TA Office Hours

Monday	19:00-20:00	吴昊一	wuhy1@	SIST 1A-313
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	20:00-21:00	刘威	liuwei4@	SIST 1A-313

Text Representation

SLP3 Ch 6; INLP Ch 14

Word representations

- In traditional NLP, we regard words as discrete symbols
- Problems
  - no natural notion of similarity
    - motel vs hotel
  - may rely on a thesaurus, e.g., WordNet, to measure similarity, but:
    - Compiling or updating a thesaurus requires a lot of work, thus:
      - Unavailable for low-resource languages / domains
      - Not always up-to-date
      - Many words and phrases are missing

Word vectors

- Representing each word in a vector space
  - also known as word embeddings

one-hot vectors

$$\begin{bmatrix}0\\1\\0\\0\\0\\0\end{bmatrix} \quad \begin{bmatrix}0\\0\\1\\0\\0\\0\end{bmatrix} \quad \begin{bmatrix}0\\0\\0\\1\\0\\0\end{bmatrix}$$

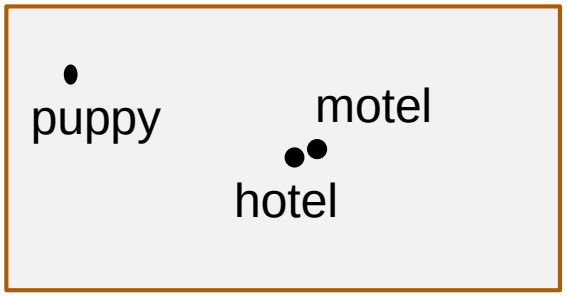
motel hotel puppy

Equivalent to discrete representation!

general vectors

$$\begin{bmatrix}0.27\\-0.16\\0.13\\0.22\end{bmatrix} \quad \begin{bmatrix}0.32\\-0.11\\0.09\\0.21\end{bmatrix} \quad \begin{bmatrix}-0.19\\-0.21\\0.03\\0.09\end{bmatrix}$$

motel hotel puppy



Word vectors

- Representing each word in a vector space
  - also known as word embeddings
- Advantages
  - Can capture similarity between words with vector similarity
  - Easier to process in many ML systems
  - Can be learned from data!

Word vectors

- Vector similarity measure
  - Cosine similarity

$$cos(\theta) = \frac{A \cdot B}{|A||B|}$$

  - Dot product similarity $A \cdot B$

Word vectors

- Sparse vector representations
  - Co-occurrence matrices
- Dense vector representations
  - Singular value decomposition
  - Word2vec
  - Contextualized word embedding (will be discussed later)

Static word embedding

Phrase/Sentence/Document Representation

- Discrete representation
  - Sequence of discrete symbols
  - Again, not straightforward to measure similarity
- Vector representation
  - Can be computed in a similar way to word vectors
  - Can be built on top of word vectors

Sparse vector representations -- co-occurrence matrices

Term-document matrix

- Each cell: count of word w in a document
- Each document is a **count vector in** $\mathbb{N}^V$ (a column below)

Shakespeare's plays

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	1	8	15
soldier	2	2	12	36
fool	37	58	1	5
clown	6	117	0	0

Term-document matrix

- Each cell: count of word w in a document
- Each document is a **count vector in** $\mathbb{N}^V$ (a column below)
- Two documents are similar if their vectors are similar

Shakespeare's plays

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	1	8	15
soldier	2	2	12	36
fool	37	58	1	5
clown	6	117	0	0

Term-document matrix

- Each word is a **count vector in** $\mathbb{N}^{D}$ (a row below)

Shakespeare's plays

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	1	8	15
soldier	2	2	12	36
fool	37	58	1	5
clown	6	117	0	0

Term-document matrix

- Each word is a **count vector in** $\mathbb{N}^{D}$ (a row below)
- Two words are similar if their vectors are similar

Shakespeare's plays

	As You Like It	Twelfth Night	Julius Caesar	Henry V
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soldier	2	2	12	36
fool	37	58	1	5
clown	6	117	0	0

Term-document matrix

- Each word is a **count vector in** $\mathbb{N}^{D}$ (a row below)
- Two words are similar if their vectors are similar
- Caution: row vectors in a term-document matrix are not ideal word vectors
  - Not good if document number is small
  - High-frequency words appear in all documents and their vectors may not be distinguishable

Word-word matrix

- A word-word matrix is |V|x|V|
- Each cell: count of word co-occurrence
- Instead of co-occurrence in entire documents, use smaller contexts
  - Paragraph
  - Window of ±4 words
- A word is now defined by a vector over counts of context words

Word-word matrix (context of ±7 words)

sugar, a sliced lemon, a tablespoonful of **apricot** preserve or jam, a pinch each of,
their enjoyment. Cautiously she sampled her first **pineapple** and another fruit whose taste she likened
well suited to programming on the digital **computer**. In finding the optimal R-stage policy from
for the purpose of gathering data and **information** necessary for the study authorized in the

	aardvark	computer	data	pinch	result	sugar	…
apricot	0	0	0	1	0	1	
pineapple	0	0	0	1	0	1	
digital	0	2	1	0	1	0	
information	0	1	6	0	4	0	
…	…						

