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Assignment Title:
Principles of Analogue Amplitude and Frequency Modulation

INTRODUCTION

In analogue modulation, a baseband message signal would be transformed into a signal which is transmitted over a assigned channel. To encode information into a transmitted signal, many techniques have developed for analogue communication systems. The main purpose of this report is to research some important properties of several analogue modulation techniques, particularly analogue amplitude modulation and frequency modulation, by analysing theoretically and simulating computationally with MATLAB.

THEORETICAL ANALYSIS

Symbol/Abbreviation	Definition
DSB	Double-sideband
AM	Amplitude modulation
FM	Frequency modulation
m(t)	Message signal
$\mathrm{M(f)}$	Fourier transform of the message signal
$\mathrm{u(t)}$	AM signal
$\mathrm{U(f)}$	Fourier transform of the AM signal
$\mathrm{v(t)}$	FM signal
$\mathrm{V(f)}$	Fourier transform of the FM signal
B	Bandwidth of message signal
FT	Fourier transform
$\mathrm{B_{FM}}$	Bandwidth of the FM signal
A_{Cmin}	The minimum A_C for correct envelope detection
a_{mod}	AM Modulation index
f_c	Carrier frequency
β	FM Modulation index
k_f	Frequency shift index
Δf_{max}	The maximum deviation in instantaneous frequency
max	Maximum
min	Minimum

Table1. Definition of Abbreviation

Amplitude Modulation is a process of varying the instantaneous amplitude of carrier signal in terms of the instantaneous amplitude of the message signal. Several variants of AM, like DSB suppressed carrier modulation and conventional AM are used in practice.

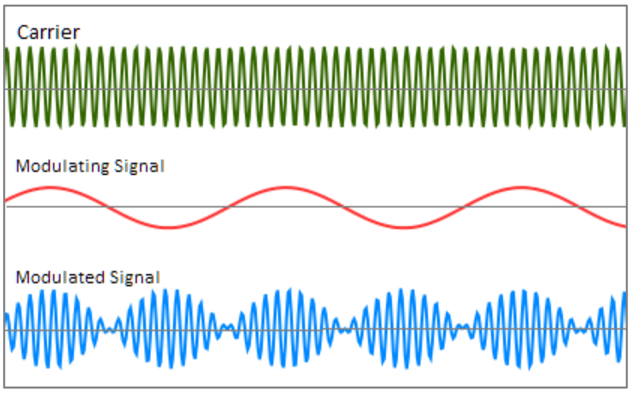


Figure1. Amplitude Modulation

For Double-sideband Suppressed Carrier Modulation, the message m(t) modulates the signal u(t) as below:

$$u_{DSB}(t) = Am(t)cos(2\pi f_c t)$$

By using Fourier Transform, it can be obtained that,

$$U_{DSB}(f) = \frac{A}{2}\left(M(f - f_c) + M(f + f_c)\right)$$

For Conventional Modulation, a carrier component is added, so that the message m(t) modulates the signal u(t) as below:

$$u_{AM}(t) = Am(t)\cos(2\pi f_c t) + A_c \cos(2\pi f_c t)$$

On taking Fourier Transform, it can be obtained that,

$$U_{AM}(f) = \frac{A}{2}(M(f - f_c) + M(f + f_c)) + \frac{A_c}{2}(\delta(f - f_c) + \delta(f + f_c))$$

The envelope is given by,

$$e(t) = |Am(t) + A_c|$$

The modulation index a_{mod} is defined as,

$$a_{mod} = \frac{AM_0}{A_c} = \frac{A|\min_t m(t)|}{A_c}$$

Envelope can be seen as a DC-shifted and scaled variant of the message, so that the message can be recovered accurately if $a_{mod} \leq 1$ by using envelope detection.

Frequency Modulation is a process of varying the instantaneous frequency of carrier signal in terms of the instantaneous amplitude of the message signal.

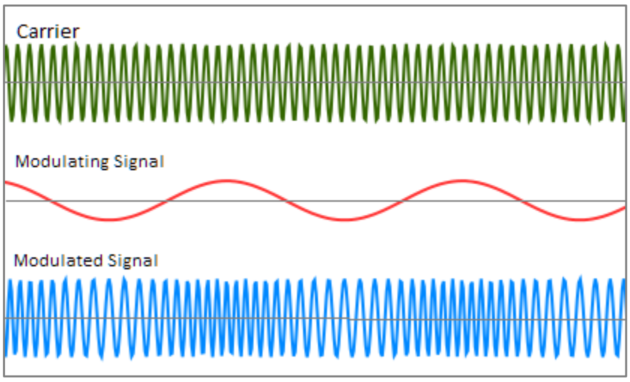


Figure2. Frequency Modulation

In Frequency Modulation, the information is connected with the phase $\theta(t)$.

