FNLP
Classification – Log-linear Models

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Recall in WSD

We have considered the following clues as useful:

- neighbouring words around the target
- Part-of-Speech tags of the target, and neighbouring words
- syntax clues
- ...

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(1) Design those hints
(2) Weight those hints
(3) Do the scoring
(4) Rank the candidates!

New View: Features

Features: pieces of evidences describing some aspects of observed data x, usually with respect to the predicted label y

- computer vision
  - the shape, color, texture, size.....of an object
  - other objects nearby, relative postions
  - number of objects available
  - ...
- natural language processing
  - the target word itself, POS, prefix, suffix, capital or not, ...
  - context: words before/after the target, their morphology, POS, ...
  - number of those indications
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- dense vector representations
  - word embedding
  - tree embedding, graph embedding, ...
  - ...

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  - embedding anything ...
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$\rightarrow$ **discriminative models**

Features in NLP

- A feature is **a function of x and y,** $f_i(x, y) \in \mathcal{R}$
- more often, it is a binary or indicator function:

Example

- when we do WSD for the target word *bank*,

$$f_i = \begin{cases} 1 & \text{if } w_{-1} = \text{transfer and } y = \textbf{FINANCIAL}, \\ 0 & \text{otherwise} \end{cases}$$

if the previous word is *transfer*, the current target should have the sense of **FINANCIAL**.

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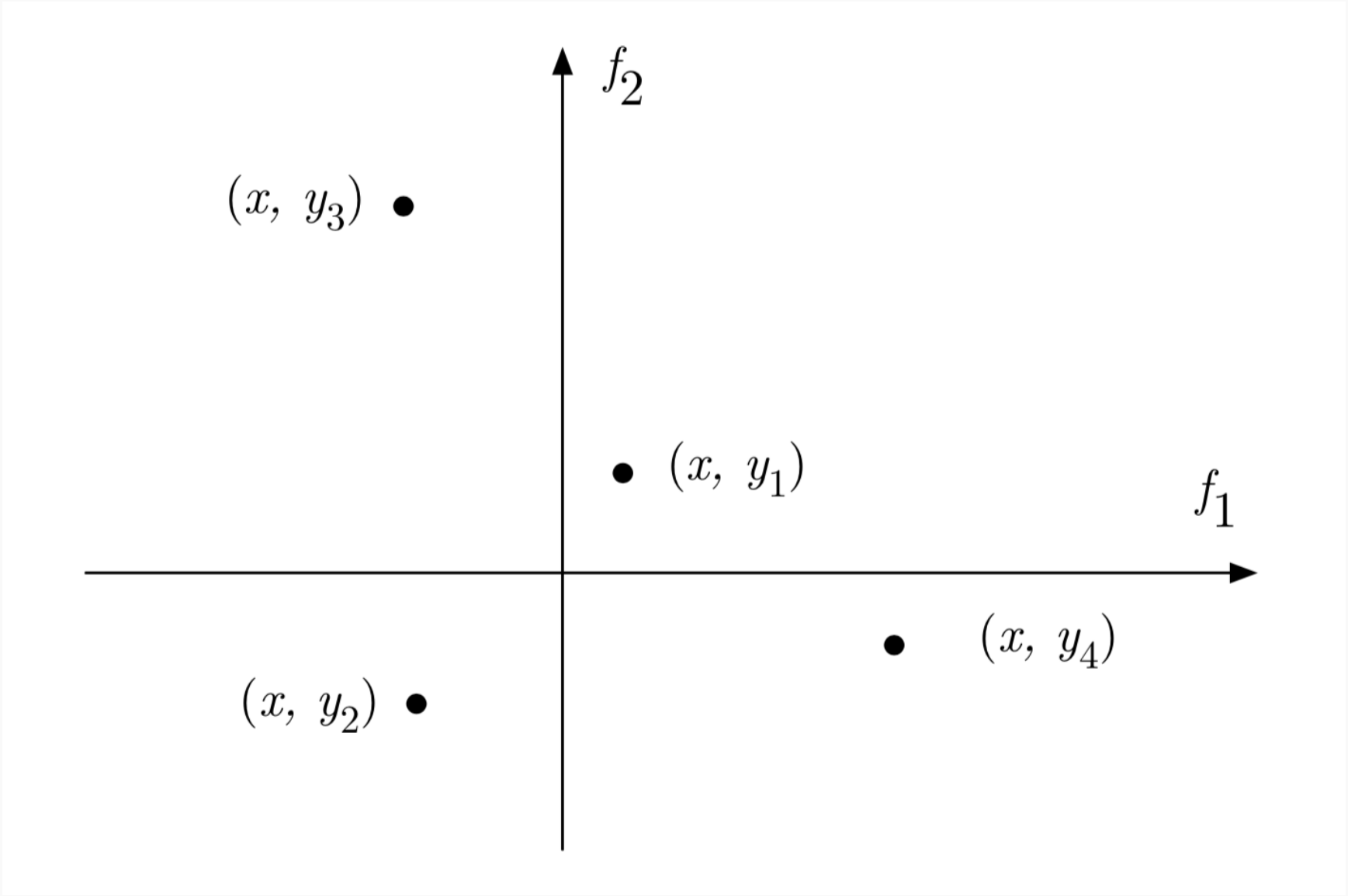
- if we have m aspects to describe an instance, i.e., m features, then:
  - **a feature vector** for each instance, (x, y)
  - $[f_1(x, y), f_2(x, y), f_3(x, y), ..., f_m(x, y)]$
  - $[1, 0, 0, ..., 1, 0]$

