4 Electromagnetic waves in material

> Let's let everything move!

4.1 Displacement current

Conservation of charge

$$\nabla \cdot \mathbf{J_s} = -\frac{\partial \rho_f}{\partial t} \tag{1}$$

However, this is incompatible with

$$\nabla \times \mathbf{H} = \mathbf{J}_f \tag{2}$$

Lets take the divergence for both sides, we get

$$\nabla \cdot (\nabla \times \mathbf{H}) = 0 = \nabla \cdot \mathbf{J} + \frac{\partial}{\partial t} \underbrace{\nabla \cdot \mathbf{D}}_{\rho_\mathbf{f}} \tag{3}$$

$\Rightarrow$ add an additional term to the current density $\mathbf{J_f} = \nabla \times \mathbf{H}$

$$\Rightarrow \nabla \times \mathbf{H} = \mathbf{J_f} + \underbrace{\frac{\partial \mathbf{D}}{\partial t}}_{\text{displacement current}} \tag{4}$$

LHS: $\nabla \times \left(\frac{\mathbf{B}}{\mu_\mathbf{0}} - \mathbf{M}\right) = \frac{1}{\mu_0} \nabla \times \mathbf{B} - \underbrace{\nabla \times \mathbf{M}}_{\mathbf{J_b}}$

Where $\mathbf{J_b}$ is the bound current density.

RHS $= \mathbf{J_s} + \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J_s} + \frac{\partial \mathbf{P}}{\partial t} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$

$\Rightarrow$ we could write $\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J_f} + \mathbf{J_b} + \mathbf{J_p}\right) + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$

where J_p is the polarization current density. which equals $\frac{\partial \mathbf{P}}{\partial t}$

Note that $\nabla \cdot \mathbf{J_p} = -\frac{\partial \rho_p}{\partial t}$ from conservation of charge.

Thus the polarization current responds to changes to bound charge, and hence in $\mathbf{P}$

4.2 Maxwell's equations in insulating linear dielectrics

Since it is insulating linear dielectrics, we have $\mathbf{J}_f = 0$ and $\mathbf{J}_b = 0$

Hence, we could get Maxwell's equation

$$\begin{aligned} \nabla \cdot \mathbf{D} = 0 \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \end{aligned} \tag{5}$$

remember, $\mathbf{D} = \epsilon_0 \epsilon_r \mathbf{E} + \mathbf{P}$ and $\mathbf{B} = \mu_0 \mu_r \mathbf{H}$

which gives

$$\nabla \cdot \mathbf{E} = 0 \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \mu_0\mu_r\epsilon_0\epsilon_r\frac{\partial \mathbf{E}}{\partial t} \tag{6}$$

Consider

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2\mathbf{E} = -\frac{\partial}{\partial t}\nabla \times \mathbf{B} \tag{7}$$

$$\Rightarrow \nabla^2\mathbf{E} = \underbrace{\mu_0\mu_r\epsilon_0\epsilon_r}_{\frac{1}{v^2}}\frac{\partial^2\mathbf{E}}{\partial t^2} \tag{8}$$

which is wave equation.

$\Rightarrow v = \frac{c}{n}$ where c = $\frac{1}{\sqrt{\epsilon_0\mu_0}}$ and n = $\sqrt{\epsilon_r\mu_r}$ where n is also called *refractive index*

Plane waves solutions

Lets choose propagation parallel to z, and hence
$\frac{\partial \mathbf{\Psi}}{\partial \mathbf{x}} = \frac{\partial \mathbf{\Psi}}{\partial \mathbf{y}} = 0$

remember that $\nabla \cdot \mathbf{E} = 0 \Rightarrow \frac{\partial E_z}{\partial z} = 0$
similarly, $\nabla \cdot \mathbf{B} = 0 \Rightarrow \frac{\partial B_z}{\partial z} = 0$

we also have $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \Rightarrow \frac{\partial B_z}{\partial t} = 0$

And $\nabla \times \mathbf{B} = \mu_0\mu_r\epsilon_0\epsilon_r\frac{\partial \mathbf{E}}{\partial t} \Rightarrow \frac{\partial E_z}{\partial t} = 0$

Hence, E_z and B_z are constant in z and t, they are not part of wave motion

now analyze the x,y components of curl:

$$-\frac{\partial E_y}{\partial z} = -\frac{\partial B_x}{\partial t}, -\frac{\partial B_y}{\partial z} = \frac{1}{v^2}\frac{\partial E_x}{\partial t} \tag{9}$$

$\Rightarrow E_x, B_y$ are solutions

Lets then take

$$\frac{\partial^2 E_x}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 E_x}{\partial t^2} \tag{10}$$

$$\Rightarrow E_x(z,t) = E_{x0}e^{i(\pm kz - \omega t)}\hat{\mathbf{x}} \tag{11}$$

Then, we could have

$$\Rightarrow B(z,t) = B_0e^{i(\pm kz - \omega t)}\hat{\mathbf{y}} \tag{12}$$

And then we could get the wave travelling in $\pm\mathbf{z}$ direction

$$\begin{aligned} \Rightarrow \mp kE_0 &= -\omega B_0 \\ \text{and } \pm kB_0 &= \frac{\omega}{v^2}E_0 \\ \Rightarrow \frac{E_0}{B_0} &= \pm\frac{\omega}{k} = \pm v \end{aligned}$$

Define Impedance Z as

$$Z = \left|\frac{E_0}{H_0}\right| = \sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r}} \tag{13}$$

remember that $H_0 = \frac{B_0}{\mu_0 \mu_r}$

The motivation of doing so is that $v = -\int \mathbf{E} \cdot d\mathbf{l}$ and $I = \oint \mathbf{H} \cdot d\mathbf{l}$

So dimension would work

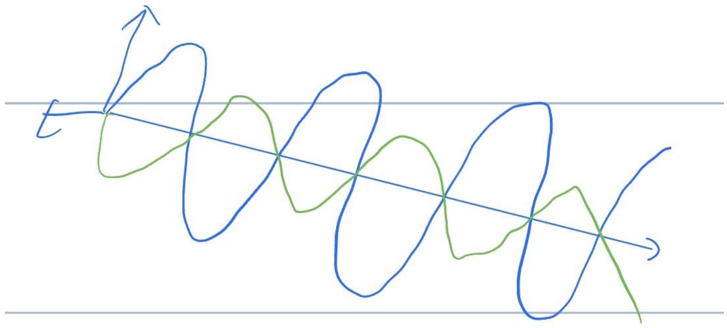
For free space, then, $\epsilon_r = \mu_r = 1$ and $Z = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\Omega$

Remember that $\nabla \times \mathbf{E} = -\dot{\mathbf{B}}$, and use E, B $\propto e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$, we could get

$$\begin{aligned} i\mathbf{k} \times \mathbf{E} &= -(-i\omega)\mathbf{B} \\ &= i\omega\mu_0\mu_r\mathbf{H} \\ \Rightarrow Z = \left|\frac{\mathbf{E}}{\mathbf{H}}\right| &= \sqrt{\frac{\mu_0\mu_r\omega}{k}} \end{aligned}$$

