ADMIS Lab
先进数据与机器智能系统实验室

查询优化
Query Optimization

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先进数据与机器智能系统实验室（ADMIS）
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课程概要

- **Part 0: Overview**
  - Ch1: Introduction
- **Part 1 Relational Databases**
  - Ch2: Relational model
  - Ch3: Introduction to SQL
  - Ch4: Intermediate SQL
  - Ch5: Advanced SQL
- **Part 2 Database Design**
  - Ch6: Database design based on E-R model
  - Ch7: Relational database design
- **Part 3 Application Design & Development**
  - Ch8: Complex data types
  - Ch9: Application development
- **Part 4 Big data analytics**
  - Ch10: Big data
  - Ch11: Data analytics
- **Part 5 Data Storage & Indexing**
  - Ch12: Physical storage system
  - Ch13: Data storage structure
  - Ch14: Indexing
- **Part 6 Query Processing & Optimization**
  - Ch15: Query processing
  - **Ch16: Query optimization**
- **Part 7 Transaction Management**
  - Ch17: Transactions
  - Ch18: Concurrency control
  - Ch19: Recovery system
- **Part 8 Parallel & Distributed Database**
  - Ch20: Database system architecture
  - Ch21-23: Parallel & distributed storage, query processing & transaction processing
- **Part 9**
  - DB Platform: **OceanBase**, MongoDB, Neo4J

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- 概述
- 关系表达式的转换
- 表达式执行代价估计
- 执行计划选择
- 物化视图*
- 查询优化中的高级话题*

查询优化概述

- Alternative ways to execute (执行) a given query
  - equivalent expressions
  - different algorithms for each operation
- The cost difference between a good way and a bad way of executing a query can be enormous
- **Estimate the cost of operations**
  - depend on the statistical information about relations which the database should maintain
  - need to estimate the statistics for intermediate results to compute the cost of complex expressions

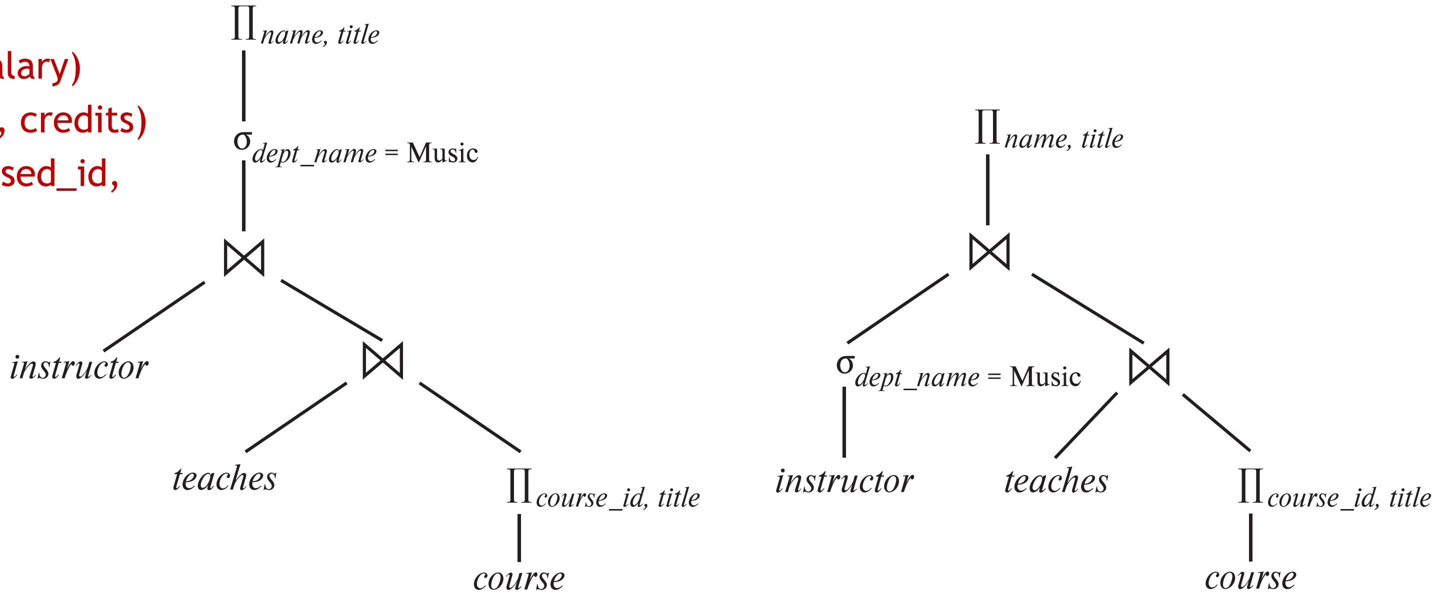
查询优化概述（续）

- **查询**：找出Music系所有教师的名字以及每位教师所教授课程的名称

instructor (ID, name, salary)

course (course_id, title, credits)