The transmitted signal can be expressed as,

$$u(t) = A_c \cos(2\pi f_c t + \theta(t))$$

The maximum deviation in instantaneous frequency in terms of $m(t)$ is given by

$$\Delta f_{max} = k_f \max_t |m(t)|$$

The modulation index is defined as,

$$\beta = \frac{\Delta f_{max}}{B} = \frac{k_f \max_t |m(t)|}{B}$$

where B is the bandwidth of m(t).

For $\beta < 1$, narrowband FM is used, and for $\beta < 1$, wideband FM is used.

Since the FM signal can be expressed as,

$$u_{FM}(t) = A_c \cdot \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) + \theta_0\right)$$

It can be obtained that,

$$v(t) = \frac{du_{FM}(t)}{dt} = -A_c(2\pi f_c t + 2\pi k_f m(t)) \sin\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau + \theta_0\right)$$

And the envelope of v(t) is $2\pi A_c|f_c + k_f m(t)|$.

Since $k_f m(t)$ is the instantaneous frequency deviation, and the magnitude of it is much smaller than f_c, which means

$$f_c + k_f m(t) > 0 \text{ for all } t$$

Carson’s rule as follows, is utilized to estimate the bandwidth of the FM signal

$$B_{FM} \approx 2B + 2\Delta f_{max} = 2B(\beta + 1)$$

RESULTS

TaskA

i) By the Fourier transform,

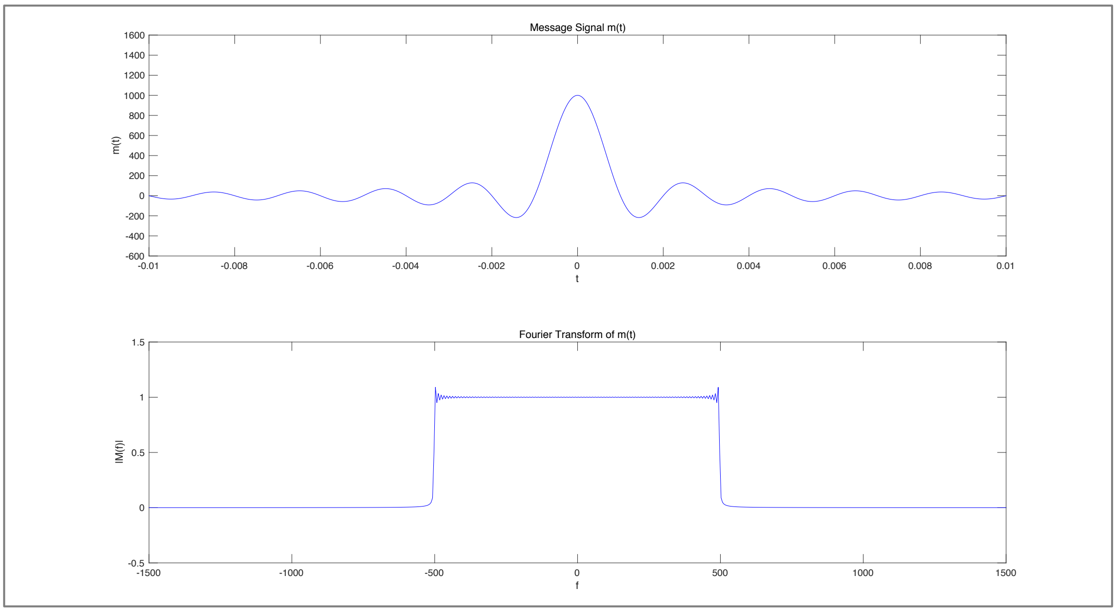
$$\mathrm{sinc}(\mathrm{Wt}) \xrightarrow{\mathrm{FT}} \frac{1}{\mathrm{W}}\mathrm{rect}\left(\frac{\mathrm{f}}{\mathrm{W}}\right)$$

Since $m(t) = (1/\mathrm{T})\,\mathrm{sinc}(t/T)$ and T=1msec,

$$\mathrm{m(t)} = 1000 \cdot \mathrm{sinc(1000t)} \xrightarrow{\mathrm{FT}} \mathrm{M(F)} = \mathrm{rect}\left(\frac{\mathrm{f}}{1000}\right)$$

It can be obtained that the bandwidth of m(t) is 500Hz.

ii) By programming on MATLAB, the plot for the FT of m(t) can be obtained as shown in *Plot1*. And it can be seen from the plot that the bandwidth of m(t) is 500Hz, which is the same as the theoretical result in Task A) i).