Which gives the same answer because $v = \frac{c}{n} = \frac{\omega}{k} = \frac{1}{\sqrt{\epsilon_0\epsilon_r\mu_0\mu_r}}$

which is this wave



4.3 conductors

> Remember that
> $\nabla \cdot \mathbf{J_f} = -\frac{\partial \rho_f}{\partial t}$
> $\nabla \cdot \mathbf{J_p} = -\frac{\partial \rho_p}{\partial t}$
> $\nabla \cdot \mathbf{J_b} = 0$
> The last one is 0 because $\nabla \cdot (\nabla \times \mathbf{M}) = 0$

For conductors, we have

$\rho_f = 0$ since there are no free charges in equilibrium
$\mathbf{J}_f = \sigma\mathbf{E}$ from Ohm's law where σ is the conductivity
$\mathbf{D} = \epsilon_0\epsilon_r\mathbf{E}$ and $\mathbf{B} = \mu_0\mu_r\mathbf{H}$ from linearity

Then we could get Maxwell's equation in conductors

$\nabla \cdot \mathbf{E} = 0$ $\qquad\qquad$ $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

$\nabla \cdot \mathbf{B} = 0$ $\qquad$ $\nabla \times \mathbf{B} = \underbrace{\mu_0\mu_r\sigma\mathbf{E}}_{\text{conduction } \mathbf{J}} + \underbrace{\mu_0\mu_r\sigma_0\sigma_r\frac{\partial \mathbf{E}}{\partial t}}_{\text{dispalcement } \mathbf{J}}$

Free charge will decay to zero in a short time τ, and it is easy to prove (said Blundell)

$\nabla \cdot \mathbf{J} = \frac{\partial \rho}{\partial t}$

Where $\nabla \cdot \mathbf{J}$ is equal to $\sigma \cdot \nabla \cdot \mathbf{E}$ from Ohm's law

and $\nabla \cdot \mathbf{E}$ is equal to $\frac{\rho}{\epsilon_0\epsilon_r}$ from Gauss's law

$\Rightarrow \rho(f) = \rho(0)e^{-\frac{t}{\tau}}$

Where $\tau = \frac{\epsilon_0 \epsilon_r}{\sigma}$

If the metal has great conductivity, then τ is very small, and hence $\rho(f)$ is very small.

Let's consider the electromagnetic wave having frequency ω, so we would like to compare $\frac{1}{\omega}$ with τ:

Condition	Conductor Type	Charge Response	Conductivity
$\omega\tau \ll 1$	Good conductor	Charges respond very quickly	$\sigma \gg \epsilon_0\epsilon_r\omega$ Conduction current dominates
$\omega\tau \gg 1$	Bad conductor	Charges respond very slowly	$\sigma \ll \epsilon_0\epsilon_r\omega$ Displacement current dominates

Take real life examples

	$\sigma(\Omega m)$	ϵ_r	$\frac{\sigma}{\epsilon_0\epsilon_r}(S^{-1})$
metal	10^7	1	10^{19}
Silicon	$4 \cdot 10^{-4}$	11.7	10^7
Glass	10^{-10}	5	10

Note that visible light would have frequency ~ $5 \cdot 10^{14}$ Hz

Let's now do some electromagnetism

$$\begin{aligned} \nabla \times (\nabla \times \mathbf{E}) &= \underbrace{\nabla(\nabla \cdot \mathbf{E})}_{0} - \nabla^2 \mathbf{E} \\ &= -\frac{\partial}{\partial t} \nabla \times \mathbf{B} \\ \nabla^2 \mathbf{E} &= \mu_0 \mu_r \sigma \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mu_r \sigma_0 \sigma_r \frac{\partial^2 \mathbf{E}}{\partial t^2} \end{aligned}$$

Again, this would yield transverse plane waves with $\mathbf{E}, \mathbf{B} \perp$ to each others

$\Rightarrow \mathbf{E} = \mathbf{E}_0 \hat{\mathbf{x}} e^{i(\widetilde{\mathbf{k}} \cdot \mathbf{z} - \omega t)}$

with $\widetilde{\mathbf{k}} = \mathbf{k} + i\kappa$

$\Rightarrow \widetilde{k}^2 = i\underbrace{\mu_0\mu_r\sigma\omega}_{\mu} + \underbrace{\mu_0\mu_r}_{\mu}\underbrace{\epsilon_0\epsilon_r}_{\epsilon}\omega^2$

$$
\begin{aligned}
(k+i\kappa)^2 &= k^2-\kappa^2+2ik\kappa \\
k^2-\kappa^2 &= \mu_0\epsilon_0\omega^2 \\
2k\kappa &= \mu\sigma\omega \Rightarrow k=\frac{\mu\sigma\omega}{2\kappa} \\
0 &= \left(\frac{\mu\sigma\omega}{2}\right)^2\frac{1}{\kappa^2}-\kappa^2-\mu\epsilon_0\omega^2 \\
0 &= (\kappa^2)^2+\mu\epsilon\omega^2(\kappa^2)-\left(\frac{\mu\sigma\omega}{2}\right)^2 \\
\kappa^2 &= -\frac{\mu\epsilon\omega^2}{2}\pm\sqrt{\left(\frac{\mu\epsilon\omega^2}{2}\right)^2+\left(\frac{\mu\sigma\omega}{2}\right)^2} \\
\kappa^2 &= \frac{\mu\epsilon\omega^2}{2}\left[\pm\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}-1\right]
\end{aligned}
$$

Taking the positive root

$\Rightarrow \kappa=\sqrt{\frac{\mu\epsilon}{2}}\omega\sqrt{\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}-1}$

Sub into original equation

$k=\frac{\mu\sigma\omega}{2\kappa}=\sqrt{\frac{\mu\epsilon}{2}}\omega\sqrt{\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}+1}$

$\Rightarrow \mathbf{E}=\mathbf{E}_0\hat{\mathbf{x}}\underbrace{e^{-\kappa z}}_{e^{-\frac{z}{\delta}}}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$

Where $\delta=\frac{1}{\kappa}$ is the *skin depth*

Reminder. Good conductors have $\sigma\gg\epsilon\omega$

$k=\kappa=\sqrt{\frac{\mu\epsilon}{2}}\omega\sqrt{\frac{\sigma}{\epsilon\omega}}=\sqrt{\frac{\mu\omega\sigma}{2}}$

We could therefore have

$\nabla^2\mathbf{E}=\mu\sigma\frac{\partial\mathbf{E}}{\partial t}+\mu\epsilon\frac{\partial^2\mathbf{E}}{\partial t^2}$

