O 13. Qxf6 Bh6

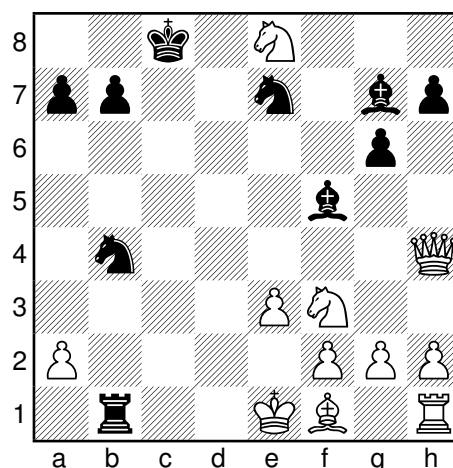


A stunning move, offering up a piece on h6. Accepting would be disastrous for White, as Black pieces mobilise quickly via Ned5. The h8 rook can also potentially come to e8, and this justifies the material investment.

14. e3 Rhe8 15. Qh4 Bg7 16. Nb5 Rxd2

The fireworks continue...

17. Rxc7+ Qxc7 18. Nxc7 Rxb2 19. Nxe8 Rb1+



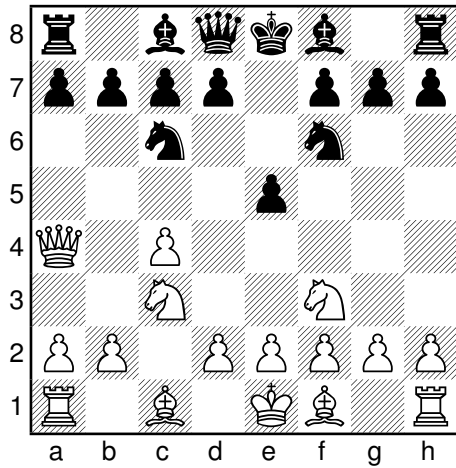
Leading to a draw by perpetual check.

1/2-1/2

The next game is less tactically rich, but rather interesting from the perspective of showcasing differences in opening play and the overall approach, when castling is not possible in the first ten moves.

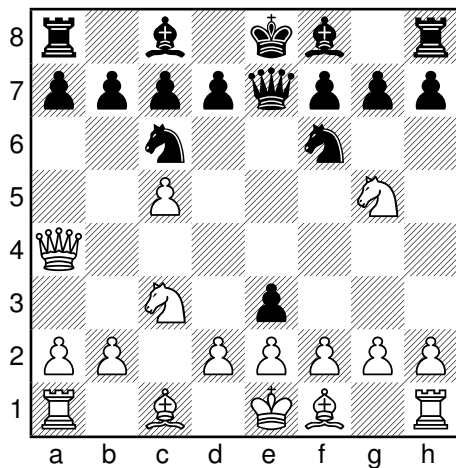
**Game AZ-5: AlphaZero No-castling (10) vs AlphaZero No-castling (10)** The first ten moves for White and Black have been sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

1. c4 e5 2. Nc3 Nf6 3. Nf3 Nc6 4. Qa4

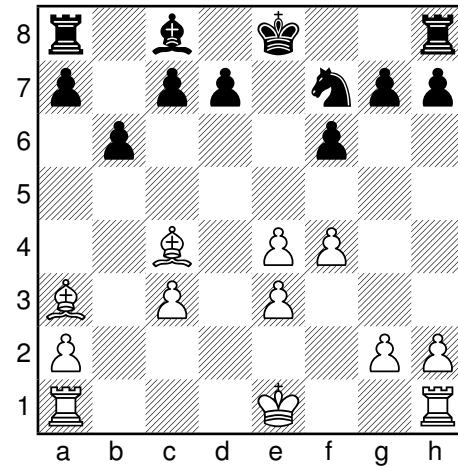


This is a slightly unusual move, showcasing that the style of play in this variation of chess involves opting for moves that do not necessarily achieve as much immediately and are somewhat less direct, potentially trying to wait for the right time to castle, when possible. In this game, however, castling does not end up being critical.

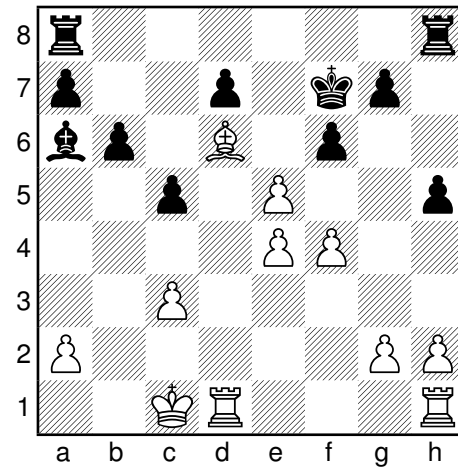
4... e4 5. Ng5 Qe7 6. c5 e3



7. dxe3 Qxc5 8. Nge4 Nxe4 9. Qxe4+ Qe5 10. Qxe5 Nxe5 11. e4 Bb4 12. f4 Nc4 13. e3 Nd6 14. Bd3 Bxc3 15. bxc3 f6 16. Ba3 Nf7 17. Bc4 b6



18. Bd5 c6 19. Bb3 Rb8 20. Bxf7+ Kxf7 21. Bd6 Ra8 22. e5 c5 23. O-O-O Ba6 24. e4 h5



And the game eventually ended in a draw.

1/2–1/2

### B.3. Pawn one square

#### B.3.1. MOTIVATION

Restricting the pawn movement to one square only is interesting to consider, as the double-move from the second (or seventh rank) seems like a “special case” and an exception from the rule that pawns otherwise only move by one square. In addition, slowing down the game could make it more strategic and less forcing.

#### B.3.2. ASSESSMENT

The assessment of the Pawn one square chess variant, as provided by Vladimir Kramnik:

“ The basic rules and patterns are still mostly the same as in classical chess, but the opening theory changes and becomes completely different. Intuitively it feels that it ought to be more difficult



for White to gain a lasting opening advantage and convert it into a win, but since new opening theory would first need to be developed, this would not pertain to human play at first. In most AlphaZero games one can notice the rather typical middlegame positions arise after the opening phase.

This variation of chess can be a good pedagogical tool when teaching and practicing slow, strategic play and learning about how to set up and commit to pawn structures. Since the pawns are unable to advance very fast, many attacking ideas that involve rapid pawn advances are no longer relevant, and the play is instead much slower and ultimately more positional. Additionally, this variation of chess could simply be of interest for those wishing for an easy way of side-stepping opening theory.

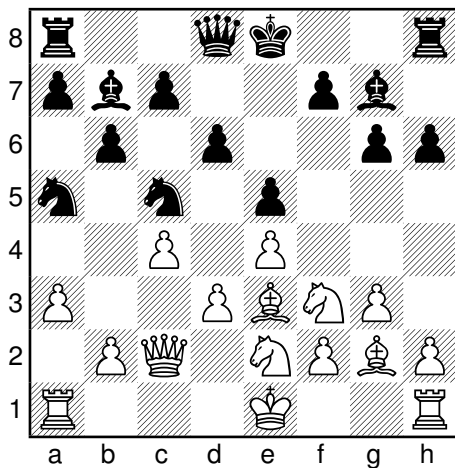
”

### B.3.3. MAIN LINES

Here we discuss “main lines” of AlphaZero under Pawn one square chess, when playing with roughly one minute per move from a particular fixed first move. Note that these are not purely deterministic, and each of the given lines is merely one of several highly promising and likely options. Here we give the first 20 moves in each of the main lines, regardless of the position.

**Main line after e3** The main line of AlphaZero after 1. e3 in Pawn one square chess is:

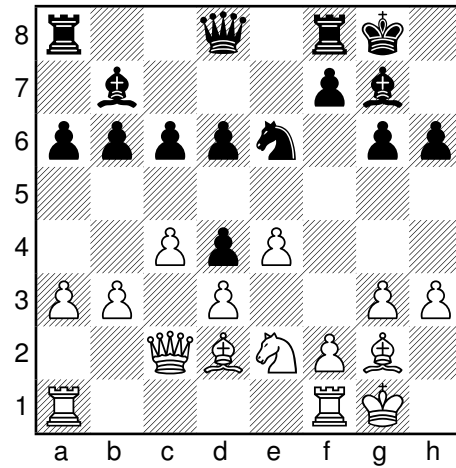
1. e3 (book) Nf6 2. d3 d6 3. Nf3 h6 4. e4 b6 5. c3 Bb7 6. Qc2 e6 7. c4 e5 8. g3 g6 9. Nc3 Bg7 10. Bg2 Nc6 11. Be3 Nd7 12. Ne2 Nc5 13. a3 Na5



An instructive position, as it looks optically like Black is blundering material. In this variation of chess, however, b2-b4 is not a legal move, because pawns can only move

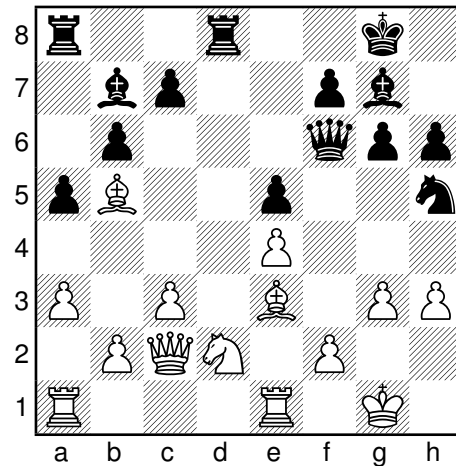
one square. This justifies the move sequence.

14. Nd2 Nc6 15. b3 a6 16. Nf3 Ne6 17. h3 O-O 18. O-O Ncd4 19. Nfxd4 exd4 20. Bd2 c6



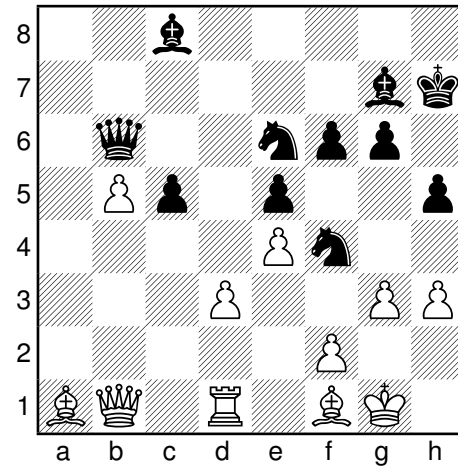
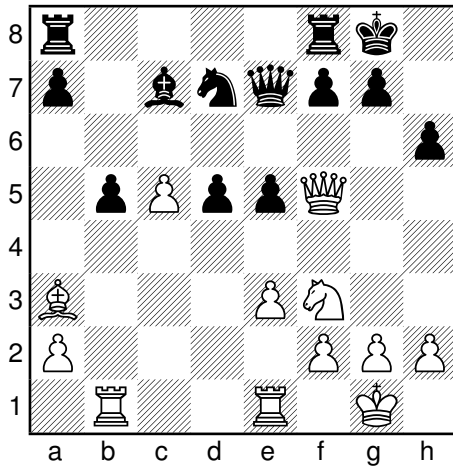
**Main line after d3** The main line of AlphaZero after 1. d3 in Pawn one square chess is:

1. d3 (book) d6 2. e3 Nf6 3. Nd2 e6 4. Ngf3 Nbd7 5. d4 g6 6. Bd3 Bg7 7. O-O O-O 8. h3 e5 9. c3 Re8 10. e4 b6 11. Re1 Bb7 12. a3 a6 13. Qc2 h6 14. Nb3 a5 15. dxe5 Nxe5 16. Nxe5 dxe5 17. Be3 Nh5 18. Bb5 Rf8 19. Nd2 Qf6 20. g3 Rfd8



**Main line after c3** The main line of AlphaZero after 1. c3 in Pawn one square chess is:

1. c3 (book) d6 2. d3 Nf6 3. Nf3 h6 4. d4 Bf5 5. c4 e6 6. Nc3 c6 7. e3 d5 8. Bd3 dxc4 9. Bxc4 Bd6 10. O-O Nbd7 11. Re1 Ne4 12. Bd3 Nxc3 13. bxc3 Bxd3 14. Qxd3 Qe7 15. c4 e5 16. Qf5 O-O 17. Rb1 b6 18. c5 Bc7 19. Ba3 b5 20. d5 cxd5

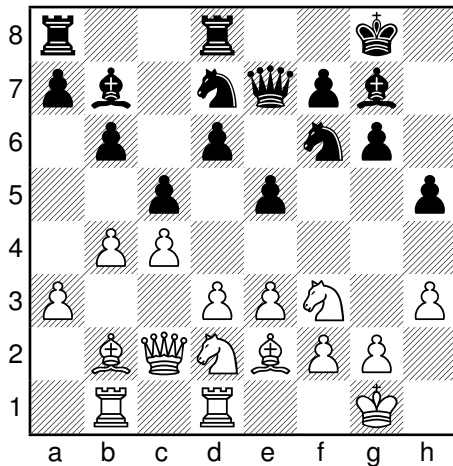


#### B.3.4. INSTRUCTIVE GAMES

Here we present some examples of AlphaZero play in Pawn one square chess.

**Game AZ-6: AlphaZero Pawn One Square vs AlphaZero Pawn One Square** The first ten moves for White and Black have been sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

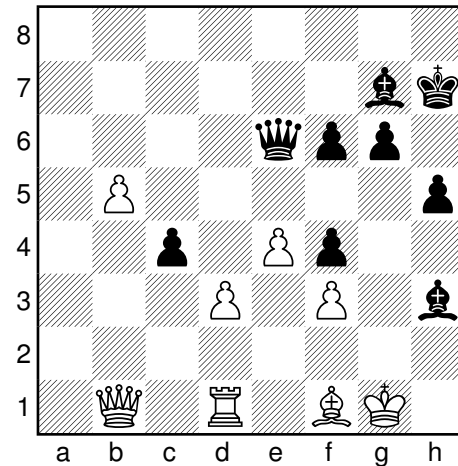
1. d3 Nf6 2. Nd2 d6 3. e3 e6 4. Ngf3 g6 5. h3 Bg7 6. c3 O-O 7. c4 Nbd7 8. Rb1 e5 9. b3 c6 10. Bb2 Qe7 11. Be2 b6 12. b4 Bb7 13. a3 h6 14. O-O h5 15. Qc2 Rfd8 16. Rfd1 c5



Here we have a rather normal middlegame position. The game continued:

17. Ne4 Rac8 18. b5 d5 19. cxd5 Nxd5 20. Nfd2 f6 21. Nc4 Nf8 22. a4 Kh7 23. a5 Ne6 24. Ra1 Nb4 25. Qb1 Rb8 26. axb6 axb6 27. Nc3 Qe8 28. Ra7 Nc7 29. Na3 Rd7 30. Ba1 Nbd5 31. Na4 Ne6 32. e4 Ndf4 33. Bf1 Bc8 34. Rxd7 Qxd7 35. Nc4 Qa7 36. Naxb6 Rxb6 37. Nxb6 Qxb6 38. g3

38... c4 39. gxf4 Nxf4 40. Bc3 Bxh3 41. Bd2 Qe6 42. Bxf4 exf4 43. f3

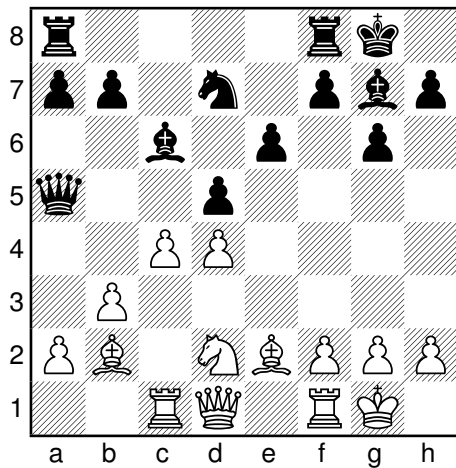


43... Bg4 44. Bg2 Bxf3 45. Bxf3 Qh3 46. dxc4 Qxf3 47. Qd3 Qg4+ 48. Kf2 Qh4+ 49. Ke2 Qh2+ 50. Kf1 Qh1+ 51. Kf2 Qh4+ 52. Ke2 Qh2+ 53. Ke1 Bf8 54. Qf3 Bc5 55. Kf1 Qg1+ 56. Ke2 Qh2+ 57. Kf1 Qg1+ 58. Ke2 Qh2+ 59. Kf1

1/2-1/2

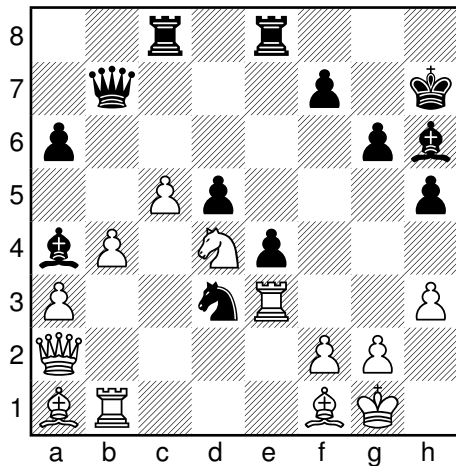
**Game AZ-7: AlphaZero Pawn One Square vs AlphaZero Pawn One Square** The first ten moves for White and Black have been sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

1. d3 c6 2. e3 d6 3. c3 g6 4. d4 Nf6 5. Nf3 Bf5 6. Be2 e6 7. O-O Nbd7 8. c4 Bg7 9. b3 O-O 10. Ba3 Ne4 11. Nfd2 c5 12. Nxe4 Bxe4 13. Nd2 Bc6 14. Rc1 Qa5 15. Bb2 cxd4 16. exd4 d5



This is a very normal-looking position, and one would be hard-pressed to guess that it originated from a different variation of chess, as it looks pretty “classical”.

17. Re1 Rfe8 18. h3 Bh6 19. Bc3 Qc7 20. Bf1 b6 21. Bb2 Qb7 22. a3 Rac8 23. Rc2 Bg7 24. Qc1 Bh6 25. Qd1 Bg7 26. Qc1 h6 27. c5 bxc5 28. dxc5 e5 29. Qb1 h5 30. Qa2 a6 31. b4 Ba4 32. Rcc1 Bh6 33. Ba1 e4 34. Rb1 Ne5 35. Nb3 Kh7 36. Nd4 Nd3 37. Re3



A very instructive position, reminiscent of a famous classical game between Petrosian and Reshevsky from Zurich in 1953, where Petrosian was playing Black. The positional exchange sacrifice allows White easy play on the dark squares.

37... Bxe3 38. fxe3 f6 39. Be2 Rc7 40. Rf1 Rf7 41. Qd2 Ne5 42. Qe1 Bb5 43. Nxb5 axb5 44. a4 Nd3 45. Qh4 bxa4 46. Bxh5 Re5 47. Be2+ Kg7 48. Qg3 Qc7 49. Bxe5 Qxe5 50. Qxe5 fxe5 51. c6 Rxf1+ 52. Bxf1 a3 53. c7 a2 54. c8=Q a1=Q 55. Qb7+ Kh6 56. Qxd5 Qe1 57. Qf7 Qxb4 58. Qa2 Qc5 59. Qd2 Nb4 60. Kf2 Nd5 61. g3 Qf8+ 62. Kg1 Qc5 63. Kf2 Qf8+ 64. Ke1 Nb4 65. Bc4 Kh7 66. Qd7+ Kh6 67. Qd2 Kg7 68. Qf2 Qe7 69. Kf1 Nd3 70. Qe2 Qf6+ 71. Kg2 Qc6 72. Bb3 Qc5 73. h4 Qc1 74. Kh2 Ne1 75. Bd1

Nf3+ 76. Kg2 Kh6 77. Kf1 g5 78. hxg5 Kxg5 79. Kf2 Qd2 80. Bc2 Qxe2+ 81. Kxe2 Kf5 82. Kf2 Ng5 83. Kg2 Nh7 84. Kh3 Nf6 85. Bb3 Kg5 86. Be6 Kh5 87. Bb3 Kg5 88. Be6 Kh5 89. g4+ Kg5 90. Kg3 Nh7 91. Kh3 Nf6 92. Kg3 Nh7 93. Kh3 Nf6  
1/2–1/2

#### B.4. Stalemate=win

In this variation of chess, achieving a stalemate position is considered a win for the attacking side, rather than a draw.

##### B.4.1. MOTIVATION

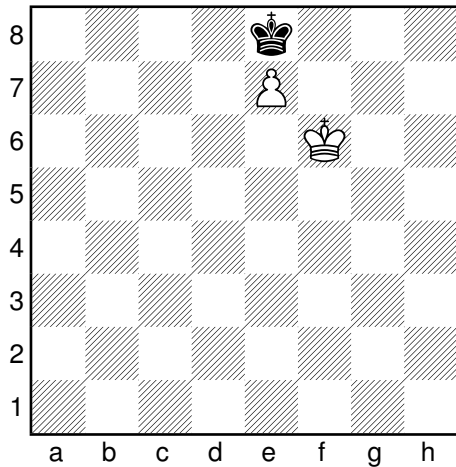
The stalemate rule in classical chess allows for additional drawing resources for the defending side, and has been a subject of debate, especially when considering ways of making the game potentially more decisive. Yet, due to its potential effect on endgames, it was unclear whether such a rule would also discourage some attacking ideas that involve material sacrifices, if being down material in endgames ends up being more dangerous and less likely to lead to a draw than in classical chess.

##### B.4.2. ASSESSMENT

The assessment of the Stalemate=win chess variant, as provided by Vladimir Kramnik:

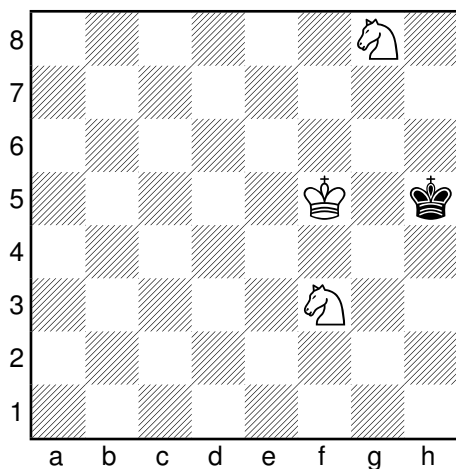
“I was at first somewhat surprised that the decisive game percentage in this variation was roughly equal to that of classical chess, with similar levels of performance for White and Black. I was personally expecting the change to lead to more decisive games and a higher winning percentage for White.

It seems that the openings and the middlegame remain very similar to regular chess, with very few exceptions, but that there is a significant difference in endgame play since some basic endgame like K+P vs K are already winning instead of being drawn depending on the position.



In the position above, with White to move, in classical chess the position would be a draw due to stalemate after Ke6. Yet, the same move wins in this variation of chess, so the defending side needs to steer away from these types of endgames.

Similarly, the stalemates that arise in K+N+N vs K are now wins rather than draws, for example:



Looking at the games of AlphaZero, it seems that there are enough defensive resources in most midgame positions that certain types of inferior endgame positions, now possible under this rule change, could be avoided and defended. A strong player can in principle learn to navigate to these positions to take advantage of them, or find ways to escape them.

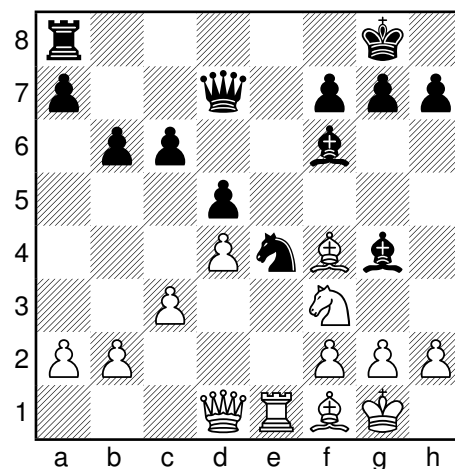
In terms of the anticipated effect on human play, I would still expect this rule change to lead to a higher percentage of wins in endgames where one side has a clear advantage, but probably not as much as one would otherwise have been expecting. This may be a nice variation of chess for chess enthusiasts with an interest in endgame patterns.

### B.4.3. MAIN LINES

Here we discuss “main lines” of AlphaZero under Stalemate=win chess, when playing with roughly one minute per move from a particular fixed first move. Note that these are not purely deterministic, and each of the given lines is merely one of several highly promising and likely options. Here we give the first 20 moves in each of the main lines, regardless of the position.

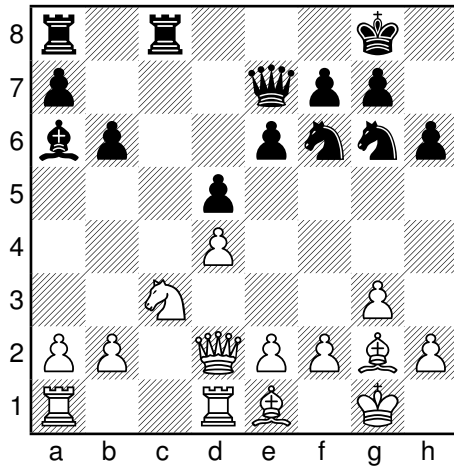
**Main line after e4** The main line of AlphaZero after 1. e4 in Stalemate=win chess is:

1. e4 (book) e5 2. Nf3 Nc6 3. Bb5 Nf6 4. O-O Nxe4 5. Re1 Nd6 6. Nxe5 Be7 7. Bf1 Nxe5 8. Rxe5 O-O 9. d4 Bf6 10. Re1 Re8 11. c3 Rxe1 12. Qxe1 Ne8 13. Bf4 d5 14. Nd2 Bf5 15. Qe2 Nd6 16. Re1 Qd7 17. Qd1 c6 18. Nb3 b6 19. Nd2 Ne4 20. Nf3 Bg4



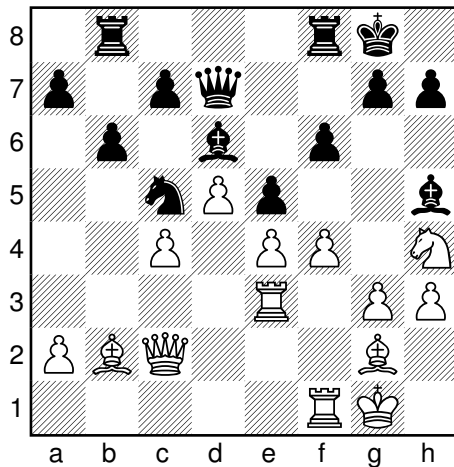
**Main line after d4** The main line of AlphaZero after 1. d4 in Stalemate=win chess is:

1. d4 (book) Nf6 2. c4 e6 3. g3 Bb4+ 4. Bd2 Be7 5. Qc2 c6 6. Bg2 d5 7. Nf3 b6 8. O-O O-O 9. Bf4 Bb7 10. Rd1 Nbd7 11. Ne5 Nh5 12. Bd2 Nhf6 13. cxd5 cxd5 14. Nc6 Qe8 15. Nxe7+ Qxe7 16. Qc7 Ba6 17. Nc3 Rfc8 18. Qf4 Nf8 19. Be1 h6 20. Qd2 Ng6

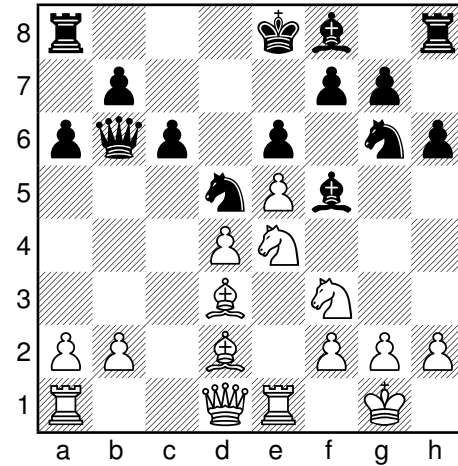


**Main line after c4** The main line of AlphaZero after 1. c4 in Stalemate=win chess is:

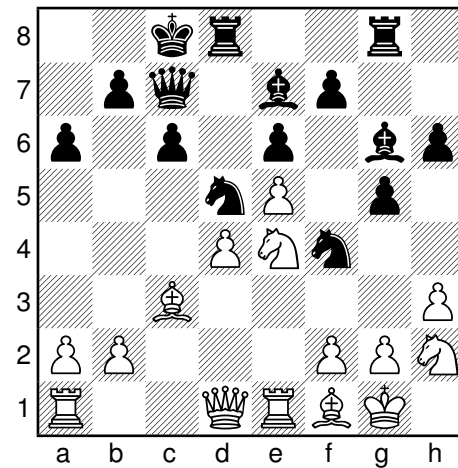
1. c4 (*book*) e5 2. g3 Nf6 3. Bg2 Bc5 4. d3 d5 5. cxd5 Nxd5 6. Nf3 Nc6 7. O-O O-O 8. Nc3 Nxc3 9. bxc3 Rb8 10. Bb2 Re8 11. d4 Bd6 12. e4 Bg4 13. h3 Bh5 14. Qc2 f6 15. d5 Na5 16. c4 b6 17. Nh4 Nb7 18. Rae1 Rf8 19. f4 Nc5 20. Re3 Qd7



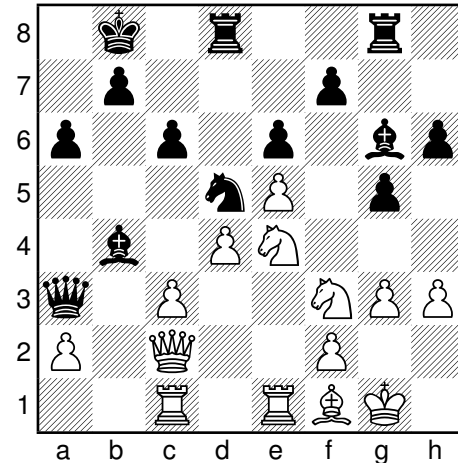
1. e4 c6 2. d4 d5 3. e5 Bf5 4. Be2 h6 5. Nf3 e6 6. O-O a6 7. c4 dxc4 8. Bxc4 Nd7 9. Bd3 Ne7 10. Nc3 Nb6 11. Re1 Nbd5 12. Ne4 Ng6 13. Bd2 Qb6



14. h3 Be7 15. Bc4 Ngf4 16. Bf1 g5 17. Ng3 Rg8 18. Bc3 Qc7 19. Nh2 Bg6 20. Ne4 O-O-O



21. g3 Nxc3 22. bxc3 Nd5 23. Rc1 Qa5 24. Qb3 Qa3 25. Qc2 Kb8 26. Nf3 Bb4



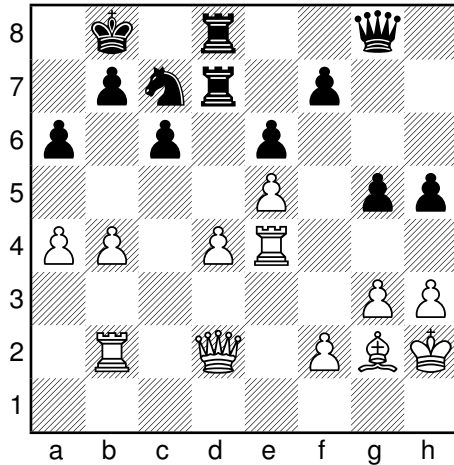
#### B.4.4. INSTRUCTIVE GAMES

The games in Stalemate=win chess are at the first glance almost indistinguishable from those of classical chess, as the lines are merely a subset of the lines otherwise playable and plausible under classical rules.

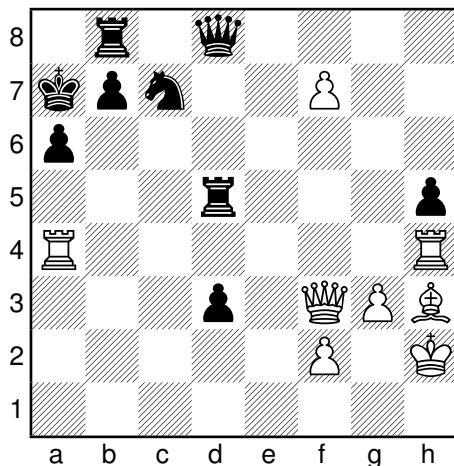
**Game AZ-8: AlphaZero Stalemate=win vs AlphaZero Stalemate=win<sup>6</sup>** The first ten moves for White and Black have been sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

<sup>6</sup>Game AZ-8 was labelled wrongly in arXiv:2009.04374v1, and this game replaces it.

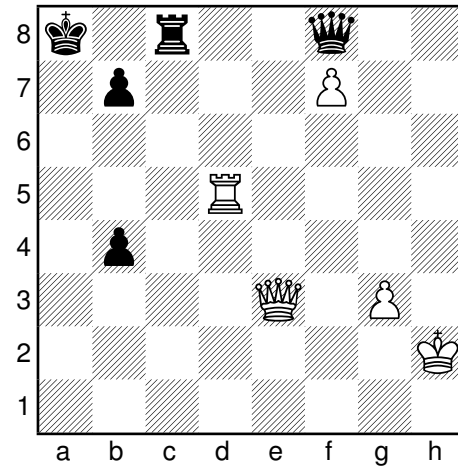
27. cxb4 Qxf3 28. Rb1 Bxe4 29. Rxe4 Qf5 30. Bg2 Nc7  
31. Rb2 Rd7 32. a4 Rgd8 33. Qc3 h5 34. Kh2 Qh7 35. Qd2  
Qg8



36. h4 gxh4 37. Rxh4 f5 38. exf6 e5 39. b5 cxb5 40. Qe3  
exd4 41. Qf4 Ka7 42. Qf3 Rb8 43. Rb4 Qd8 44. Bh3 Rd5  
45. f7 bxa4 46. Rxa4 d3



47. Qe3+ Ka8 48. Rad4 Qf8 49. Be6 Rxd4 50. Rxd4 h4  
51. Bc4 hxg3+ 52. fxg3 Nb5 53. Bxb5 axb5 54. Rxd3 b4  
55. Rd5 Rc8



White is clearly winning here, and Ra5+ is good and tempting. AlphaZero is only optimised for achieving an end result. Even though a slower approach achieves the same outcome, a win is a win! This game ultimately finishes with checkmate.

56. Rf5 Kb8 57. g4 Ka8 58. Rf2 b3 59. Qxb3 Qe7 60. Ra2+  
Kb8 61. Qg3+ Rc7 62. Rf2 Ka7 63. f8=Q Qh7+ 64. Qh3  
Qb1 65. Qf5 Qa1 66. Qf1 Qh8+ 67. Qh5 Qg7 68. Qh4  
Rc5 69. Kh1 Ra5 70. Qg3 Ra4 71. Rf3 Ra2 72. Qff2+  
Rxf2 73. Qxf2+ Ka6 74. Qg3 b5 75. Qf4 Qg8 76. Qf6+  
Ka5 77. Qf5 Ka4 78. Qf8 Qh7+ 79. Kg2 b4 80. Qe8+ Ka5  
81. Qh5+ Qxh5 82. gxh5 Ka4 83. Rf1 Ka3 84. Kf2 Kb2  
85. Ke1 b3 86. Kd1 Kc3 87. Kc1 Kd4 88. h6 b2+ 89. Kxb2  
Ke5 90. Re1+ Kf6 91. Rd1 Kg6 92. Rc1 Kxh6 93. Rc3  
Kg5 94. Rc5+ Kh4 95. Rc7 Kg3 96. Rb7 Kh4 97. Ra7 Kh3  
98. Kc3 Kg3 99. Ra1 Kh4 100. Ra3 Kh3 101. Kd4+ Kg4  
102. Ra5 Kf4 103. Ra7 Kg5 104. Ra1 Kf5 105. Ra3 Kg4  
106. Ke5 Kh5 107. Ra5 Kg5 108. Rd5 Kh6 109. Rd7 Kg5  
110. Rc7 Kh6 111. Kf5 Kh5 112. Rh7# 1-0

### B.5. Torpedo

In the variation of chess that we've named Torpedo chess, the pawns can move by either one or two squares forward from anywhere on the board rather than just from the initial squares, which is the case in Classical chess. We will refer to the pawn moves that involve advancing them by two squares as "torpedo" moves.

We have also looked at a Semi-torpedo variant in our experiments, where we only add a partial extension to the original rule and have the pawns be able to move by two squares from the 2nd/3rd and 6th/7th rank for White and Black respectively. In this section we will focus on the universal motifs of full Torpedo chess, and cover the sub-motifs and sub-patterns that correspond to Semi-torpedo chess in its own dedicated section in Appendix B.6.



## B.5.1. MOTIVATION

In a sense, having the pawns always be able to move by one or two squares makes the pawn movement more consistent, as it removes a “special case” of them only being able to do the “double move” from their initial position. Increasing pawn mobility has the potential of speeding up all stages of the game. It adds additional attacking motifs to the openings and changes opening theory, it makes middlegames more complicated, and changes endgame theory in cases where pawns are involved.

## B.5.2. ASSESSMENT

The assessment of the Torpedo chess variant, as provided by Vladimir Kramnik:

“The pawns become quite powerful in Torpedo chess. Passed pawns are in particular a very strong asset and the value of pawns changes based on the circumstances and closer to the endgame. All of the attacking opportunities increase and this strongly favours the side with the initiative, which makes taking initiative a crucial part of the game. Pawns are very fast, so less of a strategical asset and much more tactical instead. The game becomes more tactical and calculative compared to standard chess.

There is a lot of prophylactic play, which is why some games don't feature many “torpedo” moves – “torpedo” moves are simply quite powerful and the play often proceeds in a way where each player positions their pawn structure so as to disincentivise “torpedo” moves, either by the virtue of directly blocking their advance, or by placing their own pawns on squares that would be able to capture “en passant” if “torpedo” moves were to occur.

This seems to favour the “classical” style of play in classical chess, which advocates for strong central control rather than conceding space to later attack the center once established. It seems like it is more difficult to play openings like the Grunfeld or the King's Indian defence.

In summary, this is an interesting chess variant, leading to lots of decisive games and a potentially high entertainment value, involving lots of tactical play.”

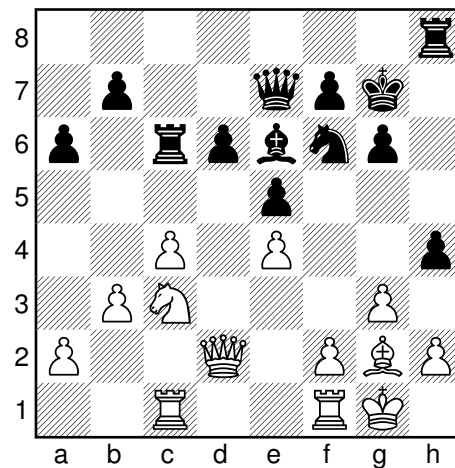
## B.5.3. MAIN LINES

Here we discuss “main lines” of AlphaZero under Torpedo chess, when playing with roughly one minute per move from a particular fixed first move. Note that these are not

purely deterministic, and each of the given lines is merely one of several highly promising and likely options. Here we give the first 20 moves in each of the main lines, regardless of the position.