Features in NLP



[from Noah Smith]

Examples of Features in NLP

WSD for **bank**:

- if the previous word is transfer, it should be of **FINANCIAL**.
- if the next word is note, it should be of **FINANCIAL**.
- if the sentence is more than 10 words long, it should be of **FINANCIAL**.
- if **bank** is the first word of the sentence, it should be of **FINANCIAL**.
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Any differences compared to what we have done last week?
Naïve Bayes?

Examples of Features in NLP

Text Classification for news reports:

- if the document contains Kobe, its category should be of **Sports**
- if the title contains NBA, its category should be of **Sports**
- if football appear in the upper half, its category should be of **Sports**
- ...

An Example in WSD

I cash a check in that **bank** *and transfer 100 dollars to my mom.*

simply written as: $f(x, y)$

i.e., $f(x, Fi)$, $f(x, Sp)$, $f(x, Po)$, $f(x, En)$,...

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Feature templates

- words before target words: $w_{-1} =$ *, $w_{-2} =$ *, $w_{-3} =$ *, ...
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Apply those feature templates to the input sentence

- $w_{-1} =$ that, $w_{-2} =$ in, $w_{-3} =$ check, ...
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when we consider a candidate label Fi for this case:

- $f_{w-1=that,Fi} = 1$, $f_{w-2=in,Fi} = 1$, $f_{w-3=check,Fi} = 1$, ...
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try more candidate labels ... Sp, Po, En, **...**

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Take an hour to research the TF-IDF weighting scheme

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Bag-of-words

The so-called `Bag-of-words` format

[by students@CMU LTI]

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What are those with very small values or even zeros?

- Take an hour to research the the stop-word list.

A Simple Features based Discriminative Model

Imagine a linear classifiers with the form like, $\lambda_{f(x,y)} f(x,y)$, where λs are weights,

- build a linear function to map $f(x,y)$ to label y
- possibly need a weight $\lambda_{f_i(x,y)}$ for each feature $f_i(x,y)$
- then, for each possible label y of instance x, we can compute a score:

$$score(x,y) = \sum_i \lambda_{f_i(x,y)} f_i(x,y)$$

- the classifier should choose y^*:

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That is, for each y, compute its score, and select the y^* that gives the largest score.

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Note that it may not be a probabilistic model.

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How to figure out those λs?

Our Expectations

We want:

- nicely fit to our linear story
- (at least, looks like) simple
- (may be) easy to drive
- (hopefully) probabilistic
- ...

Features based Linear Models: Algorithms

The key is to choose/learn proper weights λs for different features

- the Perceptron algorithm
- Margin-based models (the Support Vector Machines, SVM)
- Exponential Models:
  - log-linear models, maximum entropy models, logistic models, ...

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  - basically, produce a probabilistic model according to $score(x, y)$

$$p(y|x) = \frac{\exp score(x, y)}{\sum_{y'} \exp score(x, y')} = \frac{\exp \sum_i \lambda_{f_i(x,y)} f_i(x, y)}{\sum_{y'} \exp \sum_i \lambda_{f_i(x,y')} f_i(x, y')}$$

  - numerator: positive score for label y
  - denominator: normalization over all labels

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  - a very powerful tool!

Log-Linear Models

For a data sample (x, y);

- We care:

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Log-Linear Models

For a data sample (x, y);

- We care:

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  - linear: $\boldsymbol{\lambda} \cdot \boldsymbol{f}(x, y)$
  - Normalization: $\log \sum_{y'} \exp(\boldsymbol{\lambda} \cdot \boldsymbol{f}(x, y'))$

Maximum-Likelihood Estimations

How likely do we observe the data given the current parameters?

- Given the training data $\{(x_1, y_1), (x_2, y_2), ..., (x_k, y_k)\}$: the Likelihood of $\boldsymbol{\lambda}$ is:

$$L(\boldsymbol{\lambda}) = \prod_k p(y_k|x_k; \boldsymbol{\lambda}) = \prod_k \frac{\exp(\boldsymbol{\lambda} \cdot \boldsymbol{f}(x_k, y_k))}{\sum_{y'} \exp(\boldsymbol{\lambda} \cdot \boldsymbol{f}(x_k, y'))}$$

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After $\log$:

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  - The first component: **Empirical Counts**
  - The second component: **Expected Counts**