We could also neglect the last term, since $\frac{\partial^2\mathbf{E}}{\partial t^2}\ll\frac{\partial\mathbf{E}}{\partial t}$

$$
\begin{aligned}
&\Rightarrow \widetilde{k}^2=i\mu\sigma\omega \\
&\Rightarrow \widetilde{k}=\frac{1+i}{\sqrt{2}}\sqrt{\mu\sigma\omega} \\
&\Rightarrow \widetilde{k}=\kappa=\sqrt{\frac{\mu\sigma\omega}{2}}
\end{aligned}
$$

Hence, we could have

$\delta=\frac{1}{k}=\sqrt{\frac{2}{\mu\sigma\omega}}$

For a typical metal, δ is

$$
\begin{cases}
\text{few nm - visible light} \\
\text{few}\mu m\text{ - microwave} \\
\text{few mm - radio waves}
\end{cases}
$$

Lets go to poor conductors

Poor conductors has $\sigma\ll\epsilon\omega$, hence

$k\approx\sqrt{\frac{\mu\epsilon}{2}}\omega\sqrt{2}=\sqrt{\mu\epsilon}\omega$

$\kappa=\sqrt{\frac{\mu\epsilon}{2}}\omega\left(1+\frac{1}{2}\left(\frac{\sigma}{\epsilon\omega}\right)^2+\dots-1\right)^{\frac{1}{2}}$

which equals to $\frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$ which is independent to ω

In an insulating dielectric, $\sigma = 0$, hence $\kappa = 0$, and hence $k = \frac{\omega}{v}$ as expected.

Lets return to previous equation

Lets consider the curl equation in the conductor

$$\frac{\partial \mathbf{E}_x}{\partial z} = \frac{\partial B_y}{\partial t}$$
$$i(k+k)\mathbf{E}_0 = i\omega \mathbf{B}_0$$
$$z = \frac{\mu E_0}{B_0} = \frac{\mu\omega}{k+i\kappa}$$
$$\widetilde{k} = \frac{\mu\omega}{\sqrt{k^2+\kappa^2}}e^{i\phi}$$

If we expand, ϕ would be

$\phi = \tan^{-1}\left(\frac{\sqrt{1+(Q/\epsilon\omega)^2}-1}{\sqrt{1+(Q/\epsilon\omega)^2}+1}\right)^{\frac{1}{2}}$

For a good conductor, $\sigma \gg \epsilon\omega$, hence $\phi \to \tan^{-1} 1 = \frac{\pi}{4}$

So this means that B lags behind E in a metal

4.4 Poynting vectors

Work done on charge

$\delta q = \rho = \delta\tau$

$$\delta F = \delta q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$
$$\delta \mathbf{F} \cdot \mathbf{dl} = \delta q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \mathbf{v}\delta t$$
$$= \mathbf{E} \cdot \mathbf{J_f}\delta\tau\delta t$$

where $\mathbf{J}_f$ eaquals to $\rho\mathbf{v}$

Rate of work on charges

$$\mathbf{F} = \frac{dw}{dt}$$
$$= \mathbf{E} \cdot \mathbf{J_f} d\tau$$
$$= \frac{d}{dt}\int \underbrace{u_{mech}}_{\text{Energy density}} d\tau$$

From Maxwell's equation:
$\mathbf{J}_f = \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t}$
We have

$\mathbf{E} \cdot \mathbf{J_f} = \mathbf{E} \cdot \nabla \times \mathbf{H} - \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial \mathbf{t}}$

By dotting everything, and then

$\nabla \cdot (\mathbf{E} \times \mathbf{H}) = \mathbf{H} \cdot \nabla \times \mathbf{E} - \mathbf{E} \cdot \nabla \times \mathbf{H}$
$\mathbf{E} \cdot \mathbf{J_f} = \mathbf{H} \cdot \underbrace{\nabla \times \mathbf{E}}_{-\frac{\partial \mathbf{B}}{\partial t}} - \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} - \nabla \cdot (\mathbf{E} \times \mathbf{H})$

Where we call $\mathbf{H} \cdot \underbrace{\nabla \times \mathbf{E}}_{-\frac{\partial \mathbf{B}}{\partial t}} - \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t}$ "$\frac{\partial}{\partial t} u_{EM}$" (remember that $u_{EM} = \frac{1}{2}(\mathbf{B} \cdot \mathbf{H} + \mathbf{E} \cdot \mathbf{D})$ whcih equals to Energy stored in EM field per unit volume) and $\nabla \cdot (\mathbf{E} \times \mathbf{H})$ as "$\mathbf{S} = \mathbf{E} \times \mathbf{H}$" or poyting vector.

> How is this working?

Assume that we are using a linear media:

$$\mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t}=\frac{1}{2}\mathbf{E}\cdot\frac{\partial(\mathbf{E}\cdot\mathbf{D})}{\partial t}$$
$$\mathbf{H}\cdot\frac{\partial\mathbf{B}}{\partial t}=\frac{1}{2}\mathbf{H}\cdot\frac{\partial(\mathbf{B}\cdot\mathbf{H})}{\partial t}$$

Remember that

$$\left.\begin{aligned}\frac{1}{2}\epsilon_0E^2&=\frac{1}{2}DE\\ \frac{B^2}{2\mu_0}&=\frac{1}{2}BH\end{aligned}\right\}\text{In free space}$$

Bringing everything together, we could get

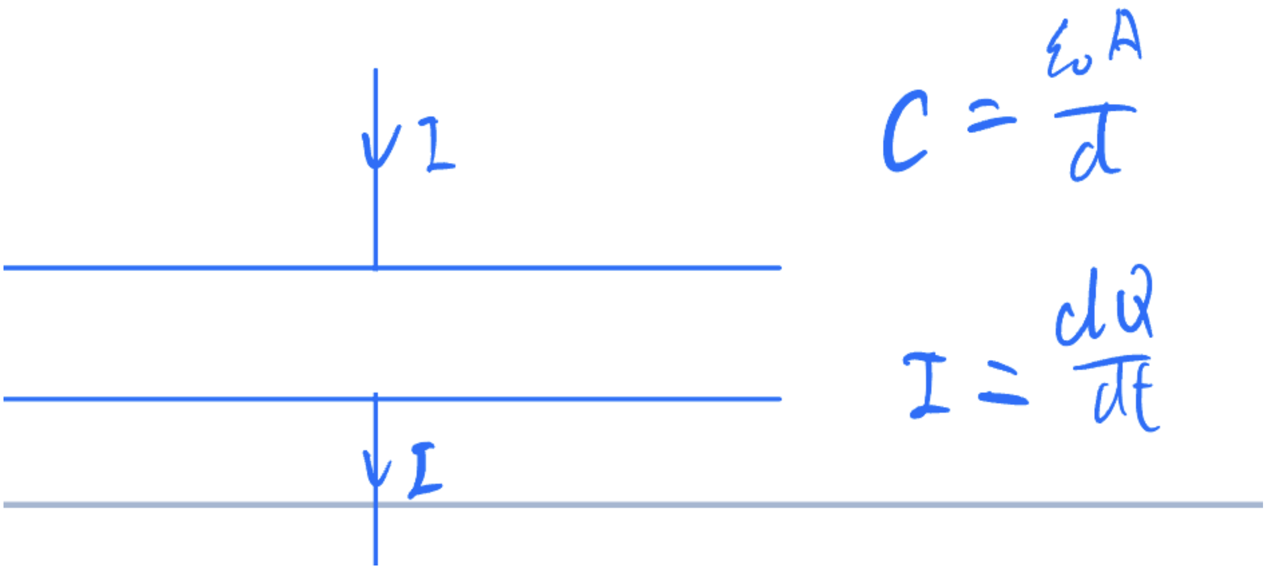
$\Rightarrow \frac{d}{dt}(u_{\text{mech}}+u_{\text{EM}})+\nabla\cdot\mathbf{S}=0$

