

# Physical Experiment II

# Physics Lab Report 9

Experiment Title:	Measurement of the Apex Angle of a Prism and the Wavelengths of Mercury Lights Using a Spectrometer
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#### **Abstract** (About 100 words, 3 points)

This purpose of this experiment is to measure the apex angle of a prism and the wavelengths of mercury. First of all, I used the autocollimation to make the telescope perpendicular toward the apex. And then, I switched the telescope to make it toward the other side of the telescope perpendicularly. With some knowledge about the grating diffraction I have learned, I used the grating to diffract the light. After that, by observing and computing, I obtained the apex angle of a prism with uncertainty and the wavelengths of the light with relative error.



## **Introduction** (3 points)

By reflection of the prism, the cross can be seen and the position of the cross follows the symmetry so that we can get the isosceles triangle. Move the telescope to make it perpendicular to the AC and AB and read the angle and then we can find the angle  $\alpha$  by using the formula so that we can determine the apex angle of a glass prism.



### Experimental Procedure (State main steps in order of

performance, 3 points)

#### The first experiment:

- 1. Turn on the power of all relative components;
- 2. Adjust the place of the triangular prism on the center of the spectrometer table;
- 3. Rotate the telescope toward one side of the triangular prism and adjust the cross to the node of the crosshairs;
- 4. Record the scale and switch the telescope to the other side and adjust the cross to the node of the crosshairs again for preparing to record another scale;
  - 5. Repeat the three steps above for 3 times.
  - 6. Analyze the data to determine the results.

#### The second experiment:

- 1. Turn on the power of all relative components;
- 2. Adjust to make sure the cross to the node of the crosshairs at any position when switching the telescope;
- 3. Put the grating on the table and switch the telescope to record the scale when it turns to the two yellow vertical lines and two green lines;
  - 4. Repeat the two steps above for 2 times;
  - 5. Analyze the data to determine the results.

**Results** (Data tables and figures, 2 points)

# **Data Tables**

DATA TABLE 2-1 (*purpose*: to measure the apex angle of a prism)

Instrument error: 1'

Position of telescope	Left side (	position 1)	Right side (position 2)			
	Vernier 1	Vernier 2	Vernier 1	Vernier 2		
Trial	θ <sub>1</sub> (°, ')	θ <sub>1</sub> '(°, ')	$\theta_2(^{\circ}, ^{\cdot})$	$\theta_2$ '(°, ')		
1	30°40'	210°42'	270°38'	90°38'		
2	30°41'	210°44'	270°40'	90°42'		
3	30°41'	210°43'	2270°39'	90°43'		
Averaged	30°41'	210°43'	270°39'	90°41'		

DATA TABLE 2-2 (*purpose*: to measure the wavelengths of lines in the spectra of mercury)

Diffraction order	k=-1(left side)							<i>k</i> =+1(right side)					
Lines	Yellow 2 Yellow 1			ow 1	Green		Green		Yellow 1		Yellow 2		
Trial	$arphi_{ ext{Y-L21}}$	$arphi_{ ext{Y-L22}}$	φ <sub>Y-L11</sub>	$\varphi_{ ext{Y-L12}}$	$arphi_{ ext{G-L1}}$	$arphi_{ ext{G-L2}}$	$arphi_{ ext{G-R1}}$	$arphi_{ ext{G-R2}}$	φ <sub>Y-R11</sub>	φ <sub>Y-R12</sub>	φ <sub>Y-R21</sub>	$arphi_{ ext{Y-R22}}$	
1	333°45'	153°45'	333°42'	153°42'	333°8'	153°8'	314°12'	134°12	313°42'	133°42'	313°37'	133°37'	
2	297°1'	117°1'	297°0'	117°0'	300°10'	120°10'	320°0'	140°0'	317°31'	137°46'	317°30'	137°45'	
Averaged	315°25'	135°25'	315°22'	135°22'	315°9'	135°9'	317°6'	137°6'	315°44'	134°35'	315°41'	134°34'	

**Discussion** (More than 150 words, 5 points)

Based on some geometric and physical knowledge, I measured the apex angle of a prism by autocollimation method and the wavelengths of lines in the spectra of mercury by diffraction grating. We are supposed to place the polished slide surface towards the objective lens and adjust the focus knob to see the cross sharply in order to minimize the error. And then rotate the telescope to position 1 and position 2 to read four angles. After that we can find  $\alpha = \frac{1}{2}[|\theta_1 - \theta_2| + |\theta_1' - \theta_2'|]$ . In the second experiment, changing slit to the prism one green line and two yellow line in the first-order mercury and aligning the cross wires with the line are helpful to minimize the errors. Besides, I also learnt how to observe the reading of the calipers and the method to use it correctly in this experiment. And the result of measuring wavelengths of the mercury lights I got was really close to the theoretical value (even less than 1%), which means the error I minimized successfully by following the right steps.

