Name: _____

Student ID: _____

School: _____

Year of Entrance: _____

ShanghaiTech University Midterm Examination Cover Sheet

Academic Year : 2023 Term: Spring

Course-offering School: SIST

Instructor: Rui Fan

Course Name: Algorithm Design and Analysis / 算法设计与分析

Course Number: CS 240

Exam Instructions for Students:

1. All examination rules must be strictly observed throughout the entire test, and any form of cheating is prohibited.

2. Other than allowable materials, students taking closed-book tests must place their books, notes, tablets and any other electronic devices in places designated by the examiners.

3. Students taking open-book tests may use allowable materials authorized by the examiners. They must complete the exam independently without discussion with each other or exchange of materials.

For Marker's Use:

Section	1	2	3	4	5						Total
Marks											
Recheck											

Marker's Signature: **Rechecker's Signature:**

Date: **Date:**

Instructions for Examiners:

1. The format of the exam papers and answer sheets shall be determined by the school and examiners according to actual needs. All pages should be marked by the page numbers in order (except the cover page). All text should be legible, visually comfortable and easy to bind on the left side. A4 double-sided printing is recommended for the convenience of archiving (There are all-in-one printers in the university).

2.The examiners should make sure that exam questions are correct and appropriate, If errors are found in exam questions during the exam, the examiners should be responsible to respond on site, which will be taking into account in the teaching evaluation.

Instructions for Students

In all problems in which you are asked to design algorithms, you should ***clearly describe*** *how your algorithm works, provide code or pseudocode when asked to, and argue why your algorithm is correct.*

Do not *write your answers on the exam paper. Instead, write them on* separate pieces of paper. ***Write*** *your name and student ID at the top of* ***each piece*** *of paper.*

All answers must be written neatly and legibly in English. If there are ***brief parts*** *of your answer which you cannot express clearly in English, you may write them in Chinese.*

Problem 1

(a) Order the following functions in order of increasing asymptotic size.

$$f_1 = \log(n^n) \qquad f_2 = n^{n^{\frac{1}{2}}} \qquad f_3 = (n!)^{\frac{1}{n}} \qquad f_4 = n \log n \log\log n$$

(b) Solve the following recurrences by giving tight Θ-notation bounds.

(i) $T(n) = 4T\left(\frac{n}{2}\right) + n^2 \log n$

(ii) $T(n) = 8T\left(\frac{n}{2}\right) + n \log n$

(c) Answer true or false to the following statements.

(i) $n = O((\log n)^{\log n})$

(ii) For any two functions f and g, either $f = O(g)$ or $g = O(f)$.

(5 parts, 4 points / part, 20 points total)

Problem 2

You are given n Boolean values $x_1, \ldots, x_n$, where each x_i is T or F. Your task is to find some i such that $x_i = T$, or return 0 if all values are F. However, you cannot read the value of any x_i directly. Instead, you can call a function $OR(i, j)$, which takes as input two indices $1 \le i \le j \le n$, and returns T if any value in $x_i, \ldots, x_j$ is T, or returns F if all these values are F. Design a divide and conquer algorithm for this problem which minimizes the number of calls to the OR function, and analyze how many calls you make.

(20 points)

Problem 3

Suppose there are n objects with values $v_1, \ldots, v_n$. You want to pick a subset of these objects with the maximum total value. However, if you pick object i, then you cannot pick any of the next f_i objects, for some natural number f_i. I.e., if you pick object i and $f_i > 0$, you cannot pick objects $i + 1, \ldots, i + f_i$; if $f_i = 0$, then picking object i does not prevent you from picking any other objects. Design a dynamic programming algorithm to maximize the total value of the objects you pick, and analyze your algorithm's time complexity.

For example, if the objects have values 3, 5, 1, 6, 7, and the corresponding f values are 1, 2, 0, 1, 1, then you should pick the objects with values 5, 7.

(20 points)

Problem 4

You are given a set of positive integers $S = \{x_1, \ldots, x_n\}$. Your goal is to partition S into two subsets, such that the sums of the values in the subsets are as close as possible. That is, you want to find two subsets S_1, S_2 with $S_1 \cup S_2 = S$, $S_1 \cap S_2 = \emptyset$, such that the following value is minimized.

$$\left| \sum_{x_i \in S_1} x_i - \sum_{x_i \in S_2} x_i \right|$$

Design an algorithm for this problem and analyze its time complexity.

(20 points)

Problem 5

A list of numbers $x_1, \ldots, x_n$ is called *wiggly* if $x_1 \leq x_2 \geq x_3 \leq x_4 \geq \cdots x_n$ (note that $x_{n-1} \leq x_n$ if n is even, and $x_{n-1} \geq x_n$ if n is odd). Given an unsorted list of numbers $y_1, \ldots, y_n$, give an efficient algorithm to permute the list into a wiggly list, and analyze the time complexity of your algorithm. Note that there may be multiple valid solutions to a given input.

For example, given the input list 4, 1, 3, 2, one possible output is 1, 3, 2, 4.

(20 points)

END OF EXAM