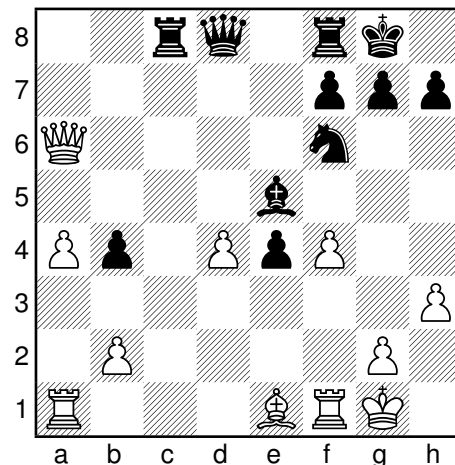
**Main line after e4** The main line of AlphaZero after 1. e4 in Torpedo chess is:

1. e4 (book) c5 2. Nf3 d6 3. d4 Nf6 4. Nc3 cxd4 5. Nxd4 a6 6. g3 h6 7. Bg2 e5 8. Nde2 Be7 9. Be3 Be6 10. Nd5 Nbd7 11. c4 Rc8 12. b3 Ng4 13. O-O Nxe3 14. Nxe3 h4 15. Nf5 Kf8 16. Qd2 Nf6 17. Nc3 g6 18. Nxe7 Qxe7 19. Rad1 Rc6 20. Rc1 Kg7



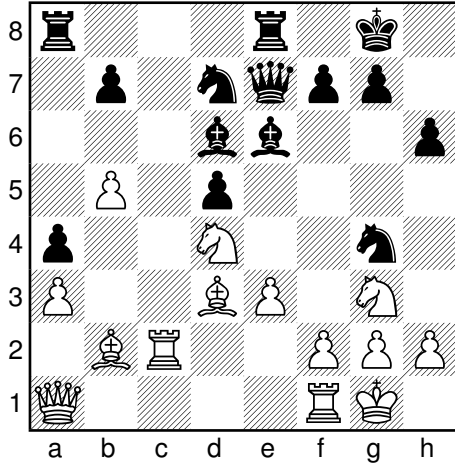
**Main line after d4** The main line of AlphaZero after 1. d4 in Torpedo chess is:

1. d4 (book) d5 2. c4 e6 3. Nf3 Nf6 4. Nc3 a6 5. e3 b6 6. Bd3 Bb7 7. O-O Bd6 8. cxd5 exd5 9. Ne5 O-O 10. a3 Nbd7 11. f4 Ne4 12. Bd2 c5 13. Be1 cxd4 14. exd4 b5 15. h3 Rc8 16. Qe2 Ndf6 17. a4 b4 18. Nxe4 dxe4 19. Bxa6 Bxa6 20. Qxa6 Bxe5



**Main line after c4** The main line of AlphaZero after 1. c4 in Torpedo chess is:

1. c4 (*book*) c5 2. e3 e6 3. d4 d5 4. Nc3 Nc6 5. Nf3 Nf6 6. a3 h6 7. dxc5 Bxc5 8. cxd5 exd5 9. b4 Bd6 10. Bb2 O-O 11. Be2 a5 12. b5 Ne7 13. O-O Re8 14. Rc1 Be6 15. Bd3 Ng6 16. Ne2 a4 17. Rc2 Qe7 18. Qa1 Nf8 19. Nfd4 N8d7 20. Ng3 Ng4

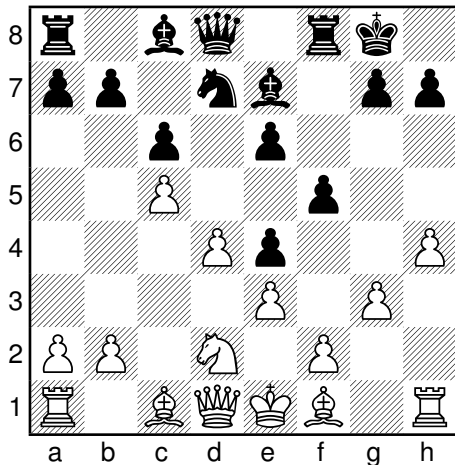


#### B.5.4. INSTRUCTIVE GAMES

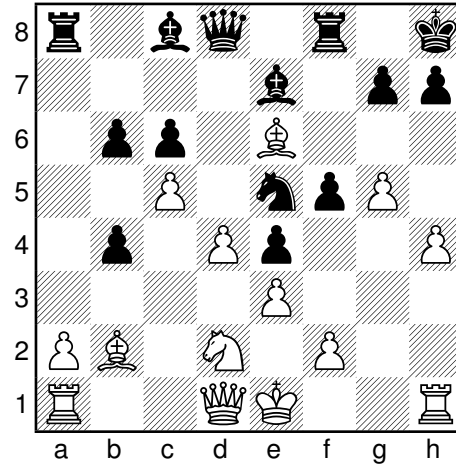
Here we showcase several instructive games that illustrate the type of play that frequently arises in Torpedo chess, along with some selected extracted game positions in cases where particular (endgame) move sequences are of interest.

**Game AZ-9: AlphaZero Torpedo vs AlphaZero Torpedo** The first ten moves for White and Black have been sampled randomly from AlphaZero’s opening “book”, with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

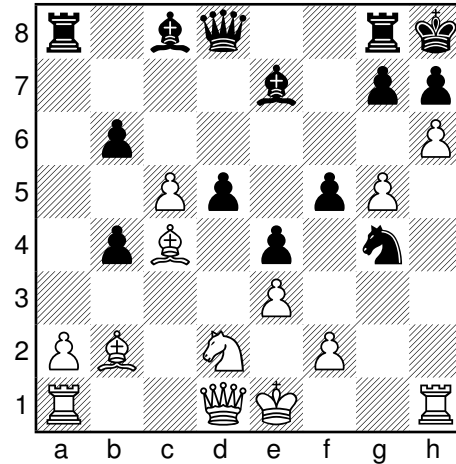
1. d4 d5 2. Nf3 Nf6 3. c4 e6 4. Nc3 c6 5. e3 Nbd7 6. g3 Ne4 7. Nxe4 dxe4 8. Nd2 f5 9. c5 Be7 10. h4 O-O



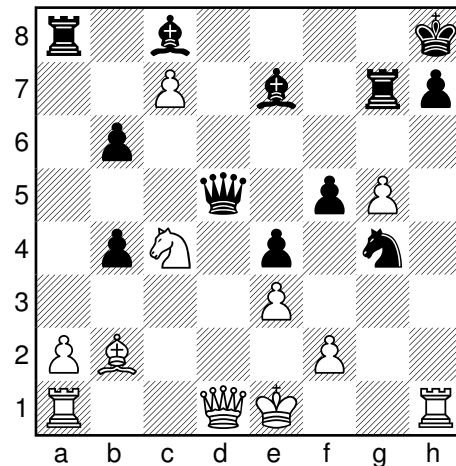
11. g5 b6 12. b4 a5 13. Bc4 axb4 14. Bxe6+ Kh8 15. Bb2 Ne5



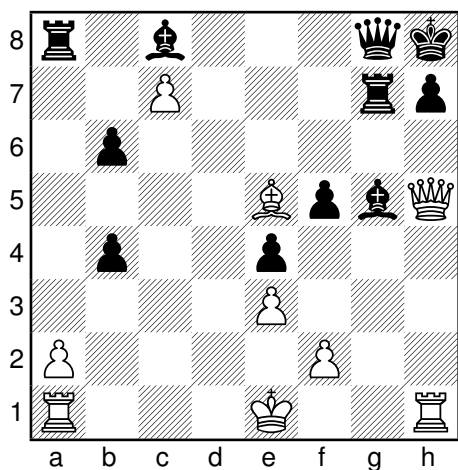
16. Bc4 Ng4 17. d6 cxd5 18. h6 Rg8



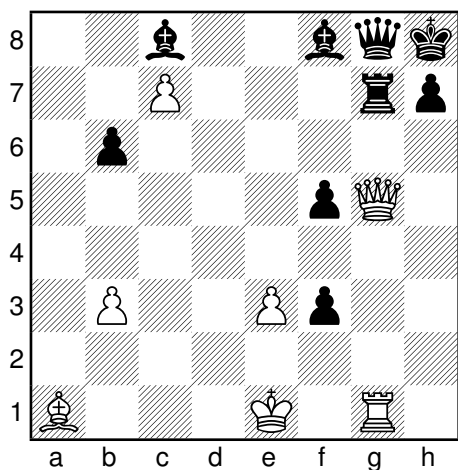
19. hxg7+ Rxg7 20. c7 Qd7 21. Bxd5 Qxd5 22. Nc4



22... Qg8 23. Ne5 Nxe5 24. Bxe5 Bxg5 25. Qh5



25... b2 26. axb3 Rxa1+ 27. Bxa1 Be7 28. f4 exf3 29. Rg1 Bf8 30. Qg5

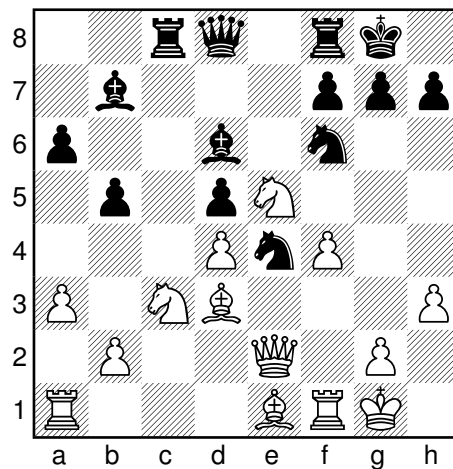


30... h6 31. Qxh6+ Qh7 32. Bxg7+ Bxg7 33. Qxh7+ Kxh7 34. Kf2 Be5 35. Rd1 Bb7 36. Rc1 Bc8 37. Kxf3 Kg6 38. e4 b4 39. Rc4 Kf6 40. Rc6+ Kg5 41. Ke3 f4+ 42. Kf2 Bd4+ 43. Kg2 Be5 44. Rc5 Kf6 45. Kf3 Ke6 46. Rb5 Bd7 47. Rxb4 Bxc7 48. Rd4 Ke7 49. Rd2 Be8 50. Rh2 Bd6 51. Rh7+ Ke6 52. Rh6+ Ke7 53. Rh2 Kf6 54. Rh8 Ke7 55. Rh2 Kf6 56. Rh6+ Ke7 57. Rh1 Kf6 58. Rh8 Ke7 59. Rh6 Be5 60. b4 Kd7 61. Kf2 Bc3 62. b6 Kc8 63. Rd6 Kb7 64. Kf3 Be5 65. Rd5 Bb8 66. Rd8 Bc6 67. Rf8 Ba4 68. Ke2 Be5 69. Rf5 Bb8 70. Rf6 Be5 71. Rf5 Bb8 72. e5 Kxb6 73. Rxf4 Bb5+ 74. Kd1 Bxe5 75. Rf5 Bh2 76. Rxb5+ Kxb5 **1/2–1/2**

**Game AZ-10: AlphaZero Torpedo vs AlphaZero Torpedo** The first ten moves for White and Black have been sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

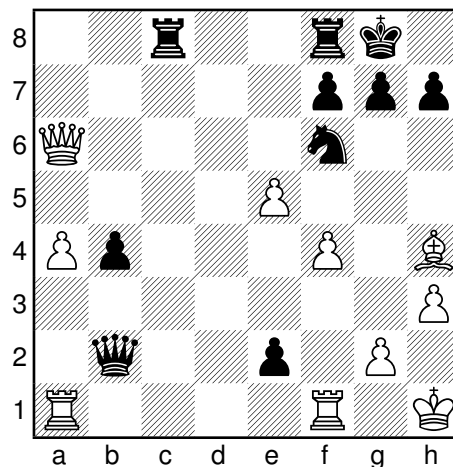
1. d4 d5 2. c4 e6 3. Nf3 Nf6 4. Nc3 a6 5. e3 b6 6. Bd3 Bb7 7. O-O Bd6 8. cxd5 exd5 9. a3 O-O 10. Ne5 c5 11. f4 Nbd7

12. Bd2 cxd4 13. exd4 Ne4 14. Be1 b5 15. h3 Rc8 16. Qe2 Ndf6

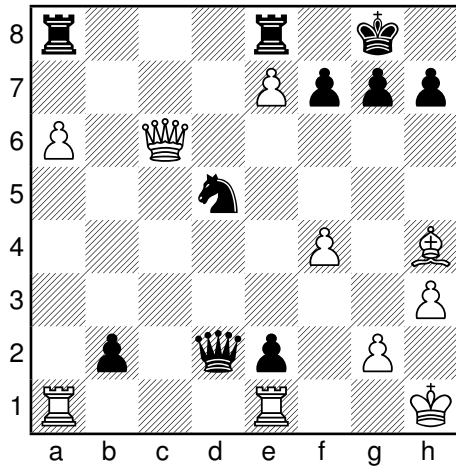


A normal-looking position arises in the middlegame (this is one of AlphaZero's main lines in this variation of chess), but the board soon explodes in tactics.

17. a4 b4 18. Nxe4 dxe4 19. Bxa6 Bxa6 20. Qxa6 Bxe5 21. dxe5 Qd4+ 22. Kh1 Qxb2 23. Bh4 e2

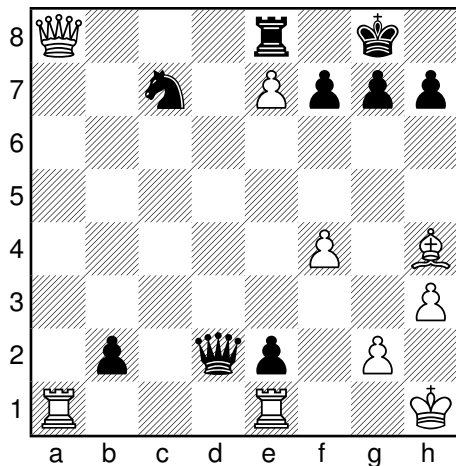


24. Rfe1 Nd5 25. e7 Rfe8 26. Qd6 Qd2 27. a6 Ra8 28. Qc6 b2

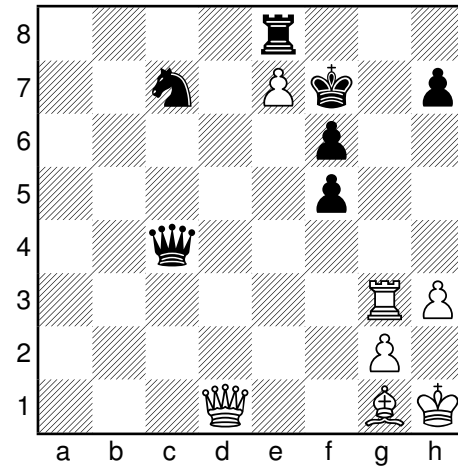


A series of consecutive torpedo moves had given rise to this incredibly sharp position, with multiple passed pawns for White and Black, and the threats are culminating, as demonstrated by the following tactical sequence.

29. Qxe8+ Rxe8 30. a8=Q Nc7

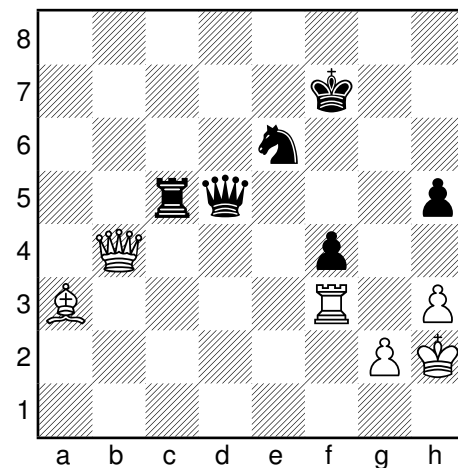


31. Qa5 bxa1=Q 32. Qxd2 Qa4 33. Rxe2 f6 34. Re3 Kf7 35. Bf2 Qb5 36. Qd6 Qf1+ 37. Bg1 Qc4 38. f5 g6 39. Rg3 gxf5 40. Qd1

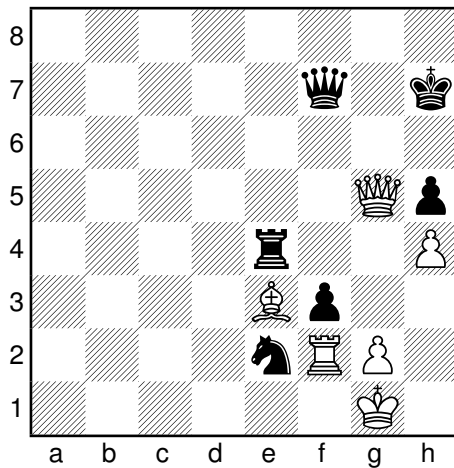


Here Black utilizes a torpedo move to give back the pawn and protect h5 via d5.

40... f3 41. Qxf3 Qd5 42. Qg4 Ne6 43. Be3 Rb8 44. Qa4 Kxe7 45. Bc1 Kf7 46. Qc2 f5 47. Kh2 Rb5 48. Qa4 h5 49. Qa7+ Qb7 50. Qa4 Qd5 51. Ba3 f4 52. Rf3 Ra5 53. Qb4 Rc5



54. Rf2 Qf5 55. Qb2 Rd5 56. Qb7+ Kg6 57. Qc6 Kh7 58. Bb2 Rd8 59. Qb7+ Kg8 60. Rf3 Qg6 61. Be5 Qf5 62. Ba1 Rd3 63. Qb1 Rd5 64. Qb8+ Rd8 65. Qb2 Nd4 66. Rf2 Ne6 67. Qb3 Kh7 68. Qb7+ Kg8 69. Qa6 Kh7 70. Qa7+ Kg6 71. Qb7 Rd1 72. Qa8 Rd8 73. Qc6 Kh7 74. Qb7+ Kg6 75. Bb2 Rd1 76. Qb8 Re1 77. Bc3 Re3 78. Qg8+ Kh6 79. Qh8+ Kg6 80. Qe8+ Kh7 81. Qd7+ Kg6 82. Qc6 Kh7 83. Qb7+ Kg8 84. Qa8+ Kh7 85. Qb7+ Kg8 86. Qc8+ Kh7 87. Qh8+ Kg6 88. Qg8+ Kh6 89. Qc8 Kh7 90. Qd7+ Kg6 91. Qe8+ Kh7 92. Bd2 Rd3 93. Qb8 Rd8 94. Qb7+ Kg8 95. Qc6 Qg6 96. Qc3 Rd4 97. Qa5 Kh7 98. Kg1 Re4 99. h4 Nd4 100. Qc7+ Qg7 101. Qa5 Qf7 102. Qa6 f3 103. Qh6+ Kg8 104. Qg5+ Kh7 105. Be3 Ne2+

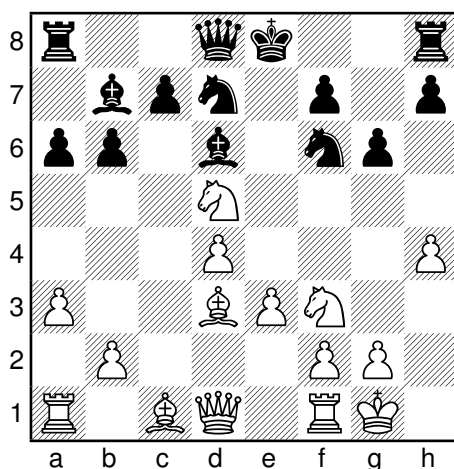


And the game soon ends in a draw.

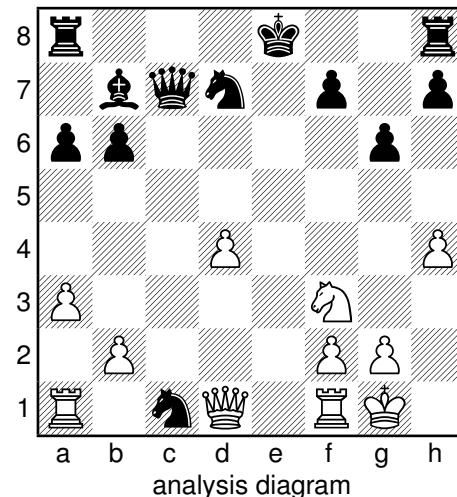
106. Kh2 Qc7+ 107. Kh1 Ng3+ 108. Kg1 Ne2+ 109. Kf1 Ng3+ 110. Kg1 Ne2+ 111. Rxe2 fxe2 112. Qxh5+ Kg8 113. Qxe2 Rxh4 114. Qa6 Qe7 115. Qc8+ Kh7 116. Qf5+ Kg7 117. Qg5+ Qxg5 118. Bxg5 Re4 119. Kf2 Kg6 120. Kf3 Re8 121. Bf4 Re7 122. g4 Rf7 123. Kg3 Rg7 124. Bb8 Kg5 125. Bd6 Rb7 126. Bf4+ Kg6 127. Kf3 Ra7 128. Bb8 Rb7 129. Bf4 Ra7 130. Bb8 Rb7 131. Bf4 1/2-1/2

**Game AZ-11: AlphaZero Torpedo vs AlphaZero Torpedo** The first ten moves for White and Black have been sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

1. d4 d5 2. Nf3 Nf6 3. c4 e6 4. Nc3 a6 5. e3 b6 6. Bd3 Bb7 7. a3 g6 8. cxd5 exd5 9. h4 Nbd7 10. O-O Bd6 11. Nxd5



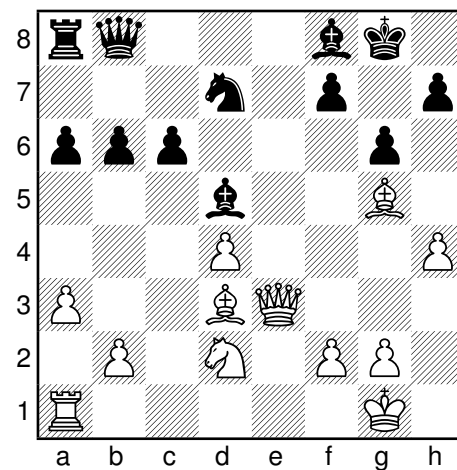
An interesting tactical motif, made possible by torpedo moves. One has to wonder, after 11... Nxd5 12. e4, what happens on 12... Nf4? The game would have followed 13. e5 Nxd3 14. exd6 Nxc1 15. dxc7 Qxc7



and here, White would have played 16. d6, a torpedo move – gaining an important tempo while weakening the Black king. 16... Qc4 17. Rxc1, followed by Re1+ once the queen has moved. AlphaZero evaluates this position as being strongly in White's favour, despite the material deficit.

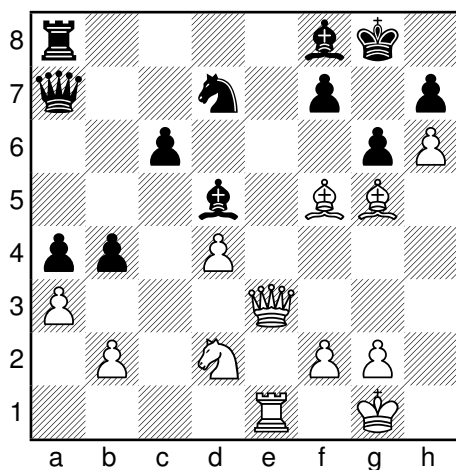
Going back to the game continuation,

11... Nxd5 12. e4 O-O 13. exd5 Bxd5 14. Bg5 Qb8 15. Re1 Re8 16. Nd2 Rxe1+ 17. Qxe1 Bf8 18. Qe3 c6

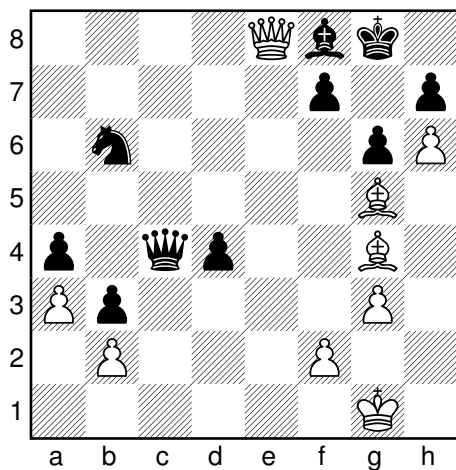


Now we see several torpedo moves taking place. First White takes the opportunity to plant a pawn on h6, weakening the Black king, then Black responds by a4 and b4, getting the queenside pawns in motion and creating counterplay on the other side of the board.

19. h6 a4 20. Re1 Qa7 21. Bf5 b4

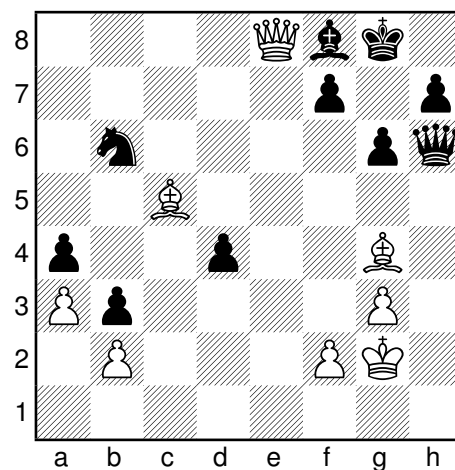


22. Bg4 Qb7 23. Bh3 Rc8 24. Qd3 Ra8 25. Qe3 Nb6 26. Rc1 Nd7 27. g3 b3 28. Bg4 Qc7 29. Re1 Nb6 30. Ne4 Bxe4 31. Qxe4 Qd6 32. Qd3 Qd5 33. Re5 Qc4 34. Qe4 c5 35. Re8 Rxe8 36. Qxe8 cxd4



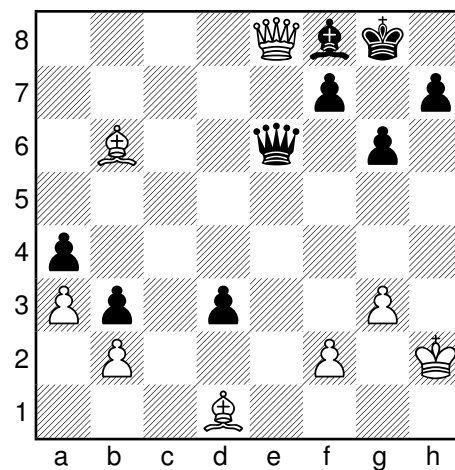
The position is getting sharp again, with Black having gained a passed pawn, and White making threats around the Black king.

37. Be7 Qc1+ 38. Kg2 Qxh6 39. Bc5



A critical moment, and a decision which shows just how valuable the advanced pawns are in this chess variation. Normally it would make sense to save the knight, but AlphaZero decides to keep the pawn instead, and rely on promotion threats coupled with checks on d5.

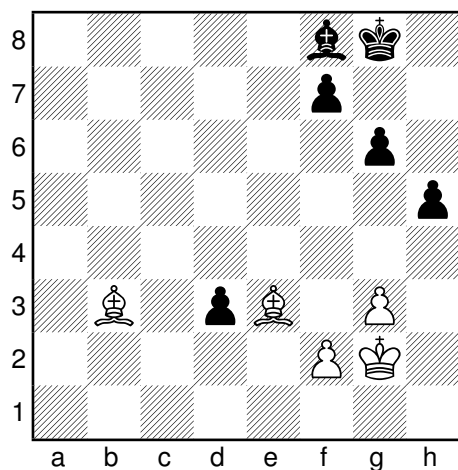
39... d3 40. Bxb6 Qg5 41. Bd1 Qd5+ 42. Kh2 Qe6



Being a piece down, Black offers an exchange of queens, an unusual sight, but tactically justified – Black is also threatening to capture on a3, and that threat is hard to meet. White can't passively ignore the capture and defend the b2 pawn with the bishop, because Black could capture on b2, offering the piece for the second time – and then follow up by an immediate a3, knowing that bxa3 would allow for b1=Q. In addition, Black could retreat the bishop instead of capturing on b2, to make room for a2 bxa3 and again b1=Q. So, it's again a torpedo move that makes a difference and justifies the tactical sequence.

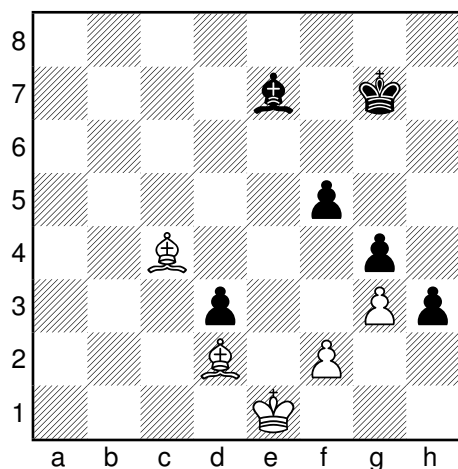
43. Qb5 Qf6 44. Kg2 h5 45. Be3 Qxb2 46. Qxa4 Qxa3 47. Qxb3 Qxb3 48. Bxb3





White is a piece up for two pawns, and has the bishop pair. Yet, Black is just in time to use a torpedo move to shut the White king out and exchange a pair of pawns on the h-file (by another torpedo move).

48... g4 49. Bd2 Kg7 50. Kf1 f5 51. Ke1 Be7 52. Bc4 h3

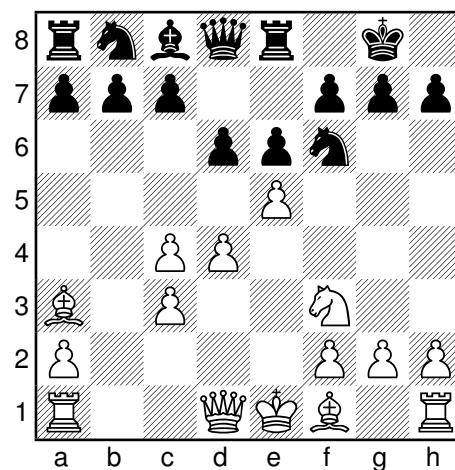


53. gxh4 Bxh4 54. Kf1 Bg5 55. Bc3+ Bf6 56. Bd2 Bg5 57. Bc3+ Bf6 58. Bxf6 Kxf6 59. Bxd3 f4 60. Be4 g3 61. Bg2 gxf2 62. Ba8 Kf5 63. Kxf2 Kg4 64. Bb7 Kh5 65. Ba6 f3 66. Kxf3

1/2-1/2

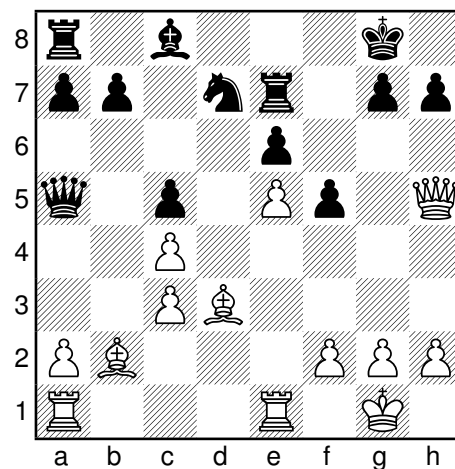
**Game AZ-12: AlphaZero Torpedo vs AlphaZero Torpedo** Playing from a predefined Nimzo-Indian opening position (the first 3 moves for each side). The remaining moves follow best play, at roughly one minute per move.

1. d4 (book) Nf6 (book) 2. c4 (book) e6 (book) 3. Nc3 (book) Bb4 (book) 4. e3 Bxc3 5. bxc3 d6 6. Nf3 O-O 7. Ba3 Re8 8. e5

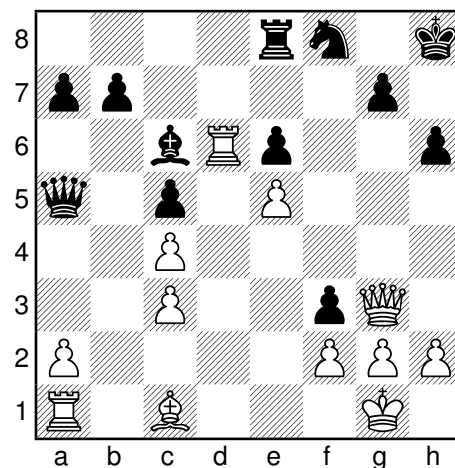


Already we see the first torpedo move, keeping the initiative.

8... dxe5 9. Nxe5 Nbd7 10. Bd3 c5 11. O-O Qa5 12. Bb2 Nxe5 13. dxe5 Nd7 14. Re1 f5 15. Qh5 Re7



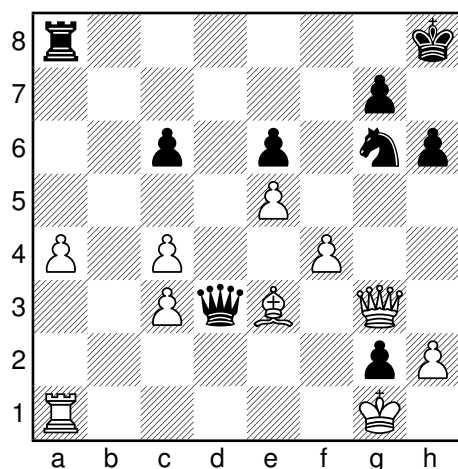
16. Qh4 Re8 17. Qg3 Nf8 18. Bc1 Kh8 19. Be2 Bd7 20. Rd1 Bc6 21. Bh5 h6 22. Bxe8 Rxe8 23. Rd6 f3



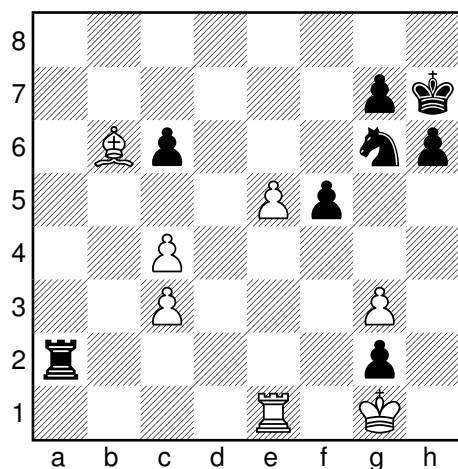
Here we see an effect of another torpedo move, after the

exchange sacrifice earlier, taking over the initiative and creating a dangerous pawn.

24. Bd2 fxg2 25. f4 Qc7 26. Be3 Qf7 27. Bxc5 Qf5 28. Rxc6 bxc6 29. Bxa7 Ng6 30. Be3 Ra8 31. a4 Qd3

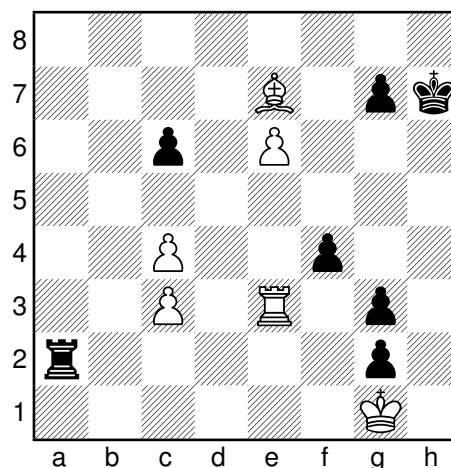


32. Re1 Kh7 33. a5 Rxa5 34. Bb6 Qxg3 35. hxg3 Ra2 36. f5 exf5



The following move shows the power of advanced pawns – 37. e6!, in order to create a threat of 38. e8=Q, so Black has to block with the knight. If instead 37. e7, Black responds by first giving the knight for the pawn – 37... Nxe7, and then after 38. Rxe7 follows it up with 38... h4!, similar to the game continuation.

37. e6 Ne7 38. Bc5 h4 39. Bxe7 hxg3 40. Re3 f4

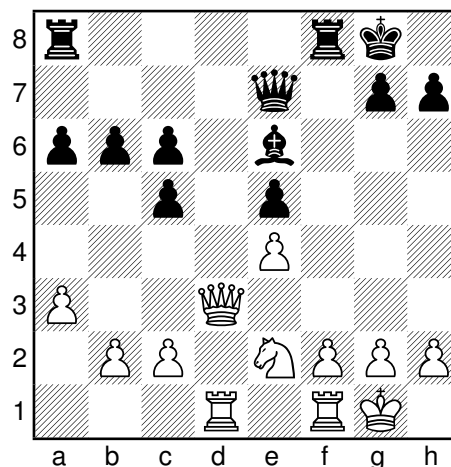


and Black manages to force a draw, as the pawns are just too threatening.

41. Rf3 Ra1+ 42. Kxg2 Ra2+ 43. Kg1 Ra1+ 44. Kg2 Ra2+ 45. Kh1 Ra1+ 46. Kg2 1/2–1/2

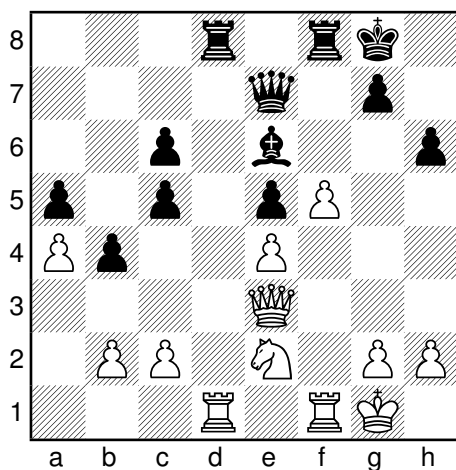
**Game AZ-13: AlphaZero Torpedo vs AlphaZero Torpedo** The game starts from a predefined Ruy Lopez opening position (the first 5 plies). The remaining moves follow best play, at roughly one minute per move.

1. e4 (book) e5 (book) 2. Nf3 (book) Nc6 (book) 3. Bb5 (book) a6 4. Bxc6 dxc6 5. O-O f6 6. d4 exd4 7. Nxd4 Bd6 8. Be3 c5 9. Ne2 Ne7 10. Nbc3 b6 11. Qd3 Be6 12. Rad1 Be5 13. Nd5 O-O 14. Bf4 c6 15. Nxe7+ Qxe7 16. Bxe5 fxe5 17. a3



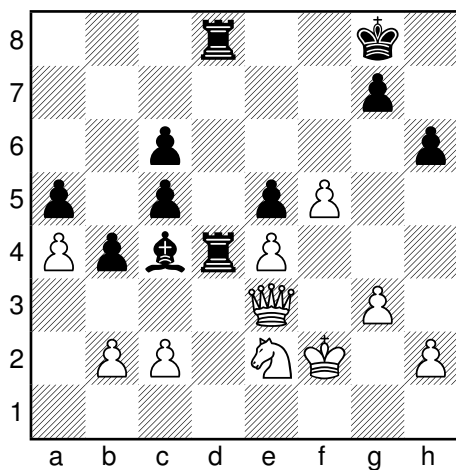
Here comes the first torpedo move (b6-b4), gaining space on the queenside.