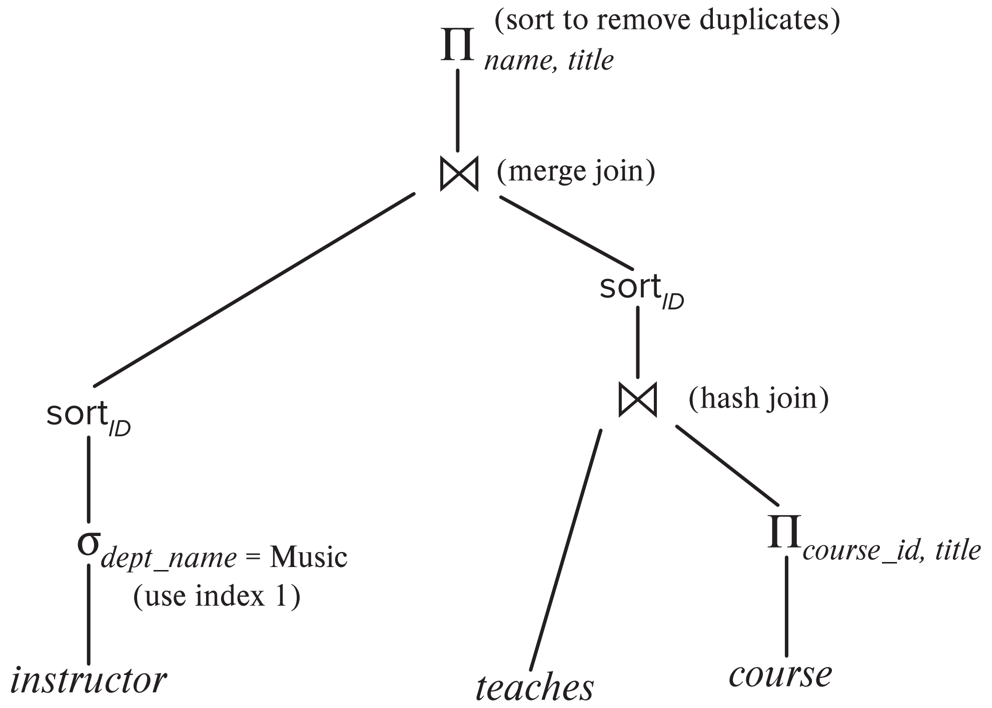
teaches (ID, course_id, sed_id, semester, year)



(a) Initial expression tree　　(b) Transformed expression tree

查询优化概述（续）

- 执行计划需明确每个运算使用的算法以及运算之间的执行如何协调



查询优化概述（续）

- **Cost-based Optimization，基于代价的优化**
  - **步骤1**：产生逻辑上与给定表达式等价的表达式
    - 使用等价规则
  - **步骤2**：对所产生的表达式以不同方式标注，产生不同的查询执行计划
  - **步骤3**：估计每个执行计划的代价，选择估计代价最小的执行计划

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关系表达式转换

- **Equivalence of two relational algebra expressions**
  - Generate the same set of tuples on every legal database instance
  - Note: the order of tuples is irrelevant

- **Equivalence rule (等价规则)**
  - The expressions of two forms are equivalent
  - Can replace expression of first form by the second, or vice versa

等价规则

- **规则1**：合取选择运算可分解为单个选择运算的序列

$$\sigma_{\theta_1 \wedge \theta_2}(E) = \sigma_{\theta_1}(\sigma_{\theta_2}(E))$$

- **规则2**：选择运算满足交换律 (commutative)

$$\sigma_{\theta_1}(\sigma_{\theta_2}(E)) = \sigma_{\theta_2}(\sigma_{\theta_1}(E))$$

- **规则3**：多个连续投影中只有最后一个运算是必需的，其余可忽略

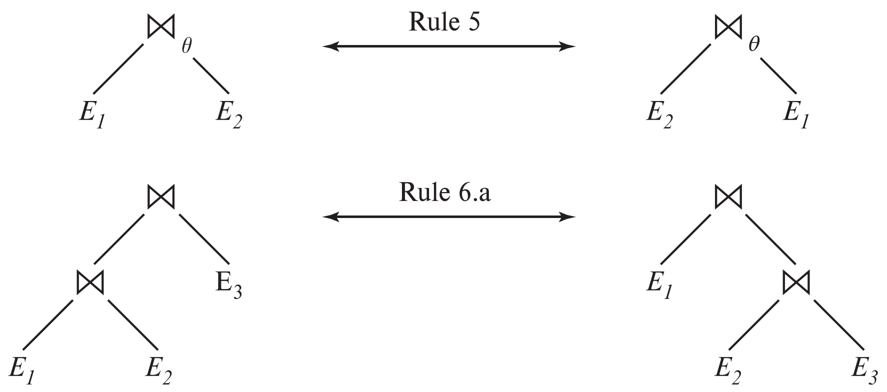
$$\Pi_{t_1}(\Pi_{t_2}(\ldots(\Pi_{tn}(E))\ldots)) = \Pi_{t_1}(E)$$