Plot1

TaskB

i) Since we have,

$$\mathrm{u_{DSB}(t)} = \mathrm{m(t)\cos(2\pi f_c t)}$$

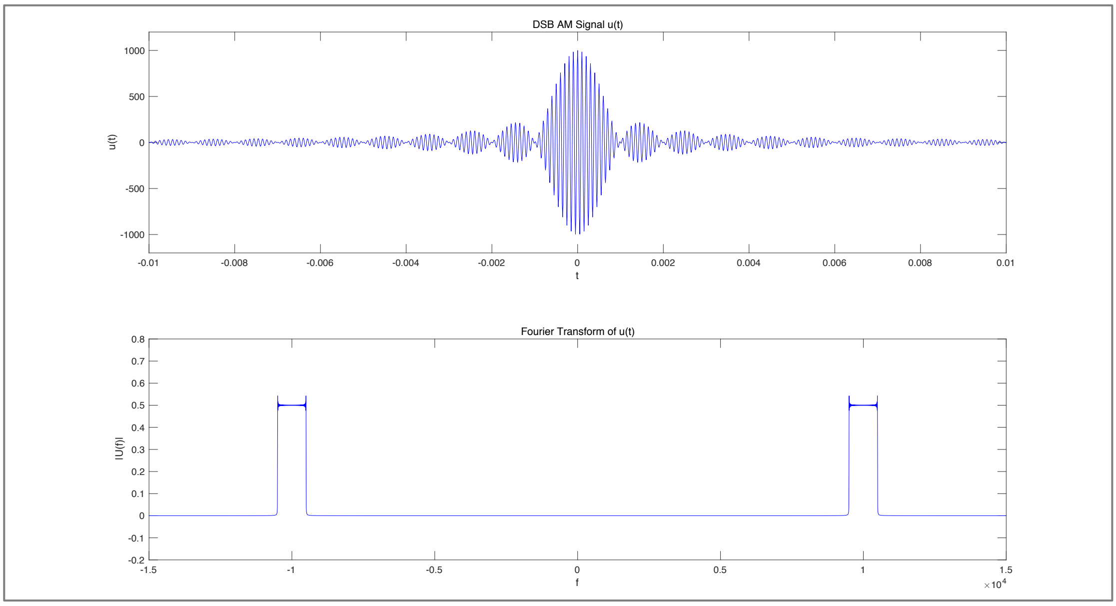
On taking Fourier Transform, it can be obtained that,

$$\mathrm{U_{DSB}(f)} = \frac{1}{2}\left(\mathrm{M(f - f_c) + M(f + f_c)}\right)$$

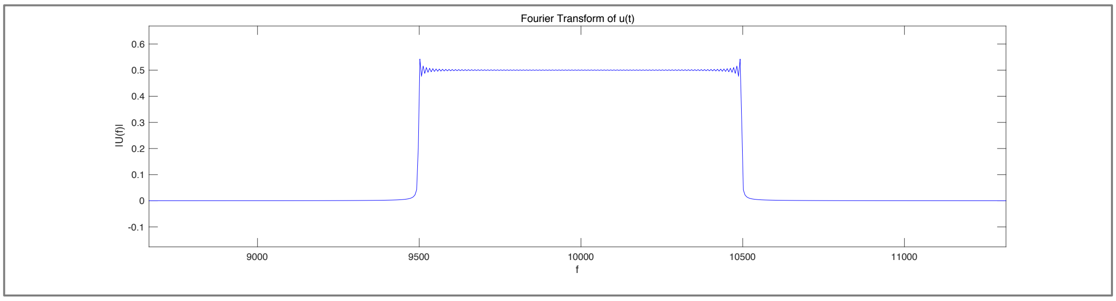
where $\mathrm{f_c} = \frac{10}{\mathrm{T}} = \frac{10}{0.01} = 1000\mathrm{Hz}.$

It can be obtained that the bandwidth of the DSB AM signal u(t) is 1000Hz.

ii) By programming on MATLAB, the plot of FT of u(t) can be obtained as shown in *Plot2* and *Plot3*(local zoom view). And it is obvious that the bandwidth of u(t) is 1000Hz, which is the same as the theoretical result in Task B) i).



Plot2



Plot3. Local Zoom View of Plot2

iii) Comparing the spectra of m(t) and u(t), it can be seen that the spectrum of u(t) is the double side shifting of m(t). Since $m(t)$ is real-valued the spectrum of $m(t)$ has conjugate symmerty. The bandwidth of u(t) is 1000Hz, which is twice that of m(t).

TaskC

i) To find the smallest possible value $A_{c,min}$ for correct envelope detection, we need to find the minimum of m(t).

$$\mathrm{m(t)} = \left(\frac{1}{\mathrm{T}}\right) \cdot \mathrm{sinc}\left(\frac{\mathrm{t}}{\mathrm{T}}\right) = 1000 \cdot \mathrm{sinc}(1000\mathrm{t})$$

To find the minimum of m(t), we need to obtain the differential of m(t) as below,

$$\frac{d[m(t)]}{dt} = 1000 \cdot \left[\left(\frac{cos(1000 \cdot \pi t)}{t}\right) - \left(\frac{sin(1000 \cdot \pi t)}{\pi t^2}\right)\right] = 0$$