17... b4 18. a4 h6 19. Qe3 Rad8 20. f3 a5 21. f5

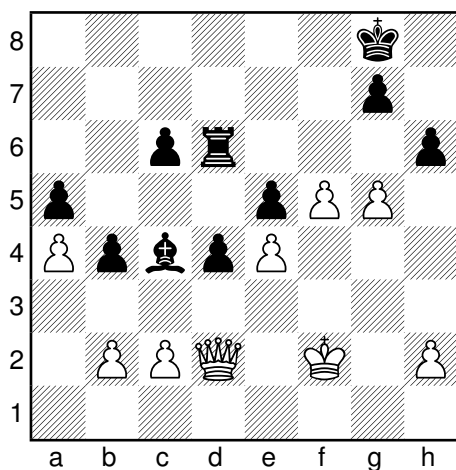


Here we see an effect of another torpedo move, f3-f5, advancing towards the Black king.

21... Bc4 22. Rde1 Qd6 23. Rd1 Qxd1 24. Rxd1 Rxd1+ 25. Kf2 Rd4 26. g3 Rfd8

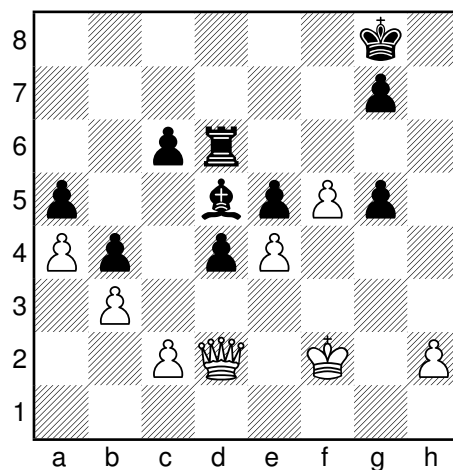


27. Nxd4 cxd4 28. Qd2 Rd6 29. g5



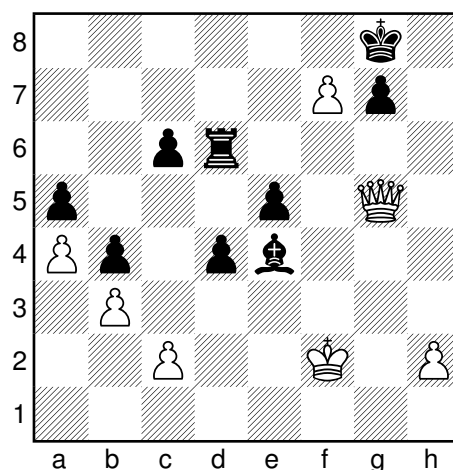
White uses a torpedo move to generate play on the kingside.

29... hxg5 30. b3 Bd5



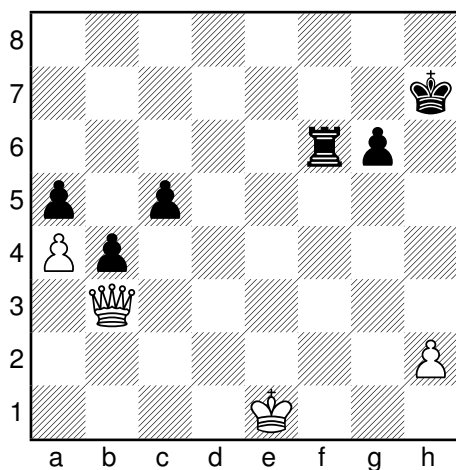
The Black bishop can't be taken, due to a torpedo threat e3+!

31. Qxg5 Bxe4 32. f7+



And yet another torpedo strike, in order to capture on e5.

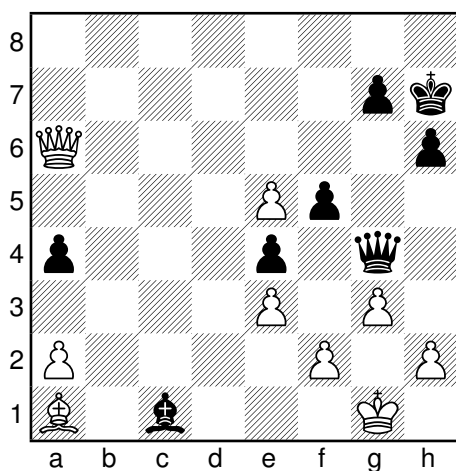
32... Kxf7 33. Qxe5 Rf6+ 34. Ke1 Bxc2 35. Qxd4 Bxb3 36. Qd7+ Kg8 37. Qd8+ Kh7 38. Qd3+ g6 39. Qxb3 c5



White ends up with the queen against the rook and two pawns, but this ends up being a draw, as the pawns are simply too fast and need to remain blocked. Normally the queen on b3 would prevent the c5 pawn from moving, but a c5-c3 torpedo move shows that this is no longer the case!

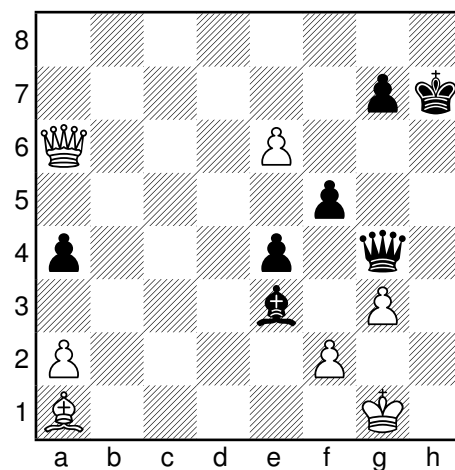
40. Kd1 c3 41. Qc4 Rf5 42. Kc2 Rf2+ 43. Kb1 Rb2+ 44. Kc1 Rd2 45. Kb1 Kh6 46. h3 Rd1+ 47. Kc2 Rd2+ 48. Kb1 Rd1+ 49. Kc2 Rh1 50. Qf4+ Kh7 51. Qc7+ Kh6 52. Qb8 Rf1 53. Qh8+ Kg5 54. Qd8+ Kh6 55. h4 Rf2+ 56. Kb1 Rf1+ 57. Kc2 Rf2+ 58. Kb1 Rf1+ 59. Kc2 **1/2-1/2**

**Game AZ-14: AlphaZero Torpedo vs AlphaZero Torpedo** The position below, with Black to move, is taken from a game that was played with roughly one minute per move:



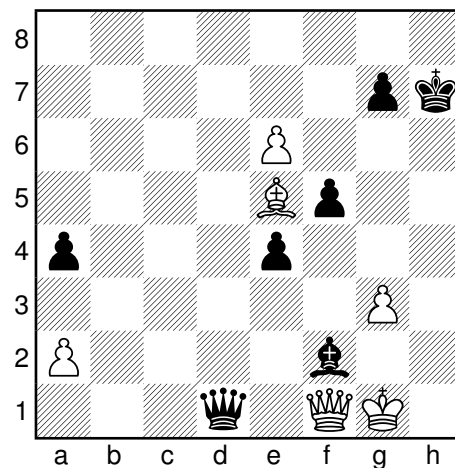
A dynamic position from an endgame reached in one of the AlphaZero games. White has an advanced passed pawn, which is quite threatening – and Black tries to respond by creating threats around the White king. To achieve that, Black starts with a torpedo move:

31... h4 32. e6 hxg3 33. hxg3 Bxe3



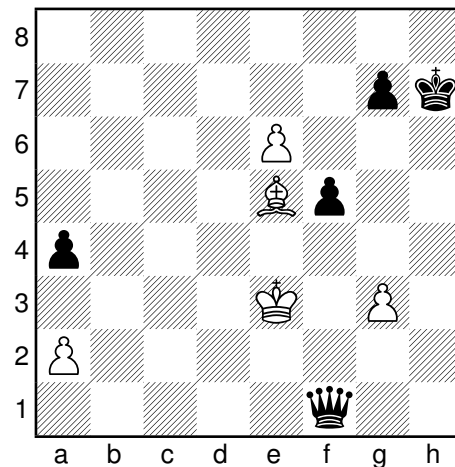
White is one torpedo move away from queening, but has to first try to safeguard the king.

34. Be5 Qd1+ 35. Qf1 Bxf2+



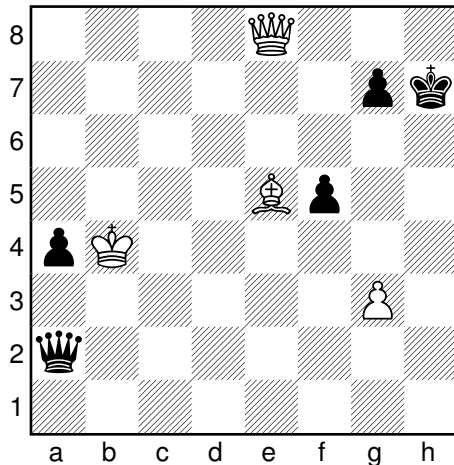
Black is in time, due to the torpedo threats involving the e-pawn.

36. Kxf2 e3+ 37. Kxe3 Qxf1



Black captures White's queen, but White creates a new one, with a torpedo move.

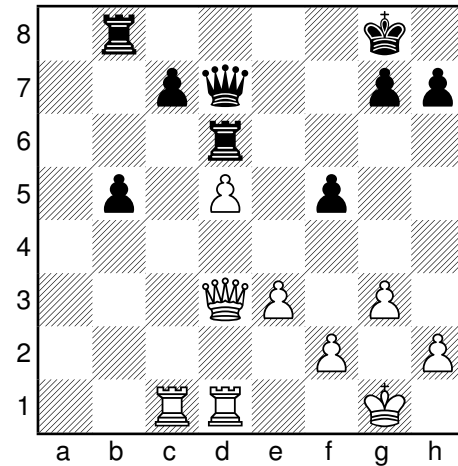
38. e8=Q Qe1+ 39. Kd3 Qb1+ 40. Kc3 Qa1+ 41. Kb4 Qxa2



An interesting endgame arises, where White is up a piece, given that Black had to give away its bishop in the tactics earlier, and Black will soon only have a single pawn in return. Yet, after a long struggle, AlphaZero manages to defend as Black and achieve a draw.

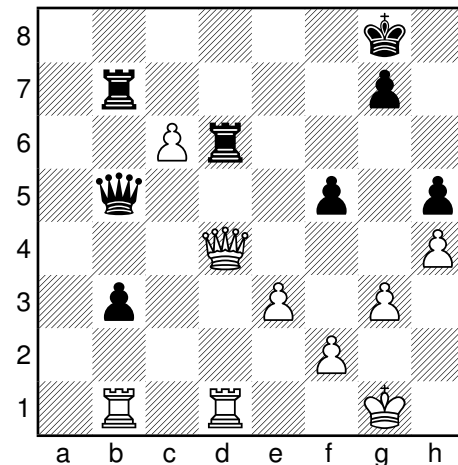
42. Qe7 Qb3+ 43. Ka5 Qg8 44. Kxa4 Kg6 45. Bf4 Qc4+ 46. Ka5 Qd5+ 47. Kb4 Qd4+ 48. Kb3 Qd3+ 49. Kb2 Qd4+ 50. Kc2 Qd5 51. Qe3 Qc4+ 52. Kd2 Qb4+ 53. Kd3 Qb3+ 54. Kd4 Qb4+ 55. Kd5 Qb5+ 56. Kd6 Qa6+ 57. Kd7 Qb5+ 58. Ke7 Qb7+ 59. Kd8 Qd5+ 60. Kc7 Qc4+ 61. Kb6 Qb4+ 62. Kc6 Qa4+ 63. Kb7 Qd7+ 64. Kb6 Kh5 65. Qf3+ Kg6 66. Kc5 Qa7+ 67. Kb4 Qb6+ 68. Kc3 Qa5+ 69. Kc2 Qa4+ 70. Kd2 Qa5+ 71. Ke2 Qb5+ 72. Kf2 Qb2+ 73. Kg1 Qb1+ 74. Kg2 Qb2+ 75. Kh3 Qc2 76. Bd6 Qc1 77. Bf4 Qc2 78. Qe3 Qc6 79. Qe1 Kf6 80. Kh4 Kg6 81. Kh3 Kf6 82. Kh2 Qc2+ 83. Bd2 Qd3 84. Bf4 Qd5 85. Qe2 Qd4 86. Bd2 Kf7 87. Bg5 Qd5 88. Bc1 Qc6 89. Bf4 Kf6 90. Qd3 Qe6 91. Bd2 Kf7 92. Kh3 Qf6 93. Qd5+ Ke7 94. g5 fxe4+ 95. Kxg4 Qe6+ 96. Qxe6 Kxe6 1/2–1/2

**Game AZ-15: AlphaZero Torpedo vs AlphaZero Torpedo** The position below, with Black to move, is taken from a game that was played with roughly one minute per move:



A position from one of the AlphaZero games, illustrating the utilization of pawns in a heavy piece endgame. The b-pawn is fast, and it gets pushed down the board via a torpedo move.

26... h5 27. h4 b3 28. Qc4 Rb7 29. Rb1 Qb5 30. Qd4 c6 31. dxc6



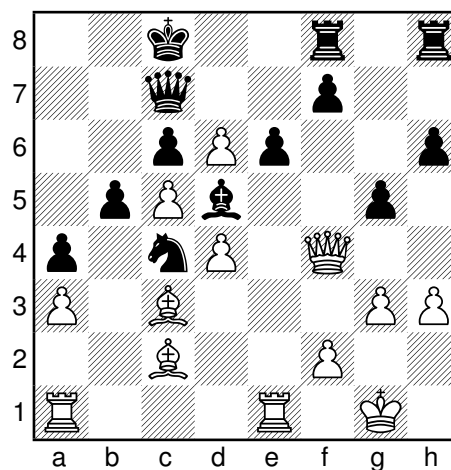
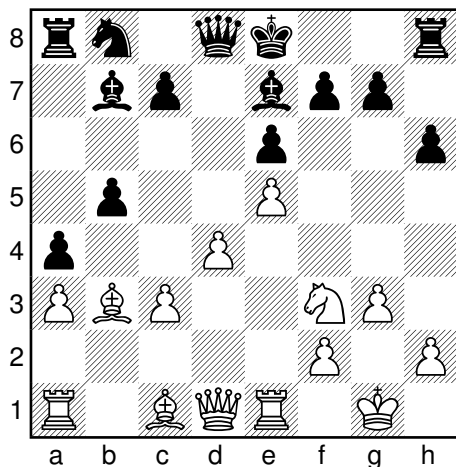
Unlike in Classical chess, this capture is possible, even though it seemingly hangs the queen. If Black were to capture it with the rook, the c-pawn would queen with check in a single move! The threat of c8=Q forces Black to recapture the pawn instead.

31... Rxc6 32. e5 fxe4 33. Qxe4 Rb8 34. Rb2 Qc4 35. Qe5 and the game soon ended in a draw. 1/2–1/2

**Game AZ-16: AlphaZero Torpedo vs AlphaZero Torpedo** The first ten moves for White and Black were sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

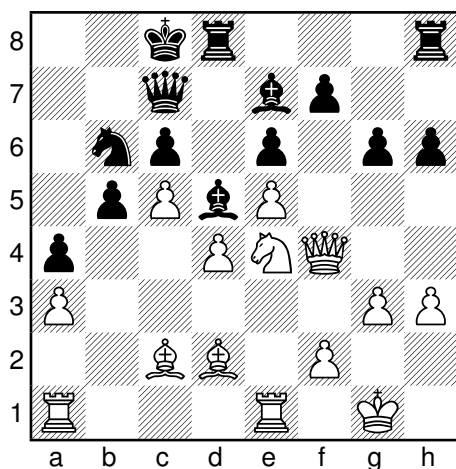
1. d4 Nf6 2. c4 e6 3. Nc3 d5 4. Nf3 a6 5. e3 b6 6. g3 dxc4

7. e5 Nd5 8. Bxc4 Be7 9. O-O Bb7 10. Re1 h6 11. a3 b5  
12. Bb3 Nxc3 13. bxc3 a4

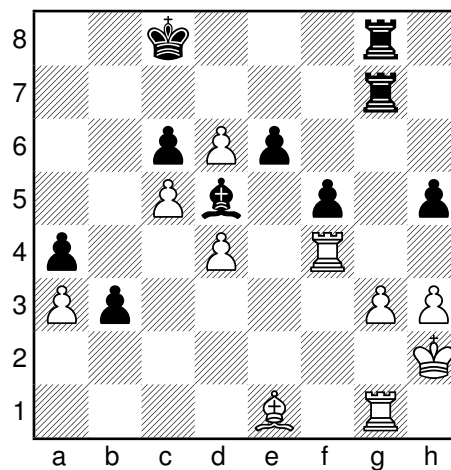


In the early stage of the game, we see White using a torpedo e3-e5 move to expand in the center and Black responding by an a6-a4 torpedo move to gain space on the queenside.

14. Bc2 Bd5 15. Qe2 c6 16. Nd2 Qa5 17. Ne4 Nd7 18. Bd2  
Qc7 19. Qg4 g6 20. h3 O-O-O 21. Qf4 Nb6 22. c5



26. Qf6 Qd8 27. Qxd8+ Rxd8 28. Bd3 Rdg8 29. f3 h5  
30. Be4 Re8 31. Bd3 Rh6 32. Rf1 f5 33. Bxc4 Bxc4 34. Rf2  
Bd5 35. Kh2 Rg6 36. Rg1 Reg8 37. Bd2 R6g7 38. Bb4 Kd7  
39. f4 gxf4 40. Rxf4 Kc8 41. Be1 b3



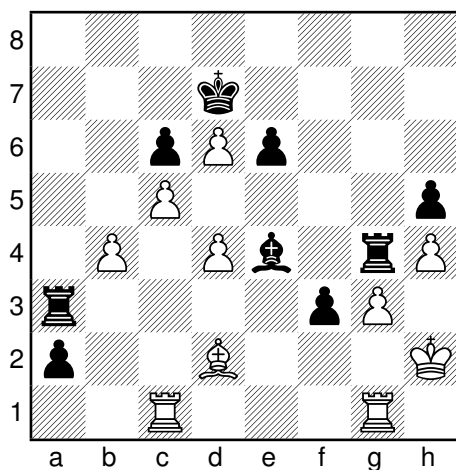
White moves forward with a c3-c5 torpedo move.

22... Nc4 23. Bc3 Rdf8 24. Nd6+ Bxd6 25. exd6 g5

Black uses two consecutive torpedo moves (b5-b3, a4-a2) on the queenside to create a dangerous passed pawn on a2.

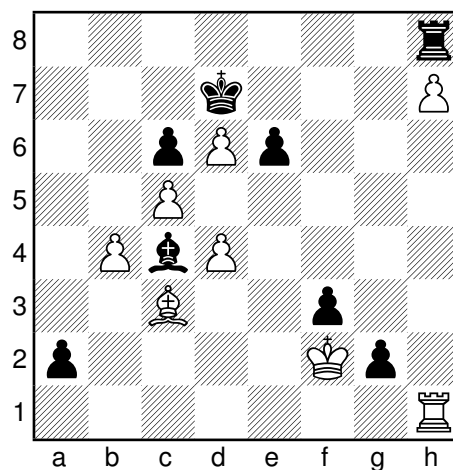
42. axb4 a2 43. Bc3 Ra7 44. R4f1 Kd8 45. Ra1 Kd7 46. Bb2  
Ra4 47. Bc3 Ra3 48. Rac1 Be4 49. h4 Rg4 50. Bd2 f3





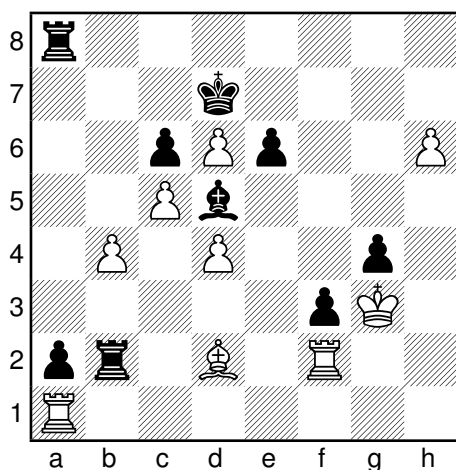
Black uses another torpedo move (f5-f3) to advance further on the kingside and create another passed pawn.

51. Rf1 Rg8 52. Ra1 Rga8 53. Rf2 Rb3 54. Kh3 Rb1 55. Bc3 Bd5 56. g4 Rb3 57. Be1 hxg4+ 58. Kg3 Rb1 59. Bc3 Rb3 60. Bd2 Rb1 61. Bc3 Rb3 62. Bd2 Rb2 63. h6



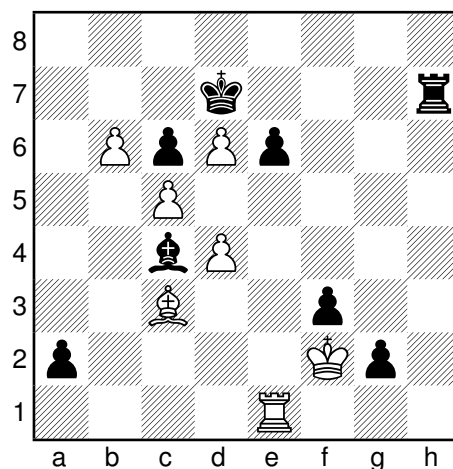
The torpedo move g4-g2 forces the White rook away from the h-file.

69. Re1 Rxh7 70. b6



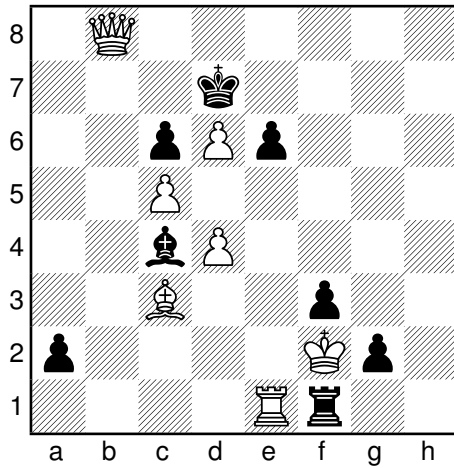
White advances the h-pawn with an h4-h6 torpedo move, seeking counterplay.

63... Rg8 64. Raf1 Bc4 65. h7 Rf8 66. Rh1 Rh8 67. Bc3 Rxf2 68. Kxf2 g2



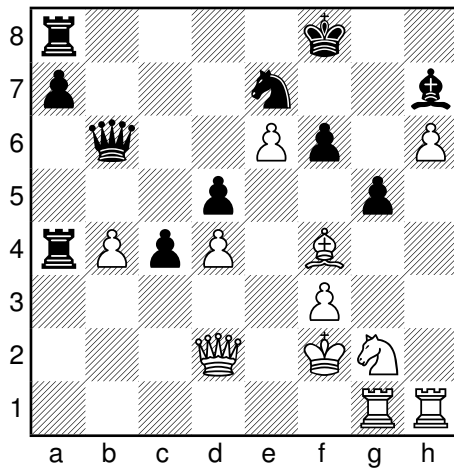
White needs to generate immediate counterplay, and does so via b4-b6, another torpedo move. White then uses a b6-b8=Q torpedo move to promote to a queen in the next move, demonstrating how fast the pawns are in this variation of chess.

70... Rh1 71. b8=Q Rf1+



72. Rxf1 gxf1=Q+ 73. Kg3 and the game eventually ended in a draw due to mutual threats and ensuing checks. **1/2–1/2**

**Game AZ-17: AlphaZero Torpedo No-castling vs AlphaZero Torpedo No-castling** This game was an experiment combining the No-castling chess with Torpedo chess, resulting in a highly tactical position. The first ten moves for White and Black were sampled randomly from AlphaZero’s opening “book”, with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

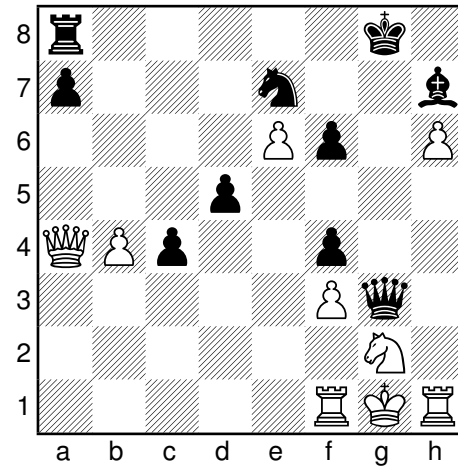


Here White executes a stunning ‘double attack’:

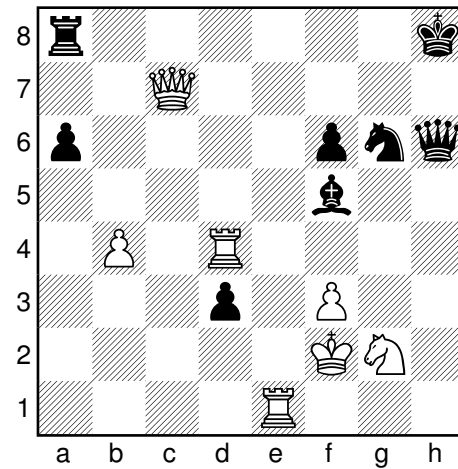
27. Qc2!! Kg8

Black can’t afford to capture the Queen, due to the powerful attack following 27... Bxc2 28. h8=Q+. White also had to assess the consequences of 27... gxf4

28. Qxa4 Qxd4+ 29. Kxg3 gxf4+ 30. Kh2 Qf2 31. Rf1 Qg3+ 32. Kg1



32... Qg6 33. Rh4 Kh8 34. Rg4 Qe8 35. Qa1 Qf8 36. Qc3 Bf5 37. Rxf4 a6 38. Re1 d3 39. Rxc4 Bxe6 40. Rd4 Bf5 41. Qc7 Ng6 42. Kf2 Qxh6



43. Qc1 Qxc1 44. Rxc1 Ne7 45. Ne3 Bg6 46. Ra1 Nc6 47. Rh4+ Kg7 48. b5 Nb8 49. Rc4 Bf7 50. Rc7 f4 51. Nd1 a4 52. Nc2 a2 53. Nxd3 Kf6 54. Rc8 Ra3 55. Nxf4 Nd7 56. Ne2 Ne5 57. b7 Rxf3+ 58. Kg2 Rb3 59. b8=Q Rxb8 60. Rxb8 and White went on to win the game easily. 1-0

## B.6. Semi-torpedo

In Semi-torpedo chess, we consider a partial extension to the rules of pawn movement, where the pawns are allowed to move by two squares from the 2nd/3rd and 6th/7th rank for White and Black respectively. This is a restricted version of another variant we have considered (Torpedo chess) where the option is extended to cover the entire board. Yet, even this partial extension adds lots of dynamic options and here we independently evaluate its impact on the arising play.

### B.6.1. MOTIVATION

As with Torpedo chess, the motivation in extending the possibilities for rapid pawn movement lies in adding dynamic,

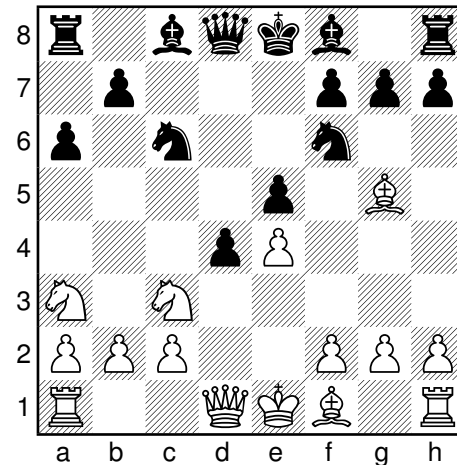
attacking options to the middlegame. Yet, given that it is only a partial extension, adding an extra rank for each side from which the pawns can move by two squares, its impact on endgame patterns is much more limited.

### B.6.2. ASSESSMENT

The assessment of the Semi-torpedo chess variant, as provided by Vladimir Kramnik:

“ Compared to Classical chess, the pawns that have been played to the 3rd/6th rank become much more useful, which manifests in several ways. First, prophylactic pawn moves to h3/h6 and a3/a6 now allow for a subsequent torpedo push. Having played h3 for example, it is now possible to play the pawn to h5 in a single move. This also means, if the goal was to push the pawn to h5 in two moves, that there are two ways of achieving it – either via h4 and h5 or via h3 and h5 – and doing the latter does not expose a weakness on the g4 square and can thus be advantageous. Secondly, fianchetto setups now allow for additional dynamic options. The g3 pawn can now be pushed to g5 in a single move, to attack a knight on f6 – and vice versa. Thirdly, openings where one of the central pawns is on the 3rd/6th rank change – consider the Meran for example – the e3 pawn can now go to e5 in a single move.

Theory might change in other openings as well, like for instance the Ruy Lopez with a7-a6, given that there would be some lines where the torpedo option of playing a6-a4 might force White to adopt a slightly different setup. AlphaZero also likes playing g6 early for Black, with a threat of g4 in some lines, aimed against a knight on f3 if White starts expanding in the center. As another example, consider a pretty standard opening sequence in the Sicilian defence: 1. e4 c5 2. Nf3 Nc6 3. d4 cxd4 4. Nxd4 Nf6 5. Nc3 e5 6. Ndb5 d6 – it turns out that here 7. Bg5 no longer keeps the advantage, because of 7... a6 8. Na3 followed up by a torpedo move 8... d4:



Here, the game could continue 9. exd5 Bxa3 10. bxa3 Nd4 11. Bd3 Qa5, and the position is assessed as equal by AlphaZero. This variation illustrates nicely how the torpedo moves provide not only an additional attacking option for White, but also additional equalizing options for Black, depending on the position.

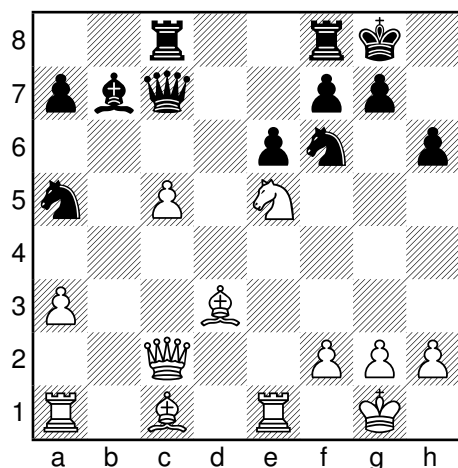
Semi-torpedo chess seems to be more decisive than Classical chess, and less decisive than Torpedo chess. It is an interesting variation, to be potentially considered by those who like the general middlegame flavor of Torpedo chess, but are unwilling to abandon existing endgame theory. ”

### B.6.3. MAIN LINES

Here we discuss “main lines” of AlphaZero under Semi-torpedo chess, when playing with roughly one minute per move from a particular fixed first move. Note that these are not purely deterministic, and each of the given lines is merely one of several highly promising and likely options. Here we give the first 20 moves in each of the main lines.

**Main line after e4** The main line of AlphaZero after 1. e4 in Semi-torpedo chess is:

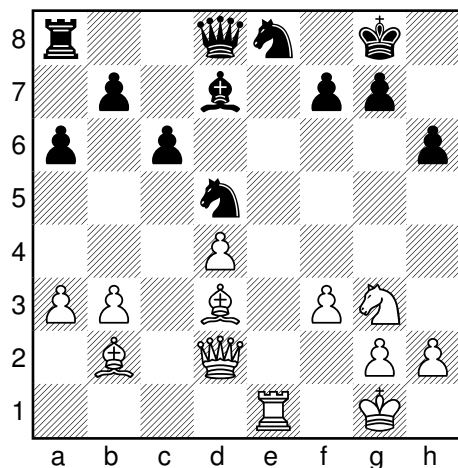
1. e4 (book) c5 2. c3 Nf6 3. e5 Nd5 4. Bc4 e6 5. Nf3 Be7 6. d4 d6 7. O-O O-O 8. Re1 Nc6 9. exd6 Qxd6 10. dxc5 Qxc5 11. Nbd2 b6 12. b4 Qd6 13. Qc2 Bb7 14. a3 Nf6 15. Ne4 Qc7 16. Bd3 h6 17. c5 bxc5 18. Nxc5 Bxc5 19. bxc5 Na5 20. Ne5 Rac8



and after 21. Bb2 White would have compensation for the pawn. There are also tactical resources in this position, for instance White could consider a more forcing line of play – 21. Bxh6!? gxh6 22. Qd2 Kg7 23. Re3 Rh8 24. Rg3+ Kf8 25. Rae1 h4 26. Rg7! Kxg7 27. Qg5+ Kf8 28. Qxf6 Rg8 29. Ng6+ Rxg6 30. Bxg6 – potentially leading to a draw by perpetual check.

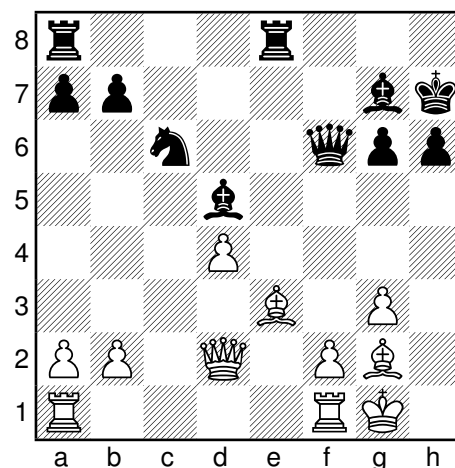
**Main line after d4** The main line of AlphaZero after 1. d4 in Semi-torpedo chess is:

1. d4 (*book*) Nf6 2. c4 e6 3. e3 d5 4. cxd5 exd5 5. Nc3 Bd6 6. Bd3 O-O 7. Nge2 a6 8. O-O Re8 9. b3 Nc6 10. Ng3 Bg4 11. f3 Bc8 12. a3 Ne7 13. Bb2 h6 14. Qd2 c6 15. e5 dxe4 16. Nxe4 Ned5 17. Nxd6 Qxd6 18. Rae1 Qd8 19. Rxe8+ Nxe8 20. Re1 Bd7



**Main line after c4** The main line of AlphaZero after 1. c4 in Semi-torpedo chess is:

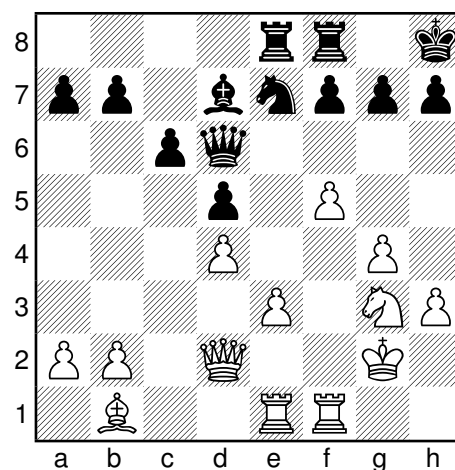
1. c4 (*book*) c5 2. g3 g6 3. Bg2 Bg7 4. e3 e6 5. d4 cxd4 6. exd4 Ne7 7. Nc3 O-O 8. Nge2 d5 9. cxd5 Nxd5 10. h4 Bd7 11. Nxd5 exd5 12. Be3 Re8 13. Nc3 Nc6 14. O-O Be6 15. h5 h6 16. hxg6 fxg6 17. Qd2 Kh7 18. Ne2 Qf6 19. Nf4 Bf7 20. Nxd5 Bxd5



#### B.6.4. INSTRUCTIVE GAMES

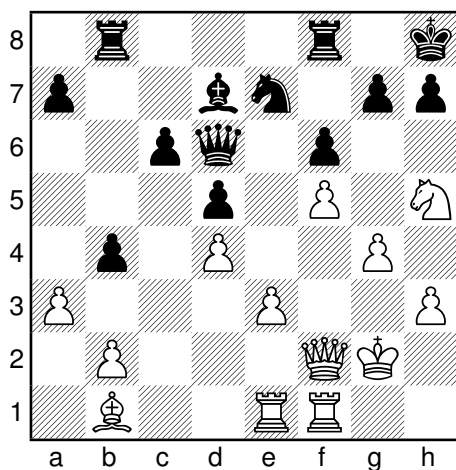
**Game AZ-18: AlphaZero Semi-torpedo vs AlphaZero Semi-torpedo** The first ten moves for White and Black have been sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

1. d4 Nf6 2. c4 e6 3. e3 d5 4. cxd5 exd5 5. Nc3 Bd6 6. Nb5 c6 7. Nxd6+ Qxd6 8. Bd3 Ne4 9. f3 Qb4+ 10. Bd2 Nxd2 11. Qxd2 Qd6 12. Ne2 O-O 13. O-O Nd7 14. g4 Nf6 15. Kg2 Bd7 16. Ng3 Kh8 17. Rae1 Rae8 18. Bb1 Ng8 19. h3 Ne7 20. f5



Here we see the first torpedo move of the game, f3-f5, claiming space before Black has the chance to play f5.

20... f6 21. a3 b6 22. Nh5 Rb8 23. Qf2 b4

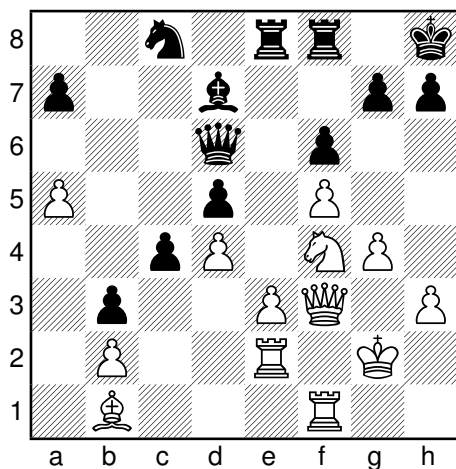


Black utilizes a torpedo move of its own, b6-b4, to initiate counterplay on the queenside.

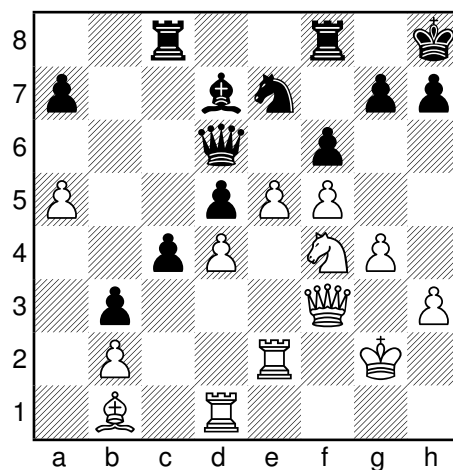
24. Qf4 Nc8 25. a4 b3 26. Qf3 Nb6 27. Nf4 Rbe8 28. Re2 c4

And c6-c4 comes as another torpedo move, speeding up the queenside expansion. White chooses not to take *en passant*, but to play a5 instead in reply.