- **规则4**：选择操作可以与笛卡尔积以及θ连接相结合

$$\sigma_\theta(E_1 \times E_2) = E_1 \bowtie_\theta E_2$$
$$\sigma_{\theta 1}(E_1 \bowtie_{\theta 2} E_2) = E_1 \bowtie_{\theta 1 \wedge \theta 2} E_2$$

等价规则（续）

- **规则5**：θ连接满足交换律
  - $E_1 \bowtie_\theta E_2 = E_2 \bowtie_\theta E_1$
- **规则6a**：自然连接满足结合律
  - $(E_1 \bowtie E_2) \bowtie E_3 = E_1 \bowtie (E_2 \bowtie E_3)$
- **规则6b**：θ连接满足下列形式的结合律：
  - $(E_1 \bowtie_{\theta 1} E_2) \bowtie_{\theta 2 \wedge \theta 3} E_3 = E_1 \bowtie_{\theta 1 \wedge \theta 3} (E_2 \bowtie_{\theta 2} E_3)$，要求其中$\theta_2$只涉及$E_2$和$E_3$的属性



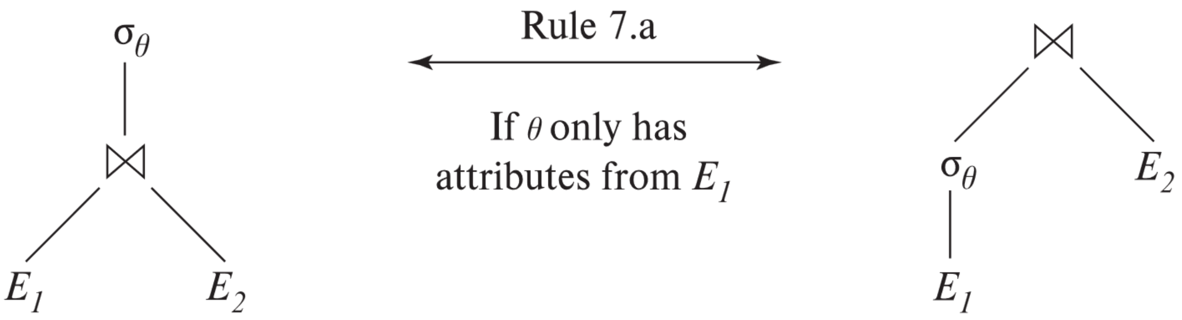
等价规则（续）

- **规则7**：选择操作在下面两个条件下对θ连接满足分配律
  - a. 当选择条件θ_0中的所有属性只涉及参与连接的表达式之一(如E_1)时：

$$\sigma_{\theta_0}(E_1 \bowtie_\theta E_2) = (\sigma_{\theta_0}(E_1)) \bowtie_\theta E_2$$

  - b. 当选择条件θ_1只涉及E_1的属性，选择条件θ_2只涉及E_2的属性时：

$$\sigma_{\theta_1 \wedge \theta_2}(E_1 \bowtie_\theta E_2) = (\sigma_{\theta_1}(E_1)) \bowtie_\theta (\sigma_{\theta_2}(E_2))$$



等价规则（续）

- **规则8**：令L_1、L_2分别代表E_1、E_2的**属性子集**，投影操作在下列条件下对θ连接满足分配率
  - a. 如果连接条件θ只涉及$L_1 \cup L_2$中的属性：

$$\Pi_{L_1 \cup L_2}(E_1 \bowtie_\theta E_2) = (\Pi_{L_1}(E_1)) \bowtie_\theta (\Pi_{L_2}(E_2))$$

  - b. 针对连接$E_1 \bowtie_\theta E_2$
    - 令L_3是E_1出现在连接条件θ中但不在$L_1 \cup L_2$中的属性
    - 令L_4是E_2出现在连接条件θ中但不在$L_1 \cup L_2$中的属性

$$\Pi_{L_1 \cup L_2}(E_1 \bowtie_\theta E_2) = \Pi_{L_1 \cup L_2}((\Pi_{L_1 \cup L_3}(E_1)) \bowtie_\theta (\Pi_{L_2 \cup L_4}(E_2)))$$

等价规则（续）

- **规则9**：集合的并和交满足交换律
  - $E_1 \cup E_2 = E_2 \cup E_1$
  - $E_1 \cap E_2 = E_2 \cap E_1$

  (set difference is not commutative)
- **规则10**：集合的并和交满足结合律
  - $(E_1 \cup E_2) \cup U_3 = E_1 \cup (E_2 \cup E_3)$
  - $(E_1 \cap E_2) \cap U_3 = E_1 \cap (E_2 \cap E_3)$
- **规则11**：选择操作对并、交、差满足分配率
  - $\sigma_\theta(E_1 - E_2) = \sigma_\theta(E_1) - \sigma_\theta(E_2)$
    - similarly for ∪ and ∩ in place of –
  - $\sigma_\theta(E_1 - E_2) = \sigma_\theta(E_1) - E_2$
    - similarly for ∩ in place of –, **but not for ∪?**
- **规则12**：投影对并的分配律
  - $\Pi_L(E_1 \cup E_2) = (\Pi_L(E_1)) \cup (\Pi_L(E_2))$

