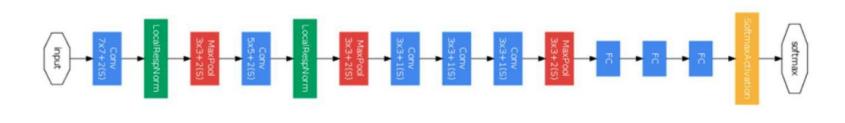
Lecture 06: CNNs III – Network Regularization & Training

Lan Xu SIST, ShanghaiTech Fall, 2022



Problem in deep network learning



$$\ell = F_2(F_1(\mathbf{u}, \Theta_1), \Theta_2)$$

Change of distribution in activation across layers



Normalize the inputs to a layer:

"you want unit gaussian activations? just make them so."

consider a batch of activations at some layer. To make each dimension unit gaussian, apply:

$$\widehat{x}^{(k)} = \frac{x^{(k)} - E[x^{(k)}]}{\sqrt{\text{Var}[x^{(k)}]}}$$

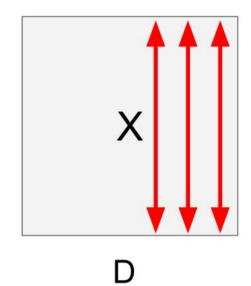
this is a vanilla differentiable function...



Layer details

Input:
$$x: N \times D$$





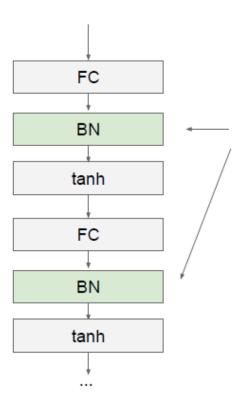
$$\mu_j = \frac{1}{N} \sum_{i=1}^N x_{i,j} \quad \text{Per-channel mean,} \\ \text{shape is D}$$

$$\sigma_j^2 = \frac{1}{N} \sum_{i=1}^N (x_{i,j} - \mu_j)^2 \quad \text{Per-channel var,} \\ \text{shape is D}$$

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}} \qquad \text{Normalized x,} \\ \text{Shape is N x D}$$



Layer details



Usually inserted after Fully Connected or Convolutional layers, and before nonlinearity.

$$\widehat{x}^{(k)} = \frac{x^{(k)} - E[x^{(k)}]}{\sqrt{Var[x^{(k)}]}}$$



Extra capacity:

Input:
$$x: N \times D$$

Learnable scale and shift parameters:

$$\gamma, \beta: D$$

Learning $\gamma = \sigma$, $\beta = \mu$ will recover the identity function!

$$\mu_j = \frac{1}{N} \sum_{i=1}^N x_{i,j} \quad \mbox{ Per-channel mean,} \\ \mbox{ shape is D}$$

$$\sigma_j^2 = \frac{1}{N} \sum_{i=1}^N (x_{i,j} - \mu_j)^2 \quad \text{Per-channel var,} \\ \text{shape is D}$$

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}} \qquad \text{Normalized x,} \\ \text{Shape is N x D}$$

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$
 Output, Shape is N x D

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Batch Normalization

Algorithm

Input: Values of
$$x$$
 over a mini-batch: $\mathcal{B} = \{x_{1...m}\}$;

Parameters to be learned: γ , β

Output: $\{y_i = \mathrm{BN}_{\gamma,\beta}(x_i)\}$

$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m} \sum_{i=1}^m x_i \qquad // \text{mini-batch mean}$$

$$\sigma_{\mathcal{B}}^2 \leftarrow \frac{1}{m} \sum_{i=1}^m (x_i - \mu_{\mathcal{B}})^2 \qquad // \text{mini-batch variance}$$

$$\widehat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} \qquad // \text{normalize}$$

$$y_i \leftarrow \gamma \widehat{x}_i + \beta \equiv \mathrm{BN}_{\gamma,\beta}(x_i) \qquad // \text{scale and shift}$$

- Improves gradient flow through the network
- Allows higher learning rates
- Reduces the strong dependence on initialization
- Acts as a form of regularization in a funny way, and slightly reduces the need for dropout, maybe