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Score

Conclusions (About 50 words, 2 points)

By analyzing the data which I got from experiments and computing with geometric and physical knowledge like:  $\alpha = \frac{1}{2}[|\theta_1 - \theta_2| + |\theta_1' - \theta_2'|]$ . I determined that the apex angle of a prism is about 54°11' and the wavelengths of yellow doublet lines are about 578.2nm and 582.0nm respectively.

Score

**References** (1 points)

Haofu, Esmond Agurgo Balfour, Introductory Physics Experiments for Undergraduates, Science Press, Beijing, 2017

### **Answers to Questions** (6 points)

- (1) According to the theory of symmetry, when the image of the cross coincides with the upper node of the crosshairs, the altitude of the base must through the center of the cross. Therefore, by moving the telescope to make it perpendicular to the other two sides means that you can see the green cross situated at the same place. And Only when the telescope toward the object vertically, the reflected beam would focus on the upper level line as the beam generator is symmetry to the upper line about the middle line.
- (2) By adjusting the knob under the eyepiece tube and focusing the knob of the telescope until the image of the cross is seen sharply on the crosshairs.

**(3)** 

i. 
$$\varphi = \theta_1 + \theta_2 = \frac{1}{2}\varphi_1 + \frac{1}{2}\varphi_2 = \frac{1}{2}(\varphi_1 + \varphi_2)$$

**ii.** The reason why there are two verniers on the dial disk is that an inconsistency between the center of the prism table and that of the dial disk, which means the center of the dial of spectrometer and the rotating spindle can't be strictly coincidence. If only read one scale, there will be system errors and if the two completely symmetrical cursor plate reading at the same time so that the absolute value of them are equal in size, it will minimize the error effectively.

## Appendix

Score

# (Calculations, 15 points)

#### **Calculations**

#### 1. Compute the apex angle of a prism and its uncertainty

(1) The apex angle: 
$$\overline{A} = 180^{\circ} - \frac{1}{2} \left[ \left| \overline{\theta_1} - \overline{\theta_2} \right| + \left| \overline{\theta_1'} - \overline{\theta_2'} \right| \right] = 180^{\circ} - \frac{1}{2} \left[ \left| 30^{\circ} 41' - 270^{\circ} 39' \right| + \left| 210^{\circ} 43' - 90^{\circ} 41' \right| \right] = 54^{\circ} 11'$$

(Note: If the difference between the two angles is about 240 degrees, the difference should be subtracted by 360°. For example,  $\overline{\theta_1} = 355^{\circ}45'$  and  $\overline{\theta_2} = 115^{\circ}43'$ , their difference is not  $355^{\circ}45' - 115^{\circ}43' = 240^{\circ}2'$  but  $360^{\circ} - (355^{\circ}45' - 115^{\circ}43') = 119^{\circ}58'$ ).

(2) The type A evaluation of uncertainty in  $\theta_1$ ,  $\theta_1$ ,  $\theta_2$ ,  $\theta_2$ .

$$\mu_{A-\theta_1} = \sqrt{\frac{1}{N(N-1)} \sum_{i=1}^{N} (x_i - \bar{x})^2}$$

$$= \sqrt{\frac{1}{3(3-1)} [(30^\circ 40^\prime - 30^\circ 41^\prime)^2 + (30^\circ 41^\prime - 30^\circ 41^\prime)^2 + (30^\circ 41^\prime - 30^\circ 41^\prime)^2]}$$

$$= 0.41^\prime$$

$$\mu_{A-\theta_1}' = \sqrt{\frac{1}{N(N-1)} \sum_{i=1}^{N} (x_i - x^2)^2}$$

$$= \sqrt{\frac{1}{3(3-1)} [(210^\circ 42' - 210^\circ 43')^2 + (210^\circ 44' - 210^\circ 43')^2 + (210^\circ 43' - 210^\circ 43')^2]}$$