After that, we can obtain that,

$$t \approx \pm 0.00143\ ;\ m(t) \approx -217.2335\ ;\ A_{c,min} = |min(m(t))| = 217.2335$$

$$\mathrm{U_{AM}(f)} = \frac{1}{2}\left(\mathrm{M(f - f_c) + M(f + f_c)}\right) + \frac{\mathrm{A_c}}{2}(\delta(\mathrm{f + f_c}) + \delta(\mathrm{f - f_c})) = \begin{cases} \frac{1}{2} & |\mathrm{f} \pm \mathrm{f_c}| \leq 500, |f| \neq \mathrm{f_c} \\ \pi \mathrm{A_c} \delta(|2\pi f| - 2\,\pi \mathrm{f_c}) + \frac{1}{2} & |f| = \mathrm{f_c} \\ 0 & otherwise \end{cases}$$

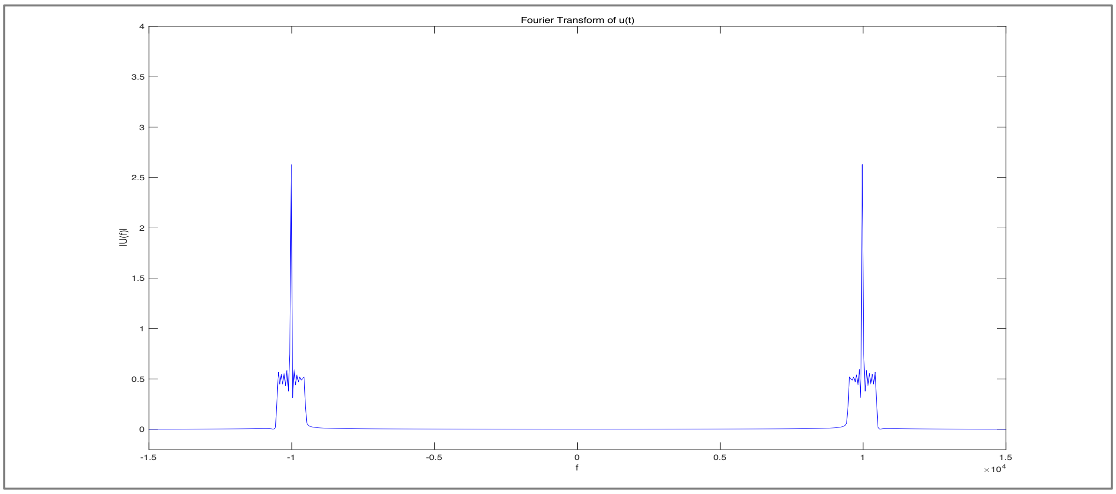
Thus, the bandwidth of $u_{AM}(t)$ is 1000Hz.

ii) By programming on MATLAB, the value $A_{c,min} = |min(m(t))|$ can be obtained as shown in *Plot4* and the plot of the FT of u(t) is obtained as shown in *Plot5* and *Plot6*(local zoom view). And it is obvious that the bandwidth of u(t) is 1000Hz, which is the same as the theoretical result in TaskC) i).