Where $\mathbf{S}$ is the poyting vector, or equiviantly,

$\frac{d}{dt}\int(u_{\text{mech}}+u_{\text{EM}})d\tau+\oint\mathbf{S}\cdot d\mathbf{a}=0$

We could say that, therefore $\mathbf{S}$ is the energy flux density, or the rate of flow of energy per unit area in the direction of S.

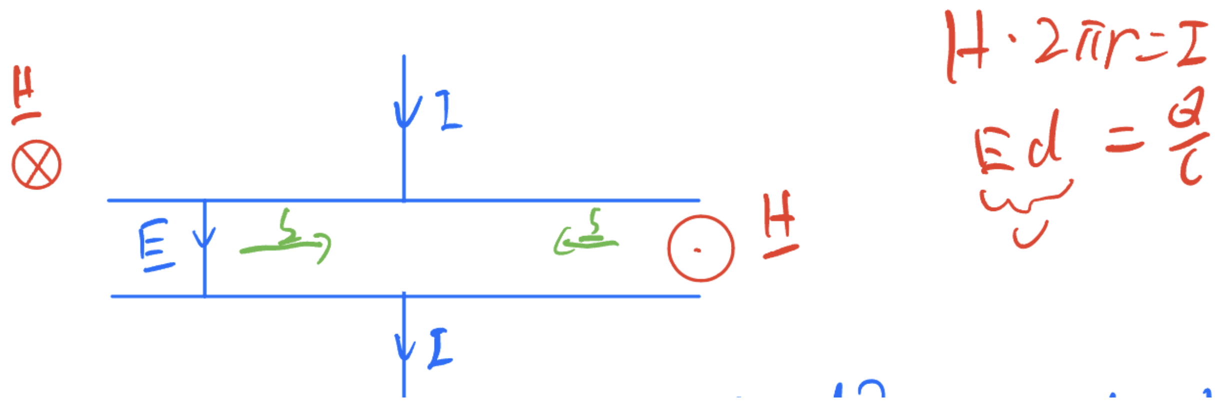
Example: a capacitor



The stored energy increase at rate

$\dot{U}=\frac{Q}{C}\frac{dQ}{dt}\quad U=\frac{Q^2}{2C}$

also:

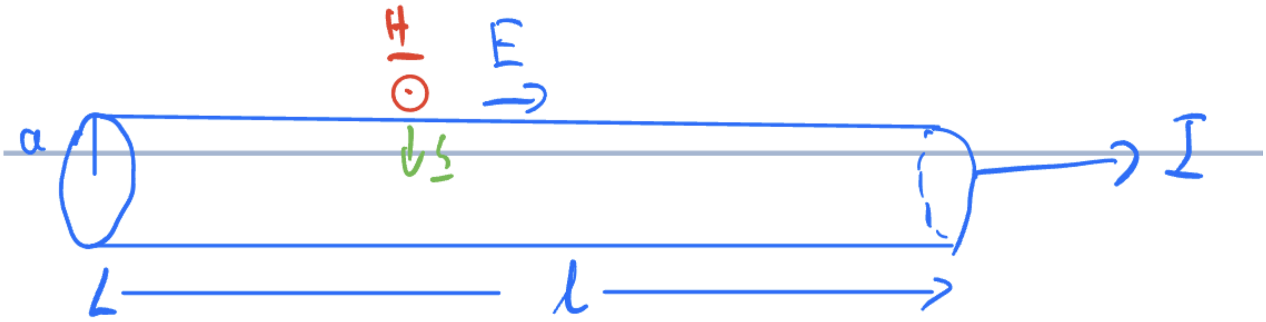


Hence, we have

$\dot{U}=S\cdot2\pi rd=\frac{Q}{c}\frac{dQ}{dt}$

where $S=EH=\frac{Q}{dc}\frac{\frac{dQ}{dt}}{2\pi r}$

There is another example



$$H \cdot S\pi a = I$$
$$E = \frac{V}{l}$$
$$\Rightarrow S = \frac{V}{l}\frac{I}{2\pi a}$$
$$\int \mathbf{S} \cdot \mathbf{da} = -IV = -I^2R$$

4.5 Radiation pressure

EM waves are made up of photons, and hecne they have momentum

$E = pc$

$\Rightarrow$ Transport of energy is appoinated by transport of momentum

$P_{rad} = \frac{<S>}{c}$

For a perfect absorber, where P_{rad} is the radiation pressure

Example For a plane EM wave in free space, we have

$$U = \frac{1}{2}\epsilon_0 E^2 + frac12\frac{B^2}{\mu_0}$$
$$\text{but } E = cB$$
$$\Rightarrow U = \epsilon_0 E^2$$
$$\mathbf{E} = \frac{1}{2}E_0\cos(kz - \omega t)\hat{\mathbf{x}}$$
$$= E_0\cos^2(kz - \omega t)$$
$$< E^2 > = \frac{1}{2}E_0^2$$
$$\Rightarrow < u > = \frac{1}{2}\epsilon_0 E_0^2$$
$$< S > = \frac{1}{2}\epsilon_0 E_0^2 c = I$$

Where I is the intensity of wave

$$\Rightarrow P_rad = \begin{cases} \frac{1}{2}\epsilon_0 E_0^2 & \text{perfect absorber} \\ \epsilon_0 E_0^2 & \text{perfect reflector} \end{cases}$$

Sunlight: I~$1kWm^{-2}$
$\Rightarrow P_{rad} = 10^{-5}Pa$
FYI, $P_{atm} = 10^5 Pa$

Example Consider a star which is growing by accretion

i.e. matter is falling onto it uniformly in all directions

The star has luminosity L (e.g. $L_{sun} = 4 \times 10^{26}$w)

Energy flux = $\frac{L}{4\pi R^2}[Wm^{-2}]$

Radiation pressure: $\frac{L}{4\pi R^2 C}$

Outward:

Force/unit mass = $k\frac{L}{4\pi R^2 C}$ where k is the opacity, which is area/unit mass, which is a constant