$$= 0.58'$$

$$\begin{split} &\mu_{A-\theta_2} = \sqrt{\frac{1}{N(N-1)} \sum_{i=1}^{N} (x_i - \overline{x})^2} \\ &= \sqrt{\frac{1}{3(3-1)} [(270^\circ 38' - 270^\circ 39')^2 + (270^\circ 40' - 270^\circ 39')^2 + (270^\circ 39' - 325^\circ 39')^2]} \\ &= 0.58' \end{split}$$

$$\mu_{A-\theta_2}' = \sqrt{\frac{1}{N(N-1)} \sum_{i=1}^{N} (x_i - \bar{x})^2}$$

$$= \sqrt{\frac{1}{3(3-1)} [(90^\circ 38' - 90^\circ 41')^2 + (90^\circ 43' - 90^\circ 41')^2 + (90^\circ 42' - 90^\circ 41')^2]}$$

$$= 1.53'$$

(3) The type B evaluation of uncertainty in  $\theta_1$ ,  $\theta_1$ ,  $\theta_2$ ,  $\theta_2$ .

$$\begin{split} \mu_{\Delta \, Instru.} &= \frac{\Delta_{Instr.}}{\sqrt{3}} = \frac{1'}{\sqrt{3}} = 0.58' \\ \mu_{\Delta \, Read.} &= \frac{\Delta_{Read.}}{\sqrt{3}} = \frac{0.2 \times 1'}{\sqrt{3}} = 0.12' \\ \mu_{B} &= \sqrt{\mu_{\Delta Instru.}^{2} + \mu_{\Delta Read.}^{2}} = \sqrt{(0.58')^{2} + (0.12')^{2}} = 0.59' \end{split}$$

(4) The combined uncertainty in  $\theta_1$ ,  $\theta_1$ ,  $\theta_2$ ,  $\theta_2$ .

$$\sigma_{\theta} = \sqrt{\mu_A^2 + \mu_B^2}$$

$$\sigma_{\theta_1} = \sqrt{\mu_{A-\theta_1}^2 + \mu_B^2} = \sqrt{0.41'^2 + 0.59'^2} = 0.72'$$

$$\sigma_{\theta_1'} = \sqrt{\mu_{A-\theta_1'}^2 + \mu_B^2} = \sqrt{0.58'^2 + 0.59'^2} = 0.83'$$

$$\sigma_{\theta_2} = \sqrt{\mu_{A-\theta_2}^2 + \mu_B^2} = \sqrt{0.58'^2 + 0.59'^2} = 0.83'$$

$$\sigma_{\theta_2'} = \sqrt{\mu_{A-\theta_2'}^2 + \mu_B^2} = \sqrt{1.53'^2 + 0.59'^2} = 1.64'$$

(5) The uncertainty in apex angle A: 
$$\sigma_A = \frac{1}{2} \left( \sqrt{\sigma_{\theta 1}^2 + \sigma_{\theta 1'}^2 + \sigma_{\theta 2}^2 + \sigma_{\theta 2'}^2} \right)$$

$$\sigma_A = \frac{1}{2} \left( \sqrt{\sigma_{\theta_1}^2 + \sigma_{\theta_1}^2 + \sigma_{\theta_2}^2 + \sigma_{\theta_2}^2} \right) = \frac{1}{2} \left( \sqrt{0.72^{12} + 0.83^{12} + 0.83^{12} + 1.64^{12}} \right) = 1.07^{12}$$

(6) The final result of the apex angle:  $A = \bar{A} \pm \sigma_A$  $A = \bar{A} \pm \sigma_A = 54^{\circ}11^{'} \pm 1^{'}$ 