29. a5 Nc8

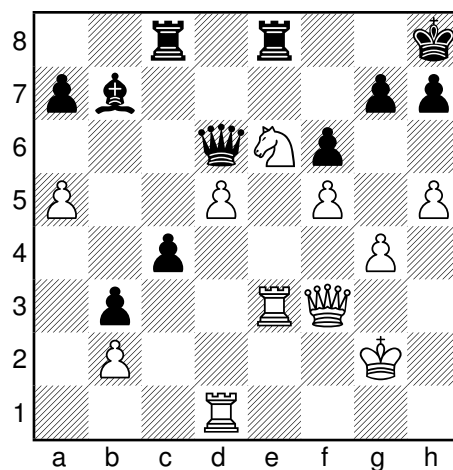


30. Rfe1 Ne7 31. Rd1 Rc8 32. e5



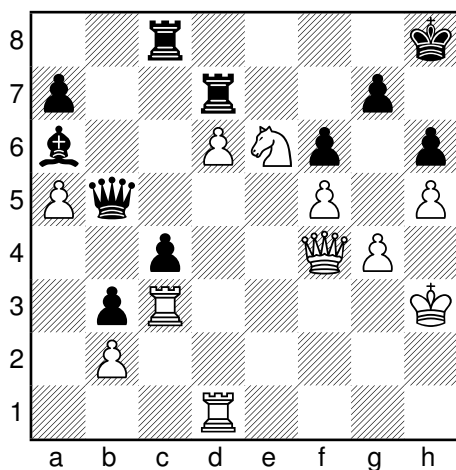
White expands in the center with another torpedo move, e3-e5.

32... dxe4 33. Bxe4 Rfe8 34. Ne6 Nc6 35. Bxc6 Bxc6 36. d5 Ba8 37. Re3 Bb7 38. h5

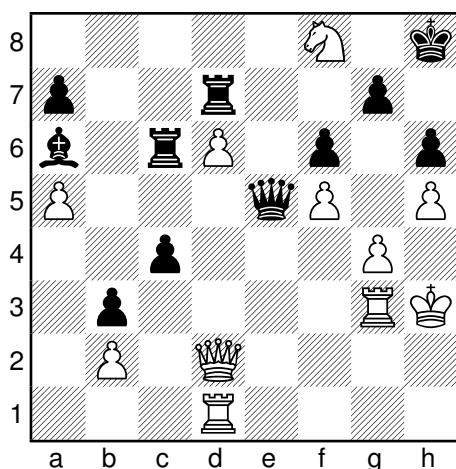


Here comes another torpedo advance, h3-h5, creating threats on the kingside.

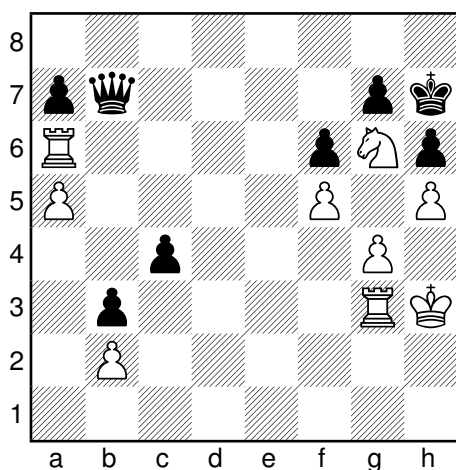
38... h6 39. Kh3 Qd7 40. Rc3 Re7 41. Qf1 Qb5 42. d6 Rd7 43. Qf4 Ba6



44. Qd2 Qe5 45. Rg3 Rc6 46. Nf8

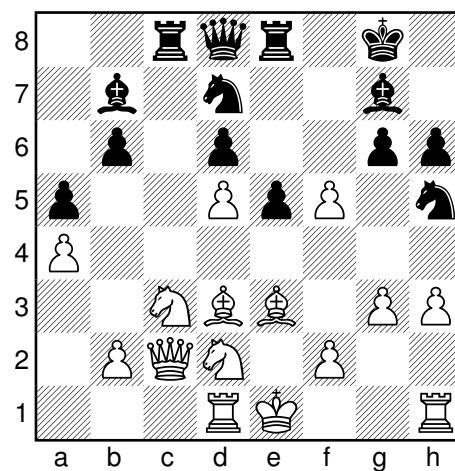


46. ... Rxd6 47. Qb4 Qb5 48. Qxd6 Rxd6 49. Rxd6 Qb8  
50. Ng6+ Kh7 51. Rxa6 Qb7

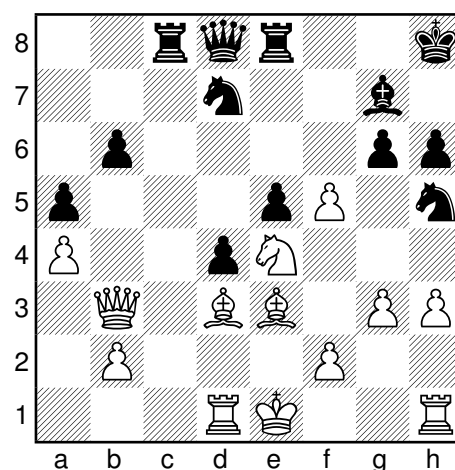


52. Re3 Qh1+ 53. Kg3 Qg1+ 54. Kf3 Qf1+ 55. Kg3 Qg1+  
56. Kf3 Qf1+ 57. Ke4 Qg2+ 58. Kf4 Qf2+ 59. Ke4 Qg2+  
60. Kf4 Qf2+ 61. Ke4 Qg2+ 1/2-1/2

**Game AZ-19: AlphaZero Semi-torpedo vs AlphaZero Semi-torpedo** The position below, with Black to move, is taken from a game that was played with roughly one minute per move:

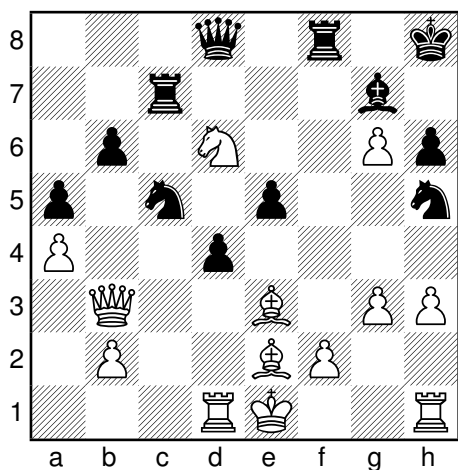


18. ... Bxd5 19. Nde4 Bxe4 20. Qb3+ Kh8 21. Nxe4 d4

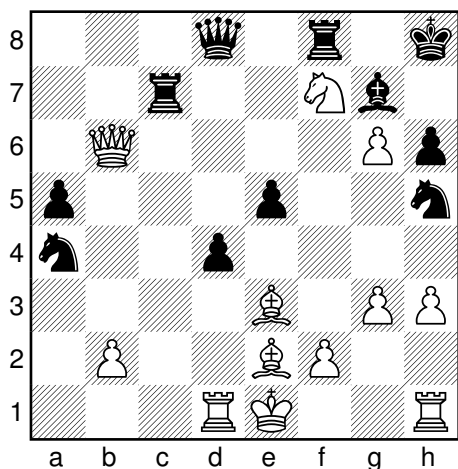


Here, a torpedo move (d6-d4) unleashes a tactical sequence.

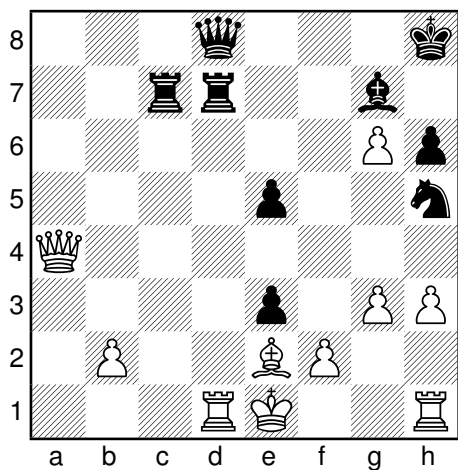
22. Nd6 Rf8 23. Be2 Rc7 24. fxe4 Nc5



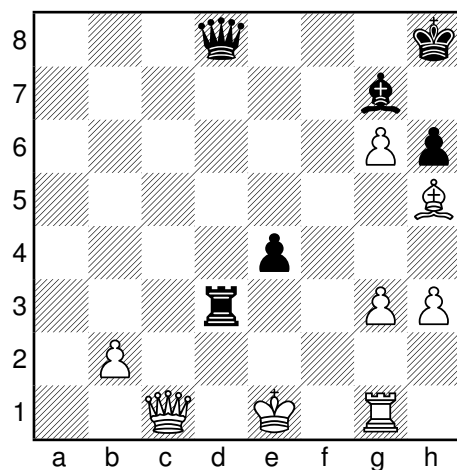
25. Qxb6 Nxa4 26. Nf7+



26. ... R8xf7 27. Qxa5 Rfd7 28. Qxa4 dxe3



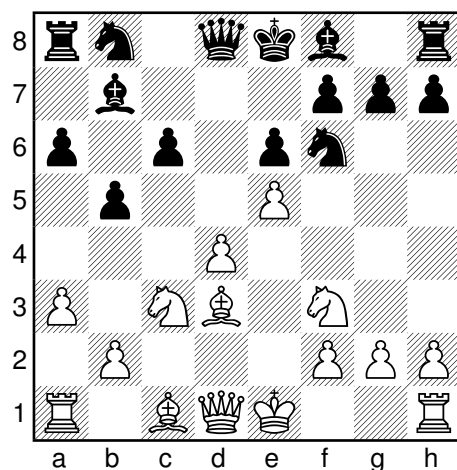
29. Bxh5 Rxd1+ 30. Qxd1 exf2+ 31. Kxf2 Rd7 32. Qc1 Rd2+ 33. Ke1 Rd3 34. Kf2 Rd2+ 35. Ke1 Rd3 36. Rg1 e4



37. Rg2 Qa5+ 38. Kf1 Qf5+ 39. Qf4 Qxf4+ 40. gxf4 Rxh3 41. Bd1 Rh4 42. Kf2 Rxf4+ 43. Ke3 Rf1 44. Bg4 Rf6 with a draw soon to follow. 1/2-1/2

**Game AZ-20: AlphaZero Semi-torpedo vs AlphaZero Semi-torpedo** The first ten moves for White and Black have been sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

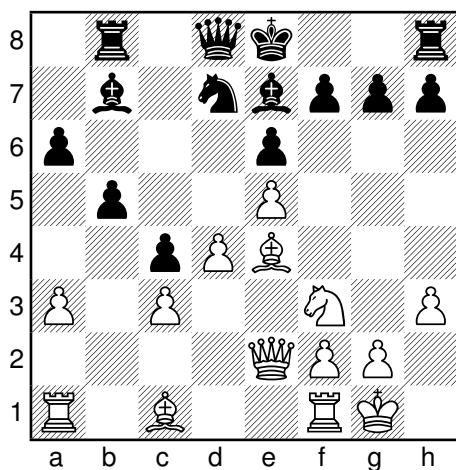
1. d4 Nf6 2. Nf3 d5 3. c4 e6 4. a3 dxc4 5. e3 c6 6. Bxc4 b5 7. Bd3 Bb7 8. Nc3 a6 9. e5



Here we see another typical central torpedo move (e3-e5), claiming space.

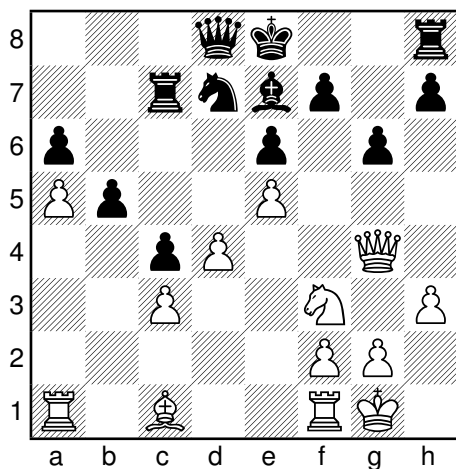
9... Nd5 10. Be4 Be7 11. h3 Nxc3 12. bxc3 Nd7 13. O-O Rb8 14. Qe2 c4





Black uses a torpedo move as a counter (c6-c4), expanding on the queenside.

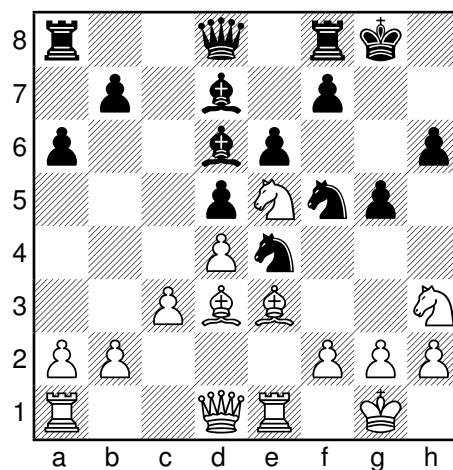
15. Bxb7 Rxb7 16. Qe4 Rc7 17. Qg4 g6 18. a5



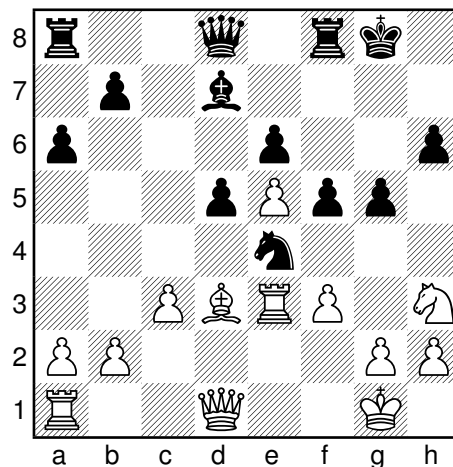
Another torpedo move follows (a3-a5), giving rise to a thematic pawn structure.

18. ... h5 19. Qg3 Nb8 20. d5 Qxd5 21. Bg5 Qd8 22. Rad1 Rd7 23. Bxe7 Qxe7 24. Ng5 O-O 25. Ne4 Rxd1 26. Rxd1 Rd8 27. Rd6 Rxd6 28. exd6 Qd8 29. Qe5 Nd7 30. Qd4 Qh4 and the game eventually ended in a draw. 1/2-1/2

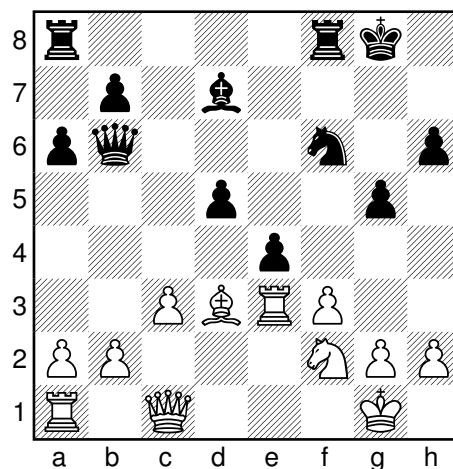
**Game AZ-21: AlphaZero Semi-torpedo vs AlphaZero Semi-torpedo** The position below, with White to move, is taken from a game that was played with roughly one minute per move:



16. f3 Bxe5 17. dxe5 Nxe3 18. Rxe3 f5

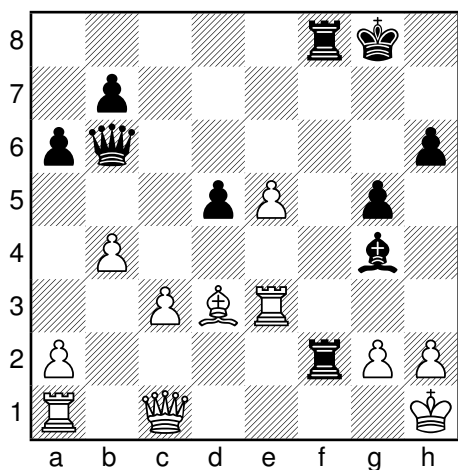


19. exf6 Qb6 20. Qc1 Nxf6 21. Nf2 e4

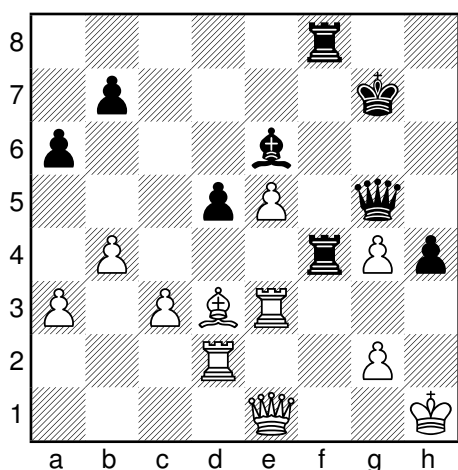


Here we see a torpedo move e6-e4 being used in a tactical sequence in center of the board.

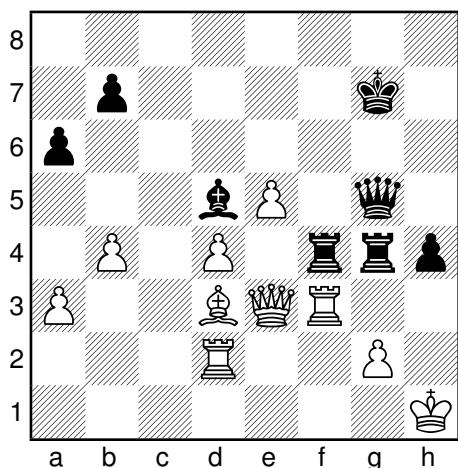
22. fxe4 Rae8 23. e5 Ng4 24. Nxe4 Bxe4 25. Kh1 Rf2 26. b4 Ref8



27. Qe1 Be6 28. Re2 R2f4 29. a3 Kg7 30. h3 Qd8 31. Re3 h5 32. Rd1 g4 33. Rd2 h4 34. hxg4 Qg5



35. Rh3 Rxc4 36. Qe3 d4 37. cxd4 R8f4 38. Rf3 Bd5



39. Rxf4 Qxf4 40. Qxf4 Rxf4 41. Kg1 Rxd4 and the game soon ended in a draw. 1/2–1/2

## B.7. Pawn-back

In the Pawn-back variation of chess, the pawns are allowed to move one square backwards, up to the 2nd/7th rank for White and Black respectively. In addition, if the pawn moves back to its starting rank, it is allowed to move by two squares again on its next move. In this particular implementation, the two-square pawn move is always allowed from the 2nd or the 7th rank, regardless of whether the pawn has moved before. A different implementation of this variation of chess might consider disallowing this, though it is unlikely to make a big difference. Because the pawns are allowed to move backwards and pawn moves are now reversible in this implementation of chess, the 50 move rule is modified so that 50 moves without captures lead to a draw, regardless of whether any pawn moves were made in the meantime.

### B.7.1. MOTIVATION

In Classical chess, pawns that move forwards leave weaknesses behind. Some of these remain long-term weaknesses, resulting in squares that can be easily occupied by the opponent's pieces. If the pawns could move backwards, they could come back to help fight for those squares and therefore reduce the number of weaknesses in a position. Allowing the pawns to move backwards would therefore make it easier to push them forward, as the effect would not be irreversible. This might make advancing in a position easier, but equally, it could provide defensive options for the weaker side, such as retreating from a less favourable situation and covering a weaknesses in front of the king.

### B.7.2. ASSESSMENT

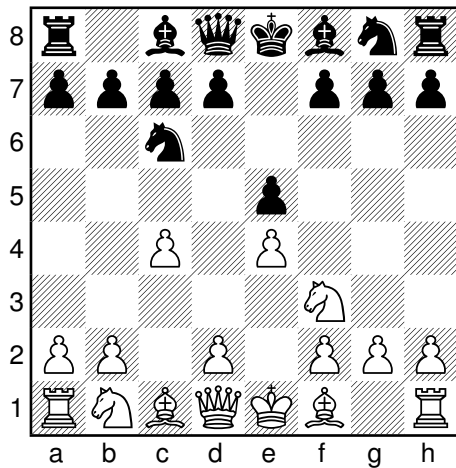
The assessment of the Pawn-back chess variant, as provided by Vladimir Kramnik:

“There are quite a few educational motifs in this variation of chess. The backward pawn moves can be used to open the diagonals for the bishops, or make squares available for the knights. The bishops can therefore become more powerful, as they are easier to activate. The pawns can be pushed in the center more aggressively than in classical chess, as they can always be pulled back. Exposing the king is not as big of an issue, as the pawns can always move back to protect. Weak squares are much less important for positional assessment in this variation, given that they can almost always be protected via moving the pawns back.

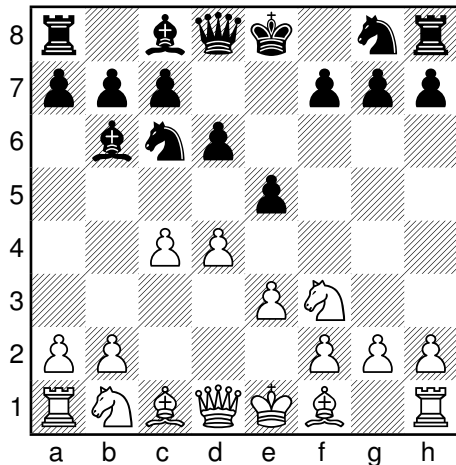
It was interesting to see AlphaZero's strong preference for playing the French defence under these rules, the point being that the light-squared bishop is no longer bad, as it can be developed

via c8-b7 followed by a timely d5-d6 back-move.

Other openings change as well. After the standard 1. e4 e5 2. Nf3 Nc6, there comes a surprise: 3. c4!



It is followed by 3... Bc5 4. e3 (a back-move!) Bb6 5. d4 d6



Who would have guessed that we are on move 5, after the game having started with e4 e5?

The Pawn-back version of chess allows for more fluid and flexible pawn structures and could potentially be interesting for players who like such strategic manoeuvring. Given that Pawn-back chess offers additional defensive resources, winning with White seems to be slightly harder, so the variant might also appeal to players who enjoy defending and attackers looking for a challenge. ”

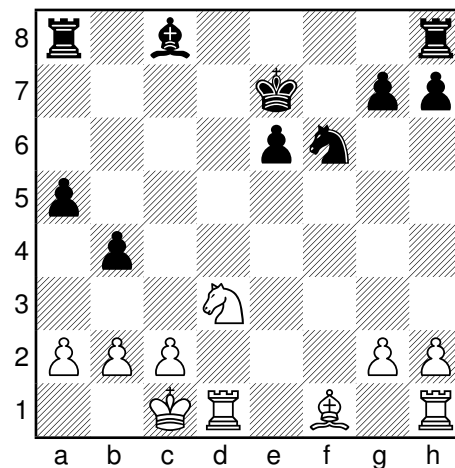
### B.7.3. MAIN LINES

Here we discuss “main lines” of AlphaZero under Pawn-back chess, when playing with roughly one minute per move

from a particular fixed first move. Note that these are not purely deterministic, and each of the given lines is merely one of several highly promising and likely options. Here we give the first 20 moves in each of the main lines, regardless of the position.

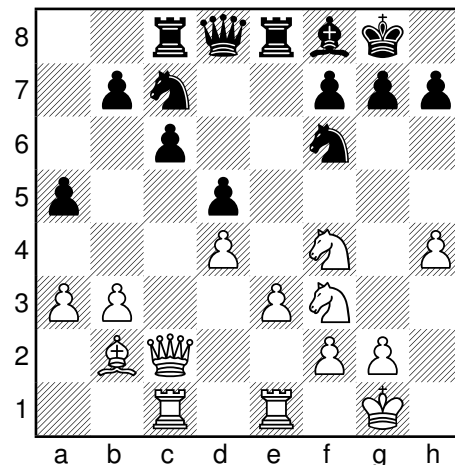
**Main line after e4** The main line of AlphaZero after 1. e4 in Pawn-back chess is:

1. e4 (book) e6 2. Nc3 d5 3. d4 Nf6 4. e5 Nfd7 5. f4 c5 6. Nf3 a6 7. Be3 b5 8. f5 Nc6 9. fxe6 fxe6 10. e4 cxd4 11. Nxd4 Nxd4 12. Qxd4 b4 13. Ne2 Nf6 14. exd5 Qxd5 15. Nf4 Qxd4 16. Bxd4 Bd6 17. Nd3 a5 18. Be5 Ke7 19. Bxd6+ Kxd6 20. O-O-O Ke7



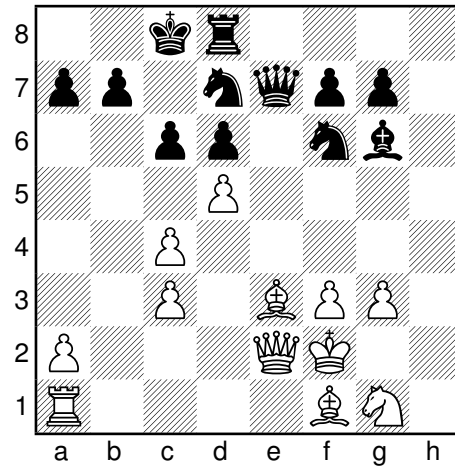
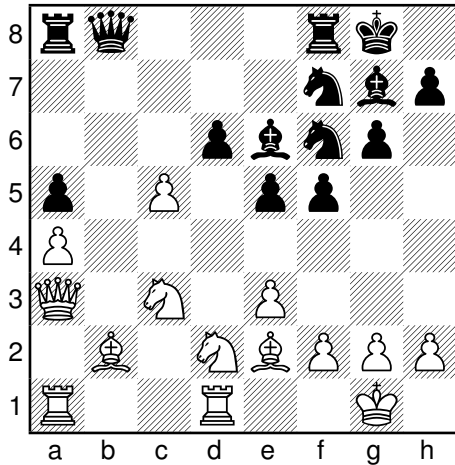
**Main line after d4** The main line of AlphaZero after 1. d4 in Pawn-back chess is:

1. d4 (book) d5 2. e3 Nf6 3. Nf3 e6 4. c4 Be7 5. b3 O-O 6. cxd5 exd5 7. Bd3 Re8 8. Bb2 a5 9. O-O Bf8 10. Nc3 c6 11. Qc2 b6 12. Ne2 Ra7 13. Rac1 Rc7 14. Rfe1 Bb4 15. Nc3 Ba6 16. Bxa6 Nxa6 17. h4 b7 18. a3 Bf8 19. Ne2 Rc8 20. Nf4 Nc7



**Main line after c4** The main line of AlphaZero after 1. c4 in Pawn-back chess is:

1. c4 (*book*) e5 2. e3 c5 3. Nc3 Nc6 4. Nf3 f5 5. d4 e4 6. Nd2 Nf6 7. d5 Ne5 8. Be2 g6 9. d4 Nf7 10. dxc5 Bxc5 11. a3 Bf8 12. b4 Bg7 13. Bb2 O-O 14. O-O d6 15. a4 Be6 16. Qb3 a5 17. Rfd1 b6 18. bxa5 bxa5 19. Qa3 e5 20. c5 Qb8



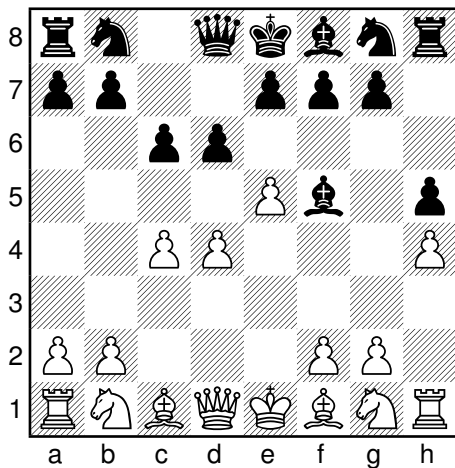
Black is putting pressure on d5, so White uses the back-move d5-d4 option to reconfigure the central pawn structure, rather than release the tension.

17. d4 d5 18. c5

#### B.7.4. INSTRUCTIVE GAMES

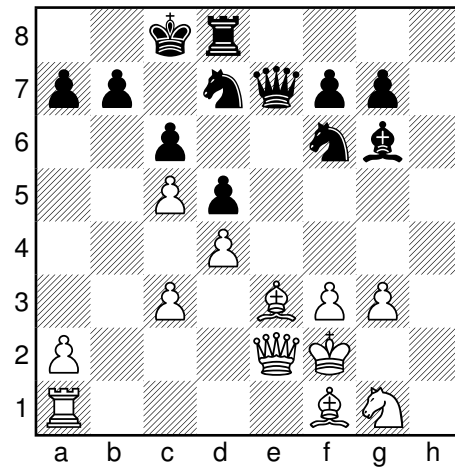
**Game AZ-22: AlphaZero Pawn-back vs AlphaZero Pawn-back** The first ten moves for White and Black have been sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

1. e4 c6 2. d4 d5 3. e5 Bf5 4. h4 h5 5. c4 d6



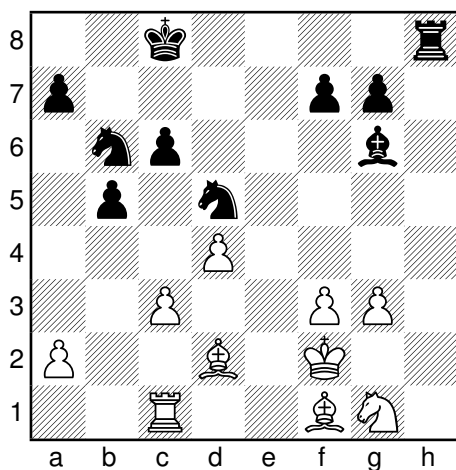
Here we see d5-d6 as the first back-move of the game, challenging White's (over)extended center – an option that would not have been available in classical chess.

6. exd6 exd6 7. d5 Be7 8. Nc3 Bxh4 9. Be3 Qe7 10. g3 Bf6 11. Rxh5 Rxh5 12. Qxh5 Bg6 13. Qe2 Bxc3+ 14. bxc3 Nd7 15. f3 O-O-O 16. Kf2 Ngf6



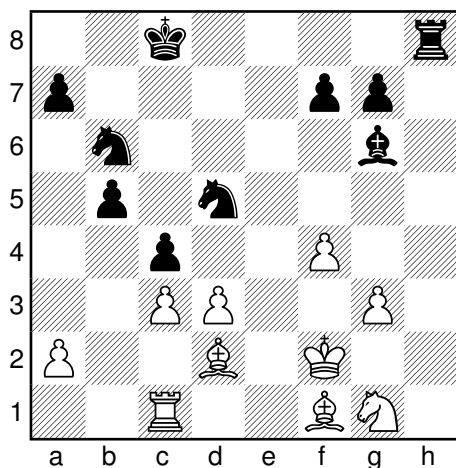
Black and White repeat back-moves a couple of times. Each time that Black challenges the c5 pawn via a d5-d6 back-move, White responds by c5-c4, refusing to exchange on that square.

18. . . d6 19. c4 d5 20. Rc1 Rh8 21. c5 d6 22. c4 d5 23. Bf4 Qxe2+ 24. Bxe2 dxc4 25. Bxc4 b5 26. Bf1 Nd5 27. Bd2 N7b6



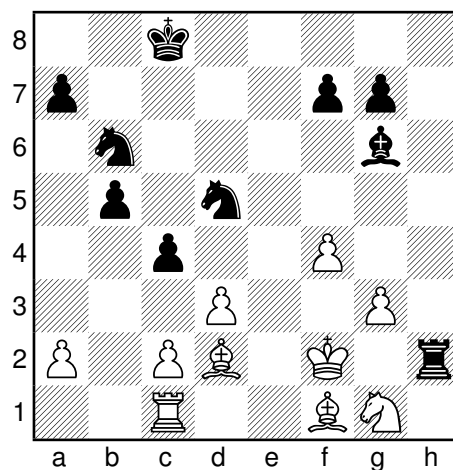
Here we see an example of how back-moves can help cover weak squares. Black is threatening to invade on the light squares on the queenside at an opportune moment, but White utilizes a back-move d4-d3 and protects c4. This, however, enables Black to go forward and Black takes the opportunity to play c6-c5.

28. d3 c5 29. f4 c4



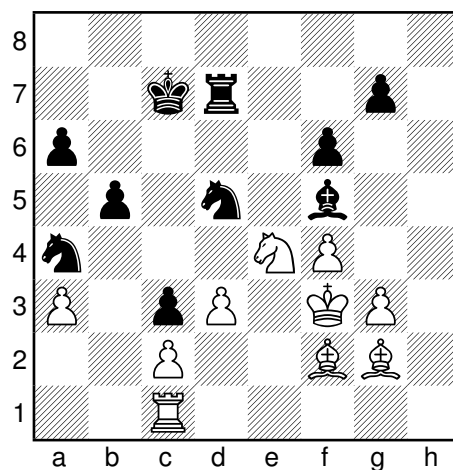
White decides to keep retreating here and not give up the light squares with a back-move c3-c2.

30. c2 Rh2+

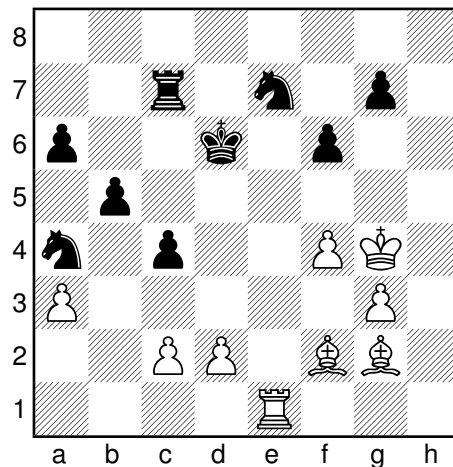


At this point it should come as no surprise how White should respond to the rook invasion using a back-move g3-g2!

31. g2 c3 32. Be1 Bf5 33. Nf3 Rh6 34. g3 Rd6 35. Bg2 a6 36. a3 Na4 37. Ng5 f6 38. Ne4 Rd7 39. Kf3 Kc7 40. Bf2



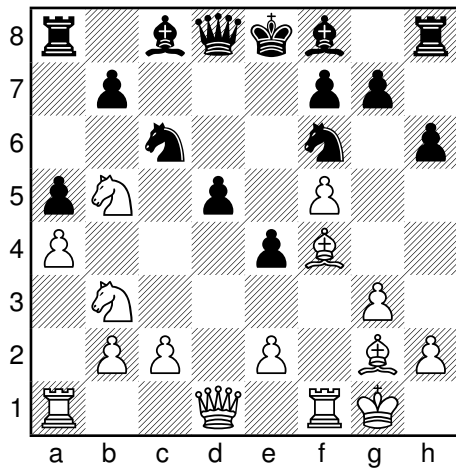
40. ... Bxe4 41. Kxe4 Kd6 42. Re1 Rc7 43. Kf5 Ne7+ 44. Kg4 c4 45. d2



Here we see both Black and White having retreated from the interaction on the queenside, Black via a back-move c3-c4 and White by playing the d-pawn back to d2. The game soon ended in a draw.

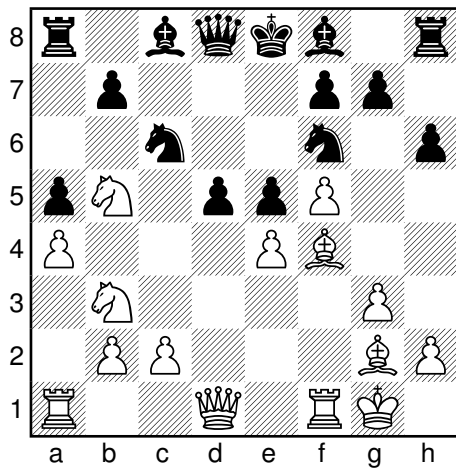
45... f5+ 46. Kf3 c3 47. d3 c4 48. d2 c3 49. d3 c4 50. g4 cxd3 51. cxd3 Rc3 52. gxf5 Rxd3+ 53. Kg4 Rd2 54. Re6+ Kd7 55. Kf3 Rd3+ 56. Kg4 Rd2 57. Kf3 Rd3+ 58. Kg4 Rd2 1/2-1/2

**Game AZ-23: AlphaZero Pawn-back vs AlphaZero Pawn-back** The position below, with Black to move, is taken from a game that was played with roughly one minute per move:



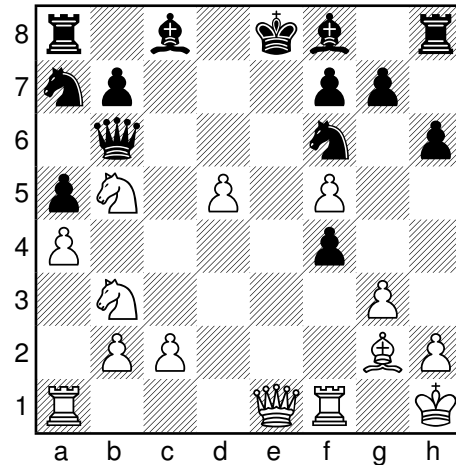
White is targeting c7 with the bishop and the knight, but here Black plays a back-move, e4-e5. It initiates a long forced tactical sequence, showcasing that things can indeed get quite tactical in this variation of chess, depending on the line of play.

13... e5 14. e4

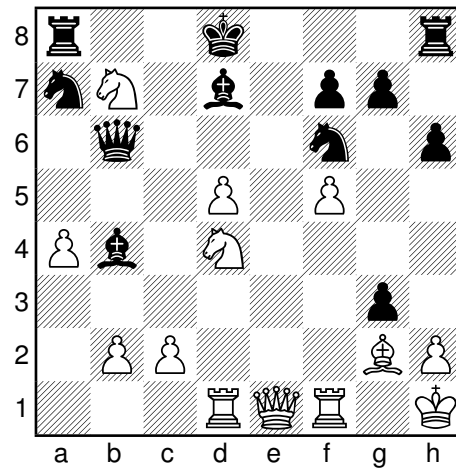


AlphaZero decides to sacrifice a piece for the initiative!

14... exf4 15. exd5 Qb6+ 16. Kh1 Na7 17. Qe1+

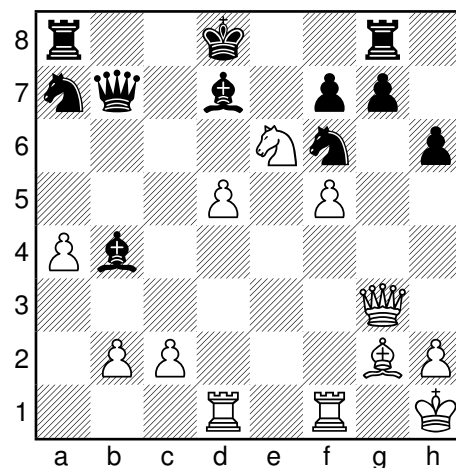


17... Kd8 18. N5d4 Bd7 19. Nxa5 fxg3 20. Rd1 Bb4 21. Nxb7+



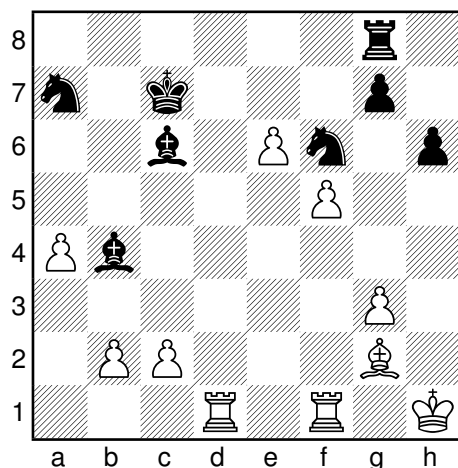
Sacrificing another piece!