Test time

Input: $x: N \times D$

Learnable scale and shift parameters:

$$\gamma, \beta: D$$

During testing batchnorm becomes a linear operator! Can be fused with the previous fully-connected or conv layer

$$\mu_j={}^{ ext{(Running)}}$$
 average of values seen during training

$$\sigma_j^2 = {}^{ ext{(Running)}}$$
 average of values seen during training

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

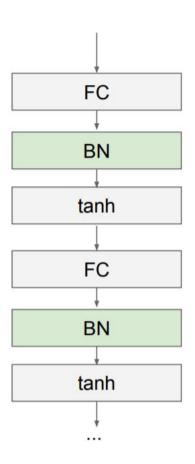
$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Per-channel mean, shape is D

Per-channel var, shape is D



Benefits



- Makes deep networks much easier to train!
- Improves gradient flow
- Allows higher learning rates, faster convergence
- Networks become more robust to initialization
- Acts as regularization during training
- Zero overhead at test-time: can be fused with conv!
- Behaves differently during training and testing: this
 is a very common source of bugs!



ConvNets

Batch Normalization for **fully-connected** networks

$$\mathbf{x} : \mathbf{N} \times \mathbf{D}$$
Normalize
$$\mu, \sigma : \mathbf{1} \times \mathbf{D}$$

$$\gamma, \beta : \mathbf{1} \times \mathbf{D}$$

$$y = \gamma(\mathbf{x} - \mu) / \sigma + \beta$$

Batch Normalization for convolutional networks (Spatial Batchnorm, BatchNorm2D)

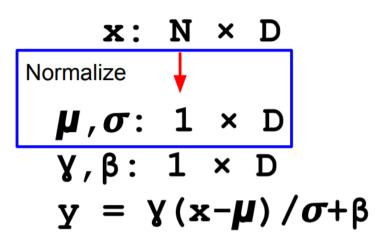
Normalize
$$\mathbf{x}: \mathbf{N} \times \mathbf{C} \times \mathbf{H} \times \mathbf{W}$$
 $\mu, \sigma: \mathbf{1} \times \mathbf{C} \times \mathbf{1} \times \mathbf{1}$
 $\gamma, \beta: \mathbf{1} \times \mathbf{C} \times \mathbf{1} \times \mathbf{1}$
 $\gamma = \gamma(\mathbf{x} - \mu) / \sigma + \beta$



Layer Normalization

Batch Normalization vs. Layer Normalization

Batch Normalization for fully-connected networks



Layer Normalization for fully-connected networks Same behavior at train and test! Can be used in recurrent networks

Normalize
$$\mu, \sigma: N \times D$$

$$\mu, \sigma: N \times 1$$

$$\gamma, \beta: 1 \times D$$

$$y = \gamma(x-\mu)/\sigma + \beta$$

Instance Normalization

Batch Normalization for convolutional networks

Normalize
$$\mathbf{x}: \mathbf{N} \times \mathbf{C} \times \mathbf{H} \times \mathbf{W}$$
 $\mu, \sigma: \mathbf{1} \times \mathbf{C} \times \mathbf{1} \times \mathbf{1}$
 $\mathbf{y}, \beta: \mathbf{1} \times \mathbf{C} \times \mathbf{1} \times \mathbf{1}$
 $\mathbf{y} = \mathbf{y}(\mathbf{x} - \boldsymbol{\mu}) / \sigma + \beta$

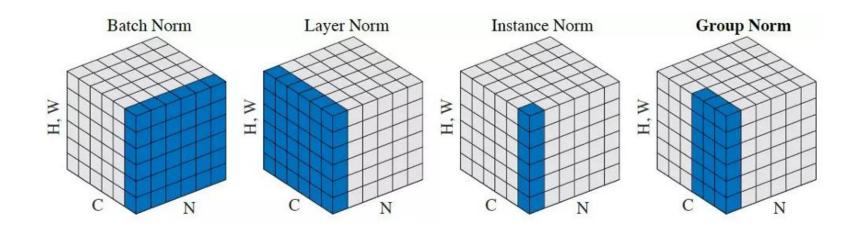
Instance Normalization for convolutional networks
Same behavior at train / test!

$$x: N\times C\times H\times W$$
Normalize
 $\mu, \sigma: N\times C\times 1\times 1$
 $y, \beta: 1\times C\times 1\times 1$
 $y = y(x-\mu)/\sigma + \beta$



Batch-like Normalization

- Layer normalization (Ba, Kiros, Hinton, 2016)
- Instance normalization (Ulyanov, Vedaldi, Lempitsky, 2016)
- Group normalization (Wu and He, 2018)