例1: Selections First

- Find the names of all instructors in Music department, along with the titles of the courses that they teach

$$\Pi_{name,\ title}(\sigma_{dept\_name=\text{'Music'}}(instructor \bowtie (teaches \bowtie \Pi_{course\_id,\ title}\ (course))))$$

- Transformation using rule 7a

$$\Pi_{name,\ title}((\sigma_{dept\_name=\text{'Music'}}(instructor)) \bowtie (teaches \bowtie \Pi_{course\_id,\ title}\ (course)))$$

例2: Multiple Transformations

- Find the names of all instructors in Music department who have taught a course in 2021, along with the titles of the courses that they taught

$$\Pi_{name,\ title}(\sigma_{dept\_name=\text{"Music"} \land year = 2021}(instructor \bowtie (teaches \bowtie \Pi_{course\_id,\ title}(course))))$$

- Rule 6a:

$$\Pi_{name,\ title}(\sigma_{dept\_name=\text{"Music"} \land year = 2021}((instructor \bowtie teaches) \bowtie \Pi_{course\_id,\ title}(course)))$$

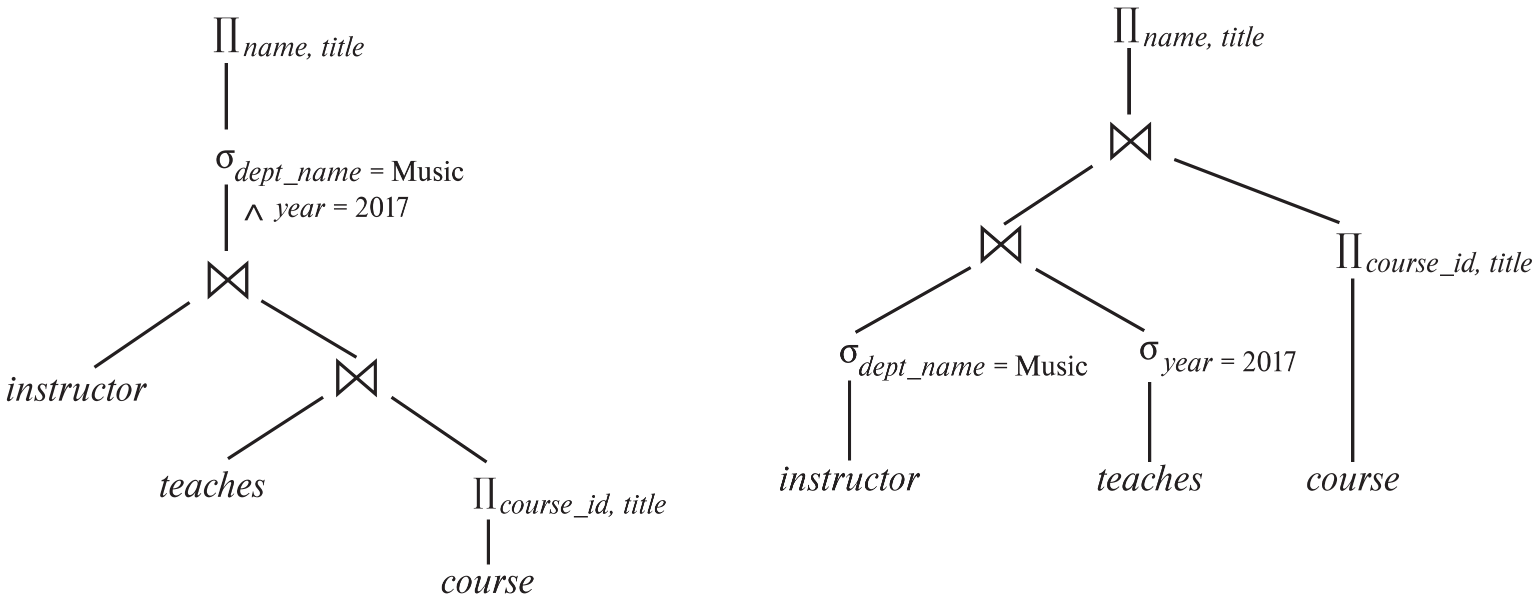
- Rule 7a:

$$\Pi_{name,\ title}((\sigma_{dept\_name=\text{"Music"} \land year = 2021}(instructor \bowtie teaches)) \bowtie \Pi_{course\_id,\ title}(course))$$

- Rule1 & 7a:

$$\Pi_{name,\ title}((\sigma_{dept\_name=\text{"Music"}}(instructor) \bowtie \sigma_{year = 2021}(teaches)) \bowtie \Pi_{course\_id,\ title}(course))$$