21... Qxb7 22. Qxg3 Rg8 23. Ne6+



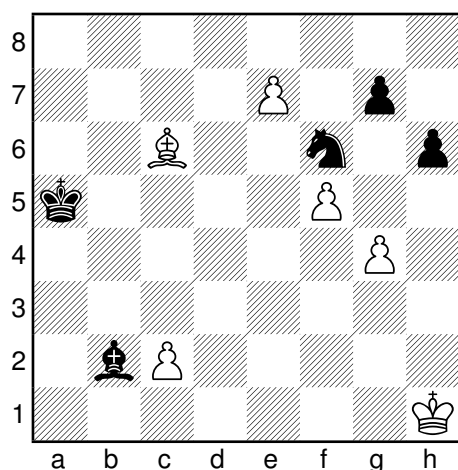
Third consecutive piece sacrifice by White!

23... fxe6 24. dxe6 Qc7 25. Bxa8 Qxg3 26. hxg3 Kc7  
27. Bg2 Bc6



It's time to take stock – White has a rook and 4 pawns for 3 pieces, a very unusual material imbalance.

28. Rf4 Rb8 29. Rc4 Bd6 30. Rb1 Kb6 31. Re1 Nh5 32. g4  
Nf6 33. Re3 Rc8 34. Rec3 Be5 35. a5+ Ka6 36. Rxc6+ Rxc6  
37. Rxc6+ Nxc6 38. Bxc6 Bxb2 39. e7 Kxa5

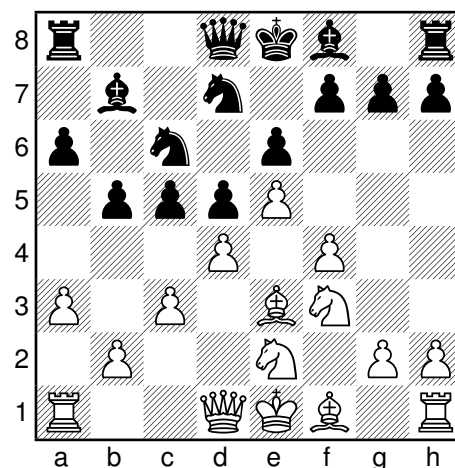


And the game soon ended in a draw.

1/2–1/2

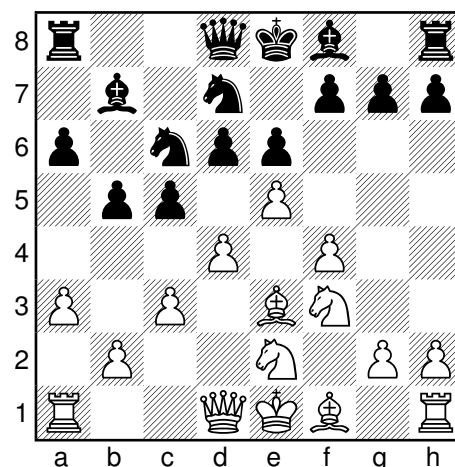
**Game AZ-24: AlphaZero Pawn-back vs AlphaZero Pawn-back** The first ten moves for White and Black have been sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

1. e4 e6 2. Nc3 d5 3. d4 Nf6 4. e5 Nfd7 5. f4 c5 6. Nf3 a6  
7. a3 Nc6 8. Be3 b5 9. Ne2 Bb7 10. c3



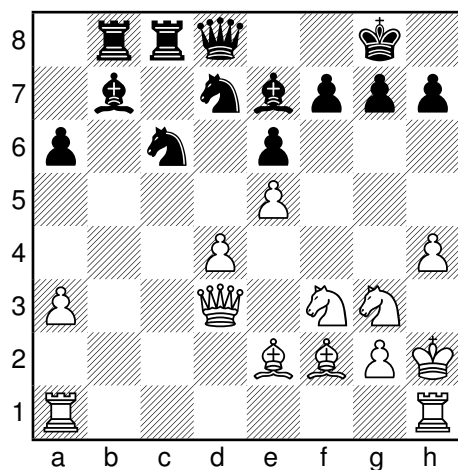
This looks like a pretty normal French position, but here comes Black's main equalizing resource, a back move d5-d6! Maybe that's all that was needed to make the French an undeniably good opening for Black?

10... d6



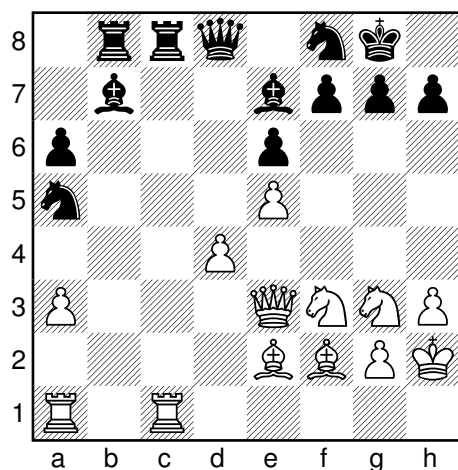
This completely changes the nature of the position, as the center is suddenly not static and Black's light-squared bishop can find good use on the a8-h1 diagonal.

11. Ng3 dxe5 12. fxe5 Qb6 13. Bf2 Rd8 14. Qb1 cxd4  
15. cxd4 b4 16. Be2 bxa3 17. bxa3 Qa5+ 18. Kf1 Rb8  
19. h4 Be7 20. Kg1 O-O 21. Qd3 Rfc8 22. Kh2 Qd8



Here AlphaZero prefers a solid back-move h4-h3 to a further expansion with h5.

23. h3 Na5 24. Rhc1 Nf8 25. Qe3

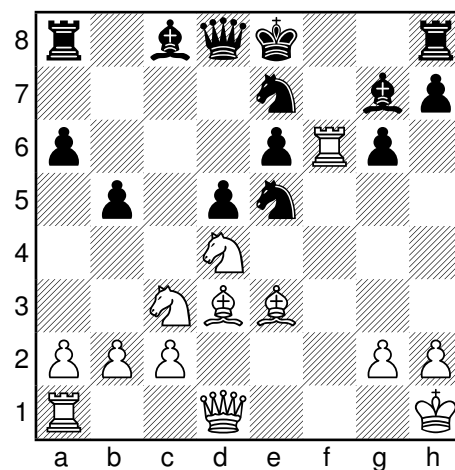


The a6 pawn is under pressure from the e2 bishop, and simply moves back to a7. The game soon fizzles out to a draw.

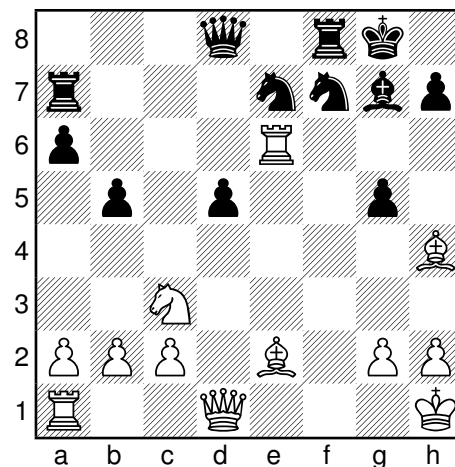
25... a7 26. a4 Ng6 27. Rab1 Rxc1 28. Qxc1 Rc8 29. Qd1 Ba8 30. Ba6 Rb8 31. Bf1 Rxb1 32. Qxb1 Bc6 33. Bb5 Qb8 34. Qd3 Qb7 35. Ne2 h6 36. Bg3 Be4 37. Qe3 Bb4 38. Bf2 Bd5 39. Qd3 Be4 40. Qe3 Bc6 41. Qd3 Be7 42. Bg3 Be4 43. Qe3 Bb4 44. Bf2 **1/2-1/2**

**Game AZ-25: AlphaZero Pawn-back vs AlphaZero Pawn-back** The first ten moves for White and Black have been sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

1. e4 e6 2. Nc3 d5 3. d4 Nf6 4. e5 Nfd7 5. Be3 c5 6. f4 a6 7. Nf3 b5 8. f5 Nc6 9. fxe6 fxe6 10. Bd3 g6 11. O-O cxd4 12. Nxd4 Ndx5 13. Kh1 Ne7 14. Rf6 Bg7



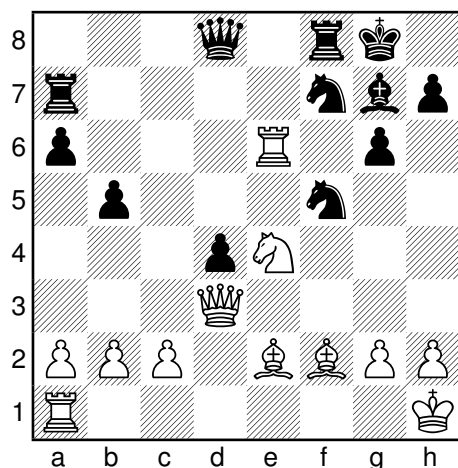
15. Nxe6 Bxe6 16. Rxe6 O-O 17. Bg5 Ra7 18. Be2 Nf7 19. Bh4 g5



Here we see that moves like g5, that would potentially otherwise be quite weakening, are perfectly playable, given that the g-pawn can (and soon will) move back to g6, and in the meantime the threatening bishop is forced to move back and unpin the Black knight on e7.

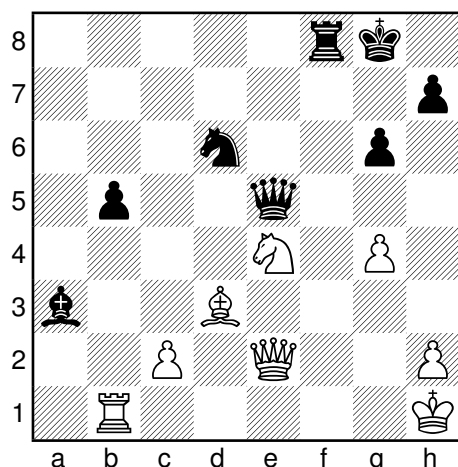
20. Bf2 d4 21. Ne4 Nf5 22. Qd3 g6





After moving the pawn back to g6 with a back-move, Black safeguards the kingside, justifying the previous g5 pawn push, which was helpful in achieving development.

23. g4 Ne3 24. Bxe3 dxe3 25. Qxe3 Re7 26. Rxe7 Qxe7 27. a4 Nd6 28. Bd3 Bxb2 29. Rb1 Qe5 30. axb5 axb5 31. Qe2 Ba3

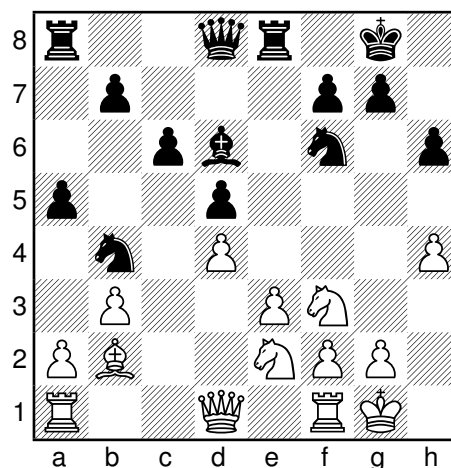


As a mirror-motif to Black's g5-g6, here White plays g4-g3 to improve the safety of its king.

32. g3 Nxe4 33. Qxe4 Qxe4 34. Bxe4 b4 and the game soon ended in a draw. 1/2-1/2

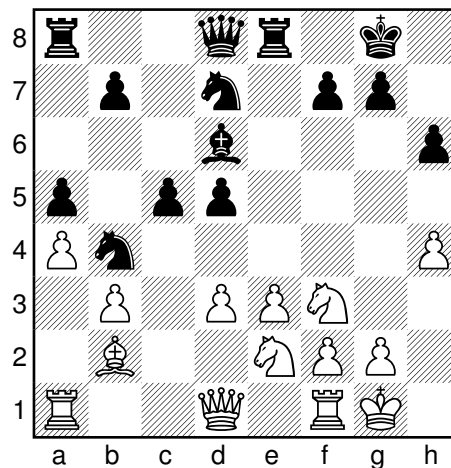
**Game AZ-26: AlphaZero Pawn-back vs AlphaZero Pawn-back** The first ten moves for White and Black have been sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

1. d4 Nf6 2. c4 e6 3. Nf3 a6 4. Nc3 d5 5. cxd5 exd5 6. b3 Bb4 7. Bd2 Be7 8. e3 O-O 9. Bc1 Bf5 10. Bd3 Bxd3 11. Qxd3 c6 12. Qc2 Re8 13. O-O a5 14. h4 Na6 15. Ne2 Nb4 16. Qd1 Bd6 17. Bb2 h6



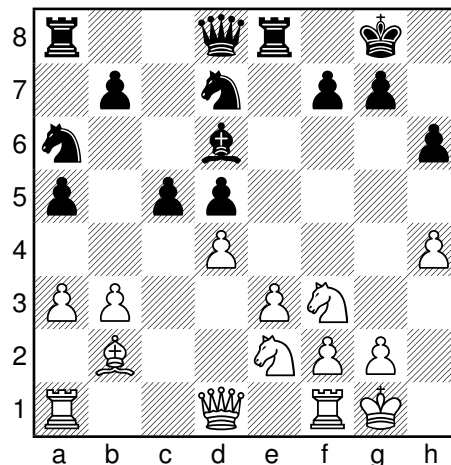
Here we see the first back-move of the game, opening the diagonal for the White bishop – d4-d3!

18. d3 Nd7 19. a4 c5



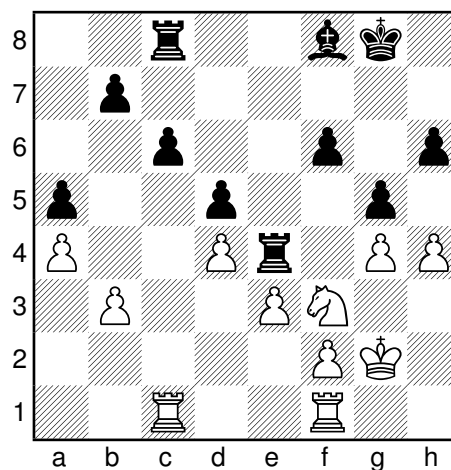
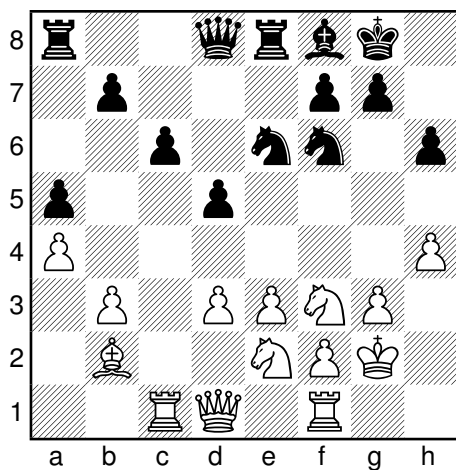
Just having played a4 on the previous move, White plays a back-move a4-a3 to challenge the b4 knight, given that the circumstances have changed due to Black having played c5.

20. a3 Na6 21. d4



White goes back to the previous plan and plays the pawn to d4 again, despite having moved it back before, showcasing the fluidity of pawn structures Black responds by moving the c-pawn back, to avoid having an isolated pawn.

21... c6 22. g3 Nc7 23. a4 Ne6 24. Kg2 Nf6 25. Rc1 Bf8  
26. d3

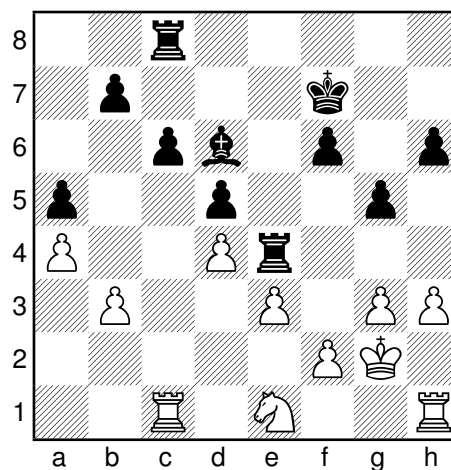
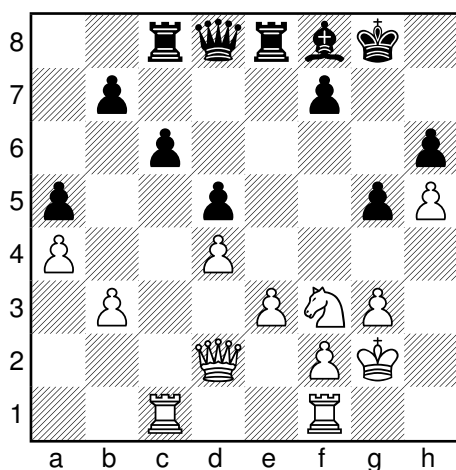


Black is attacking White's pawn on g4, so it just moves back to g3.

43. g3 Kf7 44. Rh1 g4 45. Ne1 Bd6 46. h3 g5

White opens the Bishop's diagonal with a back-move, again.

26... Rc8 27. Nfd4 Nxd4 28. Bxd4 c5 29. Ba1 Nh5 30. Ng1  
g6 31. Nf3 Ng7 32. Bxg7 Bxg7 33. d4 c6 34. Qd2 Bf8  
35. h5 g5

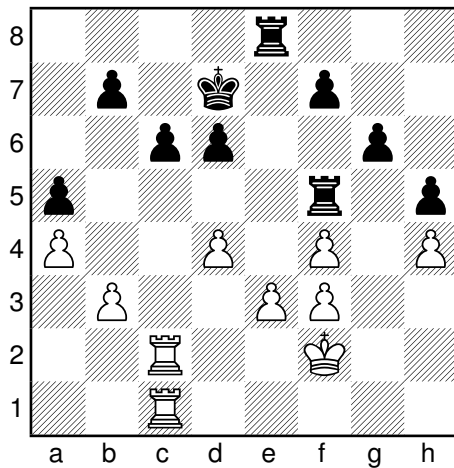


Having just played h5, White plays h5-h4 now, to attack Black's g-pawn again. They repeat once before continuing with other plans.

36. h4 g6 37. h5 g5 38. Qd3 Rc7 39. Qf5 Qc8 40. Qxc8  
Rxc8 41. h4 f6 42. g4 Re4

After having been challenged by a h4-h3 back-move, Black retreats with g4-g5 as well.

47. Nd3 Ke8 48. h4 g4 49. h3 g5 50. h4 g6 51. Kf3 Kd7  
52. Nf4 Rg8 53. Ne2 h5 54. Nf4 Bxf4 55. gxf4 b6 56. a3 f7  
57. Rc2 Ra8 58. Rb1 Re6 59. Ke2 Rf6 60. 2f3 Rf5 61. Kf2  
d6 62. a4 Re8 63. Rbc1 b7



White takes aim at the c6 pawn, but Black simply plays b6-b7, guarding it. With no clear way forward in this position, and after many more pawn structure reconfigurations, the game unsurprisingly ended in a draw. **1/2–1/2**

### B.8. Pawn-sideways

In the Pawn-sideways version of chess, pawns are allowed an additional option of moving sideways by one square, when available.

#### B.8.1. MOTIVATION

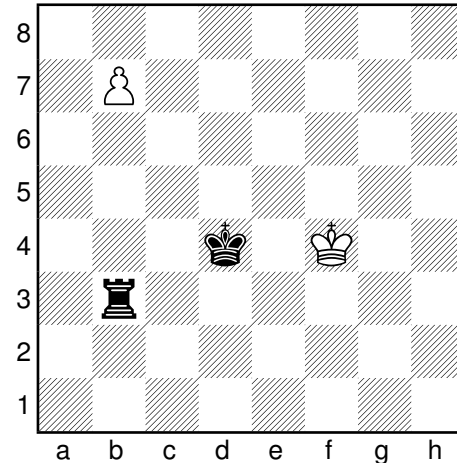
Allowing the pawns to move laterally introduces lots of new tactics into chess, while keeping the pawn structures very flexible and fluid. It makes pawns much more powerful than before and drastically increases the complexity of the game, as there are many more moves to consider at each juncture – and no static weaknesses to exploit.

#### B.8.2. ASSESSMENT

The assessment of the Pawn-sideways chess variant, as provided by Vladimir Kramnik:

“This is the most perplexing and “alien” of all variants of chess that we have considered. Even after having looked at how AlphaZero plays Pawn-side chess, the principles of play remain somewhat mysterious – it is not entirely clear what each side should aim for. The patterns are very different and this makes many moves visually appear very strange, as they would be mistakes in Classical chess.

*Lateral pawn moves change all stages of the game. Endgame theory changes entirely, given that the pawns can now “run away” laterally to the edge of the board, and it is hard to block them and pin them down. Consider, for instance, the following position, with White to move:*



*In classical chess, White would be completely lost. Here, White can play b7-a7 or b7-c7, changing files. The rook can follow, but the pawn can always step aside. In this particular position, after b7-c7, Rc3, c7-d7 – Black has no way of stopping the pawn from queening, and instead of losing – White actually wins!*

*It almost appears as if being a pawn up might give better chances of winning than being up a piece for a pawn. In fact, AlphaZero often chooses to play with two pawns against a piece, or a minor piece and a pawn against a rook, suggesting that pawns are indeed more valuable here than in classical chess.*

*This variant of chess is quite different and at times hard to understand, but could be interesting for players who are open to experimenting with few attachments to the original game!*

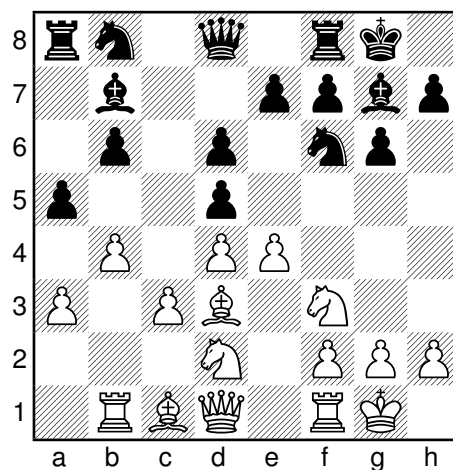
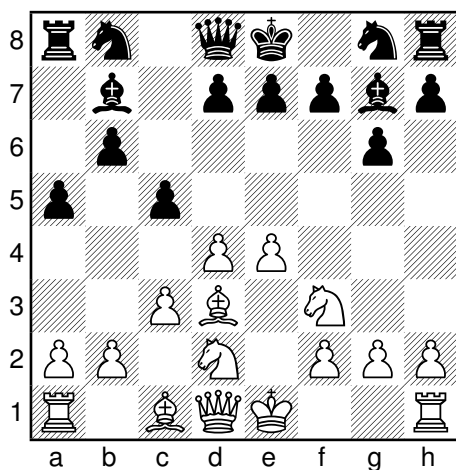
”

#### B.8.3. MAIN LINES

Here we discuss “main lines” of AlphaZero under Pawn-sideways chess, when playing with roughly one minute per move from a particular fixed first move. Note that these are not purely deterministic, and each of the given lines is merely one of several highly promising and likely options. Here we give the first 20 moves in each of the main lines, regardless of the position.

**Main line after e4** The main line of AlphaZero after 1. e4 in Pawn-sideways chess is:

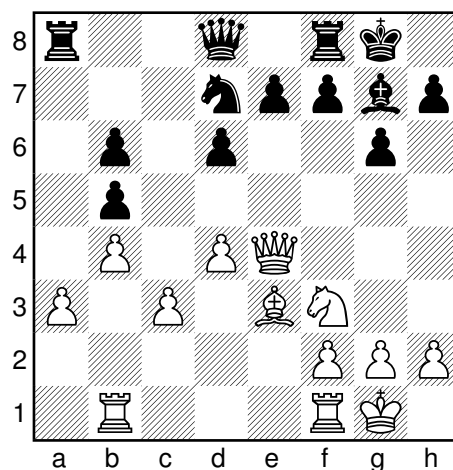
1. e4 (book) c5 2. c3 b6 3. dd4 Bb7 4. Nd2 g6 5. Bd3 Bg7 6. Ngf3 a5



11. Qc2 Nxe4 12. Nxe4 dxe4 13. Bxe4 Bxe4 14. Qxe4 Nd7  
15. Be3 ab5

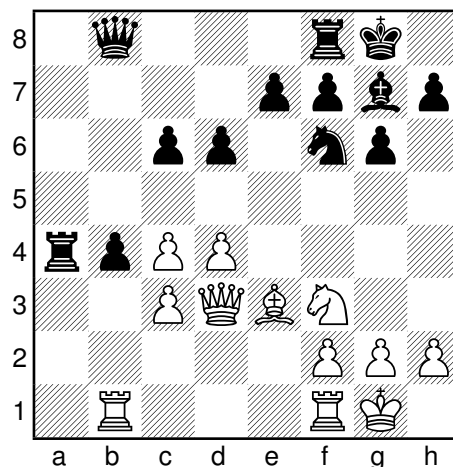
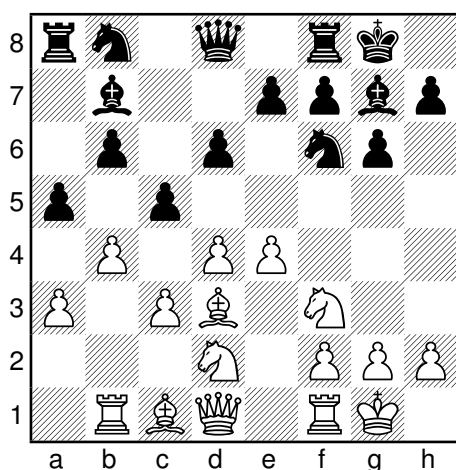
The previous move (a5) seems very unusual to a Classical chess player's eye. Black chooses to disregard the centre, while creating a glaring weakness on b5. Yet, there is method to this "madness". It seems that rushing to grab space early is not good in this setup, so White's most promising plan according to AlphaZero is to prepare b4. Apart from fighting against that advance, a5 prepares for playing a5-b5! later in this line, as we will see. Yet, this whole line of play is hard to grasp as it violates the Classical chess principles.

7. O-O d6 8. Rb1 Nf6 9. a3 O-O 10. b4



As mentioned earlier, the a5 pawn finds a new purpose – on b5! The b6 pawn will soon move to c6, in the process of reconfiguring the pawn structure.

16. ab3 Nf6 17. Qd3 bc6 18. cc4 Qb8 19. a4 b4 20. c3 Rxa4

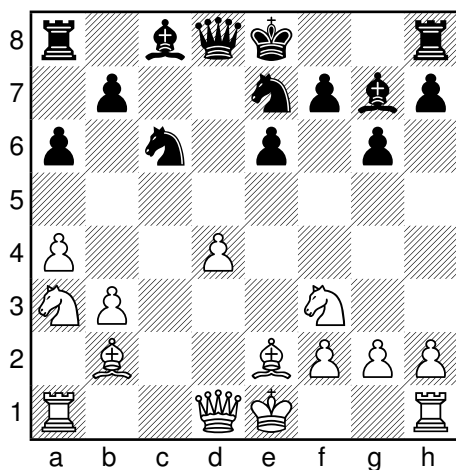


White has achieved the desired advance, to which Black responds with a lateral move – c5-d5!

10... cd5

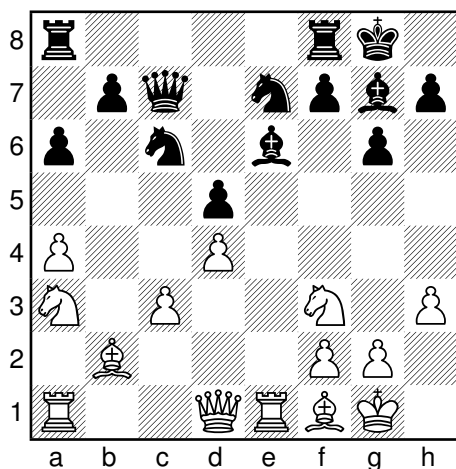
**Main line after d4** The main line of AlphaZero after 1. d4 in Pawn-sideways chess is:

1. d4 (*book*) d5 2. e3 e6 3. cc4 dxc4 4. Bxc4 a6 5. a4 c5 6. Nf3 Nc6 7. Be2 cxd4 8. exd4 g6 9. b3 Nge7 10. Bb2 Bg7 11. Na3



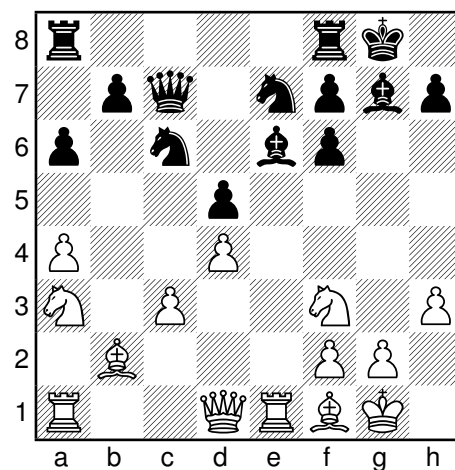
Here Black has a way of opening the light-squared bishop while safeguarding the e5 square, by playing:

11... d6 12. O-O-O 13. c3 d5 14. Re1 Qc7 15. Bf1 Be6 16. h3

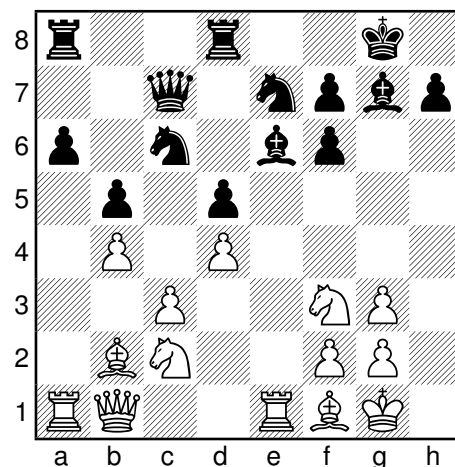


In this position, Black utilizes a rather unique defensive resource:

16... gf6

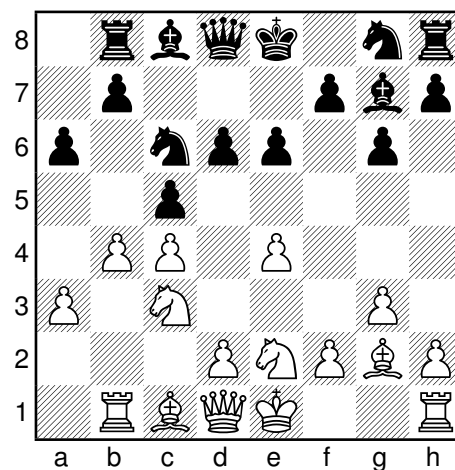


17. Nc2 Rfd8 18. Qb1 Rab8 19. hg3 b5 20. b4 Ra8



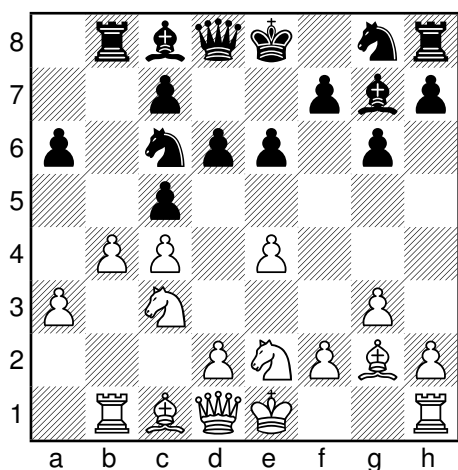
**Main line after c4** The main line of AlphaZero after 1. c4 in Pawn-sideways chess is:

1. c4 (*book*) c5 2. Nc3 Nc6 3. g3 g6 4. Bg2 e6 5. e4 a6 6. a3 Rb8 7. Nge2 Bg7 8. Rb1 dd6 9. bb4

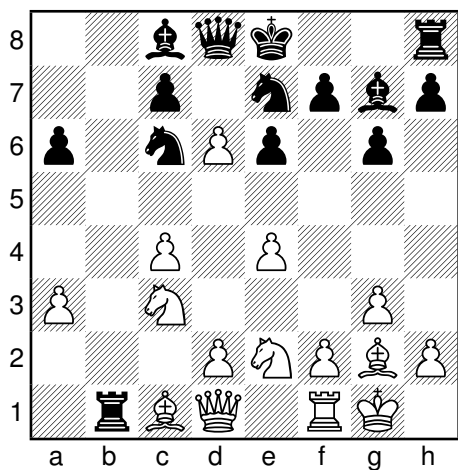


Here, Black responds with a typical lateral move.

9... c7

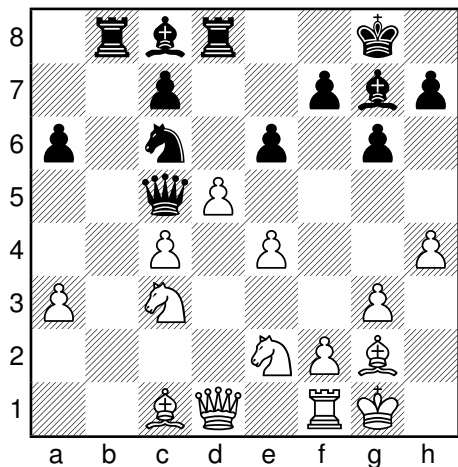


10. O-O Nge7 11. bxc5 Rxb1 12. cxd6



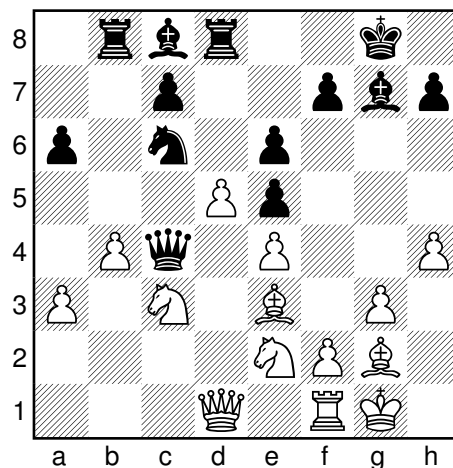
White fights for the advantage by going for this kind of a material imbalance, an exchange down.

12... Rb8 13. dxe7 Qxe7 14. dd4 O-O 15. h4 Rd8 16. d5 Qc5



Here another lateral move proves useful:

17. b4 Qc4 18. Bf4 e5 19. Bg5 gf6 20. Be3 e6

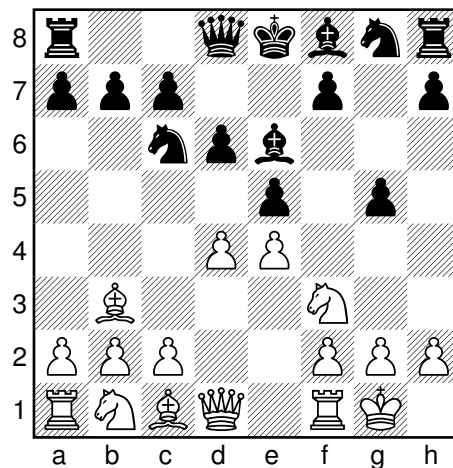


Black moves the g6 pawn first to f6 and then to e6, reaching this position. The continuation shown here is not forced, and in some of its games, AlphaZero opts for slightly different lines with Black, as this seems to be a very rich opening.

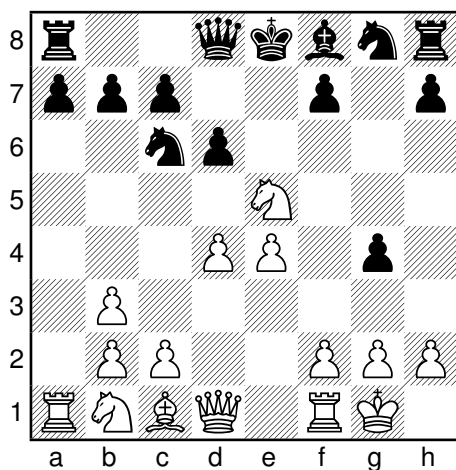
#### B.8.4. INSTRUCTIVE GAMES

**Game AZ-27: AlphaZero Pawn-sideways vs AlphaZero Pawn-sideways** The game is played from a fixed opening position that arises after: 1. e4 e5 2. Nf3 Nc6 3. Bc4. The remaining moves follow best play, at roughly one minute per move.

1. e4 (book) e5 (book) 2. Nf3 (book) Nc6 (book) 3. Bc4 (book) d6 4. O-O Be6 5. Bb3 g5 6. dd4

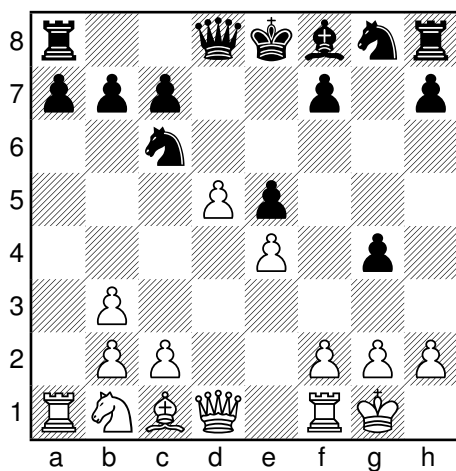


6... Bxb3 7. axb3 g4 8. Nxe5



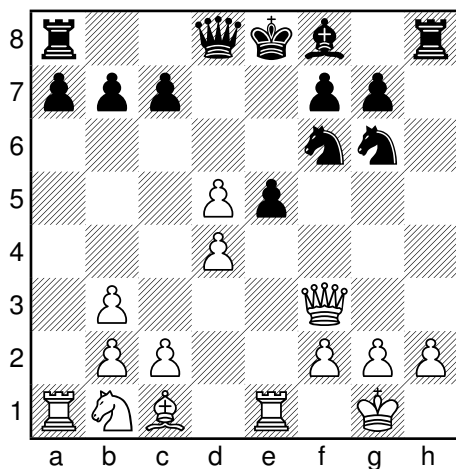
Already, things are getting very tactical and very unorthodox.