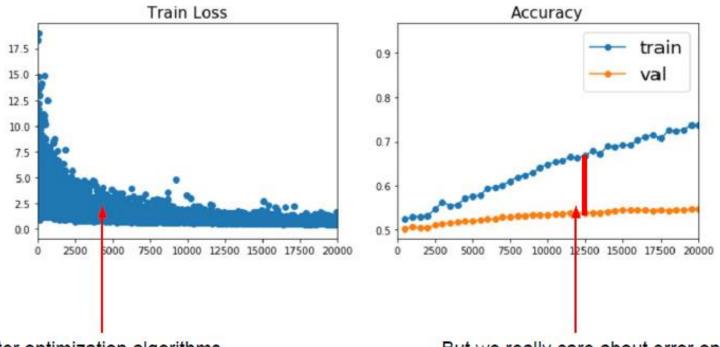


Training overview

- Two aspects of training networks
 - Optimization
 - How do we minimize the loss function effectively?
 - Generalization
 - How do we avoid overfitting?
- CNN training pipeline
 - Data processing
 - Weight initialization
 - □ Parameter updates
 - Batch normalization
- Avoid overfitting: Regularization

Beyond Training Error

- How do we generalize to unseen data?
 - Well studied but still poorly understood



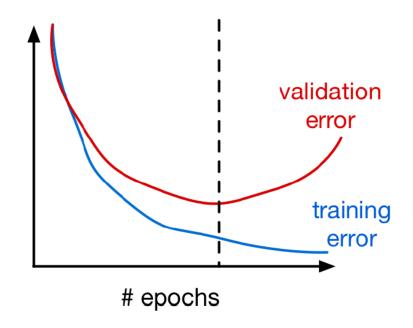
Better optimization algorithms help reduce training loss

But we really care about error on new data - how to reduce the gap?



Early Stopping

- Early stopping: monitor performance on a validation set, stop training when the validation error starts going up.
 - □ We don't always want to find a global (or even local) optimum of our cost function.

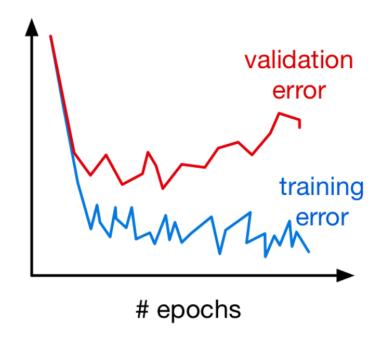


Weights start out small, so it takes time for them to grow large.
 Therefore, it has a similar effect to weight decay.



Early Stopping

- A slight catch: validation error fluctuates because of stochasticity in the updates.
 - Determining when the validation error has actually leveled off can be tricky.
 - May use temporal smoothing

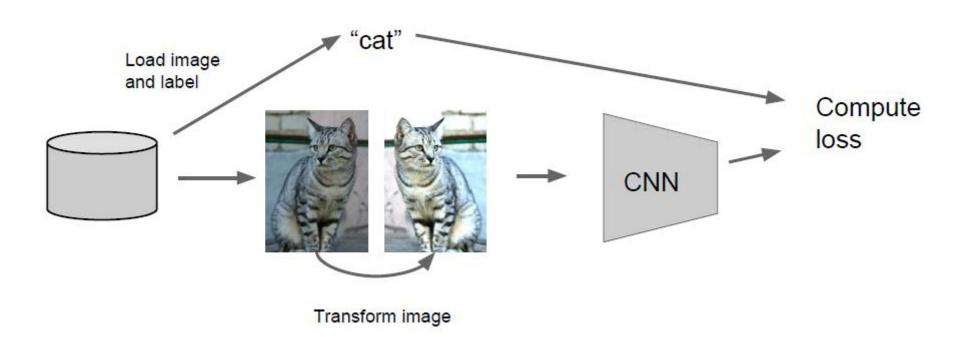


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Outline

- Regularization in CNN training
 - Data Augmentation
 - Weight Regularization & Transfer Learning
 - Stochastic Regularization
 - Hyper-parameter optimization

Create more data for regularization

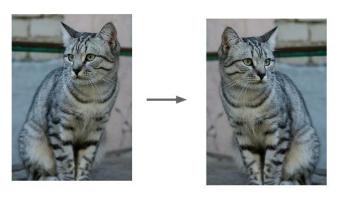


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Data Augmentation

Create more data for regularization

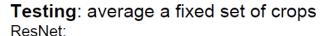
Horizontal Flips



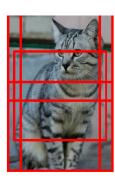
Random crops and scales

Training: sample random crops / scales ResNet:

- 1. Pick random L in range [256, 480]
- 2. Resize training image, short side = L
- 3. Sample random 224 x 224 patch



- 1. Resize image at 5 scales: {224, 256, 384, 480, 640}
- 2. For each size, use 10 224 x 224 crops: 4 corners + center, + flips

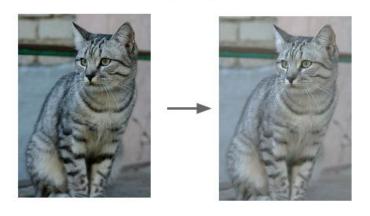




Create more data for regularization

Color Jitter

Simple: Randomize contrast and brightness



More Complex:

- Apply PCA to all [R, G, B] pixels in training set
- Sample a "color offset" along principal component directions
- Add offset to all pixels of a training image

(As seen in [Krizhevsky et al. 2012], ResNet, etc)



- Create more data for regularization
- Examples (for visual recognition)
 - translation
 - horizontal or vertical
 - □ flip
 - rotation
 - smooth warping
 - □ noise (e.g. flip random pixels)
- The choice of transformations depends on the task.
 - E.g. horizontal flip for object recognition, but not handwritten digit recognition.



- AutoAugment (Cubuk et al, Arxiv 2018)
 - An automatic way to design custom data augmentation policies for computer vision datasets,
 - □ Selecting an optimal policy from a search space of 2.9 x 10³² image transformation possibilities.
 - E.g., guiding the selection of basic image transformation operations, such as flipping an image horizontally/vertically, rotating an image, changing the color of an image, etc.
 - Using reinforcement learning strategy (More later...)

Results

- □ New state of the art: ImageNet: 83.54% top1 accuracy; SVHN: error rate 1.02%.
- AutoAugment policies are found to be transferable to other vision datasets.

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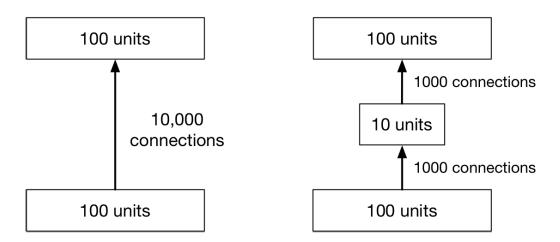
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Reducing # of Parameters

- Reducing the number of layers or the number of parameters per layer.
- Adding a linear bottleneck layer:



- The first network is strictly more expressive than the second (i.e. it can represent a strictly larger class of functions). (Why?)
- Remember how linear layers don't make a network more expressive? They might still improve generalization.



Weight Regularization

- L₂ regularization / weight decay
 - ☐ Encouraging the weights to be small in magnitude

$$\mathcal{E}_{\mathrm{reg}} = \mathcal{E} + \lambda \mathcal{R} = \mathcal{E} + \frac{\lambda}{2} \sum_{j} w_{j}^{2}$$

The gradient update can be interpreted as weight decay

$$\mathbf{w} \leftarrow \mathbf{w} - \alpha \left(\frac{\partial \mathcal{E}}{\partial \mathbf{w}} + \lambda \frac{\partial \mathcal{R}}{\partial \mathbf{w}} \right)$$
$$= \mathbf{w} - \alpha \left(\frac{\partial \mathcal{E}}{\partial \mathbf{w}} + \lambda \mathbf{w} \right)$$
$$= (1 - \alpha \lambda) \mathbf{w} - \alpha \frac{\partial \mathcal{E}}{\partial \mathbf{w}}$$



Transfer Learning

Transfer Learning with CNNs

1. Train on Imagenet



Donahue et al, "DeCAF: A Deep Convolutional Activation Feature for Generic Visual Recognition", ICML 2014 Razavian et al, "CNN Features Off-the-Shelf: An Astounding Baseline for Recognition", CVPR Workshops 2014



Transfer Learning

Transfer Learning with CNNs

1. Train on Imagenet

FC-1000 FC-4096 FC-4096 MaxPool Conv-512 Conv-512 MaxPool Conv-512 Conv-512 MaxPool Conv-256 Conv-256 MaxPool Conv-128 Conv-128 MaxPool Conv-64 Conv-64 **Image**