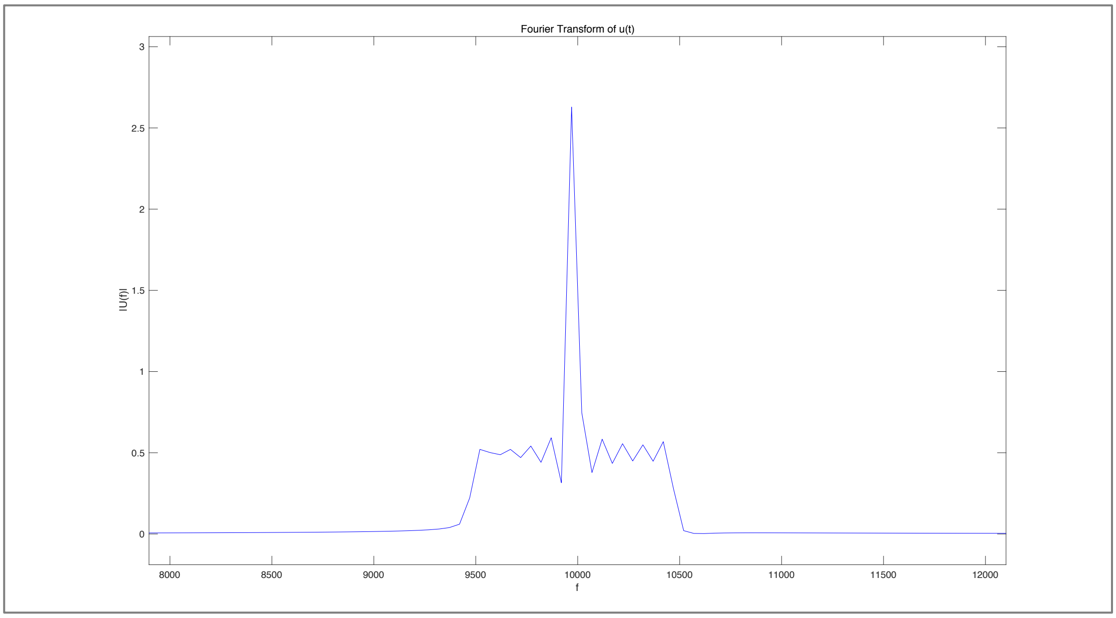
```
min =

 -217.2335
```

Plot4



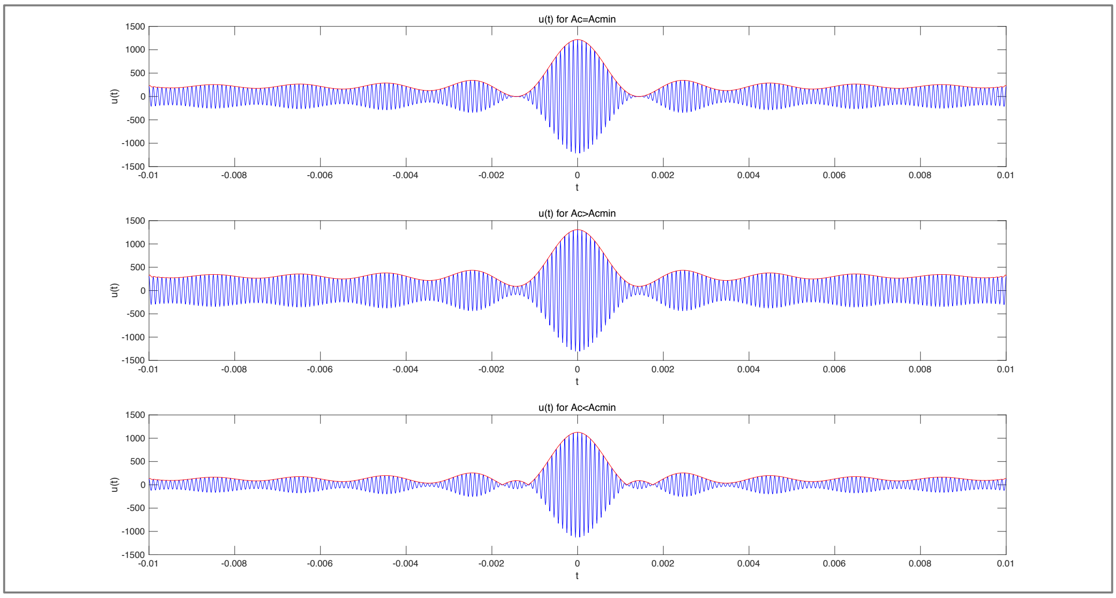
Plot5



Plot6. Local Zoom View of Plot5

iii) By comparing spectra of $m(t)$ and $u_{AM}(t)$, it can be seen that the spectrum of U(f) is the double side shifting of M(f) and $\pi A_c \delta(f)$, which are shifted to $\pm$fc by the carrier signal. The bandwidth the conventional AM signal u(t) is 1000Hz, which is twice that of m(t).

iv) By programming on MATLAB, plots for three cases of $A_{c,min} = A_c$, $A_{c,min} > A_c$ and $A_{c,min} < A_c$ are shown in *Plot7*, repectively.