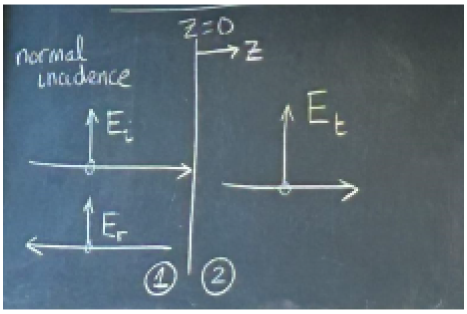
Inward: Force due to gravity/unit mass = $\frac{GM}{R^2}$

Since they balance, we have

$\Rightarrow L = \frac{4\pi CGM}{k}$

which is called Eddington Limit (Upper limit of luminosity of stars that accrete (isotopically))

4.6 EM waves - reflection and refraction



Left:
$E_i e^{i(k_1 z - \omega t)} + E_r e^{i(-k_1 z - \omega t)}$

Right:
$E_t e^{i(k_2 z - \omega t)}$

Using electromagnetic boundary conditions, we could get

$E^{\parallel}$ is continuous
$E_i^{\parallel} + E_r^{\parallel} = E_t^{\parallel}$

$H^{\parallel}$ is continuous
$\frac{E_i}{Z_1} - \frac{E_r}{Z_1} = \frac{E_t}{Z_2}$

Putting two equations together

$\frac{E_r}{E_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \quad \frac{E_t}{E_i} = \frac{2Z_2}{Z_2 + Z_1}$

Where $Z = \sqrt{\frac{\mu}{\epsilon}}$

|Poyting vector| $= S = |\mathbf{E} \times \mathbf{H}| = \frac{E^2}{Z}$

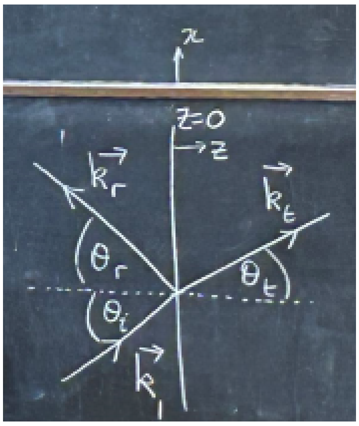
We expect $S_{\text{incident}} = S_{\text{reflected}} + S_{\text{transmitted}}$

Where they equal to

$\frac{E_i^2}{Z_1} + \frac{E_r^2}{Z_1} = \frac{E_t^2}{Z_2}$

separately

Lets now have angles
$\mathbf{E}_r e^{i(\mathbf{k_r} \cdot \mathbf{r} - \omega t)}$



$\mathbf{E_i} e^{i(\mathbf{k_i}\cdot\mathbf{r}-\omega t)}$

$\mathbf{E}_t e^{i(\mathbf{k_t}\cdot\mathbf{r}-\omega t)}$

Choose $\mathbf{k}_i$ in x-z plane

At z = 0, $E_{\parallel}$ is continuous and this holds for all x y and t
$\Rightarrow \omega$ must be the same

$\Rightarrow \mathbf{k_i}\cdot\mathbf{r} = \mathbf{k_r}\cdot\mathbf{r} = \mathbf{k_t}\cdot\mathbf{r}$ for all x, y at z=0

Take $\mathbf{r} = (0, y, 0)$

$\Rightarrow \mathbf{k_i}, \mathbf{k_r}$ and $\mathbf{k}_t$ all lie iin the xz plane
(*the plane of incidence*)

Take $\mathbf{r} = (x, 0, 0)$ so $\mathbf{k_i}\cdot\mathbf{r} = k\sin\theta_x$

$$|\mathbf{k_i}| = \mathbf{k_r}| = k_1$$
$$|\mathbf{k_t}| = k_2$$

$$\Rightarrow \underbrace{k_1\sin\theta_i = k_1\sin\theta_r}_{\theta_i=\theta_r,\text{ law of reflection}} = k_2\sin\theta_t$$

Remember that $\frac{\omega}{k} = \frac{c}{n}$, And the last two would lead to

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{k_1}{k_2} = \frac{n_2}{n_1} \quad \text{law of refraction, or snell's law}$$

Where $n = \sqrt{\epsilon_r\mu_r}$

Fresnel equations

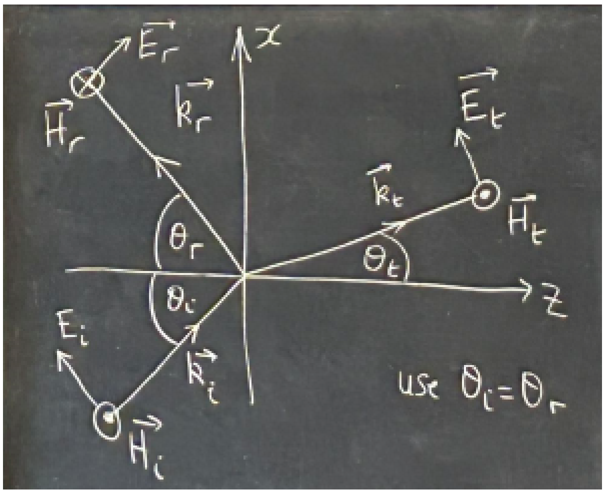
Worring about polarization directions

We work in those steps

1. **$\mathbf{E}$ in the plane of incidence**

> "parallel-like" = parallel

Remid that $\mathbf{E}, \mathbf{H}$ and $\mathbf{k}$ form a right-handed system



	incident	reflected	transmitted
E_x	$E_i \cos\theta_i$	$E_r \cos\theta_r$	$E_t \cos\theta_t$
E_z	$-E_i \sin\theta_i$	$E_r \cos\theta_r$	$-E_t \sin\theta_t$
H_y	$\frac{E_i}{Z_1}$	$-\frac{E_r}{Z_1}$	$\frac{E_t}{Z_2}$

$E_\parallel$ continuous $\Rightarrow$ E_x continuous $\Rightarrow$ $E_i \cos\theta_i + E_r \cos\theta_i = E_t \cos\theta_t$

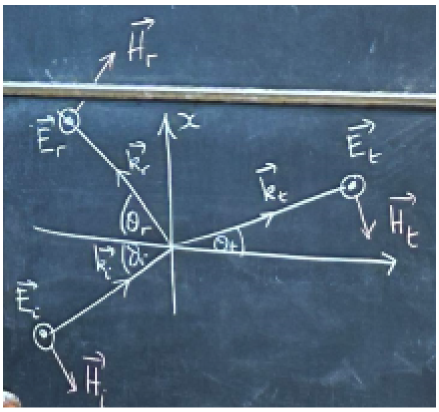
$$\frac{E_r}{E_i} = \frac{Z_2 \cos\theta_i - Z_1 \cos\theta_i}{Z_2 \cos\theta_1 + Z_1 \cos\theta_i}$$

$$\frac{E_t}{E_i} = \frac{2Z_2 \cos\theta_i}{Z_2 \cos\theta_i + Z_1 \cos\theta_i}$$