8... dxe5 9. d5



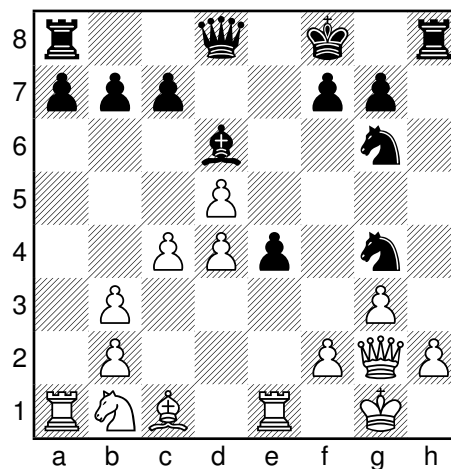
Black leaves the knight on c6 and goes on with creating counter-threats.

9... hg7 10. Qxg4 Nf6 11. Qf3 Ne7 12. Re1 Ng6 13. d4



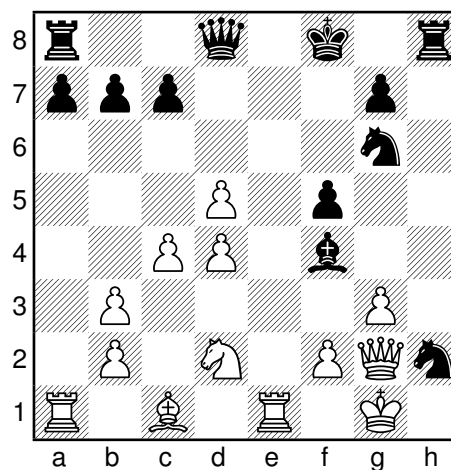
White uses a lateral move (e4-d4) to create threats on the e-file.

13... e4 14. cc4 Bd6 15. g3 Kf8 16. Qg2 Ng4



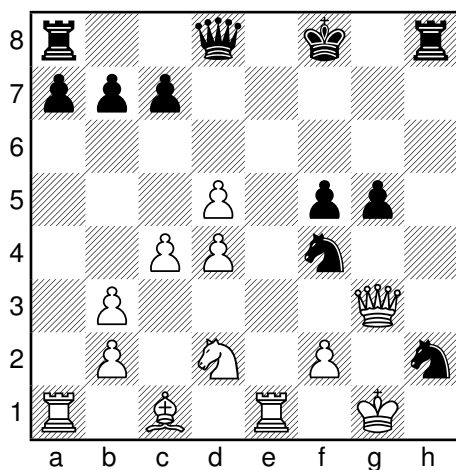
Black goes for the attack.

17. Rxe4 Nxe2 18. Nd2 f5 19. Re1 Bf4



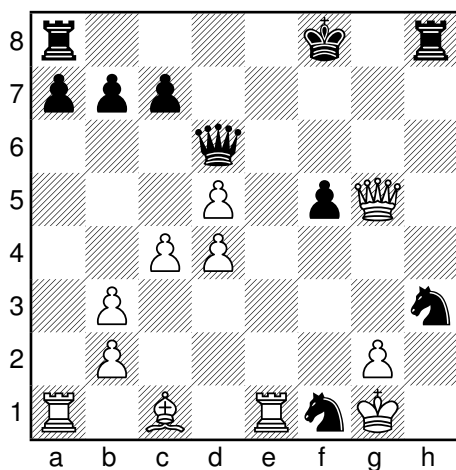
Offering a piece on f4.

20. gxf4 Nxf4 21. Qg3 gg5

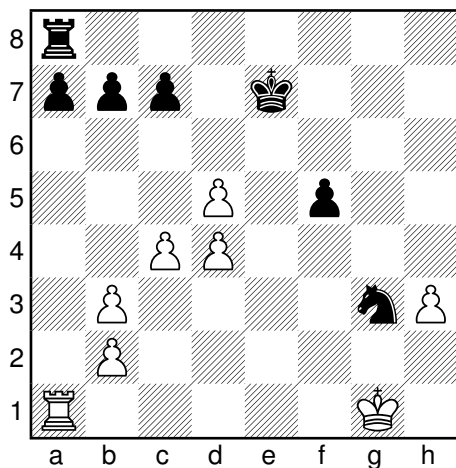


White uses a lateral pawn move to safeguard the king.

22. g2 Qd6 23. Nf1 Nxf1 24. Qxg5 Nh3+

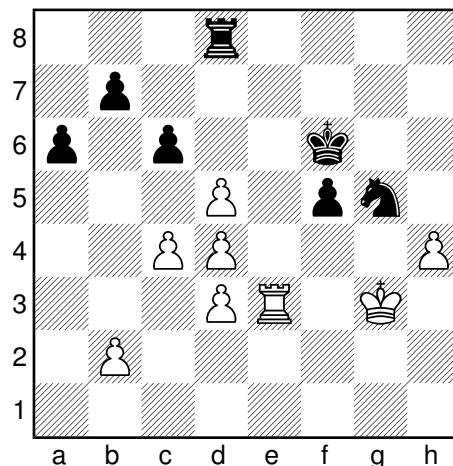


25. gxh3 Rg8 26. Re5 Rxg5+ 27. Bxg5 Ng3 28. Be7+ Qxe7  
29. Rxe7 Kxe7



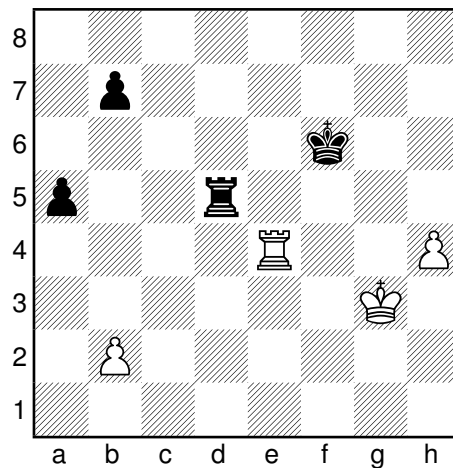
Finally the dust has settled: White having two pawns for the piece.

30. c3 a6 31. Re1+ Kd7 32. Kg2 Ne4 33. e5 Ke6 34. d3  
Rg8+ 35. Kf3 Ng5+ 36. Kg3 c6 37. Re3 Rd8 38. ed5+ Kf6  
39. h4



Here Black decides to take on d5 rather than try to move the knight, and White recaptures on d5 as well rather than taking on g5!

39... cxd5 40. cxd5 Rxd5 41. e4 fxe4 42. dxe4 Nxe4  
43. Rxe4 a5



And now the game moves towards a draw.

44. Ra4 Rd2 45. a2 ab5 46. Rf4+ Ke5 47. Rb4 c5 48. Rxb7  
Rxa2 49. h5 Ra6 50. Rb5 d5 51. Kg4 Ke4 52. Kg5 d4

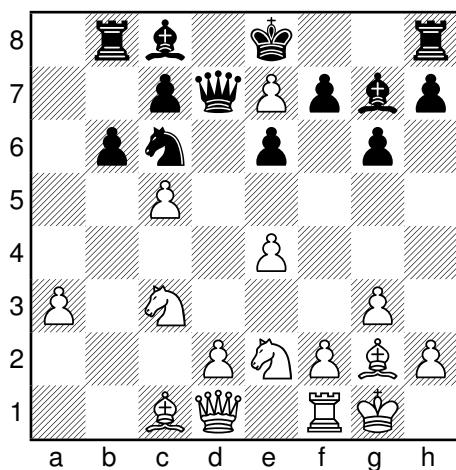
with a draw to follow soon.

1/2–1/2

**Game AZ-28: AlphaZero Pawn-sideways vs AlphaZero Pawn-sideways** The game is played from a fixed opening position that arises after 1. c4 c5. The remaining moves follow best play, at roughly one minute per move.

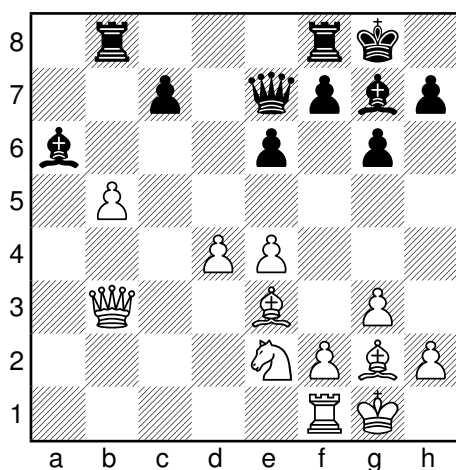
1. c4 (*book*) c5 (*book*) 2. Nc3 Nc6 3. g3 g6 4. Bg2 e6 5. e4  
a6 6. a3 dd6 7. Nge2 Bg7 8. Rb1 Nge7 9. O-O Rb8 10. bb4  
c7 11. bxc5 Rxb1 12. cxd6 Rb8 13. dxe7 Qd7 14. c5 b6





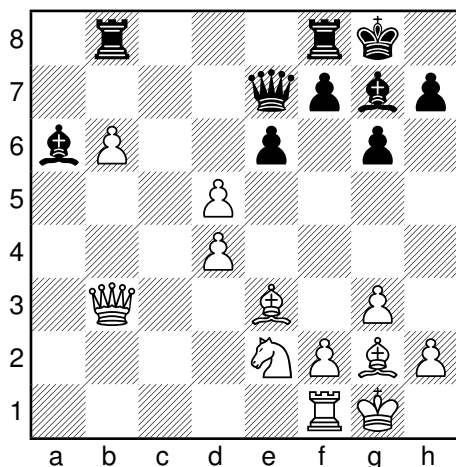
To Black's a6-b6, White responds with c5-b5, another lateral move.

15. b5 Nd4 16. Nxd4 Bxd4 17. Ne2 Bg7 18. a4 Qxe7 19. dd4 O-O 20. Qb3 a6 21. Be3 axb5 22. axb5 Ba6



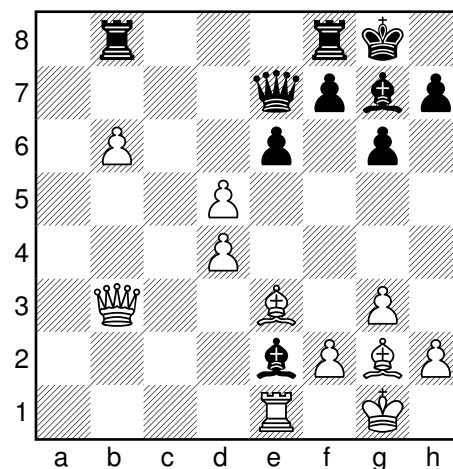
White uses a lateral move to protect the pawn

23. c4 c6 24. b6 c5 25. ed4 d5 26. cxd5



Not minding to give up the piece, for getting strong passed pawns in return.

26... Bxe2 27. Re1

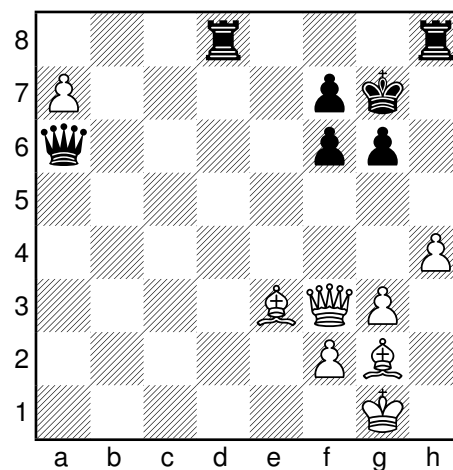


And yet, Black agrees and decides to return the piece instead.

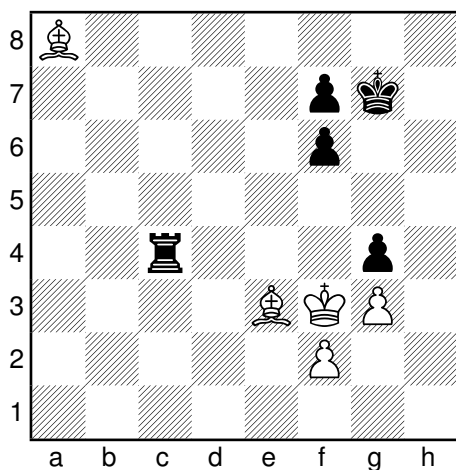
27... exd5 28. Rxe2 Bxd4 29. Bxd4 Qxe2 30. Bxd5

White opts to have the bishop pair and a pawn for two exchanges, an unbalanced position.

30... g6 31. h4 hg7 32. b7 Qa6 33. Bg2 Rfe8 34. Bc5 gg6 35. Qf3 Kg7 36. a7 Rbd8 37. Be3 Rh8



38. Qf4 Rd7 39. Qb8 Rdd8 40. Qc7 Qa1+ 41. Kh2 g5 42. Qc4 Qe5 43. Kh3 Rc8 44. Qg4 Qe6 45. Bb7 Qxg4+ 46. Kxg4 Rc4+ 47. Kf3 gxh4 48. a8=R Rxa8 49. Bxa8 g4+

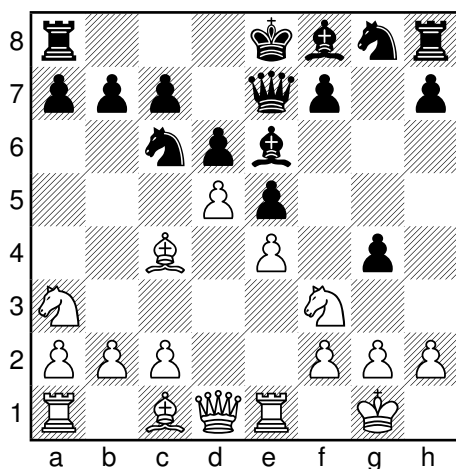


50. Ke2 e6 51. ff3 gxf3+ 52. Bxf3 f5 53. Kd3 Ra4 54. Bd1 Ra3+ 55. Ke2 ee5 56. f3 Kf6 57. Bc1 Ra2+ 58. Bd2 g5 59. Bb3 Ra3 60. Bd5 ef5 61. Be3 f4 62. Bd4+ Kf5 63. Be4+ Ke6 64. e3 fg4 65. Kf2 f5 66. Bb7 e5 67. Bc8+ Kd5 68. Bxe5 Kxe5 69. Bxg4

and the game soon ended in a draw.

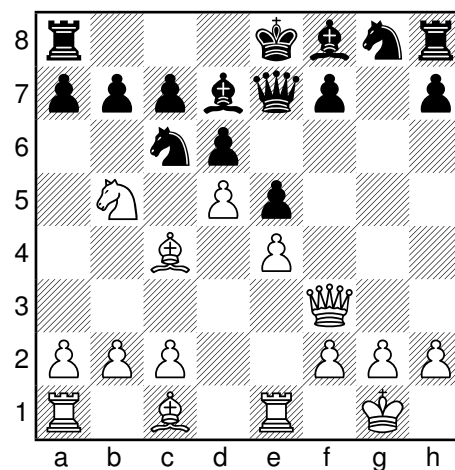
1/2–1/2

**Game AZ-29: AlphaZero Pawn-sideways vs AlphaZero Pawn-sideways** Position from an AlphaZero game played at roughly one minute per move, from a predefined position.

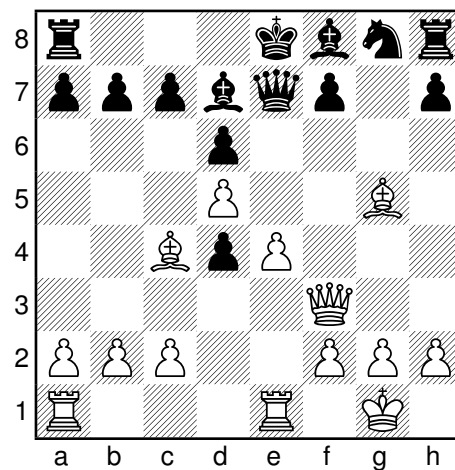


8... gxf3 9. Qxf3 Bd7 10. Nb5

Instead of capturing the knight, White has something else in mind...

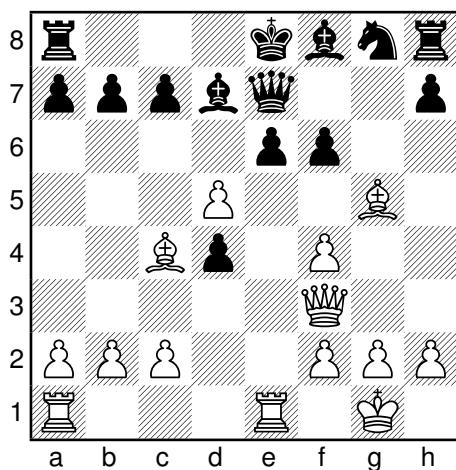


10... Nd4 11. Nxd4 exd4 12. Bg5



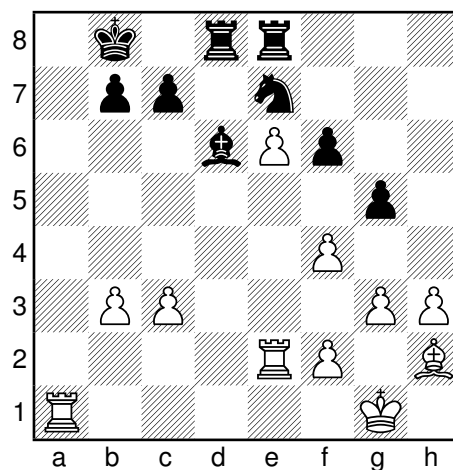
with a motif of a lateral (e4-f4) discovery! In the game, Black didn't take the bishop. So, how would have the game proceeded if Black took the bishop? Here is one possible continuation from AlphaZero: 12... Qxg5 13. f4+ Qe7 14. Rxe7+ Nxe7 15. c5 dxc5 16. Qxb7 Rc8 17. Re1 Kd8 18. Qxa7 Nc6 19. Qa4 hg7 20. c3 Rh6 21. Bb5 Rb8 22. g3 Rd6 23. d3 f6 24. h4 e6 25. h5 f7 26. Rb1 Rb6 27. Kg2. The continuation is assessed as better for White.

12... f6 13. f4 de6

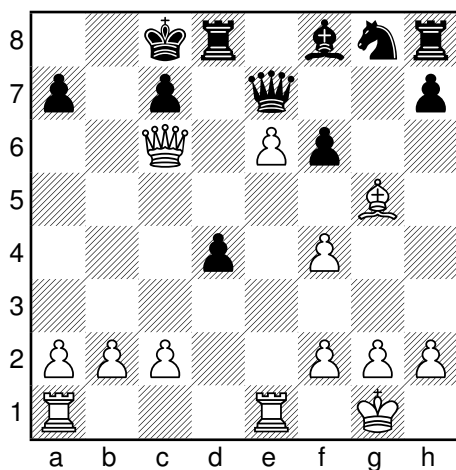


Black uses lateral moves to cover the file as well.

14. dxe6 Bc6 15. Bd5 O-O-O 16. Bxc6 bxc6 17. Qxc6

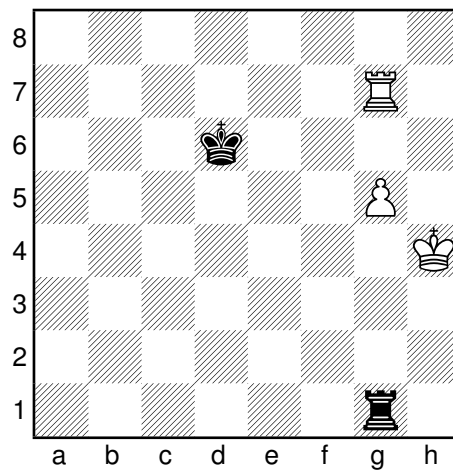


29. fxg5 fxg5 30. h4 gxh4 31. gxh4 Rh8 32. Bxd6 cxd6 33. Ra4 Rc8 34. Rg4 Nf5 35. e7 Kc7 36. Rf4 Nh6 37. g4 Kd7 38. f3 Rhe8 39. Rh2 Rxc3 40. Rxh6 Rxe7 41. Kh2 d5 42. b4 e5 43. Rf8 Rb3 44. g5 e4 45. fxe4 Rxb4 46. f4 Rb3 47. Kg1 Re2 48. Rh7+ Kd6 49. Rd8+ Kc5 50. Rd1 Rg3+ 51. Kh1 Re4 52. Rf1 Rg4 53. Rxb7 Rexf4 54. Rxf4 Rxf4 55. Kh2 Rg4 56. Rg7 Kd6 57. Kh3 Rg1 58. Kh4



White has gained several pawns for the piece, has a dangerous attack and a substantial advantage, according to AlphaZero. Yet, Black uses a lateral pawn move here to prevent immediate disaster:

17... ab7 18. Qa4 Kb8 19. Bh4 Qb4 20. Qb3 g7 21. Bg3 Bd6 22. c3 Qxb3 23. axb3 dxc3 24. bxc3 Nh6 25. h3 Nf5 26. Bh2 Rhe8 27. Re2 Ne7 28. g3 g5

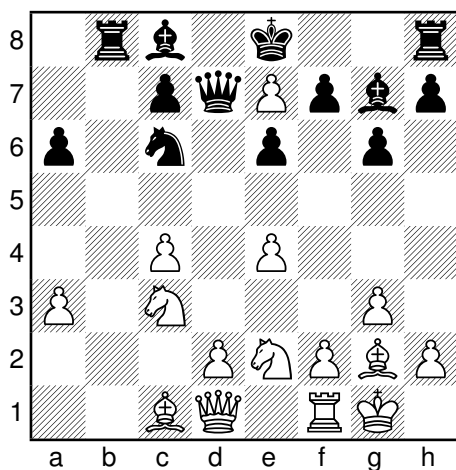


and White soon won the game.

1-0

**Game AZ-30: AlphaZero Pawn-sideways vs AlphaZero Pawn-sideways** The first ten moves for White and Black have been sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

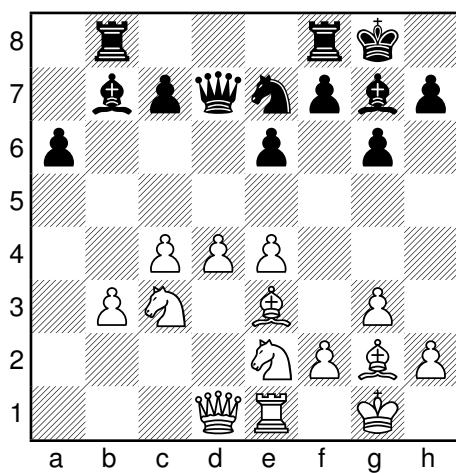
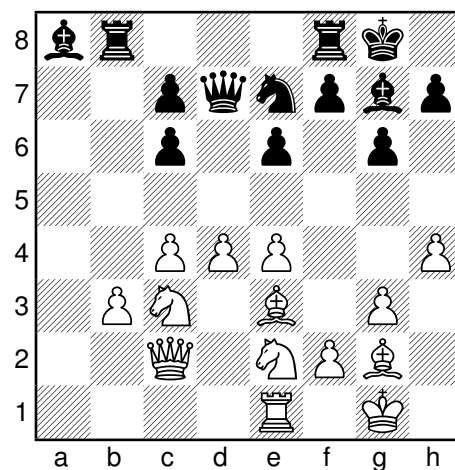
1. c4 c5 2. Nc3 Nc6 3. g3 g6 4. Bg2 e6 5. e4 dd6 6. Rb1 a6 7. a3 Bg7 8. Nge2 Rb8 9. bb4 c7 10. O-O Nge7 11. bxc5 Rxb1 12. cxd6 Rb8 13. dxe7 Qd7



In this game (unlike in the main lines section before), Black decides to recapture on e7 with the knight instead.

14. Re1 Bb7 15. b3 Nxe7 16. dd4 O-O 17. Be3

20. Bh3 Qc8 21. Rd1 Qa6 22. Bg2 Rfd8 23. Rb1 cd6 24. d5 exd5 25. Nxd5 Nxd5 26. exd5

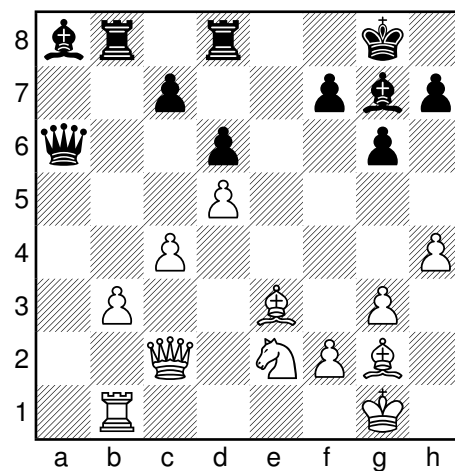


Here Black plays a lateral move (a6-b6) to improve its pawn structure:

17... b6 18. h4 Ba8 19. Qc2

But the pawn marches on, although not forward, opening the line for the rook with:

19... bc6

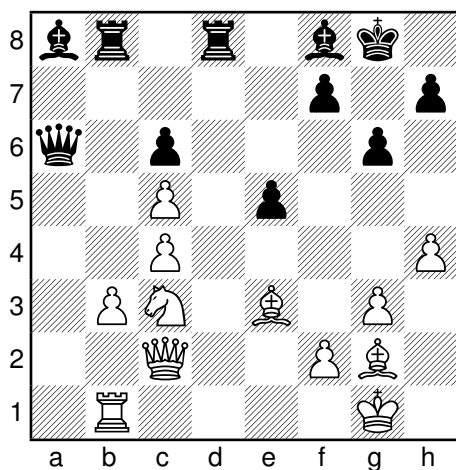


The d5 pawn is locking out the a8 bishop, so Black challenges the center with a lateral move, only to decide to push forward on the next move. This perhaps reveals a fluidity of plans as well as structures.

26... e6 27. Nf4 e5 28. Ne2 Bf8 29. Nc3 c6

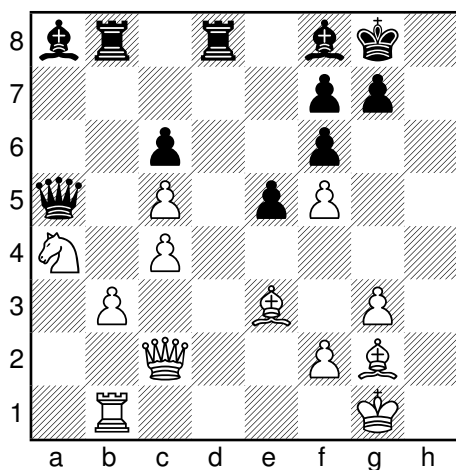
The center is challenged again, this time from the other side, but White has a lateral response to keep things locked:

30. dc5



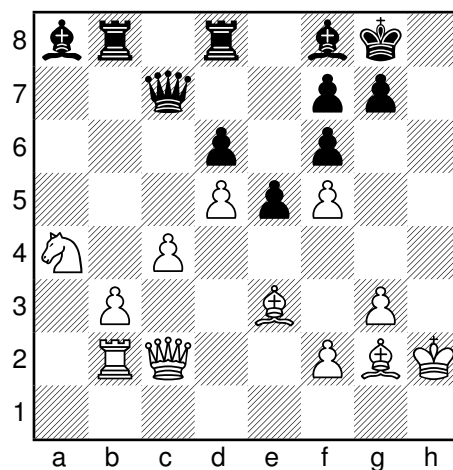
And Black responds with a lateral move as well, bringing the h-pawn towards the center.

30... hg7 31. Na4 Qa5 32. hg4 gf6 33. g5 fg6 34. f5 gf6



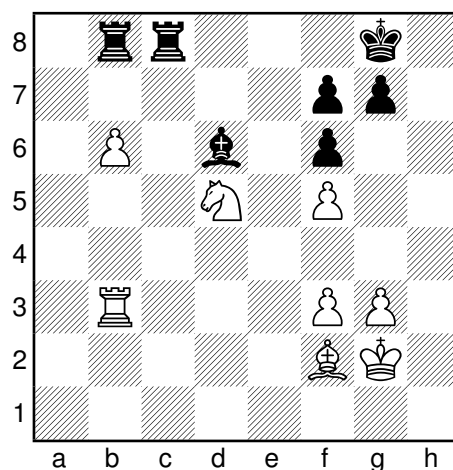
After a sequence of lateral moves, the situation has settled on the kingside.

35. Rb2 d5 36. Bf4 e5 37. Bd2 Qc7 38. Be3 Qa5 39. Ra2 Qb4 40. Rb2 Qa5 41. Kh2 d5 42. Bf4 e5 43. Bd2 Qc7 44. Be3 d6 45. d5



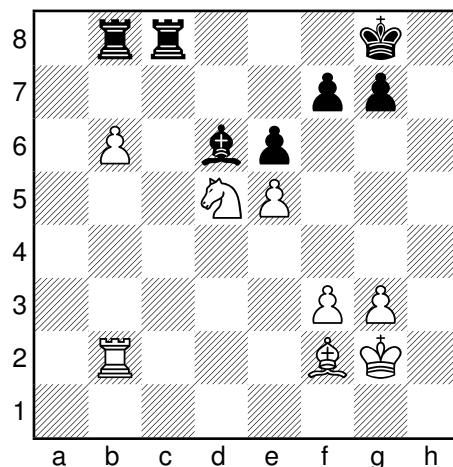
Black and White keep reconfiguring the central pawns.

45... c6 46. dc5 d6 47. cxd6 Rxd6 48. c5 Rdd8 49. b4 Bxg2 50. Kxg2 Qc6+ 51. f3 d5 52. Bd4 e5 53. Bf2 d5 54. Bd4 e5 55. Bf2 d5 56. Qb3 Qb5 57. Nc3 Qc4 58. bb5 Rdc8 59. Nxd5 Qxb3 60. Rxb3 Bxc5 61. b6 Bd6



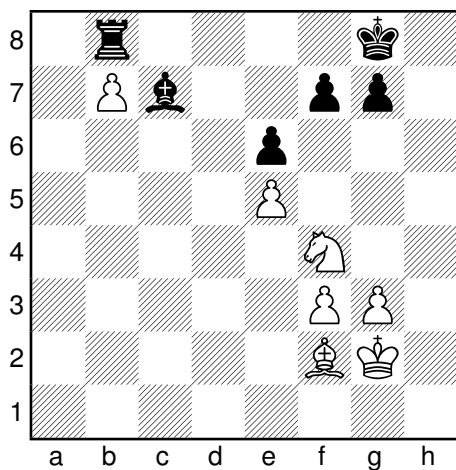
An interesting endgame arises.

62. Nc3 Bc5 63. Nd5 Bd6 64. Rb2 e6 65. e5



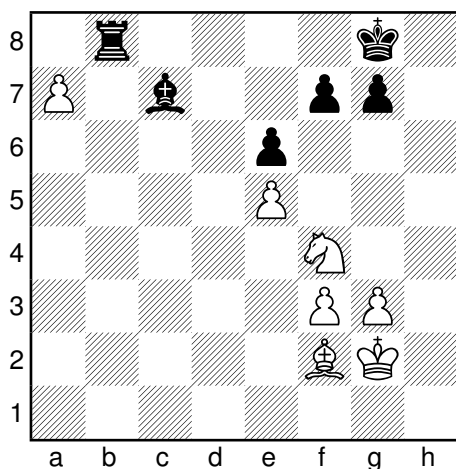
Both sides using lateral move to create threats.

65... Bf8 66. Nf4 Bc5 67. b7 Rc7 68. Rc2 Bb6 69. Rxc7 Bxc7

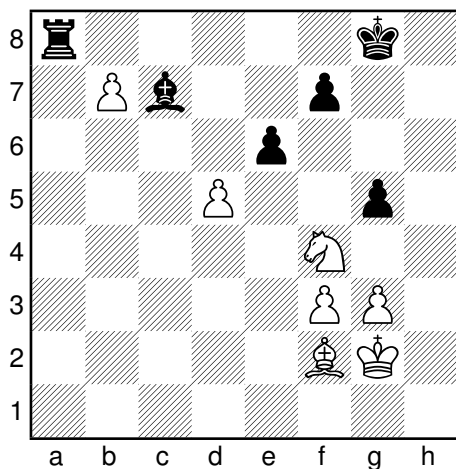


But the pawn can switch files!

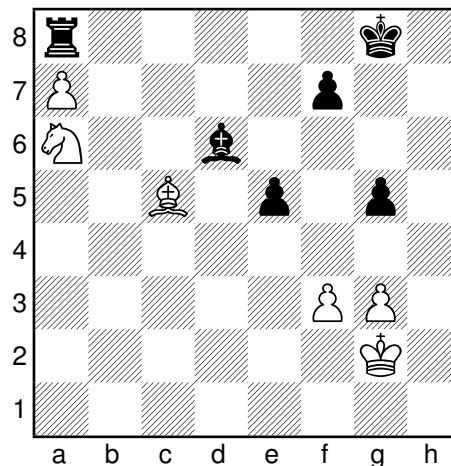
70. a7



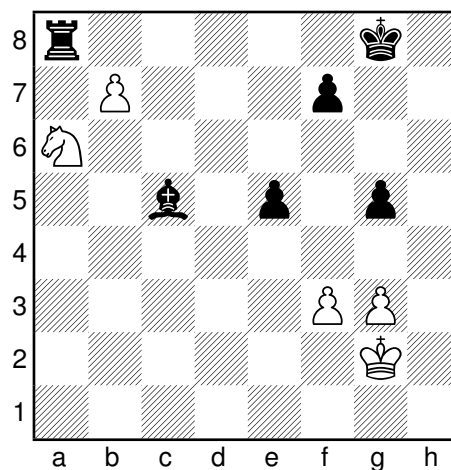
70... Ra8 71. d5 g5 72. b7



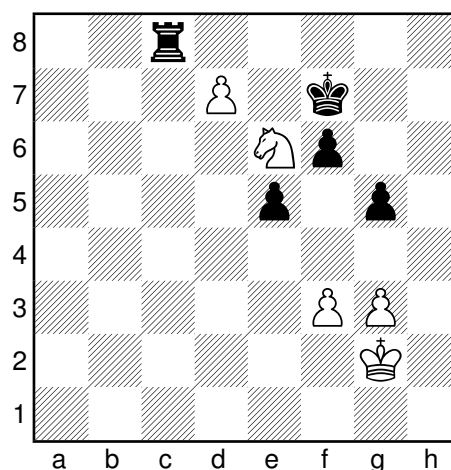
72... Rb8 73. a7 Ra8 74. Nd3 exd5 75. Nb4 e5 76. b7 Rb8 77. a7 Ra8 78. Na6 Bd6 79. Bc5



79... Bxc5 80. b7

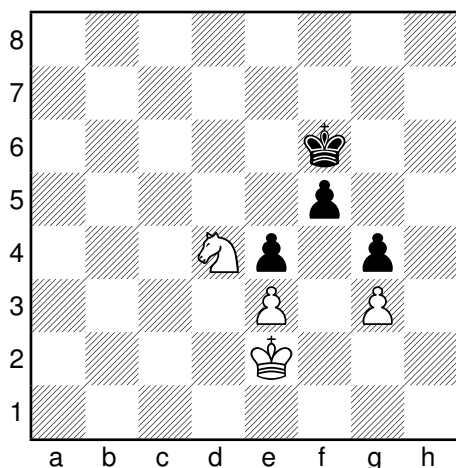


80... Rd8 81. Nxc5 f6 82. Ne6 Rb8 83. c7 Ra8 84. Nd8 Rc8 85. Ne6 Kf7 86. d7



86... Rb8 87. d8=Q Rxd8 88. Nxd8+ Ke7 89. Nc6+ Kd6

90. Nd8 Ke7 91. Nb7 ff5 92. e3 g4 93. Kf2 e4 94. Ke2 Ke6  
95. Nd8+ Ke7 96. Nc6+ Kf6 97. Nd4

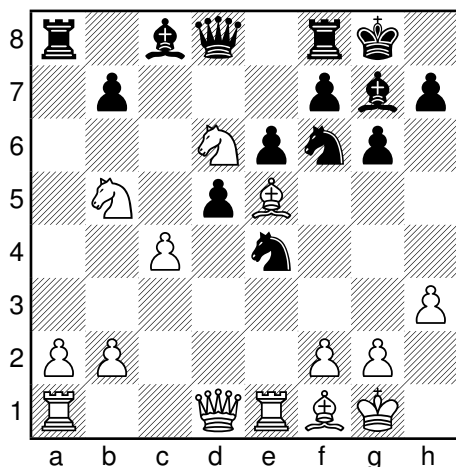


However, this position is a draw!

97... g5 98. Kf1 Ke5 99. Ne2 f5 100. Kg1 Kd5 101. Kf2 Ke5 102. Kf1 Kd5 103. Kf2 Ke5 104. Kf1 Kd5 105. Kg2 Kc4 106. Nd4 e5 107. Nc6 Kd5 108. Ne7 Ke6 109. Nc8 f5 110. Na7 Ke5 111. Nc6+ Kd5 112. Nd4 e5 113. Nf5 Ke6 114. Ng7+ Kf7 115. Nf5 Ke6 116. Nh6 f5 117. Kf1 Kf6 118. Ke1 Kg6 119. Ng8 Kf7 120. Nh6+ Kg6 121. Nxg4 fxg4 122. Kd2 Kf5 123. Kc3 ef4 124. exf4 h4 125. e4+ Kxe4 126. gxh4 Kf4 127. Kc2 Kg4 128. Kc1 Kxh4 1/2-1/2

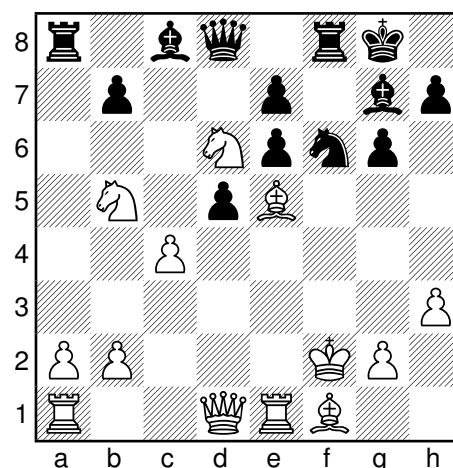
**Game AZ-31: AlphaZero Pawn-sideways vs AlphaZero Pawn-sideways** The first ten moves for White and Black have been sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

1. c4 c5 2. e3 e6 3. dd4 cxd4 4. exd4 g6 5. Nc3 Bg7 6. Nb5 bc7 7. Bf4 Na6 8. Nf3 Nf6 9. h3 d5 10. Bd3 O-O 11. O-O ab7 12. Re1 c6 13. Nd6 cc5 14. Be5 cxd4 15. Nxd4 Nc5 16. Bf1 Nce4 17. N4b5

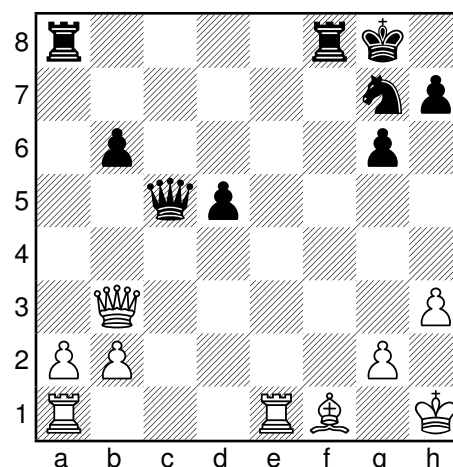


Here we see a new kind of tactic, made possible by a lateral pawn move!