2. Small Dataset (C classes)



Donahue et al, "DeCAF: A Deep Convolutional Activation Feature for Generic Visual Recognition", ICML 2014 Razavian et al, "CNN Features Off-the-Shelf: An Astounding Baseline for Recognition", CVPR Workshops 2014

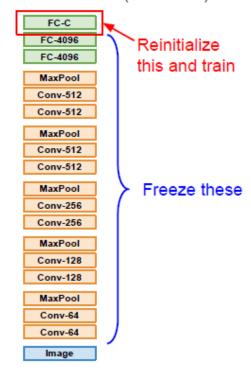
Transfer Learning

Transfer Learning with CNNs

1. Train on Imagenet

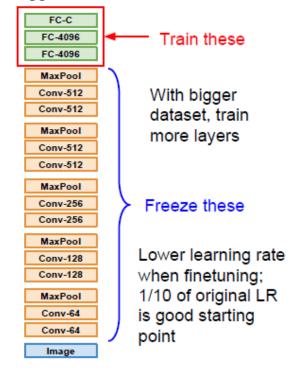
FC-1000 FC-4096 FC-4096 MaxPool Conv-512 Conv-512 MaxPool Conv-512 Conv-512 MaxPool Conv-256 Conv-256 MaxPool Conv-128 Conv-128 MaxPool Conv-64 Conv-64 **Image**

2. Small Dataset (C classes)

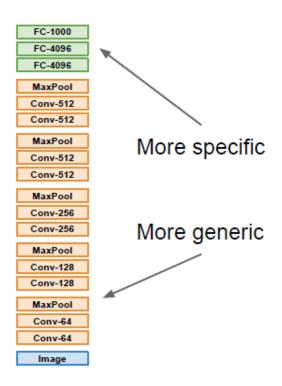


Donahue et al, "DeCAF: A Deep Convolutional Activation Feature for Generic Visual Recognition", ICML 2014 Razavian et al, "CNN Features Off-the-Shelf: An Astounding Baseline for Recognition", CVPR Workshops 2014

3. Bigger dataset







	very similar dataset	very different dataset
very little data	Use Linear Classifier on top layer	You're in trouble Try linear classifier from different stages
quite a lot of data	Finetune a few layers	Finetune a larger number of layers

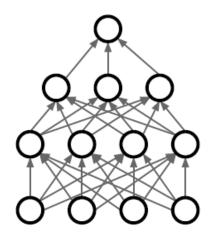
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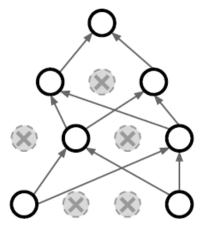
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- Regularization in CNN training
 - □ Data Augmentation
 - □ Weight Regularization & Transfer Learning
 - ☐ Stochastic Regularization
 - Hyper-parameter optimization
- Network Architectures

Stochastic Regularization

- For a network to overfit, its computations need to be really precise. This suggests regularizing them by injecting noise into the computations, a strategy known as stochastic regularization.
- Dropout is a stochastic regularizer which randomly deactivates a subset of the units





Srivastava et al, "Dropout: A simple way to prevent neural networks from overfitting", JMLR 2014

Dropout

Operations

$$h_i = m_i \cdot \phi(z_i),$$

where m_i is a Bernoulli random variable, independent for each hidden unit.

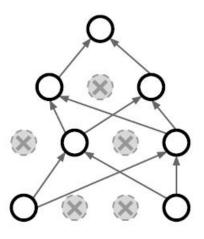
Regularization: Dropout

```
p = 0.5 # probability of keeping a unit active. higher = less dropout

def train_step(X):
    """ X contains the data """

# forward pass for example 3-layer neural network
H1 = np.maximum(0, np.dot(W1, X) + b1)
U1 = np.random.rand(*H1.shape)
```

Example forward pass with a 3-layer network using dropout

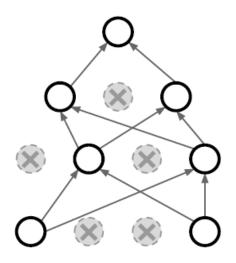




Understanding Dropout

Regularization: Dropout

How can this possibly be a good idea?



