CS5340 ASSIGNMENT 2 PART2

Name: ZHANG ENHAO Student Number:A0276557M

1 Deriving Linear Equations for Weights

Differentiating this expression with respect to w_{uj} (where j could be any parent or the bias term), we get:

1. For the bias term w_{u0}:

$$\frac{\partial}{\partial w_{u0}}\left(x_u - w_{u0} - \sum_{c\in\pi_u} w_{uc}x_{uc}\right)^2 = -2\left(x_u - w_{u0} - \sum_{c\in\pi_u} w_{uc}x_{uc}\right)$$

2. For any parent term w_{uc}:

$$\frac{\partial}{\partial w_{uc}}\left(x_u - w_{u0} - \sum_{c\in\pi_u} w_{uc}x_{uc}\right)^2 = -2x_{uc}\left(x_u - w_{u0} - \sum_{c\in\pi_u} w_{uc}x_{uc}\right)$$

Setting these derivatives to zero yields the system of linear equations for the weights. $A\mathbf{w} = \mathbf{b}$ can be represented as:

$$\begin{bmatrix} N & \sum x_{u1} & \sum x_{u2} & \dots \\ \sum x_{u1} & \sum x_{u1}^2 & \sum x_{u1}x_{u2} & \dots \\ \sum x_{u2} & \sum x_{u1}x_{u2} & \sum x_{u2}^2 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix} \begin{bmatrix} w_{u0} \\ w_{u1} \\ w_{u2} \\ \dots \end{bmatrix} = \begin{bmatrix} \sum x_u \\ \sum x_u x_{u1} \\ \sum x_u x_{u2} \\ \dots \end{bmatrix}$$

Where the weights are obtained by solving

$$A^T A\mathbf{w} = A^T\mathbf{b}$$

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2 Estimating Variance

For the node u, the predicted value is:

$$\hat{x}_u = w_{u0} + \sum_{c\in\pi_u} w_{uc}x_{uc}$$

The residual for observation i is:

$$r_i = x_{ui} - \left(w_{u0} + \sum_{c\in\pi_u} w_{uc}x_{uci}\right)$$

The variance σ_u^2 is then:

$$\sigma_u^2 = \frac{1}{N}\sum_{i=1}^{N} r_i^2$$