例2: Multiple Transformations



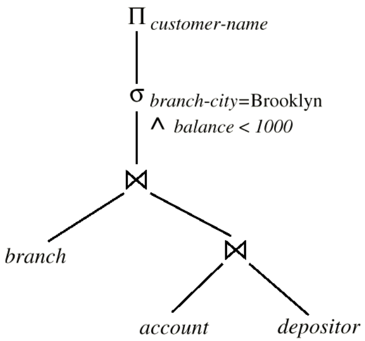
(a) Initial expression tree

(b) Tree after multiple transformations

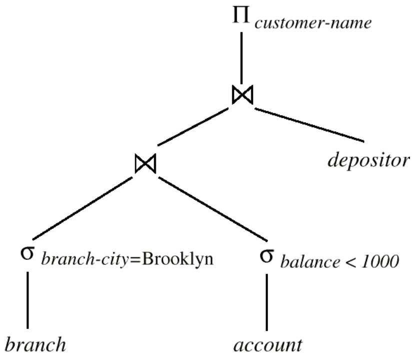
练习

- **Query:** Find the names of all the customers who have an account with balance over $1000 at the Brooklyn branch
  - $\Pi_{CN}(\sigma_{BC=\text{“}Brooklyn\text{”} \wedge\, balance>1000}(branch \bowtie (account \bowtie depositor)))$
  - CN: customer name, BC: branch city
- **Task**: Give one equivalent expression with better execution performance
- **One solution:**

$$\Pi_{CN}((\sigma_{BC=\text{"}Brooklyn\text{"}}(branch) \bowtie \sigma_{balance>1000}(account)) \bowtie depositor)$$



(a) Initial Expression Tree



(b) Tree After Multiple Transformations

连接顺序

- For three relations r_1, r_2, and r_3,

$$(r_1 \bowtie r_2) \bowtie r_3 = r_1 \bowtie (r_2 \bowtie r_3)$$

- If $r_2 \bowtie r_3$ is quite large and $r_1 \bowtie r_2$ is small, we choose

$$(r_1 \bowtie r_2) \bowtie r_3$$

so that we can compute and store a smaller temporary relation

连接顺序（续）

- Consider the expression

  $\Pi_{name,\ title}(\sigma_{dept\_name=\ \text{"Music"}}\ (instructor) \bowtie teaches) \bowtie \Pi_{course\_id,\ title}\ (course))))$

- **Solution A**
  - compute *teaches* $\bowtie \Pi_{course\_id,\ title}$ (*course*) first, and join the result with $\sigma_{dept\_name=\ \text{"Music"}}$ (*instructor*)
  - the result of the first join is likely to be a large relation
- **Solution B**
  - compute $\sigma_{dept\_name=\ \text{"Music"}}$ (*instructor*) $\bowtie$ *teaches* first
  - only a small fraction of instructors are likely to be from the Music department

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关系的统计信息

- **关系(表)的统计信息**
  - n_r: the number of tuples in relation r
  - b_r: the number of blocks of r
  - s_r: the size of a tuple of r
  - f_r: the number of tuples that fit into one block
  - $V(A, r)$: the number of distinct values that appear in r for attribute A, i.e., the size of $\Pi_A(r)$
  - If the tuples of r are stored together physically in a file, then: $b_r = \left\lceil \frac{n_r}{f_r} \right\rceil$

索引的基本信息

- F_i: the average fan-out(扇出) of internal nodes of index i
  - for tree-structured indices such as B+-tree
- HT_i: the number of levels in index i
  - i.e., the height of i
  - for a balanced tree index (e.g., B+-tree) on attribute A of relation r, $HT_i = \lceil log_{F_i}(V(A,r)) \rceil$
  - for a hash index, HT_i is 1
- LB_i: the number of lowest-level index blocks in i
  - i.e., the number of blocks at the leaf level of the index

查询代价的估计

- **Recall that**
  - Disk access is the predominant cost, and is also relatively easy to be estimated
  - The number of block transfers from disk is used as a measure of the actual cost of execution
  - It is assumed that all block transfers have the same cost

简单选择操作结果大小估计

- $\boldsymbol{\sigma_{A=a}(r)}$
  - 假设取值**均匀分布**，则可估计选择结果有$n_r/V(A,r)$个元组
- $\boldsymbol{\sigma_{A\leq V}(r)}$
  - Let c denote the estimated number of tuples satisfying the condition. If $\min(A,r)$ and $\max(A,r)$ are available in database catalog and we assume that values are uniformly distributed (值均匀分布)
    - $C = 0$, if $v < \min(A,r)$
    - $C = n_r \cdot \frac{v-\min(A,r)}{\max(A,r)-\min(A,r)}$
    - $C = n_r$, if $v \geq \max(A,r)$
  - In absence of statistical information, c is assumed to be $n_r/2$