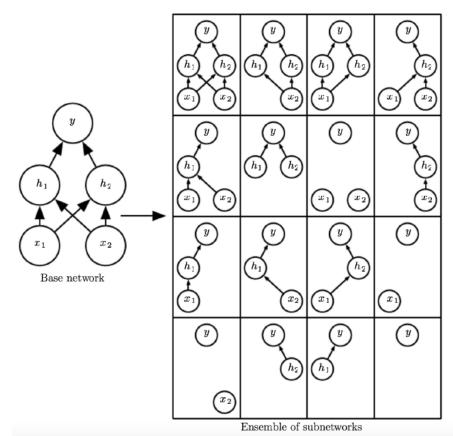
Forces the network to have a redundant representation; Prevents co-adaptation of features





Understanding Dropout

■ Dropout can be seen as training an ensemble of 2^D different architectures with shared weights (where D is the number of units):



— Goodfellow et al., Deep Learning



Dropout

Output

Input

Dropout at test time

Want to "average out" the randomness at test-time

$$y = f(x) = E_z[f(x,z)] = \int p(z)f(x,z)dz$$

But this integral seems hard ...

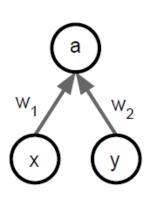


Dropout

Dropout at test time

Want to approximate the integral

$$y = f(x) = E_z[f(x,z)] = \int p(z)f(x,z)dz$$



Consider a single neuron.

At test time we have:
$$E[a] = w_1x + w_2y$$

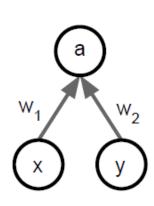


Dropout

Dropout at test time

Want to approximate the integral

$$y = f(x) = E_z[f(x,z)] = \int p(z)f(x,z)dz$$



Consider a single neuron.

At test time we have:
$$E[a]=w_1x+w_2y$$
 During training we have:
$$E[a]=\frac{1}{4}(w_1x+w_2y)+\frac{1}{4}(w_1x+0y)+\frac{1}{4}(0x+0y)+\frac{1}{4}(0x+w_2y)$$

$$=\frac{1}{2}(w_1x+w_2y)$$

At test time, **multiply** by dropout probability



Dropout at test time

```
def predict(X):
    # ensembled forward pass
H1 = np.maximum(0, np.dot(W1, X) + b1) * p # NOTE: scale the activations
H2 = np.maximum(0, np.dot(W2, H1) + b2) * p # NOTE: scale the activations
out = np.dot(W3, H2) + b3
```

At test time all neurons are active always => We must scale the activations so that for each neuron: output at test time = expected output at training time

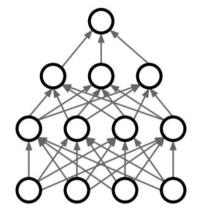
Dropout

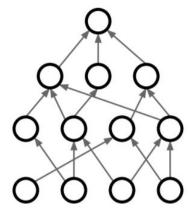
Implementation: Inverted dropout

```
p = 0.5 # probability of keeping a unit active. higher = less dropout
def train step(X):
 # forward pass for example 3-layer neural network
 H1 = np.maximum(0, np.dot(W1, X) + b1)
 U1 = (np.random.rand(*H1.shape) < p) / p # first dropout mask. Notice /p!
 H1 *= U1 # drop!
 H2 = np.maximum(0, np.dot(W2, H1) + b2)
 U2 = (np.random.rand(*H2.shape) < p) / p # second dropout mask, Notice /p!
 H2 *= U2 # drop!
 out = np.dot(W3, H2) + b3
 # backward pass: compute gradients... (not shown)
 # perform parameter update... (not shown)
                                                                      test time is unchanged!
def predict(X):
 # ensembled forward pass
 H1 = np.maximum(0, np.dot(W1, X) + b1) # no scaling necessary
 H2 = np.maximum(0, np.dot(W2, H1) + b2)
 out = np.dot(W3, H2) + b3
```



- Lots of other stochastic regularizers have been proposed:
 - DropConnect drops connections instead of activations.
 - Training: Drop connections between neurons (set weights to 0)
 - Testing: Use all the connections