Word-word matrix (context of ±7 words)

- Similar words have similar contexts & vectors

sugar, a sliced lemon, a tablespoonful of **apricot** preserve or jam, a pinch each of,

their enjoyment. Cautiously she sampled her first **pineapple** and another fruit whose taste she likened

well suited to programming on the digital **computer**. In finding the optimal R-stage policy from

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digital	0	2	1	0	1	0	
information	0	1	6	0	4	0	
…	…						

Word-word matrix

- We showed only 4x6, but the real matrix is 50,000 x 50,000
  - So it's very sparse: most values are 0.
  - That's OK: there are lots of efficient algorithms for sparse matrices.
- The size of windows depends on your goals.
  - The shorter the windows (±1-3), the more syntactic the representation
  - The longer the windows (±4-10), the more semantic the representation

Co-occurrence matrices

- The co-occurrence matrices we have seen represent each cell by co-occurrence frequencies.
- Frequency is clearly useful: if *sugar* appears a lot near *pineapple*, that's useful information.
- But raw frequency is a bad representation:
  - Overly frequent words like *the*, *it,* or *they* are not very informative about the context.
  - Yet their counts often dominate the vectors
- It's a paradox! How can we balance these two conflicting constraints?

Adjustments

- tf-idf
  - usually used for term-document matrix
- PPMI (Positive Pointwise Mutual Information)
  - usually used for term-term matrix

TF-IDF

- Term frequency
  - $tf_{t,d} = \log_{10}(\text{count}(t,d)+1)$
  - There are other variants
- Document frequency of term t
  - df_t is the number of documents that *t* occurs in.
- Inverse document frequency (idf)

$$\text{idf}_t = \log_{10}\left(\frac{N}{\text{df}_t}\right)$$

  - N is the total number of documents in the collection

Word	df	idf
Romeo	1	1.57
salad	2	1.27
Falstaff	4	0.967
forest	12	0.489
battle	21	0.246
wit	34	0.037
fool	36	0.012
good	37	0
sweet	37	0

TF-IDF

- Final tf-idf value for a word

$$w_{t,d} = \text{tf}_{t,d} \times \text{idf}_t$$

- Raw counts:

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

- tf-idf:

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	0.074	0	0.22	0.28
good	0	0	0	0
fool	0.019	0.021	0.0036	0.0083
wit	0.049	0.044	0.018	0.022

PMI

- PMI (pointwise mutual information)
  - Do events x and y co-occur more/less than if they were independent?

$$PMI(X,Y) = \log_2 \frac{P(X,Y)}{P(X)P(Y)}$$

  - Here, events are word occurrences.

PPMI (Positive Pointwise Mutual Information)

- PMI ranges from $(-\infty, +\infty)$
- But negative values are problematic
  - Things are co-occurring **less than** we expect by chance
  - Unreliable without enormous corpora
    - Imagine w1 and w2 whose probability is each 10^{-6}
    - Hard to be sure P(w1,w2) is significantly lower than 10^{-12}
  - Plus it's not clear people are good at "unrelatedness"
- So we just replace negative PMI values by 0
- Positive PMI (PPMI) between w1 and w2:

$$PPMI(w_1, w_2) = \max(\log_2 \frac{P(w_1, w_2)}{P(w_1)P(w_2)}, 0)$$

Computing PPMI on a term-context matrix

- Matrix F with W rows (words) and C columns (contexts)
- f_{ij} is # of times w_i occurs in context c_j

	computer	data	pinch	result	sugar	···
apricot	0	0	1	0	1	
pineapple	0	0	1	0	1	
digital	2	1	0	1	0	
information	1	6	0	4	0	

$$p_{ij} = f_{ij} \Big/ \sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij} \qquad p_{i*} = \sum_{j=1}^{C} f_{ij} \Big/ \sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij} \qquad p_{*j} = \sum_{i=1}^{W} f_{ij} \Big/ \sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}$$

$$\mathrm{PMI}_{ij} = \log_2 \frac{p_{ij}}{p_{i*}\, p_{*j}} \qquad \mathrm{PPMI}_{ij} = \begin{cases} \mathrm{PMI}_{ij}, & \mathrm{PMI}_{ij} > 0 \\ 0, & otherwise \end{cases}$$

Computing probabilities

$$p_{ij} = f_{ij} \Big/ \sum_{i=1}^{W} \sum_{j=1}^{C} f_{ij} \qquad p_{i*} = \sum_{j=1}^{C} f_{ij} \Big/ \sum_{i=1}^{W} \sum_{j=1}^{C} f_{ij} \qquad p_{*j} = \sum_{i=1}^{W} f_{ij} \Big/ \sum_{i=1}^{W} \sum_{j=1}^{C} f_{ij}$$