复杂选择操作结果大小估计

- **Selectivity (中选率) of the condition $\boldsymbol{\theta_i}$**
  - The probability that a tuple in the relation r satisfies θ_i
  - If s_i is the number of tuples satisfying θ_i, the selectivity of θ_i is given by s_i/n_r
- **合取: $\boldsymbol{\sigma_{\theta 1 \wedge \theta 2 \wedge \cdots \wedge \theta n}(r)}$**
  - Estimated number of tuples:

$$n_r * \frac{s_1 * s_2 * \cdots * s_n}{n_r^n}$$

- **析取: $\boldsymbol{\sigma_{\theta 1 \vee \theta 2 \vee \cdots \vee \theta n}(r)}$**
  - Estimated number of tuples:

$$n_r * \left(1 - (1 - \frac{s_1}{n_r}) * (1 - \frac{s_2}{n_r}) * \ldots * (1 - \frac{s_n}{n_r})\right)$$

- **取反: $\boldsymbol{\sigma_{\neg\theta}(r)}$**
  - Estimated number of tuples: $n_r - size(\sigma_\theta(r))$

连接操作结果大小估计

- **Cartesian product**
  - $r \times s$ contains $n_r * n_s$ tuples
- **Natural join**
  - If $R \cap S = \emptyset$, then $r \bowtie s$ is the same as $r \times s$
  - If $R \cap S$ is a key for R, then a tuple of s will join with **at most** one tuple from r, and $\text{size}(r \bowtie s) \leq n_s$
  - If $R \cap S$ is a foreign key in S referencing R, the number of tuples in $r \bowtie s$ is exactly the same as the number of tuples in s, i.e., n_s
    - E.g., for depositor ⋈ customer, customer-name in depositor is a foreign key of customer, and the result has exactly $n_{depositor}$ tuples

连接操作结果大小估计(续)

- Catalog information for join examples:
  - $n_{customer}$ = 10,000, $f_{customer}$ = 25, $b_{customer}$ =10,000/25 = 400
  - $n_{depositor}$ = 5,000, $f_{depositor}$ = 50, $b_{depositor}$ = 5,000/50 = 100
  - V(customer-name, depositor) = 2,500, which implies that, on average, each customer has two accounts
- E.g., $depositor \bowtie customer$
  - $n_{depositor}$ = 5000

连接操作结果大小估计(续)

- If $R \cap S = \{A\}$ is not a key for R or S
  - If we assume that every tuple t in R produces tuples in $R \bowtie S$, the number of tuples in $R \bowtie S$ is estimated to be:

$$n_r * \frac{n_s}{V(A,s)}$$

  - If the reverse is true, the estimate obtained will be:

$$n_s * \frac{n_r}{V(A,r)}$$

  - The lower of these two estimates is probably the more accurate one

连接操作结果大小估计(续)

- Estimate the size of depositor ⋈ customer without using the information about foreign keys:
  - V(customer-name, depositor) = 2500, and
    V(customer-name, customer) = 10000
  - The two estimates are

    5000 * 10000/2500 = 20,000 and

    5000 * 10000/10000 = 5000
- Choose the lower estimate, which is the same as the earlier estimation using foreign keys

其他操作结果集大小估计

- **投影**
  - Estimated size of $\Pi_A(r) = V(A, r)$
- **聚集**
  - Estimated size of $_A g_F(r) = V(A, r)$
- **集合操作**
  - For unions/intersections of selections on the **same relation**: rewrite and use size estimate for selections
    - E.g., $\sigma_{\theta 1}(r) \cup \sigma_{\theta 2}(r)$ can be rewritten as $\sigma_{\theta 1 \vee \theta 2}(r)$
  - For operations on different relations:
    - estimated size of $r \cup s$ = size of r + size of s
    - estimated size of $r \cap s$ = min{size of r, size of s}
    - estimated size of $r - s$ = size of r
    - All the three estimates may be quite inaccurate, but provide upper bounds for the sizes

其他操作结果集大小估计（续）

- **Outer join**
  - Estimated size of r ⟕ s = size of $r \bowtie s$ + size of r
    - Case of right outer join is symmetric
  - Estimated size of r ⟗ s = size of $r \bowtie s$ + size of r + size of s

Estimation of Distinct Values

- **Selections: $\boldsymbol{\sigma_{\theta}(r)}$**
  - If θ forces A to take a specified value:
    - If A = 3, $V(A, \sigma_{\theta}(r))$ = 1
  - If θ forces A to take one of a specified set of values
    - $V(A, \sigma_{\theta}(r))$ = the number of specified values
    - e.g., (A = 1 V A = 3 V A = 4)
  - If the selection condition θ is of the form A op v
    - Estimated $V\big(A, \sigma_{\theta}(r)\big) = V(A, r) * s$, where s is the selectivity of the selection
  - In all the other cases: use approximate estimate of $\min(V(A, r), n_{\sigma_{\theta}(r)})$
    - More accurate estimate can be obtained using probability theory

Estimation of Distinct Values (Cont.)