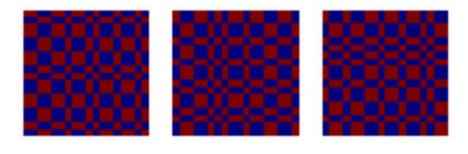




Wan et al, "Regularization of Neural Networks using DropConnect", ICML 2013

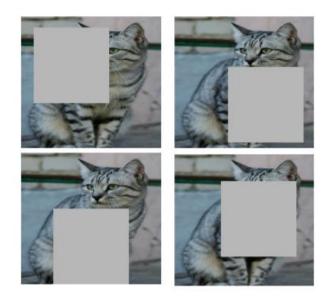


- Lots of other stochastic regularizers have been proposed:
 - □ Fractional Pooling
 - Training: Use randomized pooling regions
 - Testing: Average predictions from several regions



Graham, "Fractional Max Pooling", arXiv 2014

- Lots of other stochastic regularizers have been proposed:
 - □ Cutout
 - Training: Set random image regions to zero
 - Testing: Use full image predictions from several regions



Works very well for small datasets like CIFAR, less common for large datasets like ImageNet

DeVries and Taylor, "Improved Regularization of Convolutional Neural Networks with Cutout", arXiv 2017



- Lots of other stochastic regularizers have been proposed:
 - □ Mixup
 - Training: Train on random blends of images
 - Testing: Use original images







Randomly blend the pixels of pairs of training images, e.g. 40% cat, 60% dog

CNN Target label: cat: 0.4 dog: 0.6

Zhang et al, "mixup: Beyond Empirical Risk Minimization", ICLR 2018



- Lots of other stochastic regularizers have been proposed:
 - Training: Add random noise
 - Testing: Marginalize over the noise
- In practice
 - Consider dropout for large fully-connected layers
 - Batch normalization and data augmentation almost always a good idea
 - Try cutout and mixup especially for small classification datasets

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- Regularization in CNN training
 - □ Data Augmentation
 - □ Weight Regularization & Transfer Learning
 - ☐ Stochastic Regularization
 - ☐ Hyper-parameter optimization



(Cross-)validation strategy

coarse -> fine cross-validation in stages

First stage: only a few epochs to get rough idea of what params work Second stage: longer running time, finer search ... (repeat as necessary)

Tip for detecting explosions in the solver: If the cost is ever > 3 * original cost, break out early

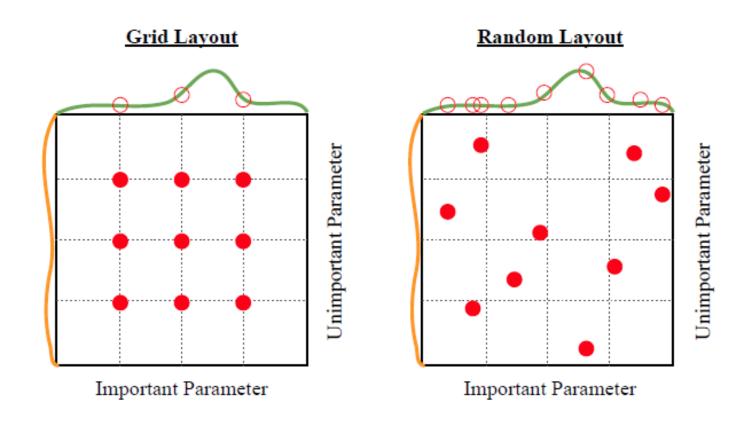
For example: run coarse search for 5 epochs

```
max count = 100
                                                           note it's best to optimize
   for count in xrange(max count):
         reg = 10**uniform(-5, 5)
         lr = 10**uniform(-3, -6)
                                                           in log space!
         trainer = ClassifierTrainer()
         model = init two layer model(32*32*3, 50, 10) # input size, hidden size, number of classes
         trainer = ClassifierTrainer()
         best model local, stats = trainer.train(X train, y train, X val, y val,
                                       model, two layer net,
                                       num epochs=5, reg=reg,
                                       update='momentum', learning rate decay=0.9,
                                       sample batches = True, batch size = 100,
                                       learning rate=lr, verbose=False)
            val acc: 0.412000, lr: 1.405206e-04, reg: 4.793564e-01, (1 / 100)
            val acc: 0.214000, lr: 7.231888e-06, reg: 2.321281e-04, (2 / 100)
            val acc: 0.208000, lr: 2.119571e-06, reg: 8.011857e+01, (3 / 100)
            val acc: 0.196000, lr: 1.551131e-05, req: 4.374936e-05, (4 / 100)
            val acc: 0.079000, lr: 1.753300e-05, reg: 1.200424e+03, (5 / 100)
            val acc: 0.223000, lr: 4.215128e-05, req: 4.196174e+01, (6 / 100)
            val acc: 0.441000, lr: 1.750259e-04, reg: 2.110807e-04, (7 / 100)
nice
            val acc: 0.241000, lr: 6.749231e-05, req: 4.226413e+01, (8 / 100)
            val acc: 0.482000, lr: 4.296863e-04, req: 6.642555e-01, (9 / 100)
            val acc: 0.079000, lr: 5.401602e-06, req: 1.599828e+04, (10 / 100)
            val acc: 0.154000, lr: 1.618508e-06, reg: 4.925252e-01, (11 / 100)
```