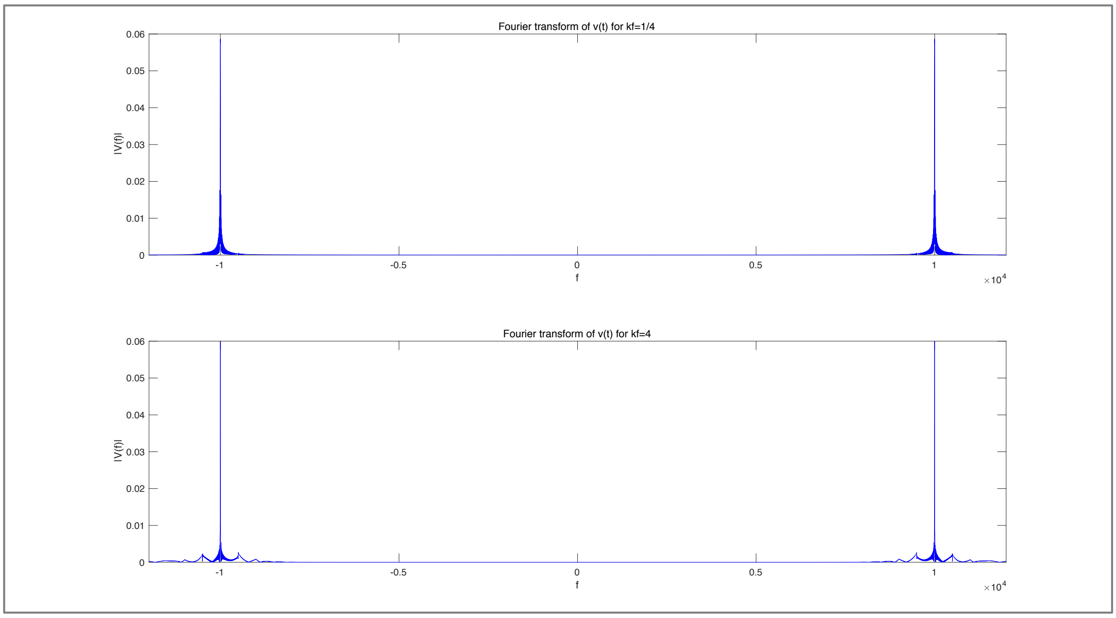


Plot7

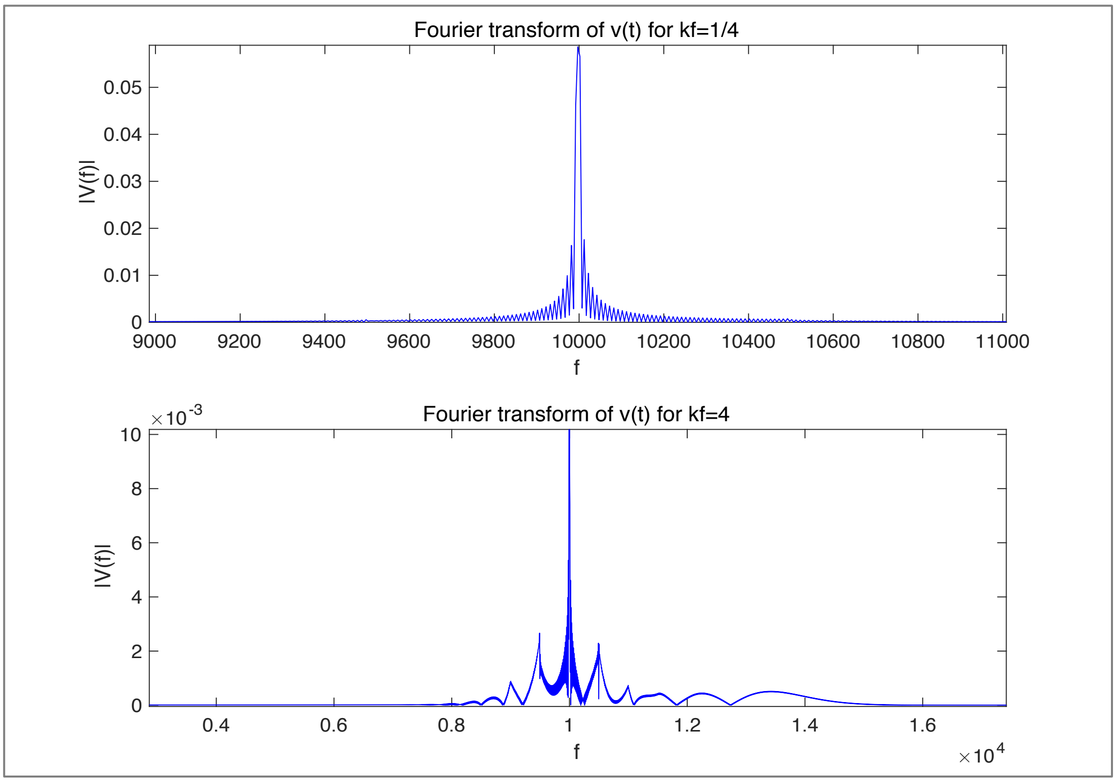
v) Comparing the time domain plots of u(t) and m(t) for all choices of Ac in Question iv when deriving m(t) from u (t) through envelop detection, we can obtain that if $A_{c,min} < A_c$, the envelop cannot be detected correctly because the envelope does not follow the shape of the message. And if $A_{c,min} \geq A_c$, the envelop can be detected correctly.

TaskD

i) By programming on MATLAB, the plots of the Fourier transform of v(t) for $k_f = 1/4$ and $k_f = 4$ are obtained as shown in *Plot8* and *Plot9*(local zoom view). And it can be seen that the bandwidth of v(t) for $k_f = 1/4$ is about 1500Hz, and the bandwidth of v(t) for $k_f = 4$ is about 8500Hz.



Plot8



Plot9. Local Zoom View of Plot8

ii) To verify the correctness of the results in i), we can compare them with the Carson's formula, that is,

$$B_{FM} \approx 2B + 2\Delta f_{max}$$

where B is the bandwidth of m(t), B=500Hz, $\max_t |m(t)| = 1000$

The maximum deviation in instantaneous frequency in terms of $m(t)$ is,

$$\Delta f_{max} = k_f \max_t |m(t)|$$

For $k_f = \frac{1}{4}$,

$$B_{FM} \approx 2B + 2\Delta f_{max} = 2B + 2k_f \max_t |m(t)| = 2 \times 500 + 2 \times \frac{1}{4} \times 1000 = 1500Hz$$