- **Joins: $\boldsymbol{r \bowtie s}$**
  - If all attributes in A are from r
    - Estimated size of $V(A, r \bowtie s) = \min(V(A,r), n_{r \bowtie s})$
  - If A contains attributes A_1 from r and A_2 from s, then
    - $V(A, r \bowtie s) = \min(V(A_1, r) * V(A_2 - A_1, s), V(A_1 - A_2, r) * V(A_2, s), n_{r \bowtie s})$
    - More accurate estimate can be obtained using probability theory
- **Projection**
  - Estimation of distinct values are straightforward for projections
  - They are the same in $\Pi_A(r)$ as in r
- **Aggregation**
  - For $\min(A)$ and $\max(A)$, the number of distinct values can be estimated as $\min(V(A,r), V(G,r))$ where G denotes grouping attributes
  - For other aggregates, assume that all values are distinct, and use $V(G,r)$

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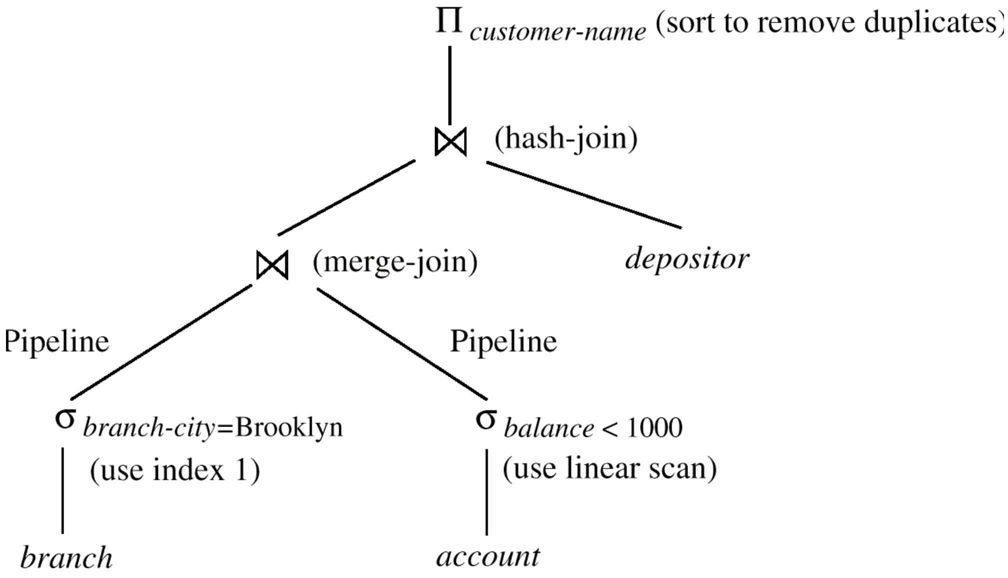
枚举等价表达式

- **Equivalent expression generation**
  - Query optimizers use equivalence rules to systematically generate expressions equivalent to the given expression
  - Generate all equivalent expressions by repeatedly executing the following step until no more expressions can be found
    - Given an expression E, if any sub-expression E_s of E matches one side of an equivalence rule, the optimizer generates a new expression where E_s is transformed to match the other side of the rule
- **Issues**
  - The above approach is very expensive in space and time
    - Space requirements can be reduced by sharing **common sub-expressions** for equivalent expressions
    - Time requirements can be reduced by not generating all expressions

执行计划

- An execution plan defines exactly what algorithm is used for each operation, and how the execution of the operations is coordinated



执行计划的选择

- **Two general approaches**
  - Search all the plans and choose the best plan in a cost-based fashion
  - Uses heuristics to choose a plan

- **Interaction of evaluation operations**
  - Choosing the cheapest algorithm for each operation independently may not yield the best overall algorithm
    - merge-join may be costlier than hash-join, but may provide a sorted output which reduces the cost for an outer level aggregation
    - nested-loop join may provide opportunity for pipelining

基于代价的优化

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- **To find the best join-order for $\boldsymbol{r_1 \bowtie r_2 \bowtie \cdots \bowtie r_n}$**
  - There are $\big(2(n-1)\big)!/(n-1)!$ different join orders for above expression (cf. Exercise 16.12)
  - E.g., with $n = 3$, the number is 12

$$r_1 \bowtie (r_2 \bowtie r_3),\ r_1 \bowtie (r_3 \bowtie r_2),\ (r_2 \bowtie r_3) \bowtie r_1,\ (r_3 \bowtie r_2) \bowtie r_1$$

$$r_2 \bowtie (r_1 \bowtie r_3),\ r_2 \bowtie (r_3 \bowtie r_1),\ (r_1 \bowtie r_3) \bowtie r_2,\ (r_3 \bowtie r_1) \bowtie r_2$$

$$r_3 \bowtie (r_1 \bowtie r_2),\ r_3 \bowtie (r_2 \bowtie r_1),\ (r_1 \bowtie r_2) \bowtie r_3,\ (r_2 \bowtie r_1) \bowtie r_3$$

  - With $n = 7$, the number is 665,280
  - With $n = 10$, the number is greater than 17.6 billion

- **No need to generate all the join orders**
  - Using dynamic programming. The least-cost join order for any subset of $\{r_1, r_2, \ldots, r_n\}$ is computed only once and stored for future use

动态规划优化

- **To find the best join tree for a set S of n relations**
  - Consider all possible plans of the form: $S_1 \bowtie (S - S_1)$ where S_1 is any non-empty subset of S
  - When the plan for any subset is computed, store it and reuse it when it is required again, instead of re-computing it
  - Recursively compute costs for joining subsets of S to find the cost of each plan. Choose the cheapest.

