

20373067-张凯歌-数学建模作业3

$$\begin{aligned} \min \quad & 20x_1 + 90x_2 + 80x_3 + 70x_4 + 30x_5 \\ \text{s.t.} \quad & \begin{cases} x_1 + x_2 + x_5 \geq 30.5 \\ x_3 + x_4 \geq 30 \\ 3x_1 + 2x_3 \leq 120 \\ 3x_2 + 2x_4 + x_5 \leq 48 \\ x_i \geq 0 \end{cases} \end{aligned}$$

(1) 写程序求解线性规划问题

程序:

```
1 import cvxpy as cp
2 from scipy.optimize import linprog
3 from numpy import array
4
5 c = array([20, 90, 80, 70, 30])
6 a = array([
7     [-1, -1, 0, 0, -1],
8     [0, 0, -1, -1, 0],
9     [3, 0, 2, 0, 0],
10    [0, 3, 0, 2, 1]
11 ])
12 b = array([-30.5, -30, 120, 48])
13 x = cp.Variable(5, pos=True)
14 obj = cp.Minimize(c @ x)
15 cons = [a @ x <= b]
16 prob = cp.Problem(obj, cons)
17 prob.solve()
18
19 print("最优解为: ", x.value)
20 print("最优值为: ", prob.value)
21 print("a @ x = ", a @ x.value)
```

结果:

```
1 最优解为: [3.04999999e+01 5.07064437e-09 6.00000062e+00 2.39999994e+01
2    1.59416166e-07]
3 最优值为: 2770.000008274157
4 a @ x = [-30.50000003 -29.99999999 103.50000084 47.99999892]
```

gurobi求解

文件:

```
1 Minimize
2 20 x1 + 90 x2 + 80 x3 + 70 x4 + 30 x5
3 Subject To
4 x1 + x2 + x5 >= 30.5
5 x3 + x4 >= 30
6 3 x1 + 2 x3 <= 120
7 3 x2 + 2 x4 + x5 <= 48
8 Bounds
9 x1 >= 0
10 x2 >= 0
11 x3 >= 0
12 x4 >= 0
13 x5 >= 0
14 End
```

结果:

```
1 Restricted license - for non-production use only - expires 2023-10-25
2 Read LP format model from file hw1-1.lp
3 Reading time = 0.00 seconds
4 : 4 rows, 5 columns, 10 nonzeros
5 Gurobi Optimizer version 9.5.2 build v9.5.2rc0 (win64)
6   Bounds range      [0e+00, 0e+00]
7   RHS range        [3e+01, 1e+02]
8 Presolve time: 0.01s
9 Presolved: 4 rows, 5 columns, 10 nonzeros
10
11 Iteration    Objective      Primal Inf.    Dual Inf.      Time
12           0    0.000000e+00    6.050000e+01    0.000000e+00    0s
13           3    2.770000e+03    0.000000e+00    0.000000e+00    0s
14
15 Solved in 3 iterations and 0.01 seconds (0.00 work units)
16 optimal objective 2.770000000e+03
17 optimal objective: 2770
```

(2) 若变量条件加上 $x_i (i = 1, 2)$ 为整数, 求解

程序:

```
1 import cvxpy as cp
2 from scipy.optimize import linprog
3 from numpy import array
4
5 c = array([20, 90, 80, 70, 30])
6 a = array([
7     [-1, -1, 0, 0, -1],
8     [0, 0, -1, -1, 0],
9     [3, 0, 2, 0, 0],
10    [0, 3, 0, 2, 1]
```

```

11 ]])
12 b = array([-30.5, -30, 120, 48])
13 # x = cp.Variable(5, pos=True)
14 x1 = cp.Variable(2, integer=True)
15 x2 = cp.Variable(3, pos=True)
16 x = cp.hstack([x1, x2])
17 obj = cp.Minimize(c @ x)
18 cons = [a @ x <= b, x >= 0]
19 prob = cp.Problem(obj, cons)
20 prob.solve()
21
22 print("最优解为: ", x.value)
23 print("最优值为: ", prob.value)
24 print("a @ x = ", a@x.value)

```

结果:

```

1 最优解为: [30.   -0.    6.25 23.75  0.5 ]
2 最优值为: 2777.5
3 a @ x = [-30.5 -30.   102.5  48. ]

```

gurobi求解

文件

```

1 Minimize
2 20 x1 + 90 x2 + 80 x3 + 70 x4 + 30 x5
3 Subject To
4 x1 + x2 + x5 >= 30.5
5 x3 + x4 >= 30
6 3 x1 + 2 x3 <= 120
7 3 x2 + 2 x4 + x5 <= 48
8 Bounds
9 x1 >= 0
10 x2 >= 0
11 x3 >= 0
12 x4 >= 0
13 x5 >= 0
14 Integer x1 x2
15 End