	P(w, context)					
	computer	data	pinch	result	sugar	**P(w)**
apricot	0.00	0.00	0.05	0.00	0.05	0.11
pineapple	0.00	0.00	0.05	0.00	0.05	0.11
digital	0.11	0.05	0.00	0.05	0.00	0.21
information	0.05	0.32	0.00	0.21	0.00	0.58
P(context)	0.16	0.37	0.11	0.26	0.11	

Computing PPMI

PPMI matrix

PPMI	computer	data	pinch	result	sugar
apricot	-	-	2.25	-	2.25
pineapple	-	-	2.25	-	2.25
digital	1.66	0.00	-	0.00	-
information	0.00	0.57	-	0.47	-

Original matrix

	computer	data	pinch	result	sugar
apricot	0	0	1	0	1
pineapple	0	0	1	0	1
digital	2	1	0	1	0
information	1	6	0	4	0

Weighting PMI

- PMI is biased toward infrequent events
  - Very rare words have very high PMI values
- Two solutions
  - Give rare words slightly higher probabilities
  - Use add-k smoothing (which has a similar effect)

Use Laplace (add-k) smoothing

Add-2 smoothed	computer	data	pinch	result	sugar	···
apricot	2	2	3	2	3	
pineapple	2	2	3	2	3	
digital	4	3	2	3	2	
information	3	8	2	6	2	

Add-2 smoothed	**P(w, context)**					
	computer	data	pinch	result	sugar	**P(w)**
apricot	0.03	0.03	0.05	0.03	0.05	0.20
pineapple	0.03	0.03	0.05	0.03	0.05	0.20
digital	0.07	0.05	0.03	0.05	0.03	0.24
information	0.05	0.14	0.03	0.10	0.03	0.36
P(context)	0.19	0.25	0.17	0.22	0.17	

Use Laplace (add-k) smoothing

PPMI	computer	data	pinch	result	sugar
apricot	-	-	2.25	-	2.25
pineapple	-	-	2.25	-	2.25
digital	1.66	0.00	-	0.00	-
information	0.00	0.57	-	0.47	-

Only shrink **non-zero and valid** PPMI values

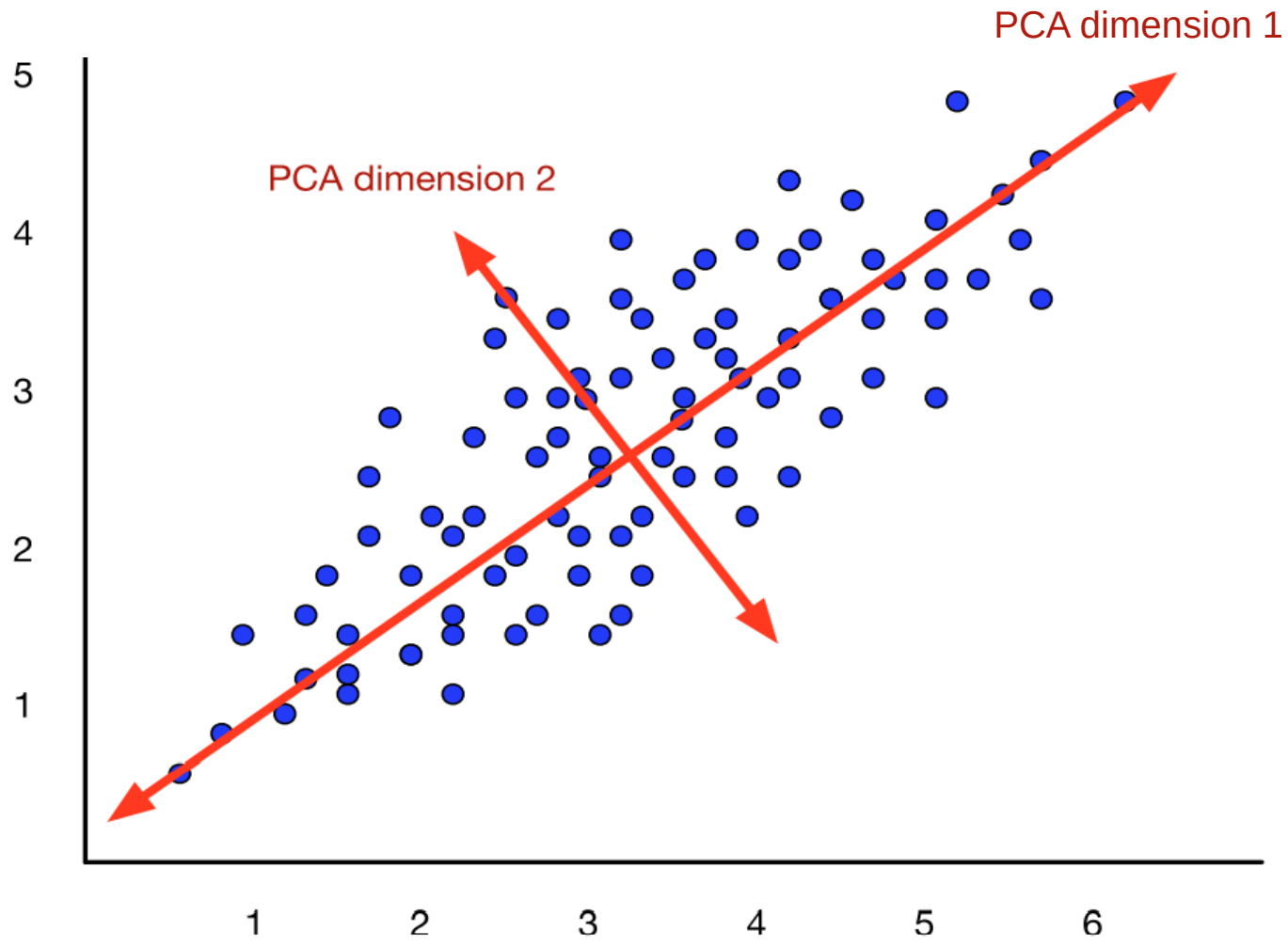
PPMI (add-2)	computer	data	pinch	result	sugar
apricot	0.00	0.00	0.56	0.00	0.56
pineapple	0.00	0.00	0.56	0.00	0.56
digital	0.62	0.00	0.00	0.00	0.00
information	0.00	0.58	0.00	0.37	0.00