Now run finer search...

```
max count = 100
                                               adjust range
                                                                               max count = 100
for count in xrange(max count):
                                                                               for count in xrange(max count):
      reg = 10**uniform(-5, 5)
                                                                                      reg = 10**uniform(-4, 0)
      lr = 10**uniform(-3, -6)
                                                                                     lr = 10**uniform(-3, -4)
                    val acc: 0.527000, lr: 5.340517e-04, reg: 4.097824e-01, (0 / 100)
                    val acc: 0.492000, lr: 2.279484e-04, reg: 9.991345e-04, (1 / 100)
                    val acc: 0.512000, lr: 8.680827e-04, reg: 1.349727e-02, (2 / 100
                    val acc: 0.461000, lr: 1.028377e-04, reg: 1.220193e-02, (3 / 100
                    val acc: 0.460000, lr: 1.113730e-04, reg: 5.244309e-02, (4 / 100)
                                                                                               53% - relatively good
                    val acc: 0.498000, lr: 9.477776e-04, reg: 2.001293e-03, (5 / 100)
                                                                                               for a 2-layer neural net
                    val acc: 0.469000, lr: 1.484369e-04, reg: 4.328313e-01, (6 / 100)
                    val acc: 0.522000, lr: 5.586261e-04, reg: 2.312685e-04, (7 / 100)
                                                                                               with 50 hidden neurons.
                    val acc: 0.530000, lr: 5.808183e-04, reg: 8.259964e-02, (8 / 100)
                    val acc: 0.489000, lr: 1.979168e-04, reg: 1.010889e-04, (9 / 100)
                    val acc: 0.490000, lr: 2.036031e-04, reg: 2.406271e-03, (10 / 100)
                    val acc: 0.475000, lr: 2.021162e-04, reg: 2.287807e-01, (11 / 100)
                    val acc: 0.460000, lr: 1.135527e-04, reg: 3.905040e-02, (12 / 100)
                    val acc: 0.515000, lr: 6.947668e-04, reg: 1.562808e-02, (13 / 100)
                    val acc: 0.531000, lr: 9.471549e-04, reg: 1.433895e-03, (14 / 100)
                    val acc: 0.509000, lr: 3.140888e-04, reg: 2.857518e-01, (15 / 100)
                    val acc: 0.514000, lr: 6.438349e-04, reg: 3.033781e-01, (16 / 100)
                    val acc: 0.502000, lr: 3.921784e-04, reg: 2.707126e-04, (17 / 100)
                    val acc: 0.509000, lr: 9.752279e-04, reg: 2.850865e-03, (18 / 100)
                    val acc: 0.500000, lr: 2.412048e-04, reg: 4.997821e-04, (19 / 100)
                    val acc: 0.466000, lr: 1.319314e-04, reg: 1.189915e-02, (20 / 100)
                    val acc: 0.516000, lr: 8.039527e-04, reg: 1.528291e-02, (21 / 100)
```

Random search vs. Grid search



Random Search for Hyper-Parameter Optimization, Bergstra and Bengio, 2012



Hyperparameters to play with:

- network architecture
- learning rate, its decay schedule, update type
- regularization (L2/Dropout strength)
- Other hyperparameter optimization methods
 - □ Shahriari, et al. "Taking the human out of the loop: A review of Bayesian optimization." Proceedings of the IEEE 104.1 (2016): 148-175.



Summary

- Bag of tricks for improving generalization
 - □ Pros: you have a toolbox to use
 - Cons: many trial and error, tedious process
- Seeking fully automatic approaches to model selection
 - □ Bayesian optimization
 - Reinforcement learning
- Next time
 - CNN in Vision
- Reference
 - □ CS231n course notes