```

结果

```

1 Restricted license - for non-production use only - expires 2023-10-25
2 Read LP format model from file hw1-2.lp
3 Reading time = 0.00 seconds
4 : 4 rows, 5 columns, 10 nonzeros
5 Gurobi Optimizer version 9.5.2 build v9.5.2rc0 (win64)
6 Thread count: 14 physical cores, 20 logical processors, using up to 20
  threads
7 Optimize a model with 4 rows, 5 columns and 10 nonzeros
8 Model fingerprint: 0x0cafa2d4
9 variable types: 3 continuous, 2 integer (0 binary)
10 Coefficient statistics:

```

```

11 Matrix range [1e+00, 3e+00]
12 Objective range [2e+01, 9e+01]
13 Bounds range [0e+00, 0e+00]
14 RHS range [3e+01, 1e+02]
15 Presolve removed 3 rows and 3 columns
16 Presolve time: 0.01s
17 Presolved: 1 rows, 2 columns, 2 nonzeros
18 Variable types: 0 continuous, 2 integer (0 binary)
19 Found heuristic solution: objective 2777.5000000
20
21 Root relaxation: cutoff, 0 iterations, 0.00 seconds (0.00 work units)
22
23 Explored 1 nodes (0 simplex iterations) in 0.01 seconds (0.00 work units)
24 Thread count was 20 (of 20 available processors)
25
26 Solution count 1: 2777.5
27
28 Optimal solution found (tolerance 1.00e-04)
29 Best objective 2.777500000000e+03, best bound 2.777500000000e+03, gap
0.0000%
30 Optimal objective: 2777.5

```

(3) 变量条件加上 $x_i (i = 1, 2, 3)$ 为整数且 x_3 是5的倍数, 求解

分析:

只要令 $x_1 = t_1, x_2 = t_2, x_3 = 5t_3, x_4 = t_4, x_5 = t_5$ 其中 $t_i (i = 1, 2, 3)$ 为整数 $t_i \geq 0$ 即可

程序:

```

1 import cvxpy as cp
2 from scipy.optimize import linprog
3 from numpy import array
4
5 c = array([20, 90, 400, 70, 30])
6 a = array([
7     [-1, -1, 0, 0, -1],
8     [0, 0, -5, -1, 0],
9     [3, 0, 10, 0, 0],
10    [0, 3, 0, 2, 1]
11 ])
12 b = array([-30.5, -30, 120, 48])
13 # x = cp.Variable(5, pos=True)
14 x1 = cp.Variable(3, integer=True)
15 x2 = cp.Variable(2, pos=True)
16 x = cp.hstack([x1, x2])
17 obj = cp.Minimize(c @ x)
18 cons = [a @ x <= b, x >= 0]
19 prob = cp.Problem(obj, cons)
20 prob.solve()
21
22 print("最优解为: ", x.value)
23 print("最优值为: ", prob.value)
24 print("a @ t = ", a@x.value)

```

结果:

```
1 最优解为: [30. -0.  2.  20.  0.5]
2 最优值为: 2815.0
3 a @ x = [-30.5 -30.  110.  40.5]
```

由程序求解得到的t得到最终解x为:

$$x_1 = 30, x_2 = 0, x_3 = 10, x_4 = 20, x_5 = 0.5$$

gurobi求解

文件:

```
1  Minimize
2  20 t1 + 90 t2 + 400 t3 + 70 t4 + 30 t5
3  Subject To
4  t1 + t2 + t5 >= 30.5
5  5 t3 + t4 >= 30
6  3 t1 + 10 t3 <= 120
7  3 t2 + 2 t4 + t5 <= 48
8  Bounds
9  t1 >= 0
10 t2 >= 0
11 t3 >= 0
12 t4 >= 0
13 t5 >= 0
14 Integer t1 t2 t3
15 End
```

结果:

```
1  Restricted license - for non-production use only - expires 2023-10-25
2  Read LP format model from file hw1-3.lp
3  Reading time = 0.00 seconds
4  : 4 rows, 5 columns, 10 nonzeros
5  Gurobi Optimizer version 9.5.2 build v9.5.2rc0 (win64)
6  Thread count: 14 physical cores, 20 logical processors, using up to 20
   threads
7  Optimize a model with 4 rows, 5 columns and 10 nonzeros
8  Model fingerprint: 0x3ae597d2
9  Variable types: 2 continuous, 3 integer (0 binary)
10 Root relaxation: cutoff, 0 iterations, 0.00 seconds (0.00 work units)
11
12 Explored 1 nodes (0 simplex iterations) in 0.01 seconds (0.00 work units)
13 Thread count was 20 (of 20 available processors)
14
15 Solution count 1: 2815
16
17 Optimal solution found (tolerance 1.00e-04)
18 Best objective 2.815000000000e+03, best bound 2.815000000000e+03, gap
   0.0000%
19 Optimal objective: 2815
```