Dense vector representations
-- singular value decomposition (SVD)

SVD-based method

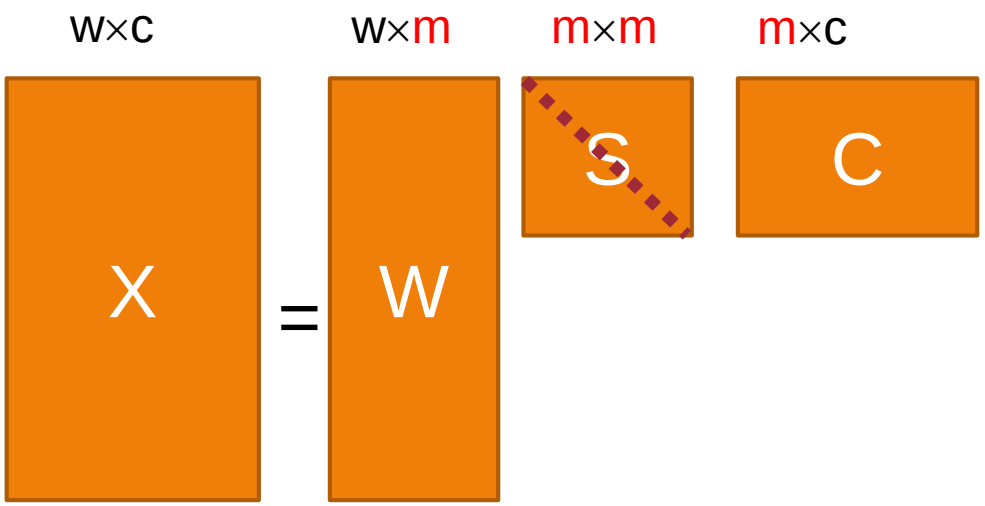
- Approximate an N-dimensional dataset using fewer dimensions
- By first rotating the axes into a new space
  - In which the highest order dimension captures the most variance in the original dataset
  - And the next dimension captures the next most variance, etc.

SVD-based method



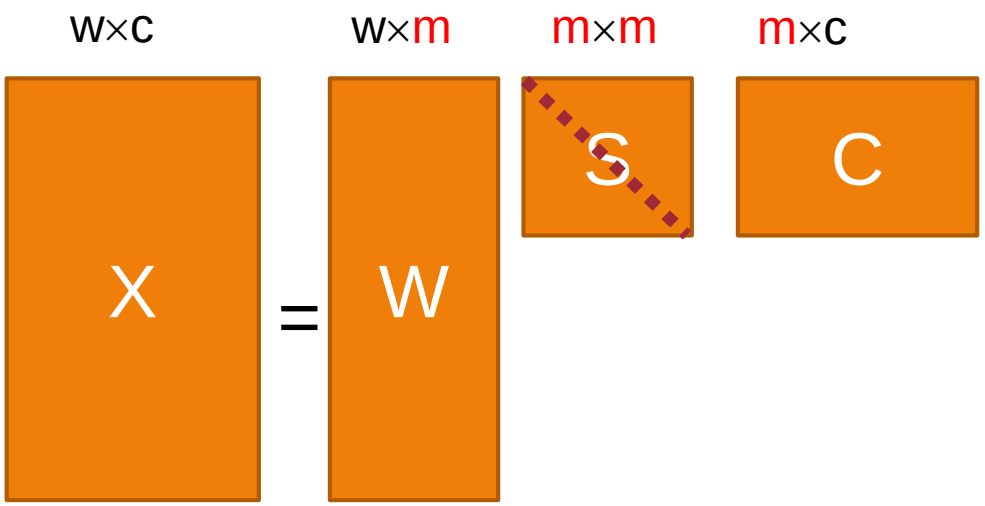
SVD

- Any *w*×*c* matrix **X** equals the product of 3 matrices:
  - **W**: rows correspond to original, but *m* columns represent dimensions in a new latent space, such that
    - *m* column vectors are orthogonal to each other
    - Columns are ordered by the amount of variance in the dataset each new dimension accounts for



