ShanghaiTech University

CS240 Algorithm Design and Analysis

Spring 2023

Problem Set 4

Due: 23:59, April 30, 2023

1. Submit your solutions to the course Blackboard.
2. If you want to submit a handwritten version, scan it clearly.
3. Late homeworks submitted within 24 hours of the due date will be marked down 25%. Homeworks submitted more than 24 hours after the due date will not be accepted unless there is a valid reason, such as a medical or family emergency.
4. You are required to follow ShanghaiTech's academic honesty policies. You are allowed to discuss problems with other students, but you must write up your solutions by yourselves. You are not allowed to copy materials from other students or from online or published resources. Violating academic honesty can result in serious penalties.

Note: When proving that a problem A is NP-complete, clearly divide your answer into three steps:

1. Prove that A is in NP.
2. Choose an NP-complete problem B. For any instance of B, construct an instance of problem A, so that the yes/no answers to the two instances are the same.
3. Show your construction runs in polynomial time.

Problem 1:

Suppose you are the manager of a clothing store which sells j different jackets and h different shirts. c customers come to the store, and each of them has a set of jackets and shirts which they like. Each customer will either buy one jacket and one shirt from the set he likes, or nothing at all. The customer is satisfied if he buys one jacket and one shirt. In addition, each jacket or shirt can be sold to at most one customer. Design an efficient algorithm which maximizes the number of satisfied customers.

Problem 2:

Consider a directed graph $G = (V, E)$. Let $X \subseteq V$ be a set of *start nodes*, and $Y \subseteq V$ be a set of *destination nodes*, and assume that X and Y are disjoint. Your goal is to find a set of paths, where each path starts at a node in X and ends at a node in Y, and none of the paths share an edge. Design an efficient algorithm to determine if such a set of paths exist, and find such a set if possible.

Problem 3:

SIST wants to hire graduate students to TA courses. Each course needs exactly two TAs, and there is a list of pairs of students who can TA a course together (i.e. if two students are not a pair from the list, they cannot TA a course together). For a given input value k, we want to determine whether there exists a *TA cycle* with k distinct students $s_1, ..., s_k$, such that s_1 and s_2 TA a course together, s_2 and s_3 TA a course together, ..., and s_k and s_1 TA a course together (note that each student in the cycle will TA two courses). Show that this problem is NP-complete.

Problem 4:

In the Knapsack problem, we have n items, and the j'th item has weight w_j and value v_j ($j = 1, \dots, n$). All w_j, v_j values are positive integers. The question is whether there exists a subset of the n items with total weight at most W and total value at least V, for input values W and V. Show that Knapsack is NP-complete.

Problem 5:

Let C be a finite set and let S be a collection of subsets of C. The Set Packing problem asks whether, for an input value k, there exist k sets from S which are pairwise disjoint (i.e. no two sets share an element). Show that Set Packing is NP-complete, using a reduction from the Independent Set problem.

Problem 6:

Suppose you are organizing a summer sports camp which offers n different sports, and you need hire camp counselors who are skilled at these sports. You receive applications from m people, where each person is skilled at some subset of the sports. For an input value $k \leq m$, you want to determine whether it is possible to hire k people, so that for each sport you hire at least one person skilled in that sport. Show that this problem is NP-complete, using a reduction from the Vertex Cover problem.