Now look for Fresnel equations for p-polarizations

2. $\mathbf{E}$ **perpendicular to the plane of incidence**

> "s-like" s = senkrecht = perpendicular



	incident	reflected	transmitted
E_y	E_i	E_r	E_t
H_x	$-\frac{E_i}{Z_1}\cos\theta_i$	$\frac{E_r}{Z_1}\cos\theta_r$	$-\frac{E_t}{Z_2}\cos\theta_t$
H_z	$\frac{E_i}{Z_1}\sin\theta_i$	$\frac{E_r}{Z_1}\sin\theta_r$	$\frac{E_t}{Z_2}\sin\theta_t$

$E_{||}$ continuous
$\Rightarrow E_y \text{continuous} \quad E_i + E_r = E_t$
$H_{||}$ continuous
$\Rightarrow H_x continuous \quad -\frac{E_i}{Z_1}\cos\theta_i + \frac{E_r}{Z_1}\cos\theta_r = -\frac{E_t}{Z_2}\cos\theta_t$

Remember that

$Z = \sqrt{\frac{\mu_r\mu_0}{\epsilon_r\epsilon_0}} = \frac{Z_0}{n} \quad n = \sqrt{\epsilon_r}\text{while } \mu_r = 1$

Let's set $\mu_r = 1$
Then, $Z = \sqrt{\frac{\mu_r\mu_0}{\epsilon_r\epsilon_0}} = \frac{Z_0}{n} \quad n = \sqrt{\epsilon_r}$

So we can replace Z_i with $\frac{1}{n_i}$ in expressions involving ratios of Z's.

e.g. Fresnel equations for p-polarization

$r = \frac{E_r}{E_i} = \frac{\frac{1}{n_2}\cos\theta_t - \frac{1}{n_1}\cos\theta_i}{\frac{1}{n_2}\cos\theta_t + \frac{1}{n_1}\cos\theta_i}$
$= \frac{n_1\cos\theta_t - n_2\cos_t heta_i}{n_1\cos\theta_t + n_2\cos\theta_i}$

Use Snell's law

$$n_1\sin\theta_i = n_2\sin\theta_t$$
$$\Rightarrow r = \frac{\sin 2\theta_t - \sin 2\theta_i}{\sin 2\theta_t + \sin 2\theta_i}$$
$$t = \frac{4\sin_t heta_t\cos\theta_i}{\sin 2\theta_t + \sin 2\theta_i}$$

For s-polarization, we have

$$r = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)}$$
$$t = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_t + \theta_i)}$$

We also have

$$n_1\sin\theta_i = n_2\sin\theta_t$$
$$\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i$$
$$\cos\theta_t = \sqrt{1 - \sin^2\theta_t} = \sqrt{1 - \frac{n_1^2}{n_2^2}\sin^2\theta_i}$$

Fresnel equations:

/ $n_1\cos\theta_i$ on top and bottom

$$\alpha = \frac{\cos\theta_t}{\cos\theta_i} = \frac{1}{\cos\theta_i}\sqrt{1 - (\frac{n_1}{n_2}\sin\theta_i)^2}$$
$$\beta = \frac{n_2}{n_1}$$

	$E^{\parallel}$**(p)**	$E^{\perp}$**(s)**
r	$\frac{\alpha-\beta}{\alpha+\beta}$	$\frac{1-\alpha\beta}{1+\alpha\beta}$
t	$\frac{2\alpha}{\alpha+\beta}$	$\frac{2}{1+\alpha\beta}$

Remember, EM waves have an energy flux given by

$S = |\mathbf{E}\times\mathbf{H}| = \frac{E^2}{Z}$

Intensify coefficients

$$T = \frac{I_r}{I_i} = |r|^2 = \begin{cases} (\frac{\alpha-\beta}{\alpha+\beta})^2 & (p) \\ (\frac{1-\alpha\beta}{1+\alpha\beta})^2 & (s) \end{cases}$$

$T = \frac{I_t}{I_i} = |t|^2 \frac{n_2}{n_1} \frac{\cos\theta_t}{cos\theta_i}$

where $\frac{n_2}{n_1}$ is due to waves at ifferent speeds. And $\frac{\cos\theta_t}{\cos\theta_i}$ is due to the wavesfronts at different angles.

$$T = |t|^2\alpha\beta = \begin{cases} \alpha\beta(\frac{2}{\alpha+\beta})^2 & (p) \\ \alpha\beta(\frac{2}{1+\alpha\beta})^2 & (s) \end{cases}$$

Lets check in certain cases

- $\theta_i = 0 \Rightarrow \alpha = 1$

$$r_s = r_p = \frac{1-\beta}{1+\beta} = \frac{n_1 - n_2}{n_1 + n_2}$$
$$t_s = t_p = \frac{2}{1+\beta} = \frac{2n_1}{n_1 + n_2}$$
$$R_s = R_p = (\frac{n_1 - n_2}{n_1 + n_2})^2$$

$$T_s = T_p - \frac{n_2}{n_1} \frac{4n_1^2}{(n_1+n_2)^2} = \frac{4n_1 n_2}{(n_1+n_2)^2}$$

Example: air/glass interface

$n_1 = 1 \quad n_2 = 1.5$

$$r_s = r_p = -0.2$$
$$T_s = T_p = 0.8$$
$$R_s = R_p = 0.04$$
$$T_s = T_p = 0.96$$

$\Rightarrow$ 4% of light is reflected, 96% is transmitted

[If n_2, for example, = 1.75, $R_s = R_p = 0.074$, which is sa problem]

Lets take another go at differnet angle

- $\theta_i = 90^\circ$, and set $\beta > 1$

$\sin\theta_i = 1 \quad \cos\theta_i = 0$

$\Rightarrow \alpha \to \infty$

$r_p = 1 \quad r_s = -1$

$t_p = 0 \quad t_s = 0$

- now consider $\beta < 1$, we cna have total interal reflection for $\theta_i > \theta_c$ where θ_c is the critical angle = $\beta = \frac{n_2}{n_1}$

At $\theta_c = \sin^{-1}\beta$, $\alpha = \frac{1}{\cos\theta_c}\sqrt{1 - (\frac{\sin\theta_c}{\beta})^2} = 0$

Hence, we have

$r_p = -1 \quad r_s = 1$

$t_p = \frac{2}{\beta} \quad t_s = 2$

$\sin\theta_c = \beta$

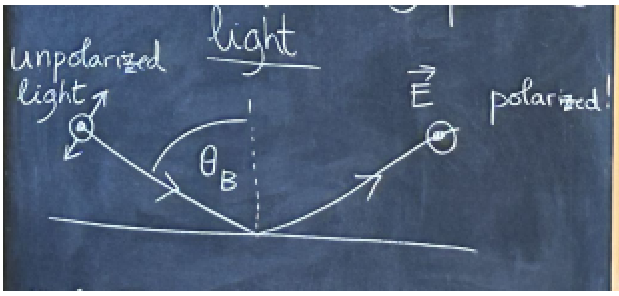
r_p vanishes at the certain angle called **"Brewster's angle"** θ_B