SVD

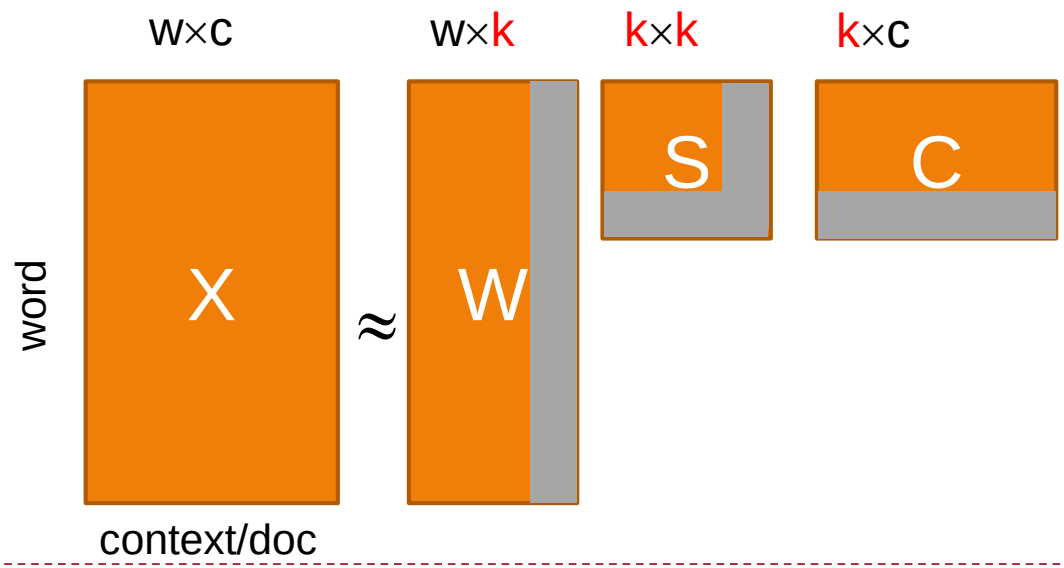
- Any *w*×*c* matrix **X** equals the product of 3 matrices:
  - **C**: columns correspond to original, but *m* rows represent dimensions in a new latent space
  - **S**: diagonal *m x m* matrix of **singular values** expressing the importance of each dimension.



Latent Semantic Analysis (LSA)

- SVD applied to term-document matrix (Deerwester et al, 1988)
  - Instead of keeping all *m* dimensions, we just keep the top *k* singular values
  - The result is a least-squares approximation to the original **X**
  - Use each row of **W** as a *k*-dimensional word representation
  - Use each column of **C** as a *k*-dimensional doc representation

Can also be applied to term-term matrix



Latent Semantic Analysis (LSA)

- Dense SVD vectors often work better than sparse vectors
  - Denoising: eliminate low-order dimensions that represent unimportant information
  - Better generalizability to unseen data
  - Fewer dimensions: smaller subsequent models, easier to train

Dense vector representations
-- Word2vec

Neural word embedding

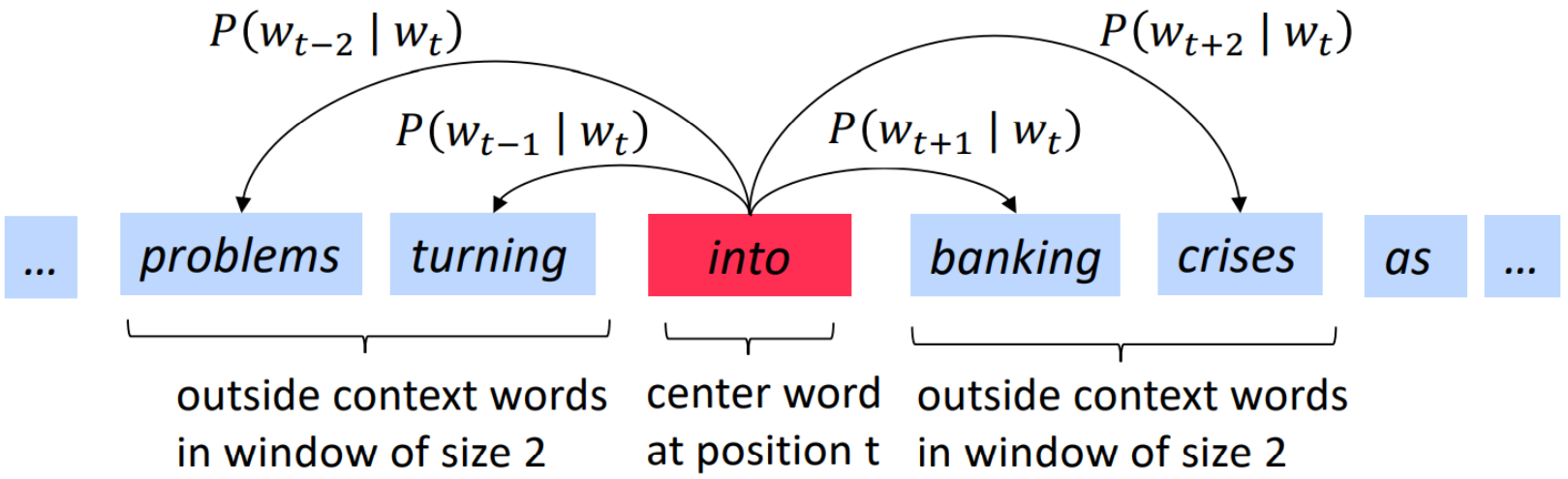
- Learning word representations using neural networks
  - Static word embedding: word2vec, GloVe
  - Contextualized word embedding (to be discussed later)