For $k_f = 4$,

$$B_{FM} \approx 2B + 2\Delta f_{max} = 2B + 2k_f \max_t |m(t)| = 2 \times 500 + 2 \times 4 \times 1000 = 9000Hz$$

Fortunately, the results are close to what we got from i), which means the results in i) is correct.

iii) The bandwidth of v(t) for $k_f = 1/4$ is 1500Hz, and the bandwidth of v(t) for $k_f = 4$ is about 9000Hz. While the bandwidth of the AM signals in B) and C) is 1000Hz. Comparing them, it can be concluded that the bandwidth of FM signal is larger than that of AM signal, and the larger $\mathrm{k_f}$ means larger bandwidth because of $\mathrm{B_{FM}} \approx 2\mathrm{B} + 2\Delta \mathrm{f_{max}}$.

iv) To derive an approximation for the maximum theoretical value $\mathrm{k_{f,max}}$ of $\mathrm{k_f}$ in order the FM signal v(t) to remain passband around the carrier frequency $\mathrm{f_c}$, we can calculate as below,

$$\mathrm{B_{FM}} \leq 2\mathrm{f_c}$$

$$\mathrm{B_{FM}} \approx 2\mathrm{B} + 2\Delta \mathrm{f_{max}} = 2\mathrm{B} + 2\mathrm{k_f}\max_t |\mathrm{m(t)}| \leq 2\mathrm{f_c}$$

Where B=500Hz, $\max_t |\mathrm{m(t)}| = 1000$, $\mathrm{f_c} = 10000\mathrm{Hz}$

$$\mathrm{k_f} \leq 9.5$$

Thus, the maximum theoretical value $\mathrm{k_{f,max}}$ is 9.5.

v) The maximum value ($k_{f\,\mathrm{max,demod}}$) of k_f for correct message detection at the output of the discriminator demodulator can be derived as below,

$$\mathrm{u_{FM}}(t) = \mathrm{A_c} \cdot \cos\left(2\pi \mathrm{f_c t} + 2\pi \mathrm{k_f}\int_0^{\mathrm{t}} \mathrm{m}(\tau) + \theta_0\right)$$

We have

$$\mathrm{v(t)} = \frac{d\mathrm{u_{FM}}(t)}{dt} = -\,\mathrm{A_c}(2\pi \mathrm{f_c t} + 2\pi \mathrm{k_f m(t)}) \sin\left(2\pi \mathrm{f_c t} + 2\pi \mathrm{k_f}\int_0^{\mathrm{t}} \mathrm{m}(\tau)\mathrm{d}\tau + \theta_0\right)$$

And the envelope of v(t) is $2\pi \mathrm{A_c}|\mathrm{f_c} + \mathrm{k_f m(t)}|$

Since $\mathrm{k_f m(t)}$ is the instantaneous frequency deviation, and the magnitude of it is much smaller than $\mathrm{f_c}$, which means

$$\mathrm{f_c} + \mathrm{k_f m(t)} > 0 \text{ for all t}$$

$$\mathrm{k_f} < -\frac{\mathrm{f_c}}{\mathrm{m(t)}}$$

$$\mathrm{k_{f\,max,demod}} = -\frac{\mathrm{f_c}}{min(m(t))} = -\frac{10000}{-217.2335} \approx 46.0334$$

Thus, the maximum value ($k_{f\,\max,\text{demod}}$) is about 46.0334, which is much larger than the maximum theoretical value $k_{f,max}$ in iv).

CONCLUSION

Overall, this report implements analogue modulation techniques and mainly focuses on the bandwidth of different kinds of signals to research some fundamental principles of DSB AM, Conventional AM and FM. And all of these signals and plots have been simulated successfully with MATLAB. By comparing different time domain plots and spectra, it helped me understand many features of amplitude and frequency modulations, for instance, the bandwidth the DSB signal is twice that of message signal and also explore how to derive the smallest A_c to detect envelop of AM signal correctly and the maximum k_f for correct detection of FM signal.

REFERENCE

[1] Upamanyu Madhow, *Introduction to Communication Systems*, Cambridge University Press, 2014;

[2] http://www.equestionanswers.com/notes/modulation-analog-digital.php.

APPENDIX

Code1-*Plot1* in Task A ii)

```
T=0.001;
Ts=0.00001;
time=[-0.10000000001:Ts:0.10000000001];
fs=1/Ts;
f=(0:length(time)-1)*fs/length(time)-fs/2;

syms t
m=(1/T)*sinc(t/T);
m=subs(m,t,time);
subplot(2,1,1);
```

```
plot(time,m,'LineWidth',0.5,'color',[0,0,1]);
axis([-0.01 0.01 -600 1600])
title('Message Signal m(t)')
xlabel('t')
ylabel('m(t)')

M=fftshift(fft(double(m)))/fs;
subplot(2,1,2);
plot(f,abs(M),'LineWidth',0.5,'color',[0,0,1]);
axis([-1500 1500 -0.5 1.5])
title('Fourier Transform of m(t)')
xlabel('f')
ylabel('|M(f)|')
```