#### 2. Compute the wavelengths of yellow doublet lines

(1) The diffraction angles of spectral lines of green, yellow doublet tint. For example, for green line,

$$\overline{\varphi_G} = \frac{1}{4}(|\varphi_{G-L1} - \varphi_{G-R1}| + |\varphi_{G-L2} - \varphi_{G-R2}|) = \frac{1}{4}(|333°8' - 314°12'| + |153°8' - 134°12'|) = \frac{1}{4}(18°56' + 18°56') = 9°28'$$

$$\begin{aligned} \overline{\varphi_{Y1}} &= \frac{1}{4} (|\varphi_{Y-L11} - \varphi_{Y-R11}| + |\varphi_{Y-L12} - \varphi_{Y-R12}|) \\ &= \frac{1}{4} (|333^{\circ}42' - 313^{\circ}42'| + |153^{\circ}42' - 133^{\circ}42'|) \\ &= \frac{1}{4} (20^{\circ}0' + 20^{\circ}0') = 10^{\circ}0' \end{aligned}$$

$$\overline{\varphi_{Y2}} = \frac{1}{4} (|\varphi_{Y-L21} - \varphi_{Y-R21}| + |\varphi_{Y-L22} - \varphi_{Y-R22}|)$$

$$= \frac{1}{4} (|333^{\circ}45' - 313^{\circ}37'| + |153^{\circ}45' - 133^{\circ}37'|)$$

$$= \frac{1}{4} (20^{\circ}8' + 20^{\circ}8') = 10^{\circ}4'$$

(2) The diffraction space d,  $d = \frac{\lambda_{green}}{\sin \varphi_{green}}$  ( $\lambda_{green} = 546.07$  nm).

$$d = \frac{\lambda_{green}}{\sin \varphi_{green}} = \frac{546.07nm}{\sin 9^{\circ}28'} = \frac{546.07nm}{0.164} = 3329.7nm$$

(3) The wavelengths of yellow doublet lines,  $\lambda_{Y1} = d \times sin \varphi_{Y1}$ ;  $\lambda_{Y2} = d \times sin \varphi_{Y2}$ 

$$\lambda_{Y1} = d \times sin \varphi_{Y1} = 3329.7 \text{nm} \times 0.17365 = 578.2 \text{nm}$$
  
 $\lambda_{Y2} = d \times sin \varphi_{Y2} = 3329.7 \text{nm} \times 0.17479 = 582.0 \text{nm}$ 

(4) The relative error of  $\lambda_{Y1}$  and  $\lambda_{Y2}$ . Find the accepted values in Fig. 3.9-15.

$$\sigma_{\lambda_{Y1}} = \left| \frac{578.2 - 577.0}{577.0} \right| \times 100\% = 0.20\%$$

$$\sigma_{\lambda_{Y2}} = \left| \frac{582.0 - 579.0}{579.0} \right| \times 100\% = 0.52\%$$

# Appendix

# (Scanned data sheets)

Data Table 3.9-1 Purpose: To measure the apex angle of a prism

	Instrument en	ror:				
Position of telescope	Left side (J	position 1)	Right side (position 2)			
m:d	Vernier 1	Vernier 2	Vernier 1	Vernier 2		
Trial	θ <sub>1</sub> (°, ′)	θ <sub>1</sub> ' (°, ΄)	θ <sub>2</sub> (°, ΄)	θ <sub>2</sub> ' (°, ')		
1	20.40	20°47.	270°38′	90°38		
2	30° <b>4</b> 1	21 <b>0°0</b> 4	27 <b>0</b> °40′	20°46		
3	30°41'	2 <b>10°43</b> ′	27039	908#°4		
Average	ASST.	206° 26'	270°39′	8		
	30°41′	210°43'		90°4		

Data Table 3.9-2 (Purpose: To measure the wavelengths of lines in the spectra of mercury

Oiffraction order	K = -1 (left side)					k = +1 (right side)						
Lines	Yell	Yellow 2 Yellow 1		Green		Green		Yellow 1		Yellow 2		
Trial	φ <sub>Y-L21</sub>	φ <sub>Y-L22</sub>	φ <sub>Y-L11</sub>	φ <sub>Y-L12</sub>	φ <sub>G-L1</sub>	φ <sub>G-L2</sub>	φ <sub>G-R1</sub>	$\varphi_{ ext{G-R2}}$	φ <sub>Y-R11</sub>	φ <sub>Y-R12</sub>	φ <sub>Y-R21</sub>	φ <sub>Y-R22</sub>
1	33345	153°45	33342	15342	333°8′	153°8′	31412	13412	313°42	133°42	3133]	133°3]
2	197°1'	#17°/	ATO	117°0	30010	ים מים מונים מ	3 <b>2</b> 00	1400	31 31	<b>33</b> ]°46	B1 30	<b>37</b> 4
Average	BUS	135°25	31522	13522	3159	135 9	3176	1376	315°44	134.35	<b>3</b>	134

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