17... Nxf2 18. Kxf2 e7



19. Kg1 exd6 20. Nxd6 Nh5 21. Bxg7 Nxg7 22. Nxc8 Qxc8 23. cxd5 Qc5+ 24. Kh1 exd5 25. Qb3 b6

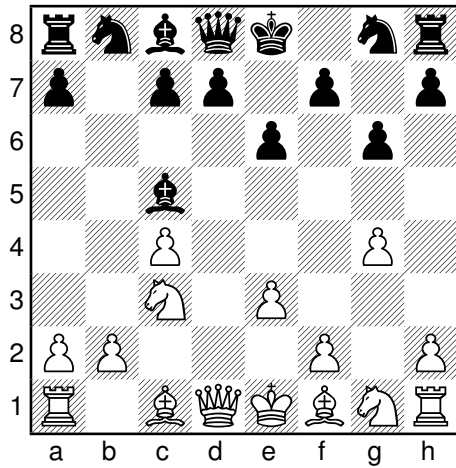


The dust has settled, and the game soon ended in a draw.

26. g4 Qd6 27. Bg2 Rad8 28. Rac1 Ne6 29. Qxd5 Qxd5 30. Bxd5 Rxd5 31. Rxe6 Rf2 32. Rxb6 Rdd2 33. g5 hg7 34. a4 Rh2+ 35. Kg1 Rdg2+ 36. Kf1 Rf2+ 37. Kg1 Rfg2+ 38. Kf1 Rf2+ 39. Ke1 Rfg2 40. Rb8+ Kh7 41. Kf1 Rf2+ 42. Kg1 Rfg2+ 43. Kf1 Rf2+ 44. Kg1 Rfg2+ 45. Kf1 1/2-1/2

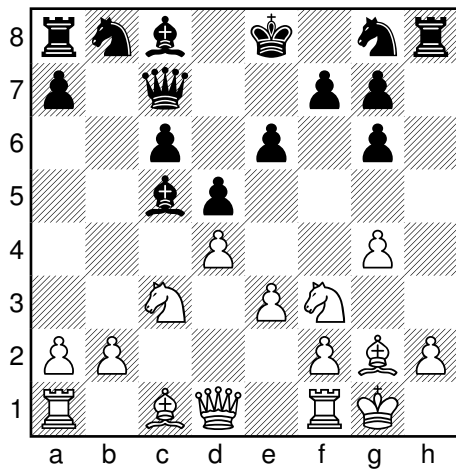
**Game AZ-32: AlphaZero Pawn-sideways vs AlphaZero Pawn-sideways** The first ten moves for White and Black have been sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

1. c4 c5 2. Nc3 g6 3. e3 e6 4. dd4 bc7 5. dxc5 Bxc5 6. g4



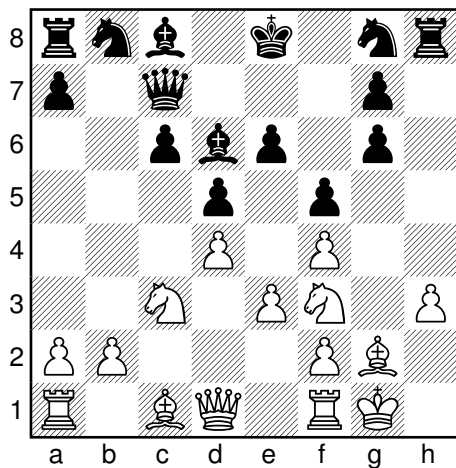
Now that is an unusual sight, the early advance of the g-pawn.

6... hg7 7. Bg2 c6 8. Nf3 d5 9. O-O Qc7 10. d4



White plays c4-d4, a lateral move, to reinforce the center.

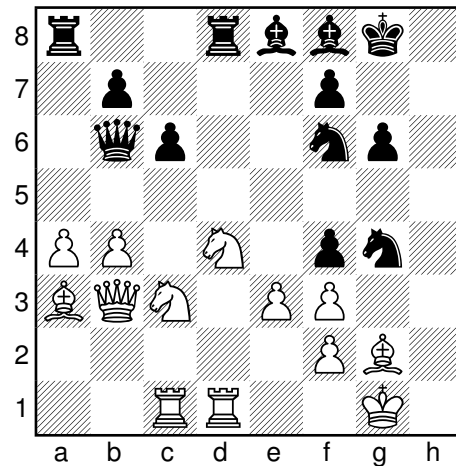
10... Bd6 11. h3 f5 12. f4



The g-pawn, advanced earlier in what seemed to be weaken-

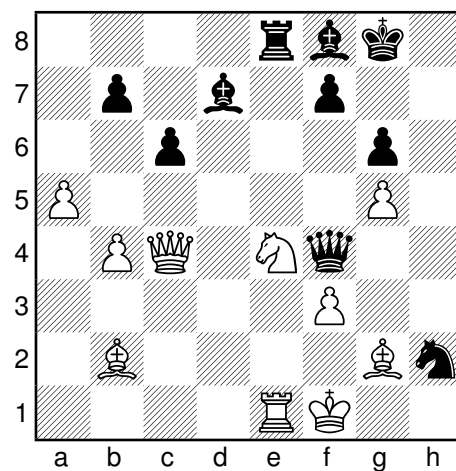
ing, now finds its place on f4, where it shuts out the activity on the b8-h2 diagonal.

12... Nf6 13. a3 b7 14. Rb1 f7 15. b4 O-O 16. Bb2 Rd8 17. Rc1 Bf8 18. Qb3 Bd7 19. dc4 dxc4 20. Qxc4 Be8 21. g3 Bg7 22. Qb3 Qb6 23. Nd4 Nbd7 24. aa4 Bf8 25. Ba3 ee5 26. fxe5 Nxe5 27. Rfd1 Neg4 28. gf3 f4



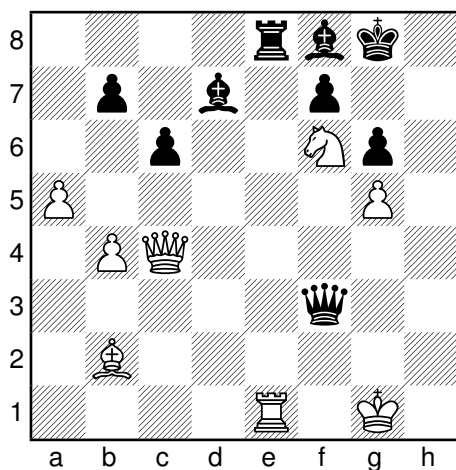
The game gets quite tactical here.

29. a5 Qc7 30. exf4 Qxf4 31. fxg4 Rxd4 32. Rxd4 Qxd4 33. g5 Ng4 34. Ne4 Qe5 35. Bb2 Qh2+ 36. Kf1 Bd7 37. f3 Qf4 38. Re1 Re8 39. Qc4 Nh2+



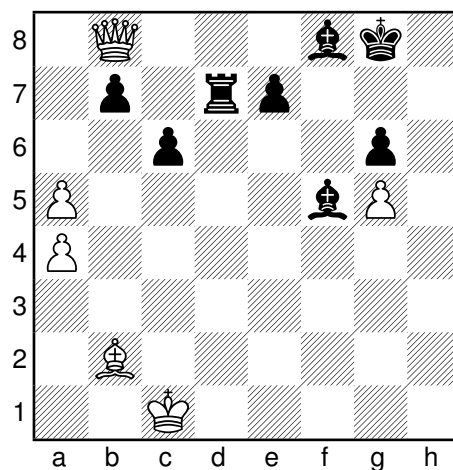
40. Kg1 Nxf3+ 41. Bxf3 Qxf3 42. Nf6+





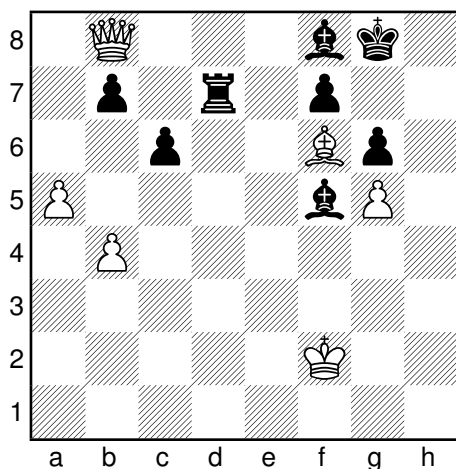
Black needs to give away its queen to stop the attack.

42... Qxf6 43. Bxf6 Rxe1+ 44. Kf2 Rd1 45. Qf4 Bf5  
46. Qb8 Rd7



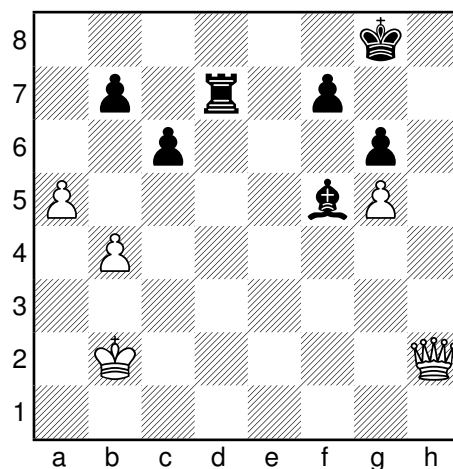
This resource is what Black was keeping in reserve, as a potential way of responding to the threats on the a3-f8 diagonal while the f8 bishop was pinned.

57. Qh2 Bg7 58. b4 Bxb2+ 59. Kxb2 f7



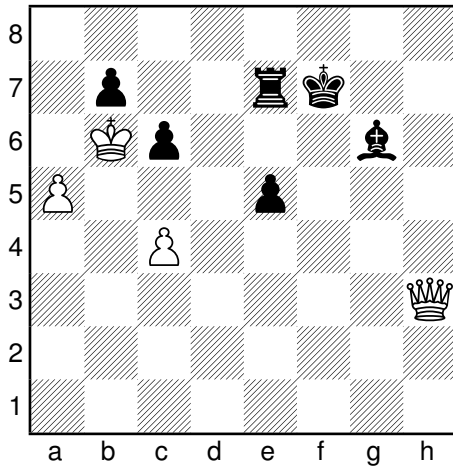
Is this a fortress? As we will see, the question is slightly more complicated by the fact that the pawn structure isn't fixed, and things will eventually open up.

47. Be5 Rd2+ 48. Ke1 Rd7 49. Bc3 Bd3 50. a4 Bc2 51. Qc8 Re7 52. Kd2 Bf5 53. Qb8 Rd7+ 54. Kc1 Rd3 55. Bf6 Rd7 56. Bb2 e7

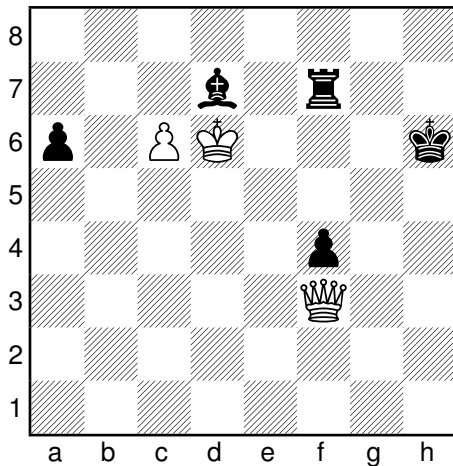


The pawn has served its purpose on e7 and moves back.

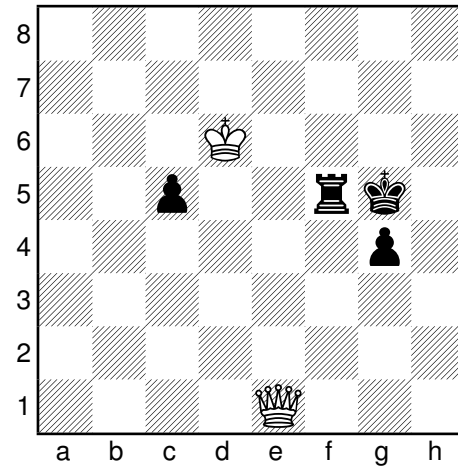
60. c4 Be6 61. Qb8+ Kh7 62. Kb3 Kg7 63. Kc3 f6 64. gxf6+ Kxf6 65. Qf8+ Bf7 66. Kb4 g5 67. Qh6+ Bg6 68. Qh8+ Kf7 69. Qh3 Re7 70. Kc5 f5 71. Kd6 Re6+ 72. Kd7 Re7+ 73. Kd8 Re8+ 74. Kc7 Re7+ 75. Kb6 e5



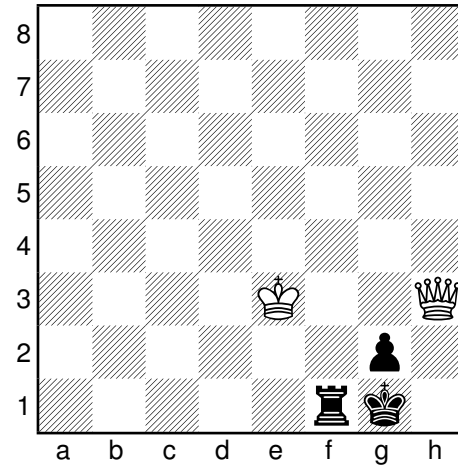
76. Qf3+ Kg7 77. a6 bxa6 78. Kxc6 e4 79. Qe3 Rf7 80. c5 f4 81. Qf3 Bf5 82. Kd6 Rf6+ 83. Ke5 Bd7 84. Qg2+ Kf7 85. Qh1 Kg7 86. Qb7 Rf7 87. Qg2+ Kh7 88. Qf3 Kg7 89. Kd6 Kh6 90. Qb3 Kg7 91. Qc3+ Kh7 92. Qd3+ Kg7 93. Qd4+ Kh7 94. Qd5 Kg7 95. Qg2+ Kh7 96. Qh2+ Kg8 97. Qg1+ Kh7 98. Qb1+ Kg7 99. Qb2+ Kg8 100. Qb8+ Kg7 101. Qb2+ Kg8 102. Qg2+ Kh7 103. Qc2+ Kg8 104. Qg6+ Kf8 105. Qh5 Kg7 106. Qe5+ Kg8 107. Qg5+ Kh7 108. Qd5 Kg7 109. Qe5+ Kg8 110. Qg5+ Kh7 111. Qh5+ Kg7 112. Qf3 Kh6 113. c6



113... Bxc6 114. Kxc6 Kg5 115. Kd6 Rf5 116. Ke6 Rf6+ 117. Ke5 Rf5+ 118. Ke4 Rf7 119. Kd4 Rd7+ 120. Kc4 b6 121. Qg2+ g4 122. Qf1 Rd6 123. Qc1+ f4 124. Qg1+ g4 125. Qe3+ Kf5 126. Qf2+ Kg5 127. Qe3+ Kf5 128. Qg3 Rf6 129. Qh4 c6 130. Kd4 d6 131. Kd5 c6+ 132. Kc5 Rg6 133. Qg3 Rf6 134. Qh4 Rg6 135. Qg3 Rf6 136. Kb6 Kg5 137. Kc7 Rf3 138. Qe5+ Rf5 139. Qe1 c5 140. Kd6



140... c4 141. Qe7+ Kf4 142. Qe2 g3 143. Ke6 Kg5 144. Qxc4 Rf6+ 145. Ke5 Rf5+ 146. Kd6 Rf6+ 147. Ke5 Rf5+ 148. Ke6 Rf6+ 149. Kd7 Rf4 150. Qe2 Kh4 151. Kd6 Kh3 152. Ke5 Rf2 153. Qh5+ Kg2 154. Ke4 Kg1 155. Ke3 g2 156. Qh4 Rf8 157. Ke2 Rf1 158. Qg3 Kh1 159. Qh3+ Kg1 160. Qh4 h2 161. Qd4+ Kh1 162. Qh4 Kg1 163. Qg5+ g2 164. Qh6 Rf2+ 165. Ke3 Rf1 166. Ke2 Rf2+ 167. Ke3 Rf1 168. Qh3



And the game ended in a draw in a couple of moves.

1/2–1/2

## B.9. Self-capture

In Self-capture chess, we have considered extending the rules of chess to allow players to capture their own pieces.

### B.9.1. MOTIVATION

The ability to capture one's own pieces could help break "deadlocks" and offer additional ways of infiltrating the opponent's position, as well as quickly open files for the attack. Self-captures provide additional defensive resources as well, given that the King that is under attack can consider

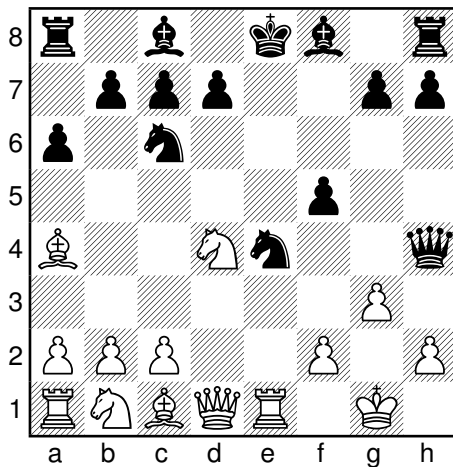
escaping by self-capturing its own adjacent pieces.

### B.9.2. ASSESSMENT

The assessment of the Self-capture chess variant, as provided by Vladimir Kramnik:

“ I like this variation a lot, I would even go as far as to say that to me this is simply an improved version of regular chess.

Self-captures make a minor influence on the opening stage of a chess game, though we have seen examples of lines that become possible under this rule change that were not possible before. For example, consider the following line 1. e4 e5 2. Nf3 Nc6 3. Bb5 a6 4. Ba4 Nf6 5. 0-0 Nxe4 6. d4 exd4 7. Re1 f5 8. Nxd4 Qh4 9. g3 in the Ruy Lopez.



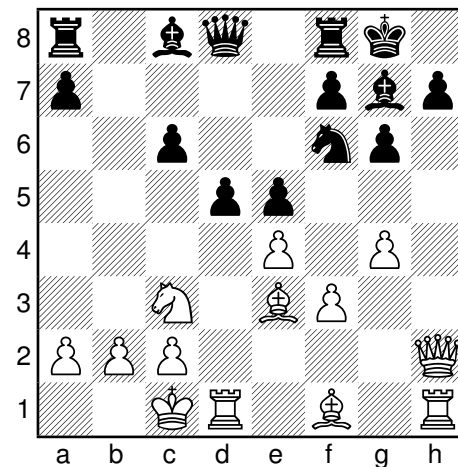
While not the main line, it is possible to play in Self-capture chess and AlphaZero assesses it as equal. In classical chess, however, this position is much better for White. The key difference is that in self-capture chess Black can respond to g3 by taking its own pawn on h7 with the queen, gaining a tempo on the open file. In fact, White can gain the usual opening advantage earlier in the variation, by playing 8. Ng5 d5 9. f3 Bd6 10. fxe4 dxe4, which AlphaZero assesses as giving the 60% expected score for White after about a minute's thought, which is usually possible to defend with precise play. In fact, there are multiple improvements for both sides in the original line, but discussing these is beyond the scope of this example. It is worth noting that AlphaZero prefers to utilise the setup of the Berlin Defence, similar to its style of play in classical chess.

Regardless of its relatively minor effect on the openings, self-captures add aesthetically beautiful motifs in the middlegames and provide

additional options and winning motifs in the endgames.

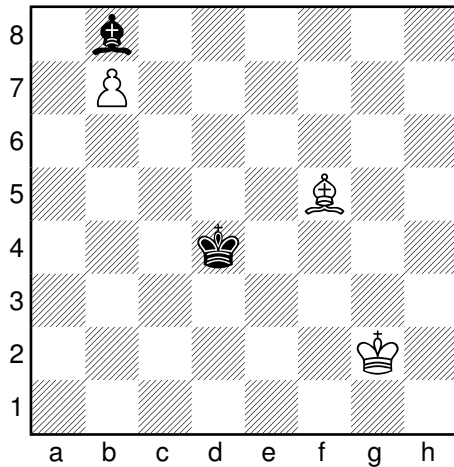
Taking one's own piece represents another way of sacrificing in chess, and material sacrifices make chess games more spectacular and enjoyable both for public and for the players. Most of the times this is used as an attacking idea, to gain initiative and compromise the opponent's king.

For example, consider the Dragon Sicilian, as an example of a sharp opening. After 1. e4 c5 2. Nf3 d6 3. d4 cxd4 4. Nxd4 Nf6 5. Nc3 g6 6. Be3 Bg7 7. f3 0-0 8. Qd2 Nc6 9. 0-0-0 d5 something like 10. g4 e5 11. Nxc6 bxc6 is possible, at which point there is already Qxh2, a self-capture, opening the file against the enemy king. Of course, Black can (and probably should) play differently.

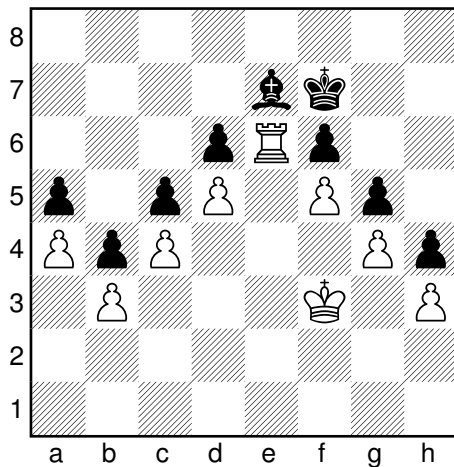


The possibilities for self-captures in this example don't end, as after 12... d4, White could even consider a self-capture 13. Nxe4, sacrificing another pawn. This is not the best continuation though, and AlphaZero evaluates that as being equal. It is just an illustration of the ideas which become available, and which need to be taken into account in tactical calculations.

In terms of endgames, self-captures affect a wide spectrum of otherwise drawish endgame positions winning for the stronger side. Consider the following examples:



*In this position, under Classical rules, the game would be an easy draw for Black. In Self-capture chess, however, this is a trivial win for White, who can play Bc8 and then capture the bishop with the b7 pawn, promoting to a queen!*



*This endgame, which represents a fortress in classical chess, becomes a trivial win in self-capture chess, due to the possibilities for the White king to infiltrate the Black position either via e4 and a self-capture on d5 or via e2, d3 and a self-capture on c4.*

*To conclude, I would highly recommend this variation for chess lovers who value beauty in the game on top of everything else.*

”

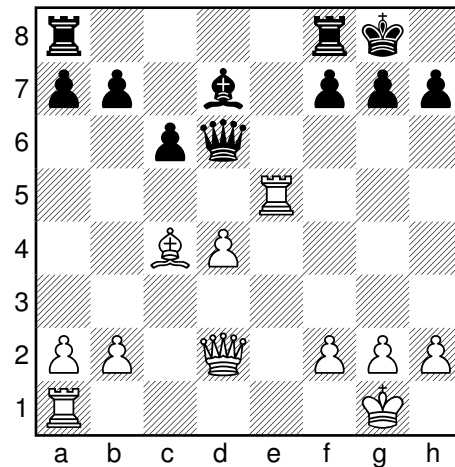
### B.9.3. MAIN LINES

Here we discuss “main lines” of AlphaZero under Self-capture chess, when playing with roughly one minute per move from a particular fixed first move. Note that these are not purely deterministic, and each of the given lines is

merely one of several highly promising and likely options. Here we give the first 20 moves in each of the main lines, regardless of the position.

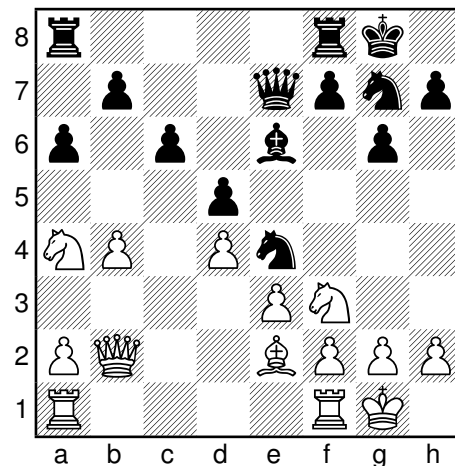
**Main line after e4** The main line of AlphaZero after 1. e4 in Self-capture chess is:

1. e4 (book) e5 2. Nf3 Nc6 3. Bb5 Nf6 4. O-O Nxe4 5. Re1 Nd6 6. Nxe5 Be7 7. Bf1 Nxe5 8. Rxe5 O-O 9. Nc3 Ne8 10. Nd5 Bd6 11. Re1 c6 12. Ne3 Be7 13. c4 Nc7 14. d4 d5 15. cxd5 Bb4 16. Bd2 Bxd2 17. Qxd2 Nxd5 18. Nxd5 Qxd5 19. Re5 Qd6 20. Bc4 Bd7



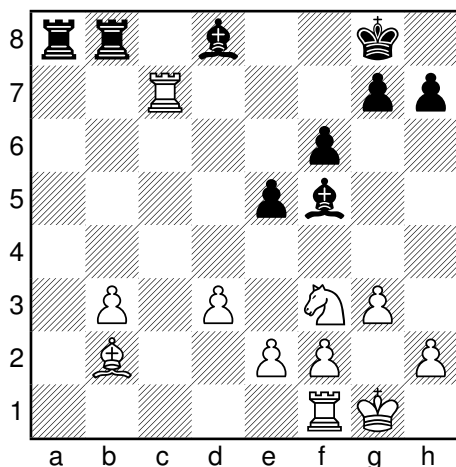
**Main line after d4** The main line of AlphaZero after 1. d4 in Self-capture chess is:

1. d4 (book) d5 2. c4 e6 3. Nc3 Nf6 4. cxd5 exd5 5. Bg5 c6 6. Qc2 Nbd7 7. e3 Be7 8. Nf3 Nh5 9. Bxe7 Qxe7 10. Be2 O-O 11. O-O Ndf6 12. Ne5 g6 13. Qa4 Be6 14. b4 a6 15. Qb3 Ng7 16. Na4 Ne4 17. Qb2 Qg5 18. Nf3 Qe7 19. Ne5 Qg5 20. Nf3 Qe7



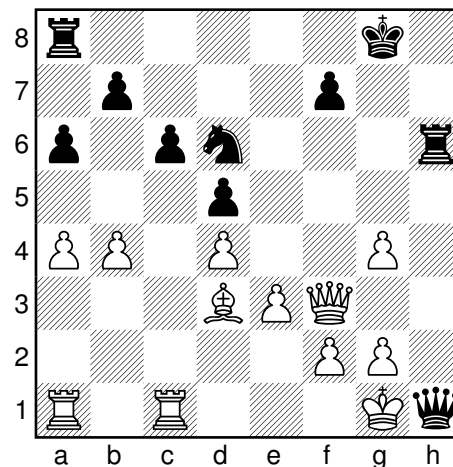
**Main line after c4** The main line of AlphaZero after 1. c4 in Self-capture chess is:

1. c4 (book) e5 2. g3 d5 3. cxd5 Nf6 4. Bg2 Nxd5 5. Nc3 Nb6 6. b3 Nc6 7. Bb2 f6 8. Rc1 Bf5 9. Bxc6+ bxc6 10. Nf3 Qd7 11. O-O Be7 12. d3 a5 13. Ne4 O-O 14. Qc2 a4 15. Qxc6 Qxc6 16. Rxc6 Nd5 17. Nc3 Nxc3 18. Rxc3 axb3 19. axb3 Rfb8 20. Rxc7 Bd8



And here we see the first self-capture of the game, creating threats down the h-file:

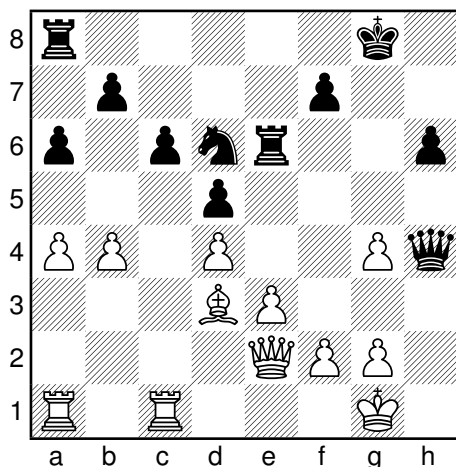
37... Rxh6 38. Qf3 Qh1+



#### B.9.4. INSTRUCTIVE GAMES

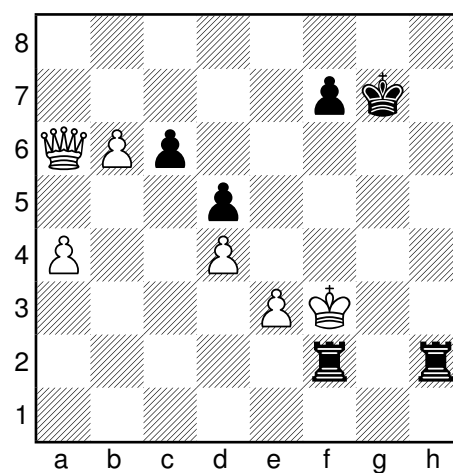
**Game AZ-33: AlphaZero Self-capture vs AlphaZero Self-capture** The first ten moves for White and Black have been sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

1. d4 Nf6 2. c4 e6 3. Nc3 d5 4. cxd5 exd5 5. Bg5 c6 6. Qc2 Nbd7 7. Nf3 h6 8. Bh4 Be7 9. e3 O-O 10. Bd3 Re8 11. O-O Ne4 12. Bxe4 Bxh4 13. Bh7+ Kh8 14. Nxh4 Qxh4 15. Bd3 Qe7 16. a3 Nf6 17. b4 Bd7 18. h3 Kg8 19. Rfb1 Rec8 20. Qd1 Be6 21. Ne2 a6 22. Nf4 Ne8 23. a4 Nd6 24. Qb3 Qd7 25. Be2 Bf5 26. Rc1 Qd8 27. Qb2 Be4 28. Rc5 Bf5 29. Rc3 Ra7 30. Rcc1 Raa8 31. Rc5 Qh4 32. Bf1 Re8 33. Rcc1 g5 34. Nd3 Bxd3 35. Bxd3 g4 36. hxg4 Re6 37. Qe2



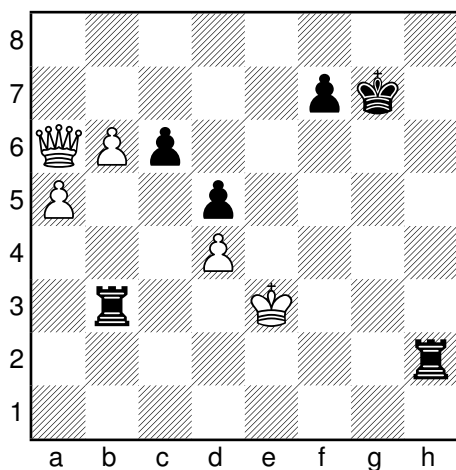
The end? Not really. In self-capture chess the king can escape by capturing its way through its own army, and hence here it just takes on f2 and gets out of check.

39. Kxf2 Qh4+ 40. Ke2 Re8 41. Rh1 Qxh1 42. Rxh1 Rxh1 43. Qf4 Ne4 44. Bxe4 Rxe4 45. Qb8+ Kg7 46. Qxb7 Rh6 47. Qxa6 Rg4 48. Kf1 Rh1+ 49. Kf2 Rg6 50. b5 Rh2 51. b6 Rg4 52. Kf3 Rf2+



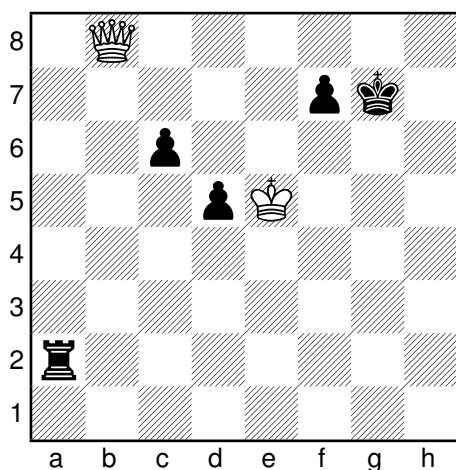
Unlike in classical chess, White can still play on here, and AlphaZero does, by advancing the king forward with a self-capture!

53. Kxe3 Rb2 54. a5 Rb3+



And, as if one pawn was not enough, White self-captures another one by taking on d4.

55. Kxd4 Ra2 56. Ke5 Rb5 57. b7 Rxa5 58. Qxa5 Rxa5 59. b8=Q Ra2



White manages to get a queen, but in the end, Black's defensive resources prove sufficient and the game eventually ends in a draw.

60. Kd6 Re2 61. Kxc6 Re6+ 62. Kd7 Rg6 63. Qa8 Re6 64. Qxd5 Kg8 65. Qa8+ Kg7

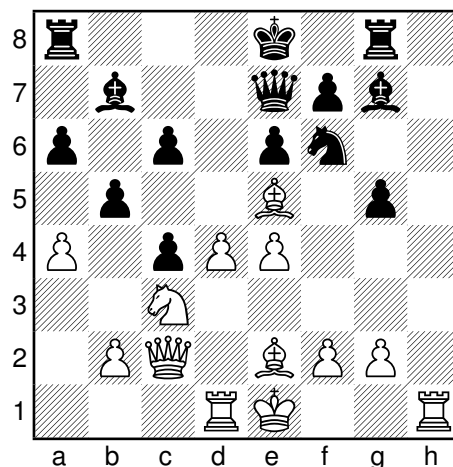
With draw soon to follow.

1/2-1/2

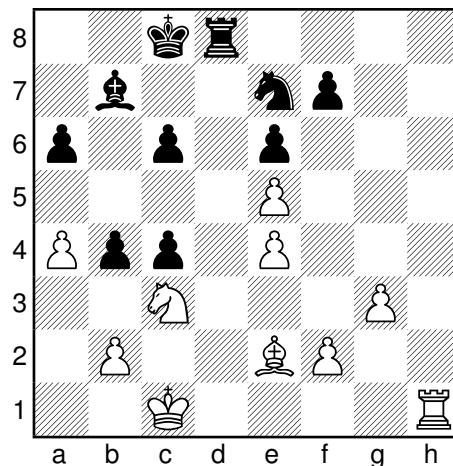
**Game AZ-34: AlphaZero Self-capture vs AlphaZero Self-capture** The first ten moves for White and Black have been sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

1. d4 d5 2. c4 e6 3. Nc3 Nf6 4. Nf3 c6 5. Bg5 h6 6. Bh4 dxc4 7. e4 g5 8. Bg3 b5 9. Be2 Bb7 10. Ne5 Nbd7 11. Qc2 Bg7

12. Rd1 Qe7 13. h4 Nxe5 14. Bxe5 a6 15. a4 Rg8 16. hxg5 hxg5

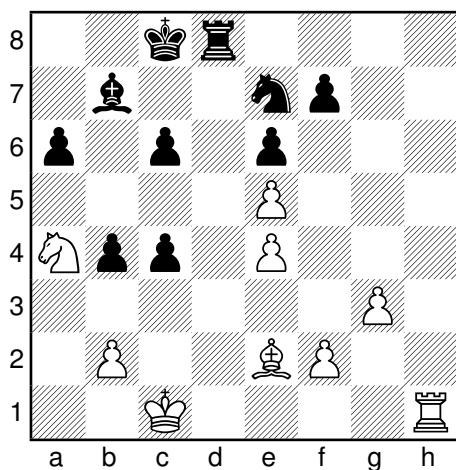


17. Qc1 O-O-O 18. Qxg5 Nd5 19. Qxe7 Nxe7 20. g3 Bxe5 21. dxe5 Rxd1+ 22. Kxd1 Rd8+ 23. Kc1 b4



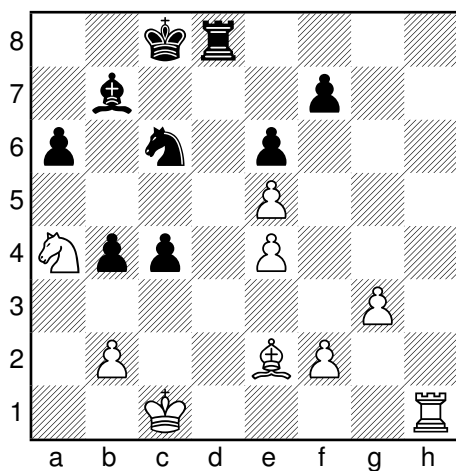
Here we come to the first self-capture of the game, White decides to give up the a4 pawn in order to get the knight to an active square.

24. Nxa4

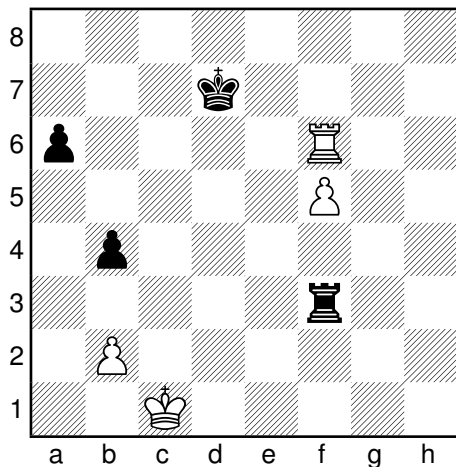


And Black responds in turn with a self-capture of its own, on c6!

24... Nxc6



25. Nb6+ Kc7 26. Nxc4 Nd4 27. Bd3 Nf3 28. Bc2 Rd4  
29. Nd6 Nxe5 30. Nxb7 Kxb7 31. f4 Nd3+ 32. Bxd3 Rxd3  
33. Rh7 Rxc3 34. Rxf7+ Kc6 35. Rf6 Kd7 36. Rf7+ Kc6  
37. Rf6 Kd7 38. f5 exf5 39. exf5 Rf3

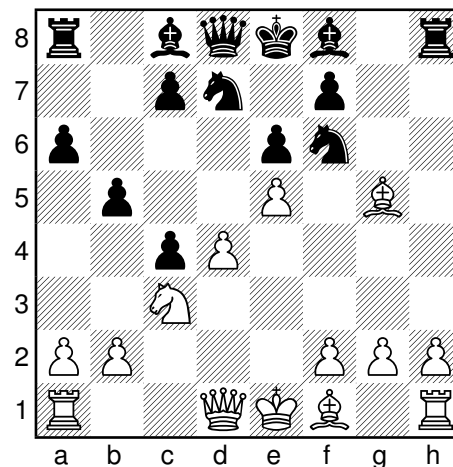


And the game eventually ended in a draw.