Code2-*Plot2&3* in Task B ii)

```
T=0.001;
fc=10/T;
Ts=0.00001;
time=[-0.10000000001:Ts:0.10000000001];
fs=1/Ts;
f=(0:length(time)-1)*fs/length(time)-fs/2;

syms t
m=(1/T)*sinc(t/T);
u=m* cos(2*pi*fc*t) ;
u=subs(u,t,time);

subplot(2,1,1);
plot(time,u,'LineWidth',0.5,'color',[0,0,1]);
axis([-0.01 0.01 -1200 1200])
title('DSB AM Signal u(t)')
xlabel('t')
```

```
ylabel('u(t)')

U=fftshift(fft(double(u)))/fs;
subplot(2,1,2);
plot(f,abs(U),'LineWidth',0.5,'color',[0,0,1]);
axis([-15000 15000 -0.2 0.8])
title('Fourier Transform of u(t)')
xlabel('f')
ylabel('|U(f)|')
```

Code3-*Plot4&5&6* in Task C ii)

```
T=0.001;
fc=10/T;
Ts=0.00001;
time=[-0.010000000001:Ts:0.010000000001];
fs=1/Ts;
f=(0:length(time)-1)*fs/length(time)-fs/2;

syms t
m=(1/T)*sinc(t/T);
u=(m+217.2335)* cos(2*pi*fc*t);
u=subs(u,t,time);
y = abs(hilbert(double(u)));
U=fftshift(fft(double(u)))/fs;
plot(f,abs(U),'LineWidth',0.5,'color',[0,0,1]);
axis([-15000 15000 -0.2 4])
title('Fourier Transform of u(t)')
xlabel('f')
ylabel('|U(f)|')

clearclct= -0.01:0.0001:0.01;
p = inline(m);
```

```
min = min(p(t))
```

Code4-*Plot4&5&6* in Task C iv)

```
T=0.001;
fc=10/T;
Ts=0.00001;
time=[-0.010000000001:Ts:0.010000000001];
fs=1/Ts;
f=(0:length(time)-1)*fs/length(time)-fs/2;

syms t
m=(1/T)*sinc(t/T);
u=(m+217.2335)* cos(2*pi*fc*t);
u=subs(u,t,time);
subplot(3,1,1);
plot(time,u,'LineWidth',0.5,'color',[0,0,1]);
title('u(t) for Ac=Acmin')
xlabel('t')
ylabel('u(t)')
hold on
yu = abs(hilbert(double(u)));
plot(time,yu,'LineWidth',0.5,'color',[1,0,0]);
axis([-0.01 0.01 -1500 1500])

u=(m+307.2335)*cos(2*pi*fc*t);
u=subs(u,t,time);
subplot(3,1,2);
plot(time,u,'LineWidth',0.5,'color',[0,0,1]);
title('u(t) for Ac>Acmin')
xlabel('t')
ylabel('u(t)')
hold on
```

```
yu = abs(hilbert(double(u)));
plot(time,yu,'LineWidth',0.5,'color',[1,0,0]);
axis([-0.01 0.01 -1500 1500])


u=(m+127.2335)* cos(2*pi*fc*t);
u=subs(u,t,time);
subplot(3,1,3);
plot(time,u,'LineWidth',0.5,'color',[0,0,1]);
title('u(t) for Ac<Acmin')
xlabel('t')
ylabel('u(t)')
hold on
yl = abs(hilbert(double(u)));
plot(time,yl,'LineWidth',0.5,'color',[1,0,0]);
axis([-0.01 0.01 -1500 1500])
```

Code5-*Plot8&9* in Task D i)

```
T=0.001;
Ts=0.00001;
time=[-0.10000000001:Ts:0.10000000001];
fc=10/T;
fs=1/Ts;
syms t t2
m=(1/T)*sinc(t2/T);
f=(0:length(time)-1)*fs/length(time)-fs/2;

kf1=1/4;
theta=2*pi*kf1*int(m,t2,[0,t]);
v1=zeros(length(time));
v1=subs(cos(2*pi*fc*t+theta),t,time);
V1=fftshift(fft(double(v1)))/fs;
subplot(2,1,1);
```

```
plot(f,abs(V1),'LineWidth',0.5,'color',[0,0,1]);
axis([-12000 12000 0 0.06]);
title('Fourier transform of v(t) for kf=1/4');
xlabel('f');
ylabel('|V(f)|');

kf2=4;
theta=2*pi*kf2*int(m,t2,[0,t]);
v2=zeros(length(time));
v2=subs(cos(2*pi*fc*t)+theta),t,time);
V2=fftshift(fft(double(v2)))/fs;
subplot(2,1,2);
plot(f,abs(V2),'LineWidth',0.5,'color',[0,0,1]);
axis([-12000 12000 0 0.06]);
title('Fourier transform of v(t) for kf=4');
xlabel('f');
ylabel('|V(f)|');
```