Word2vec overview

- Word2vec (Mikolov et al. 2013) is a framework for learning word vectors
- Idea:
  - We have a large corpus of text
  - Go through each position t in the text, which has a center word c and several context (“outside”) words o
  - Assume every word is represented by a vector
  - Use the similarity of word vectors for c and o to calculate the probability of o given c (or vice versa)
  - Keep adjusting the word vectors to maximize this probability

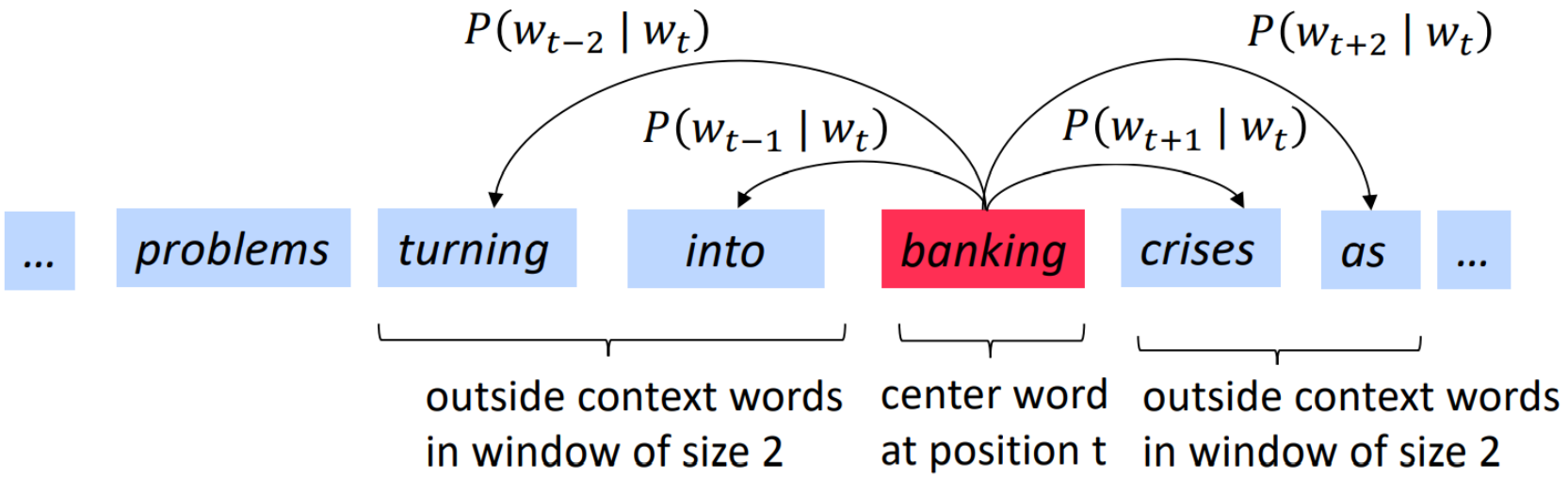
Word2vec skip-grams

- Example



Word2vec skip-grams

- Example



Word2vec skip-grams

- How to calculate $P(w_{t+j}|w_t)$
- Two vectors per word w:
  - v_w when w is a center word; u_w when w is a context word
- Then for a center word c and a context word o

$$P(o|c) = \frac{\exp(u_o^T v_c)}{\sum_{w \in V} \exp(u_w^T v_c)}$$

Dot-product similarity; exp to make it positive

Normalize over entire vocabulary

- This is called a softmax function
  - mapping arbitrary values to a probability distribution
  - "max" because it amplifies largest values
  - "soft" because it still retains smaller values

Word2vec skip-grams objective function

- For each $t = 1, \cdots, T$, predict context words within a window of fixed size m, given center word w_t.