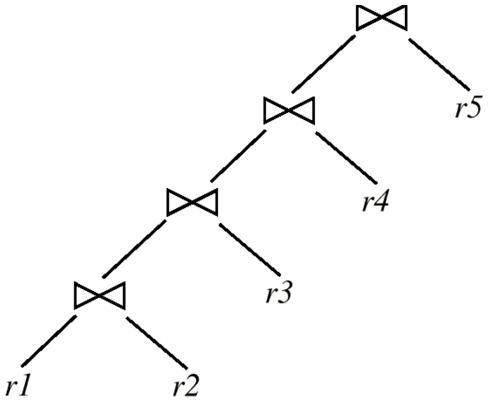
连接顺序优化算法

Dynamic-programming algorithm

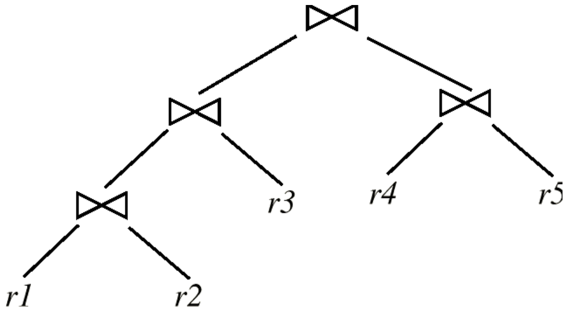
```
procedure findbestplan(S)
    if (bestplan[S].cost ≠ ∞)
        return bestplan[S]
    if (S contains only 1 relation)
        set bestplan[S].plan and bestplan[S].cost based on best way of accessing S
    else for each non-empty subset S1 of S such that S1 ≠ S
        P1= findbestplan(S1)
        P2= findbestplan(S - S1)
        A = best algorithm for joining results of P1 and P2
        cost = P1.cost + P2.cost + cost of A
        if cost < bestplan[S].cost
                bestplan[S].cost = cost
                bestplan[S].plan = "execute P1.plan;
                execute P2.plan;
                join results of P1 and P2 using A"
    return bestplan[S]
```

左深连接树

- In left-deep join trees, the right-hand-side input for each join is a relation, not the result of an intermediate join



(a) Left-deep Join Tree



(b) Non-left-deep Join Tree

Cost of Optimization

- **Complexity of dynamic programming**
  - The time complexity is $O(3^n)$. With n = 10, this number is 59000 instead of 17.6 billion
  - Space complexity is $O(2^n)$
- **Complexity for finding the best left-deep join tree**
  - Consider n alternatives with one relation as right-hand side input and the other relations as left-hand side input.
  - Using (recursively computed and stored) least-cost join order for each alternative on left-hand-side, choose the cheapest of the n alternatives.
  - If only left-deep trees are considered, time complexity of finding best join order is $O(n2^n)$
  - Space complexity remains at $O(2^n)$
- Cost-based optimization is expensive, but worthwhile for queries on large datasets (typical queries have small n, generally < 10)

启发式优化（Heuristic Optimization）

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- Cost-based optimization is expensive, even with dynamic programming
- DBMS may use **heuristics** to reduce the number of choices that must be made in a cost-based fashion
- Heuristic optimization transforms the query-tree by using **a set of rules** that typically (but not in all cases) improve execution performance:
  - Perform selection early (reduce the number of tuples)
  - Perform projection early (reduce the number of attributes)
  - Perform most restrictive selection and join operations before other similar operations
  - Some systems use only heuristics, others combine heuristics with partial cost-based optimization

启发式优化的主要步骤

- **Deconstruct** conjunctive selections into a sequence of single selection operations (Equiv. rule 1.)
- Move **selection operations** down the query tree for the earliest possible execution (Equiv. rules 2, 7a, 7b, 11)
- Execute first those **selection and join operations** that will produce the smallest relations (Equiv. rule 6)
- Replace Cartesian product operations that are followed by a selection condition by **join operations** (Equiv. rule 4a)
- Deconstruct and move as far down the tree as possible the lists of **projection** attributes, creating new projections where needed (Equiv. rules 3, 8a, 8b, 12)
- Identify those subtrees whose operations can be pipelined, and execute them using **pipelining**

作业

- 16.5 (第7版)
  - Canvas上提交，单个PDF文件
  - Deadline: 课堂完成

16.5 Consider the relations $r_1(A, B, C)$, $r_2(C, D, E)$, and $r_3(E, F)$, with primary keys A, C, and E, respectively. Assume that r_1 has 1000 tuples, r_2 has 1500 tuples, and r_3 has 750 tuples. Estimate the size of $r_1 \bowtie r_2 \bowtie r_3$, and give an efficient strategy for computing the join.