$r_p = \frac{\alpha-\beta}{\alpha+\beta} = 0$ when $\alpha = \beta = \frac{n_2}{n_1}$

$$\frac{1}{cos\theta_B}\sqrt{1-(\frac{\sin\theta_B}{\beta})^2} = \beta$$

$$\text{square both sides:}$$
$$\sec^2\theta_B - \frac{1}{\beta^2}\tan^2\theta_B = \beta^2$$
$$(1-\frac{1}{\beta^2})\tan^2\theta_B = \beta^2 - 1$$
$$\frac{(\beta^2-1)}{\beta^2}\tan^2\theta_B = (\beta^2-1)$$
$$\Rightarrow \tan\theta_B = \beta$$

Method of producing polarized light



Reflectted light polarized with $\mathbf{E}$ perpendicular to the plane of incidence

Polarizing sunglasses with transmission axis vertical reduce glare because reflected light is mainly horizontally polarized.

Total internal reflection

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$
$$\sin\theta_t = \frac{\sin\theta_i}{n_2/n_1}$$

Since $n_2 < n_1$, there will be total internal reflection when $\sin\theta_t > 1$

For transmitted wave, its like

- $e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$ in medium
  = $e^{i(k_2[x\sin\theta_t + z\cos\theta_t]-\omega t)}$
  since $z\cos\theta t = 1-\sin^2\theta_t$ = $\underbrace{e^{i(k_2 x\sin\theta_t-\omega_t)}}_{wave}\underbrace{e^{-(\sin^2\theta_t-1)^{\frac{1}{2}}k_2 z}}_{e^{-z/\delta}}$ where $\delta = \frac{\lambda_2}{2\pi}\frac{1}{(\sin^2\theta_t-1)^{\frac{1}{2}}}$

  This is called an *evanescent wave*

Lets now consider the **Plane travelling wave**

$$E_x = E_0 e^{i(kz-\omega t)}$$

$$\nabla \times \mathbf{E} = -\dot{\mathbf{B}} = i\omega B$$

$$\mathbf{B} = \frac{1}{i\omega}\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ \partial x & \partial y & \partial z \\ E_x & 0 & 0 \end{vmatrix}$$
$$= \frac{1}{i\omega}\begin{pmatrix} 0 \\ \partial_z E_x \\ -\partial_y E_x \end{pmatrix}$$
$$= \frac{1}{i\omega}\begin{pmatrix} 0 \\ ikE_x \\ 0 \end{pmatrix}$$
$$= \frac{E_x}{\omega/k}\hat{\mathbf{y}}$$
$$= \frac{E_x}{c}\hat{\mathbf{y}}$$

For s-polarization, $\theta_i > \theta_c$, we have

$$\sin\theta_t = \frac{\sin\theta_i}{n_2/n_1} > 1$$
$$\cos\theta_t = i\sqrt{\sin^2\theta_t - 1}$$
$$r_s = \frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t}$$
$$= \frac{A - iB}{A + iB}$$
$$\Rightarrow R_s = |r_s|^2 = \frac{A^2+B^2}{A^2+B^2} = 1$$
$$t_s = \frac{2n_1\cos\theta_i}{n_1\cos\theta_i + n_2\cos\theta_t} = \frac{2A}{A+iB}$$

So is complex

$\Rightarrow \mathbf{E}_t = \hat{\mathbf{y}} t_s E_i e^{i(k_2[x\sin\theta_t - \omega t])} e^{-z/\delta}$

where $\delta = \frac{\lambda_2}{2\pi}\frac{1}{(\sin^2\theta_t - 1)^{\frac{1}{2}}}$

$$\nabla \times \mathbf{E} = -\dot{\mathbf{B}} = i\omega\mathbf{B}$$

$$\mathbf{B} = \frac{1}{i\omega}\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ \partial_x & \partial_y & \partial_z \\ 0 & E_t & 0 \end{vmatrix}$$
$$= \frac{1}{i\omega}\begin{pmatrix} -\partial_z E_t \\ 0 \\ \partial_x E_t \end{pmatrix}$$
$$= \begin{pmatrix} \frac{iE_t}{\omega\delta} \\ 0 \\ k_2\sin\theta_t E_t/\omega \end{pmatrix}$$

$\Rightarrow B_z$ is in phase with E_y

$\Rightarrow$ transport of energy along x

B_x is out of phase with E_y

$\Rightarrow$ no transport of energy along z

$< \mathbf{S} >=< \mathbf{E} \times \mathbf{H}^* >$
$\mathbf{H}^*$ is the complex conjugate of H

Reminder on conductors

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} - \frac{\partial}{\partial t} \nabla \times \mathbf{B}$$

$$\mu_0 \mu_r \sigma \mathbf{E} + \mu_0 \mu_r \sigma_0 \sigma_r \frac{\partial \mathbf{E}}{\partial t} = 0 \quad \text{if good conductor}$$

$$\nabla^2 \mathbf{E} = \mu_0 \sigma \frac{\partial \mathbf{E}}{\partial t}$$

$$-\tilde{k}^2 = i\omega\mu_0\sigma$$

$$\Rightarrow \tilde{k} = k + i\kappa \quad k = \kappa = \sqrt{\frac{\mu_0 \sigma \omega}{2}}$$

$$\nabla \times \mathbf{E} = -\dot{\mathbf{B}}$$

$$-\frac{\partial B_y}{\partial t} = (\nabla \times \mathbf{E})_y = \frac{\partial E_x}{\partial z}$$

$$i\omega B_0 == i(k + i\kappa)E_0$$

$$Z = \frac{E_0}{H_0} = \frac{E_0}{B_0/\mu_0} = \frac{\mu_0 \omega}{k + i\kappa}$$

$$= \sqrt{\frac{2\mu_0\omega}{\sigma}} \frac{1}{1+i}$$

$$\Rightarrow Z = \sqrt{\frac{\mu_0 \omega}{2\sigma}}(1 - i)$$

Reflection from a meatal surface

air	metal
$Z_1 = \sqrt{\frac{\mu_0}{\epsilon_0}}$	$Z_2 = \sqrt{\frac{\mu_0 \omega}{2\sigma}}(1-i)$

$|z_2| \ll |Z_1|$ for a good conductor because $\sigma \gg \epsilon_0 \epsilon_r \omega$

write $\alpha = \frac{\sqrt{\frac{\mu_0 \omega}{2\sigma}}}{\sqrt{\frac{\mu_0}{\epsilon_0}}} = \sqrt{\frac{\omega \epsilon_0}{2\sigma}} \ll 1$

$$\begin{aligned}
r &= \frac{E_r}{E_i} \quad \text{normal incidence} \\
&= \frac{Z_2 - Z_1}{Z_2 + Z_1} \\
&= \frac{Z_2/Z_1 - 1}{Z_2/Z - 1 + 1} \\
&= \frac{\alpha(1-i) - 1}{\alpha(1-i) + 1} \\
&\approx \frac{1 - 2\alpha + \cdots}{1 + 2\alpha + \cdots} \\
&= 1 - 4\alpha + O(\alpha^2) \\
&= 1 - \frac{2\omega\delta}{c} + O(\alpha^2) \\
&= 1 - \frac{4\pi\delta}{\lambda} + O(\alpha^2)
\end{aligned}$$

where $\delta = \sqrt{\frac{2}{\omega\mu_0\sigma}}$ is the skin depth
$\Rightarrow$ most of the EM wave intensity is reflected
$\Rightarrow$ metals are shiny!