1/2–1/2

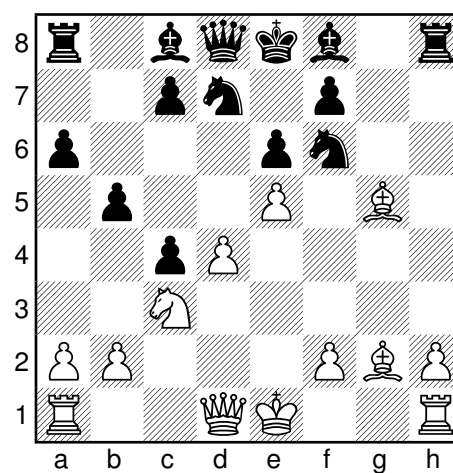
**Game AZ-35: AlphaZero Self-capture vs AlphaZero Self-capture** The first ten moves for White and Black have been sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

1. d4 e6 2. Nf3 Nf6 3. c4 d5 4. Bg5 dxc4 5. Nc3 a6 6. e4 b5  
7. e5 h6 8. Bh4 g5 9. Nxc4 hxc4 10. Bxc4 Nbd7



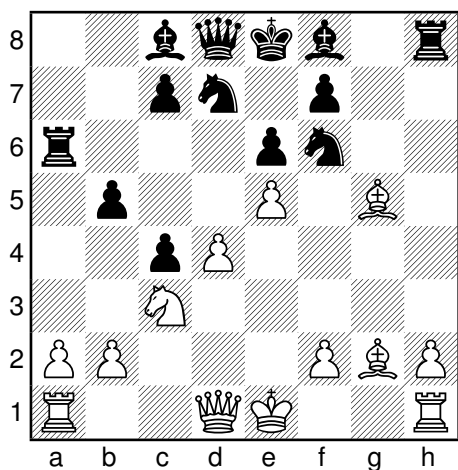
In this highly tactical position, self-captures provide additional resources, as AlphaZero quickly demonstrates, by a self-capture on g2, developing the bishop on the long diagonal at the price of a pawn.

11. Bxg2

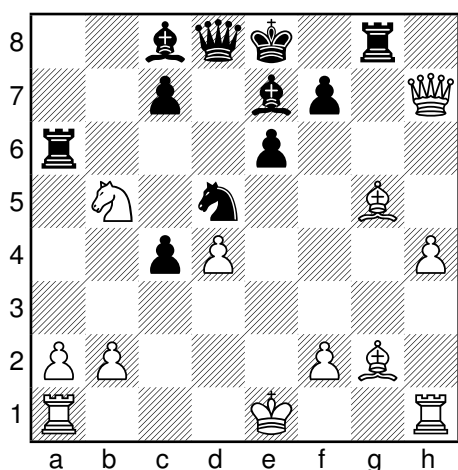


Yet, Black responds in turn by a self-capture on a6:

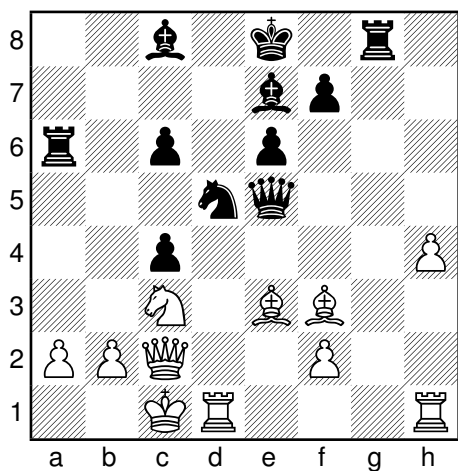
11... Rxa6



12. exf6 Rg8 13. h4 Nxf6 14. Nxb5 Be7 15. Qc2 Nd5  
16. Qh7

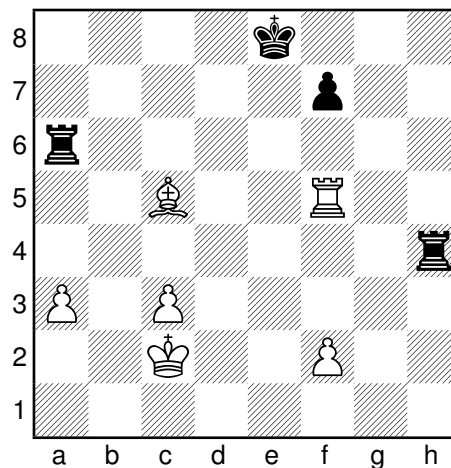


16... Rf8 17. Bh6 Nf6 18. Qc2 Rg8 19. Bf3 c6 20. Nc3  
Qxd4 21. Be3 Qe5 22. O-O-O Nd5



23. Kb1 Nxc3+ 24. bxc3 c5 25. Rhg1 Rh8 26. Rg4 Qf5  
27. Qxf5 exf5 28. Rxc4 Be6 29. Bd5 Rd6 30. Rxc5 Rb6+

31. Kc2 Bxc5 32. Bxc5 Ra6 33. a3 Bxd5 34. Rxd5 Rxh4  
35. Rxf5



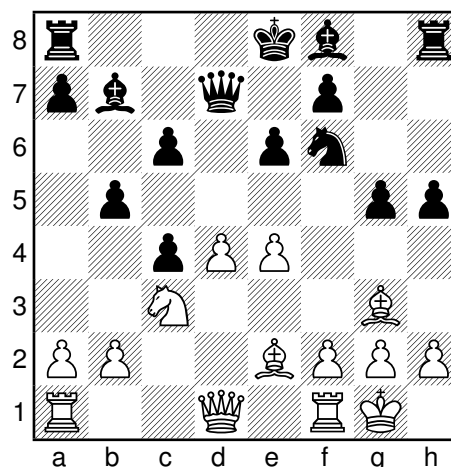
and the game eventually ended in a draw.

1/2–1/2

**Game AZ-36: AlphaZero Self-capture vs AlphaZero Self-capture** The first ten moves for White and Black have been sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

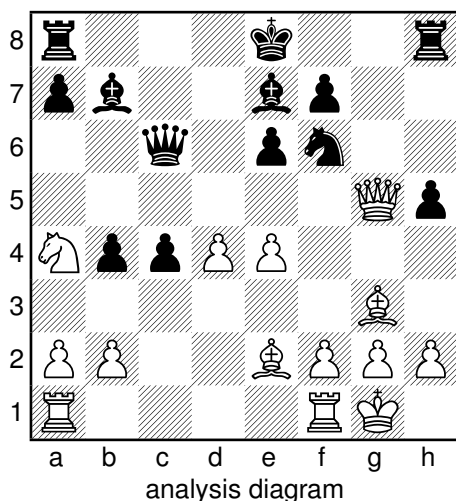
In this game, self-captures happen towards the end, but the game itself is pretty tactical and entertaining. We therefore included the full game.

1. Nf3 d5 2. d4 Nf6 3. c4 e6 4. Nc3 c6 5. Bg5 h6 6. Bh4  
dxc4 7. e4 g5 8. Bg3 b5 9. Be2 Bb7 10. O-O Nbd7 11. Ne5  
h5 12. Nxd7 Qxd7



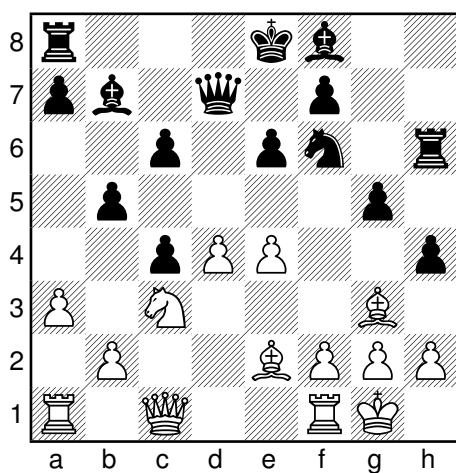
In the game, White played the pawn to a3, but it's interesting to note that potential self-captures factor in the lines that AlphaZero is calculating at this point. AlphaZero is initially considering the following line: 13. Qd2 Be7 14. Qxg5 b4 15. Na4 Qxc6



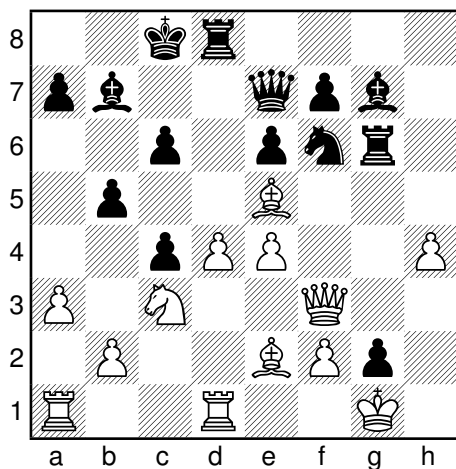


where Black has just self-captured its c6 pawn! 16. Nc5 Nxe4 17. Qe5 with exchanges to follow. Going back to the game:

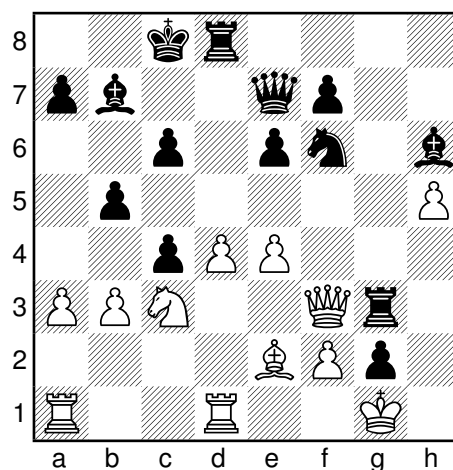
13. a3 Rh6 14. Qc1 h4



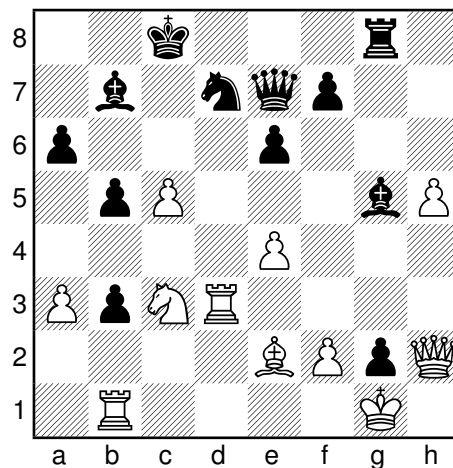
15. Be5 h3 16. Qxg5 hxg2 17. Rd1 Rg6 18. Qf4 Qe7 19. Qf3 Bg7 20. h4 O-O-O



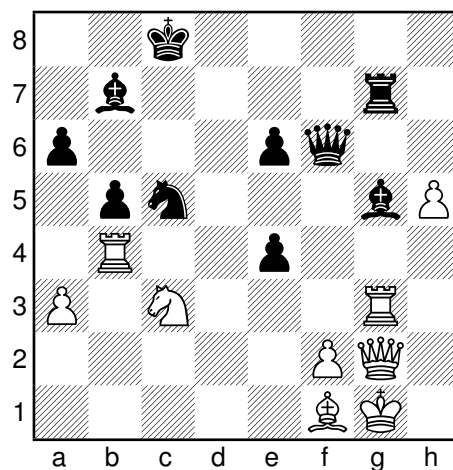
21. Bg3 Bh6 22. h5 Rgg8 23. b3 Rxc3



24. Qxc3 Rg8 25. Qh3 Bf4 26. Qh4 Bg5 27. Qh2 cxb3 28. Rd3 a6 29. Rb1 c5 30. dxc5 Nd7

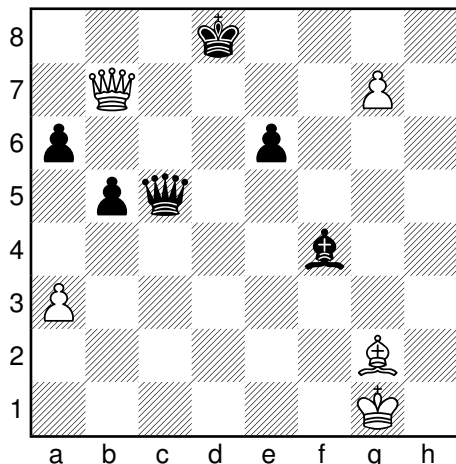


31. Rg3 Rg7 32. Qxg2 f5 33. Rxb3 Nxc5 34. Rb4 Qf6 35. Bf1 fxe4



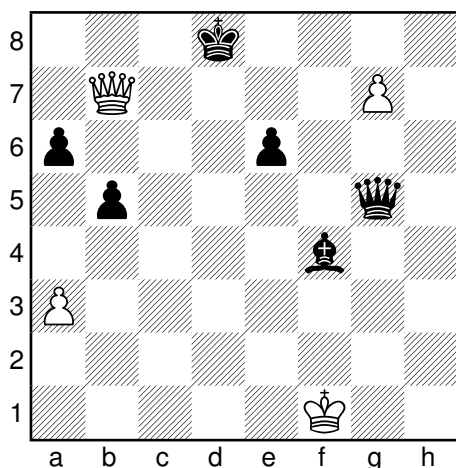
36. Nxe4 Nxe4 37. Rxe4 Bxe4 38. Qxe4 Bf4 39. Rg6 Rxg6+

40. hxc6 Qg5+ 41. Bg2 Be5 42. f4 Bxf4 43. Qb7+ Kd8  
44. g7 Qc5+



What happens next is a rather remarkable self-capture, demonstrating that it's not only the pawns that can justifiably be self-captured, as the least valuable pieces. Indeed, White self-captures the bishop on g2, in its attempt at avoiding perpetuals!

45. Kxg2 Qg5+ 46. Kf1



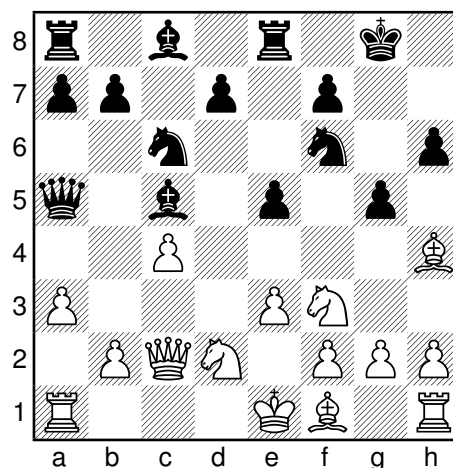
Yet, Black responds in turn, by capturing its own bishop! The game ultimately ends in a draw.

46... Qxf4+ 47. Kg2 Qg4+ 48. Kf2 Qf4+ 49. Ke2 Qe5+ 50. Kf3 Qf5+ 51. Ke3 Qe5+ 52. Kd3 Qd6+ 53. Ke4 Qxe6+ 54. Kf4 Qf6+ 55. Ke3 Qe5+ 56. Kd3 Qd6+ 57. Ke4 Qe6+ 58. Kd4 Qf6+ 59. Kd5 Qf3+ 60. Kd6 Qf6+ 61. Kc5 Qf2+ 62. Kb4 Qd2+ 63. Kb3 Qd3+ 64. Kb2 Qd2+ 65. Kb1 Qd3+ 66. Kb2 Qd2+ 67. Kb3 Qd3+ 68. Kb4 Qd2+ 69. Kxa3 Qc3+ 70. Ka2 Qc2+ 71. Ka1 Qc1+ 72. Ka2 Qc2+ 73. Ka1 Qc1+ 74. Ka2 Qc2+ 1/2-1/2

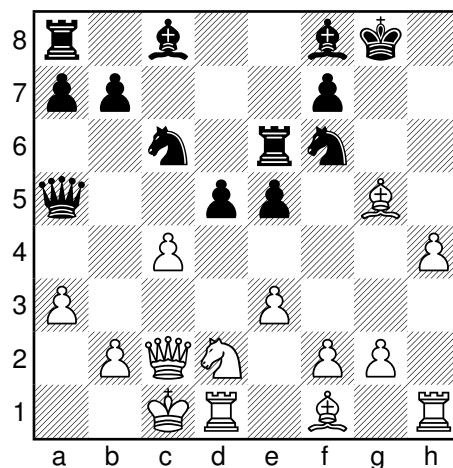
**Game AZ-37: AlphaZero Self-capture vs AlphaZero Self-capture** The first ten moves for White and Black

have been sampled randomly from AlphaZero's opening "book", with the probability proportional to the time spent calculating each move. The remaining moves follow best play, at roughly one minute per move.

1. d4 Nf6 2. c4 e6 3. Qc2 c5 4. dxc5 h6 5. Nf3 Bxc5 6. a3 O-O 7. Bf4 Qa5+ 8. Nbd2 Nc6 9. e3 Re8 10. Bg3 e5 11. Bh4 g5

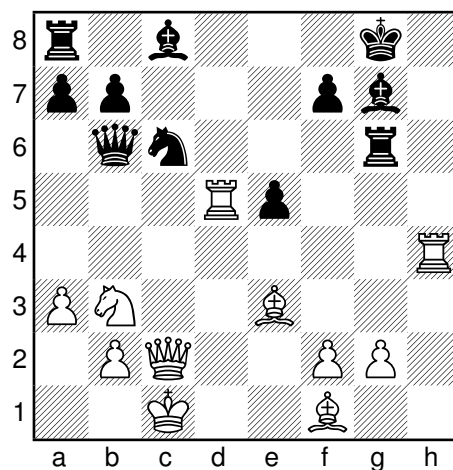
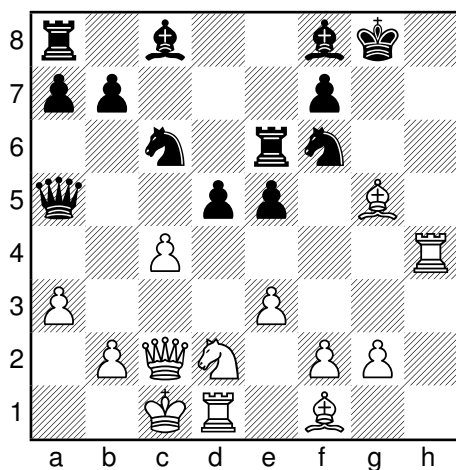


12. Nxg5 hxg5 13. Bxg5 Re6 14. O-O-O Bf8 15. h4 d5



Here we see the first self-capture move of the game, creating threats along the h-file:

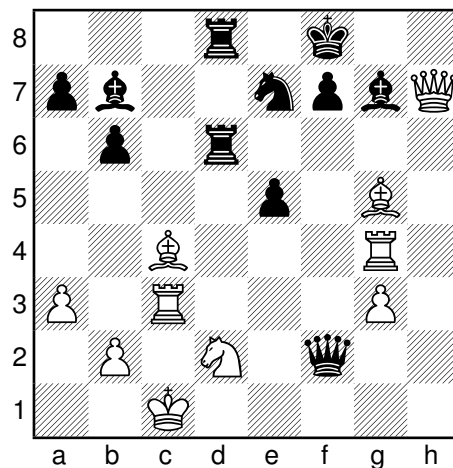
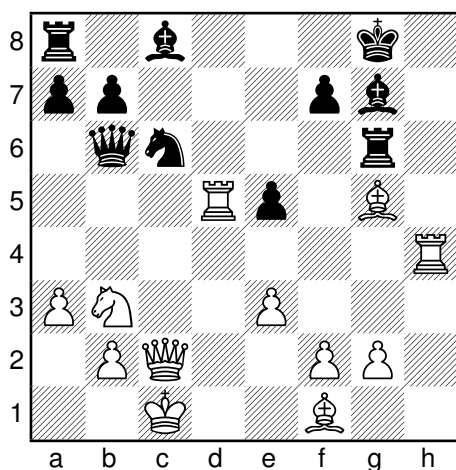
16. Rxh4



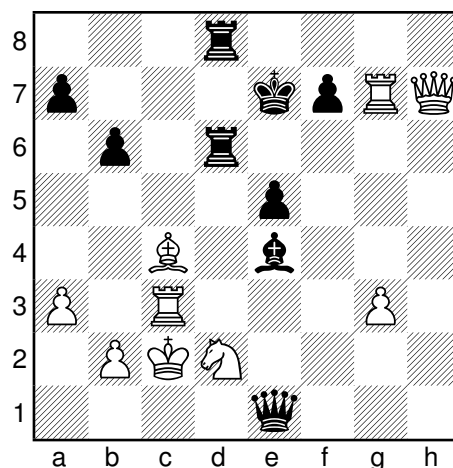
20... Qc7 21. Rc5 b6 22. Rc3 Bb7 23. Bd3 Qd8 24. g3  
Rd6 25. Bc4 Kf8 26. Qh7 Qf6 27. Nd2 Ne7 28. Rg4 Rad8  
29. Bg5 Qxf2

It's interesting to note that White could have also tried opening the h-file a move earlier, by playing 15. Rxh2 instead of 15. h4, but AlphaZero prefers provoking 15... d5 first and having its rook on the 4th rank, where it stands more active and controls additional squares.

16... Bg7 17. Nb3 Qb6 18. cxd5 Nxd5 19. Rxd5 Rg6



30. Bxe7+ Kxe7 31. Rxg7 Qe1+ 32. Kc2 Be4+

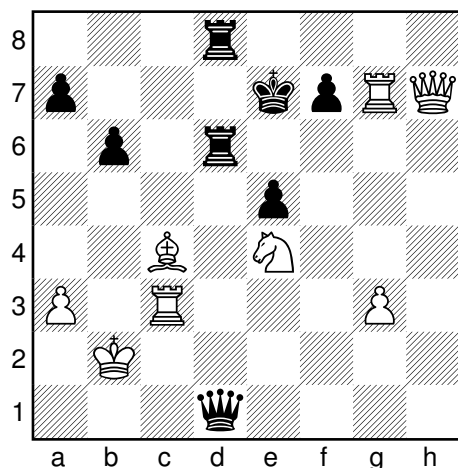


Here comes another self-capture:

20. Bxe3

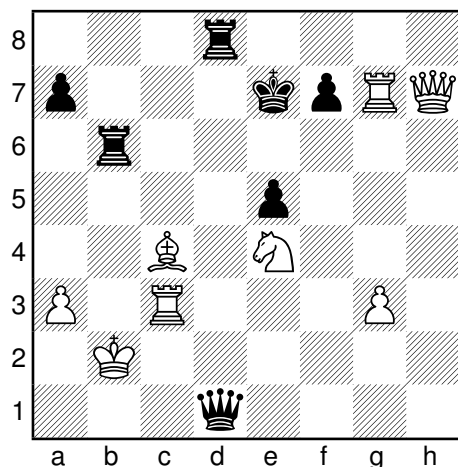
33. Nxe4 Qd1+ is what is played and made possible by a self-capture, avoiding mate:

34. Kxb2



Here Black responds by a self-capture on b6:

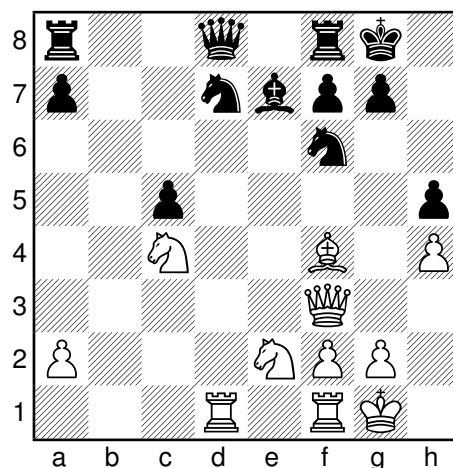
34... Rxb6+



The game soon ends in a draw.

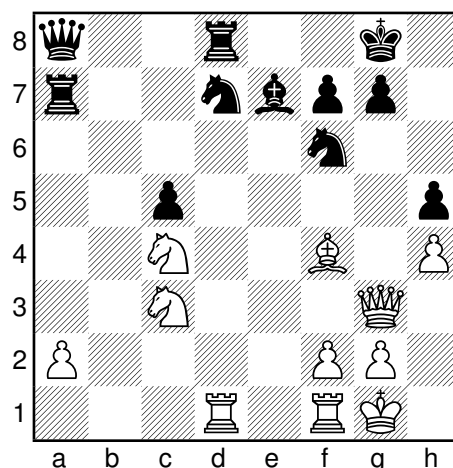
35. Rb3 Rxb3+ 36. Bxb3 Qe2+ 37. Kb1 Qf1+ 38. Ka2 Qe2+ 39. Ka1 Qe1+ 40. Ka2 Qe2+ 41. Kb1 Qe1+ 42. Kc2 Qe2+ 43. Kc3 Qe3+ 44. Kb2 Qe2+ 45. Kxa3 Qa6+ 46. Ba4 Qd3+ 47. Ka2 Qe2+ 48. Ka1 Qe1+ 49. Kb2 Qe2+ 50. Ka1 Qe1+ 51. Ka2 Qe2+ 52. Ka3 Qd3+ 53. Bb3 Qa6+ 54. Kb2 Qe2+ 55. Kb1 Qf1+ 56. Ka2 Qa6+ 57. Kb2 Qe2+ 58. Ka1 Qf1+ 59. Ka2 Qa6+ 60. Kb1 Qf1+ 61. Kb2 Qe2+ **1/2-1/2**

**Game AZ-38: AlphaZero Self-capture vs AlphaZero Self-capture** The following position, with Black to play, arose in an AlphaZero game, played at roughly one minute per move.



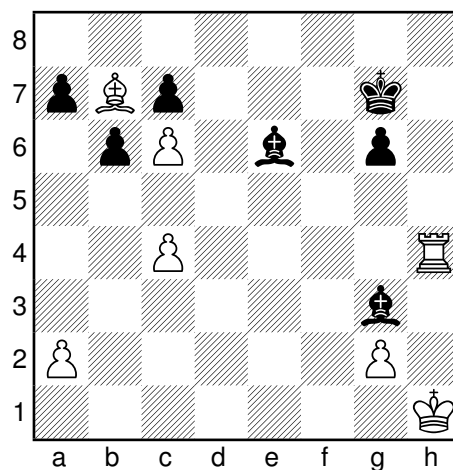
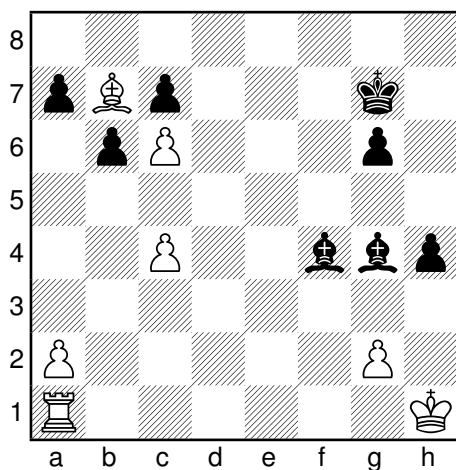
In this position, with Black to play, in classical chess Black would struggle to find a good plan and activity. Yet, here in self-capture chess, Black plays the obvious idea – sacrificing the a7 pawn to open the a-file for its rook and initiate active play!

19... Rxa7 20. Nc3 Qa8 21. Qg3 Rfd8



Black soon managed to equalize and eventually draw the game. **1/2-1/2**

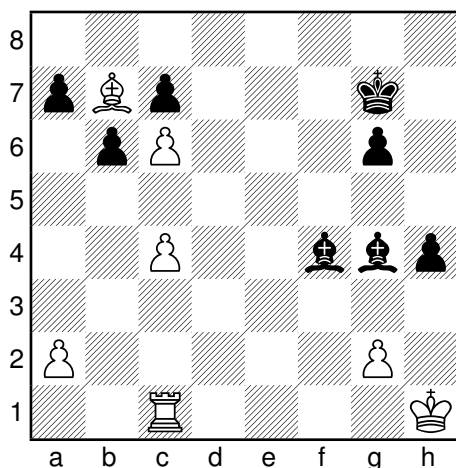
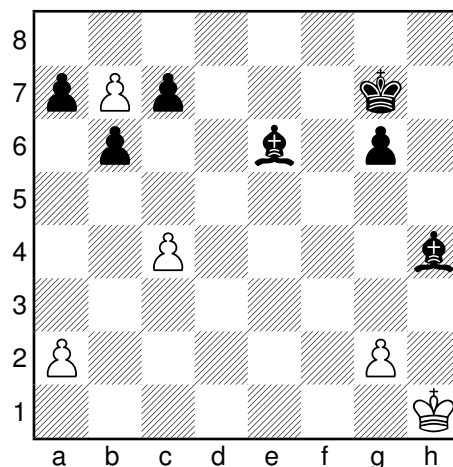
**Game AZ-39: AlphaZero Self-capture vs AlphaZero Self-capture** The following position, with White to play, arose in an AlphaZero game, played at roughly one minute per move.



38... Bxh4 39. cxb7

In the previous moves, AlphaZero had manoeuvred its light-squared bishop to b7 via a6, with a clear intention of setting up threats to self-capture on b7 and promote the pawn on b8. Yet, if attempted immediately, Black can respond in turn by playing c6, c5, or even self-capturing on c7 with the bishop. If the bishop moves away from the b8-h2 diagonal, White can proceed with the plan. This explains why White plays the following next:

34. Rc1

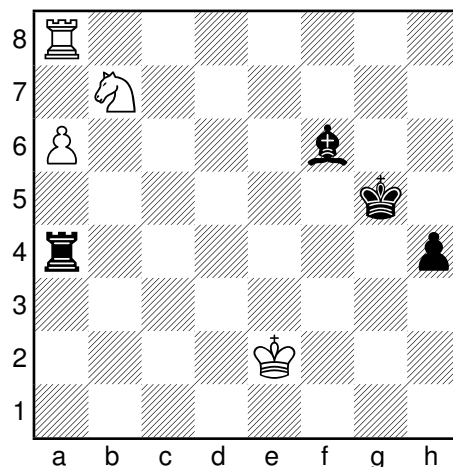


The rook can now be taken on c1, but this would allow the promotion of the c-pawn via a self-capture.

34... Be6 35. Rf1 Bd6 36. Rd1 Bf4 37. Rd4 Bg3 38. Rxh4

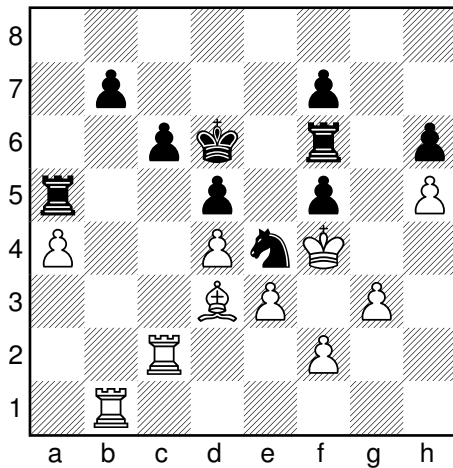
And White went on to eventually win the game. 1-0

**Game AZ-40: AlphaZero Self-capture vs AlphaZero Self-capture** The following position, with White to play, arose in an AlphaZero game, played at roughly one minute per move.



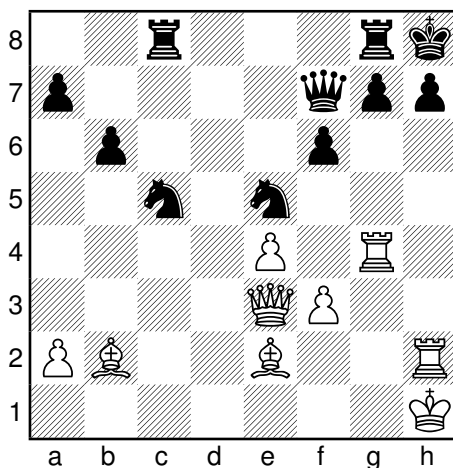
In this position, White plays a self-capture, *50. axb7*, giving away the knight, for an immediate threat of promoting on b8. This is a common pattern in endgames in this variation, where pieces can be used to help promote the passed pawns.

**Game AZ-41: AlphaZero Self-capture vs AlphaZero Self-capture** The following position, with Black to play, arose in an AlphaZero game, played at roughly one minute per move.



In this position, AlphaZero as Black plays another self-capture motif: *75... fxe4+*, self-capturing its own knight with check, while attacking White's bishop on d3. This highlights novel tactical opportunities where self-captures can be utilised not only as dynamic material sacrifices for the initiative, but rather a key part of tactical sequences where material gets immediately recovered.

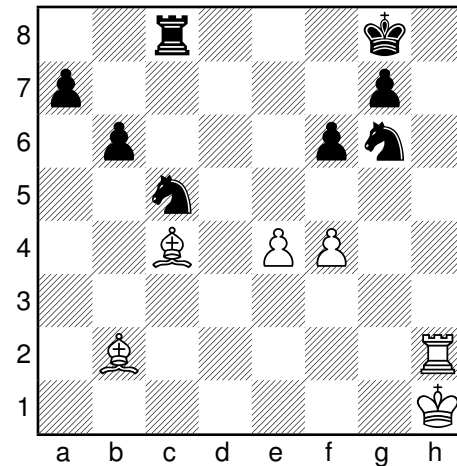
**Game AZ-42: AlphaZero Self-capture vs AlphaZero Self-capture** The following position, with White to play, arose in a fast-play AlphaZero game, played at roughly one second per move.



At the moment, White is two pawns down for the attack

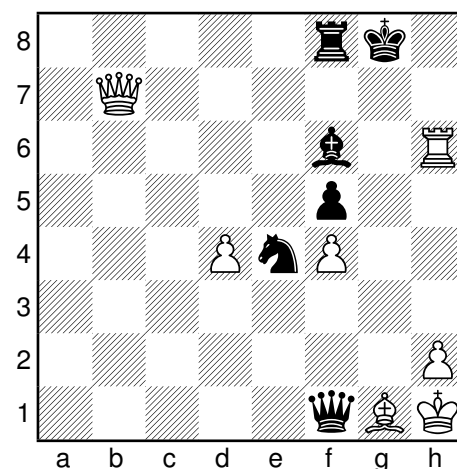
and has very strong threats against the Black king. In Classical chess, those might prove fatal, but here Black uses a self-capture as a defensive resource, as can be seen in the following forcing sequence:

*34. Rxh7+ Kxh7 35. Rh4+ Kxg8* – Black is forced to capture its own rook to avoid checkmate – *36. f4 Ng6 37. Rh2 Qxa2 38. Qc1 Qa4 39. Qc4+ Qxc4 40. Bxc4+*



And here Black uses the second self-capture in this sequence, *40... Kxg7*, to secure the king.

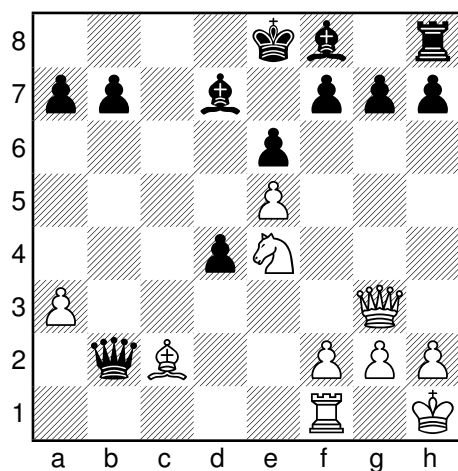
**Game AZ-43: AlphaZero Self-capture vs AlphaZero Self-capture** The following position, with White to play, arose in a fast-play AlphaZero game, played at roughly one second per move.



With White to play, in Classical chess this would result in a mate in one move, on h7. Yet, in Self-capture chess Black can escape by self-capturing its rook on f8, at which point White has to attend to its own king's safety.

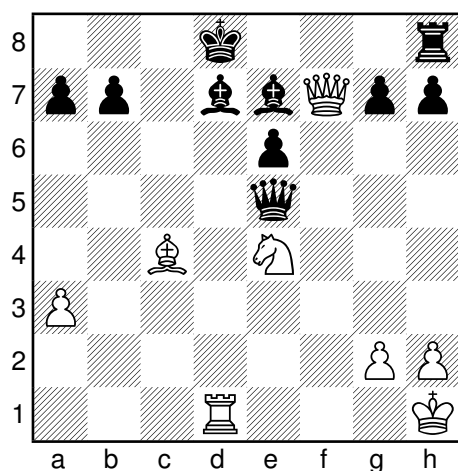
*45. Qh7+ Kxf8 46. Rxf6+ Nxf6 47. Qg6 Qf3+ 48. Qg2 Qxf4*, leading to a simplified position.

**Game AZ-44: AlphaZero Self-capture vs AlphaZero Self-capture** The following position, with White to play, arose in a fast-play AlphaZero game, played at roughly one second per move.

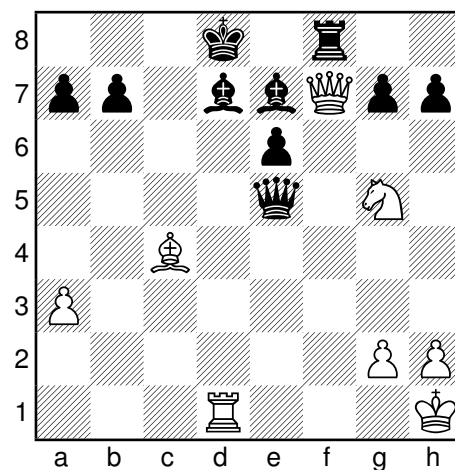


In this position, with White to move, White self-captures a pawn to open up dynamic possibilities against the Black king on the f-file.

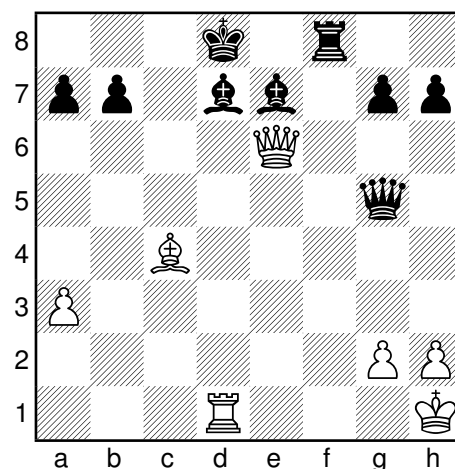
20. Qxf2 d3 21. Qxf7+ Kd8 22. Bxd3 Qxe5 23. Rd1 Be7  
24. Bc4



24... Rf8 25. Ng5

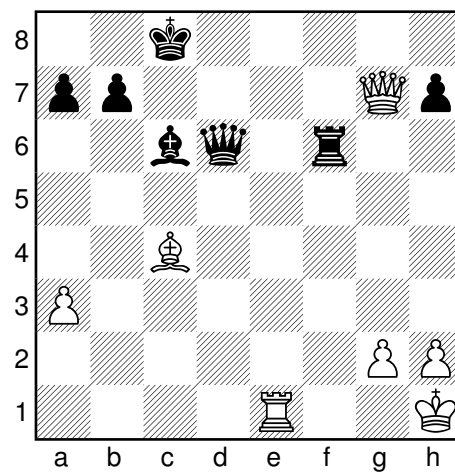


25... Qxg5 26. Qxe6



Here, Black utilizes a self-capture for defensive purposes, giving up the e7 bishop

26... Qxe7 27. Qh3 Rf6 28. Qg3 Kc8 29. Re1 Qd6 30. Qxg7 Bc6



with a roughly equal position.