Likelihood = $L(\theta) = \prod_{t=1}^{T} \prod_{\substack{-m \le j \le m \\ j \neq 0}} P(w_{t+j} \mid w_t; \theta)$

θ is all variables to be optimized

sometimes called *cost* or *loss* function

The objective function $J(\theta)$ is the (average) negative log likelihood:

$$J(\theta) = -\frac{1}{T} \log L(\theta) = -\frac{1}{T} \sum_{t=1}^{T} \sum_{\substack{-m \le j \le m \\ j \neq 0}} \log P(w_{t+j} \mid w_t; \theta)$$

Minimizing objective function $\Leftrightarrow$ Maximizing predictive accuracy

Word2vec skip-grams objective function

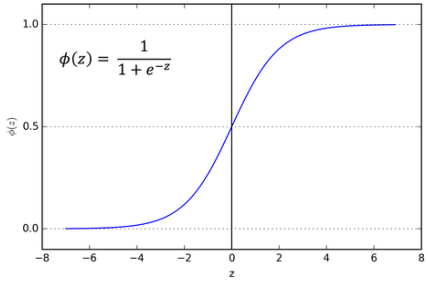
- A potential problem with the objective: normalizing over entire vocabulary during softmax
  - Expensive for large vocabulary

$$P(o|c) = \frac{\exp(u_o^T v_c)}{\sum_{w \in V} \exp(u_w^T v_c)}$$

Word2vec skip-grams: another objective

- For a center word c and a context word o, we model $P(\text{co}-\text{occur}\,|c,o)$ instead of $P(o|c)$

$$P(\text{co}-\text{occur}\,|c,o) = \sigma(\boldsymbol{u}_o^T \boldsymbol{v}_c)$$



  - Binary classification
  - No need for normalization over vocabulary
- Problem
  - What happens if we only maximize $P(\text{co}-\text{occur}\,|c,o)$ for all c/o pairs in the training corpus?
- Solution: negative sampling
  - For each c, take K negative samples of o (using word probabilities)
  - Minimize their probabilities

Word2vec skip-grams: another objective

- Objective function for each c/o pair:

$$J(\theta, c, o) = -\log P(\text{co}-\text{occur} \mid c, o) - \log \prod_{k=1}^{K} \left(1 - P(\text{co}-\text{occur} \mid c, o_k)\right)$$

$$= -\log\left(\sigma(\boldsymbol{u}_o^T \boldsymbol{v}_c)\right) - \sum_{k=1}^{K} \log\left(1 - \sigma\left(\boldsymbol{u}_k^T \boldsymbol{v}_c\right)\right)$$

$$= -\log\left(\sigma(\boldsymbol{u}_o^T \boldsymbol{v}_c)\right) - \sum_{k=1}^{K} \log\left(\sigma\left(-\boldsymbol{u}_k^T \boldsymbol{v}_c\right)\right)$$

  - Maximize probability of real outside word, and minimize probability of negative samples (*contrastive learning*)

- Overall objective:

$$J(\theta) = \sum_{t} \sum_{\substack{-m \le j \le m \\ j \neq 0}} J\left(\theta, w_t, w_{t+j}\right)$$

Optimization

- Stochastic gradient descent
  - Gradient descent
    - Perform update in the steepest downhill direction
    - Ex: $g(w_1, w_2)$
    - Update:

$$w_1 \leftarrow w_1 - \alpha * \frac{\partial g}{\partial w_1}(w_1, w_2)$$

$$w_2 \leftarrow w_2 - \alpha * \frac{\partial g}{\partial w_2}(w_1, w_2)$$

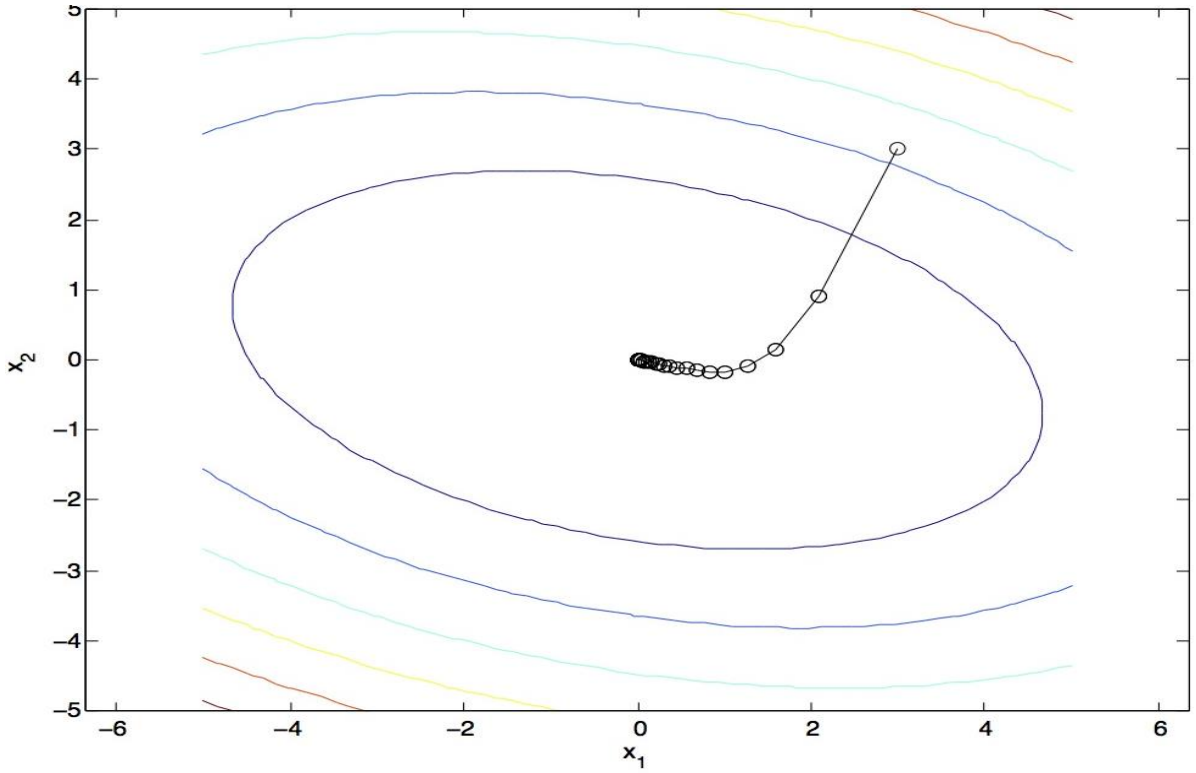
    - In vector notation:

$$w \leftarrow w - \alpha * \nabla_w g(w)$$

    - Nowadays: gradient computing using auto-differentiation

Optimization

- Stochastic gradient descent
  - Gradient descent
    - Perform update in the steepest downhill direction



Optimization

- Stochastic gradient descent
  - Gradient descent
    - Perform update in the steepest downhill direction
  - Stochastic
    - The objective is computed from many training examples
      - In skip-grams, each c/o pair is a training example
    - Perform update with gradient on one training example (or a mini-batch of examples)

Word2vec skip-grams

- We have two embeddings for a word
  - v_w when *w* is a center word
  - u_w when *w* is a context word
- We can either
  - Use v_w
  - Sum them
  - Concatenate them to make a double-length embedding

Word2vec CBOW

- Skip-grams (SG)
  - Predict context (“outside”) words (position independent) given center word
- Continuous Bag of Words (CBOW)
  - Predict center word from (bag of) context words

Phrase/Sentence/Document Embedding

- Build from word embeddings
  - Pooling: mean, max
  - Boundary words
  - Applying an LSTM or Transformer over sequence of word embeddings (to be discussed later)

Evaluating vector representations

Evaluate vector representations

- General evaluation for intermediate tasks: Intrinsic vs. extrinsic
- Intrinsic:
  - Evaluation on a specific subtask
  - Fast to compute
  - Helps to understand that system
  - Not clear if really helpful unless correlation to downstream tasks is established
- Extrinsic:
  - Evaluation on a downstream task
  - Can take a long time
  - Unclear who's responsible: intermediate system, or downstream system, or their interaction

Intrinsic evaluation of word embeddings

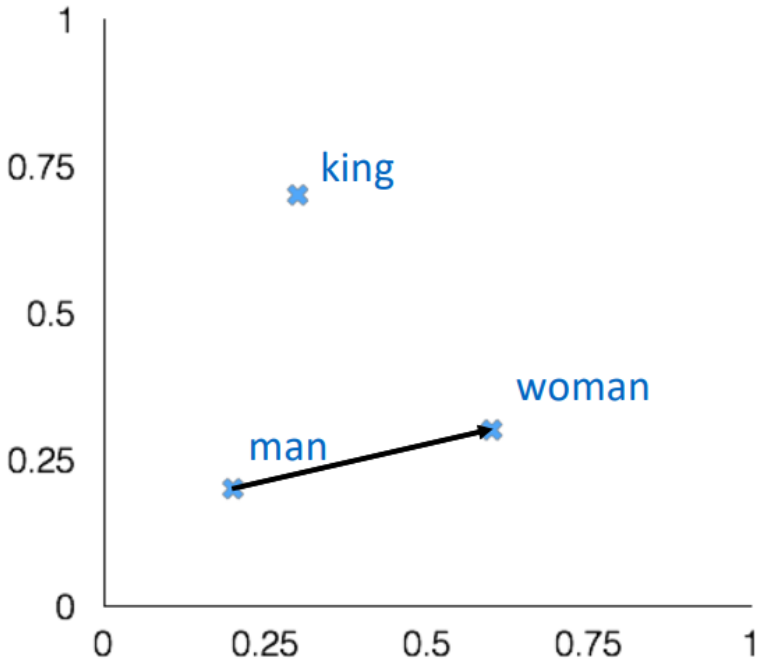
- Word Vector Analogies

a:b :: c:?

man:woman :: king:?

$$d = \arg\max_i \frac{(x_b - x_a + x_c)^T x_i}{||x_b - x_a + x_c||}$$

- Evaluate word vectors by how well their cosine distance after addition captures intuitive semantic and syntactic analogy questions
- Problem: What if the information is there but not linear



Summary

Text Representation

- Sparse vector representations
  - Co-occurrence matrices
  - Adjustments: tf-idf, PPMI
- Dense vector representations
  - Singular value decomposition
    - Latent Semantic Analysis
  - Word2vec
    - Skip-gram
- Evaluation