Gradient Ascend Methods

- Need to maximize $LL(\boldsymbol{\lambda})$ where

$$\frac{\partial LL(\boldsymbol{\lambda})}{\partial \lambda_{f_i(x,y)}} = \sum_k f_i(x_k, y_k) - \sum_k \sum_{y'} f_i(x_k, y') p(y'|x_k; \boldsymbol{\lambda})$$

- Initialize all λs to be 0
- Iterate until convergence
  - Calculate $\Delta = \frac{\partial LL(\boldsymbol{\lambda})}{\partial \lambda}$
  - Calculate $\beta_* = \arg\max_\beta LL(\boldsymbol{\lambda} + \beta \cdot \Delta)$
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- Slow!
  - Optimizations available : Conjugate Gradient Methods
  - or, Stochastic Gradient Methods

Stochastic Gradient Ascend

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- Initialize all λs, number of epochs T, learning rate α
- For $t \in \{1, ..., T\}$:
  - choose a random permutation π of $\{1, 2, ..., k, ...\}$ (the whole dataset)
  - for $i \in \{1, 2, ..., k, ...\}$:
    - calculate $\Delta = \frac{\partial LL(\boldsymbol{\lambda})_{\pi(i)}}{\partial \lambda}$
    - set $\boldsymbol{\lambda} \leftarrow \boldsymbol{\lambda} + \alpha \cdot \Delta$
- output $\boldsymbol{\lambda}$

More to Mention

Just anther way of talking this:

$$\boldsymbol{\lambda}^* = \arg\max_{\boldsymbol{\lambda}} LL(\boldsymbol{\lambda})$$

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- then, use Stochastic Gradient **Descend** instead of *Ascend* in the last slide
- all seem exciting terms :-)

Anything More?

- How do we get the probablistic thing?
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or

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Is It Done?

It looks we have everything ready to build a model!

- Say, we have a feature f_1, defined as:

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- In our training data, **NFL** is seen 100 times, with **Sports** every time
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Regularization

What can we do to prevent such cases?

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- Modified the loss function

$$LL(\boldsymbol{\lambda}) = \sum_k \boldsymbol{\lambda} \cdot \boldsymbol{f}(x_k, y_k) - \sum_k \log \sum_{y'} \exp(\boldsymbol{\lambda} \cdot \boldsymbol{f}(x_k, y')) - \frac{\alpha}{2} ||\boldsymbol{\lambda}||^2$$

$$= \sum_k \boldsymbol{\lambda} \cdot \boldsymbol{f}(x_k, y_k) - \sum_k \log \sum_{y'} \exp(\boldsymbol{\lambda} \cdot \boldsymbol{f}(x_k, y')) - \frac{\alpha}{2} \sum_i \lambda_i^2$$

- When calculating the gradients

$$\frac{\partial LL(\boldsymbol{\lambda})}{\partial \lambda_{f_i(x,y)}} = \sum_k f_i(x_k, y_k) - \sum_k \sum_{y'} f_i(x_k, y') p(y'|x_k; \boldsymbol{\lambda}) - \alpha \lambda_{f_i(x,y)}$$

- Adds a penalty for large weights

Regularization

Prevent every parameter from becoming too large in magnitude.

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where $\alpha > 0$, p could choose from 1, 2 or others.

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Gives us:

- L-2 Regularization
- L-1 Regularization
  - Sparsity! (parameters)
  - but .....

Reasons for Log-linear Models

You will find it useful in the (near) future...

- You love feature engineering;
- You hate feature engineering

In Short

- A strong classifier you would be more familiar in the future
  - connection to many dominant classifiers
- Take care of features for your tasks
  - feature selection
  - biased feature selection
  - purposely feature selection
- Take care of the learning process and also evaluation protocal.
  - regularization
  - randomness

Reading

- M. Collins, Notes on Log-Linear Models (http://www.cs.columbia.edu/ mcollins/loglinear.pdf)
- Lecture Note, Linear Regression and Gradient Ascent, CS109@Stanford, (https://web.stanford.edu/class/archive/cs/cs109/cs109.1208/lectures/2
- Griffiths, T. L., and Steyvers, M. (2004). Finding scientific topics. Proceedings of the National Academy of Sciences, 101, 5228-5235

Book Chapter 6.5, Dan's book

Further Reading

- David M. Blei, Andrew Y. Ng and Michael I. Jordan. Latent Dirichlet Allocation, Journal of Machine Learning Research 3 (2003) 993-1022
- Zhang and Oles (2010), Text Categorization Based on Regularized Linear Classification Methods (http://www.stat.yale.edu/lc436/papers/temp/Zhang Oles 2001.pdf)
- Lecture Notes 1, Discriminative Algorithms, CS229-Machine Learning@Stanford, Andrew Ng (http://cs229.stanford.edu/materials.html)
- Adam Berger, Stephen Della Pietra, and Vincent Della Pietra, A maximum entropy approach to natural language processing. Computational Linguistics, 22(1):39-71, 1996.
- Galen Andrew and Jianfeng Gao, Scalable training of L1-regularized log-linear models. In Proc. of ICML, 2007.