4.7 Plasmas

Plasmas are neutral gas of charged particles, such as ions and free electrons (like metals)

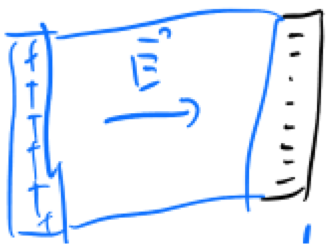
Examples: where you can find plasma

- metal
- ionosphere
- stars
- fusion reactor
- interstellar gas /intergalactic medium
- supernova remnants
- radio galaxies / quasars
- lightening
- aurorae
- fire
- plasma displays

Its density varies from $10^2 - 10^{35} kg/m^3$, temperature is $10^0 - 10^{13} k$

we would only focus on cold plasma

Lets consider a slab of plasma



having nnumber density n

the positive ions are fixed in place, and now lets move electrons by distance ξ

$\rightarrow$ E field: $E = \frac{\sigma}{\epsilon_0}$ and $\sigma = ne\xi$

$$m\ddot{\xi} = -eE \quad \text{where} E = \frac{ne\xi}{\epsilon_0}$$

$$\Rightarrow \ddot{\xi} + \omega_p^2 \xi = 0 \quad \text{which is SHM}$$

$$\text{where } \omega_p^2 = \frac{ne^2}{\epsilon_0 m}$$

This is the SHM at the plasma frequency ω_p

Now, lets drive charges with EM wave $\mathbf{E}e^{-i\omega t}$ (we could ignore B if $v \ll c$)

Hence, we could get $\xi = \xi_0 e^{-i\omega t}$

$$-m\omega^2\xi_0 = -e\mathbf{E}$$

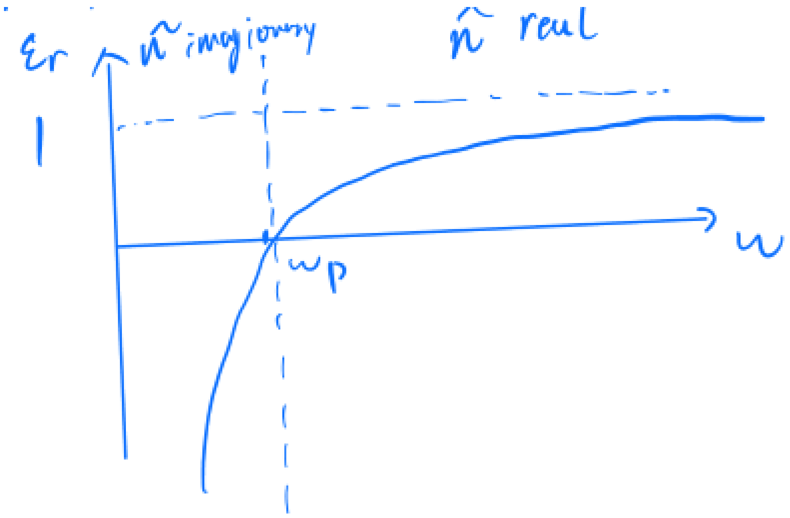
$$\mathbf{P} = -ne\xi_0 = -\frac{ne^2}{m\omega^2}\mathbf{E}$$

$$= (\epsilon_r - 1)\epsilon_0\mathbf{E}$$

$$\Rightarrow \underbrace{\epsilon_r}_{\tilde{n}^2} = 1 - \frac{\omega_p^2}{\omega^2}$$

where $\tilde{n}^2$ is the refractive index, and that can be imaginary

we could plot the relation between ω and ϵ_r



at $\omega > \omega_p$, $\tilde{n}$ is real, hence EM waves can propagate

at $\omega < \omega_p$, $\tilde{n}$ is imaginary, hence EM waves cant propagate

For example, metals are shiny, but only at optical frequencies. They will transport if it is going to much shorter wavelengths.

e.g. ionosphere

AM radios would be refracted + reflected (~ 1 MHz)

FM radio and TV radios would escape (~100MHz)

Lets look at the dispersion relation again

$\frac{\omega}{k} = \frac{c}{n}$

we could get

$$\tilde{n} = 1 - \frac{\omega_p^2}{\omega^2} = \frac{c^2k^2}{\omega^2}$$

$$\Rightarrow \omega = c^2k^2 + \omega_p^2$$

The dispersion relationship would be like

Hence, we could conclude that

- waves are dispersive in plasma
- there are no propagating waves for $\omega < \omega_p$
- waves with $\omega \approx \omega_p$ are slow
  $v_g = \frac{d\omega}{dk} \to 0$ as $\omega \to \omega_p$

We have $2\omega d\omega = 2c^2 kdk$

$\Rightarrow \frac{d\omega}{dk} = \frac{c^2 k}{\omega} = c\sqrt{1 - \frac{\omega_p^2}{\omega^2}}$

We could have end behaviors:

$$v_g \to 0 \quad \text{as} \quad \omega \to \omega_p$$
$$v_g \to c \quad \text{as} \quad \omega \to \infty$$

Let's then take

$\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$

Choose $\mathbf{k}$ to be $(0, 0, k)$, we have

$\mathbf{P} = \frac{-ne^2}{m\epsilon_0\omega^2}(\epsilon_0\mathbf{E}) = -\left(\frac{\omega_p}{\omega}\right)^2 \epsilon_0\mathbf{E}$

We could hence get Maxwell's equation

$$\nabla \times \mathbf{H} = \frac{\partial \dot{\mathbf{D}}}{\partial t} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$$
$$\nabla \times (\nabla \times \mathbf{E}) = -\mu_0 \frac{\partial}{\partial t} \nabla \times \mathbf{H}$$
$$= 0\mu_0 \frac{\partial^2}{\partial t^2}\mathbf{D}$$
$$\text{LHS} = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\mu_0 \frac{\partial^2}{\partial t^2}(\epsilon_0 \mathbf{E} + \mathbf{P})$$

For the wave, we have

$\nabla \to i\mathbf{k}$

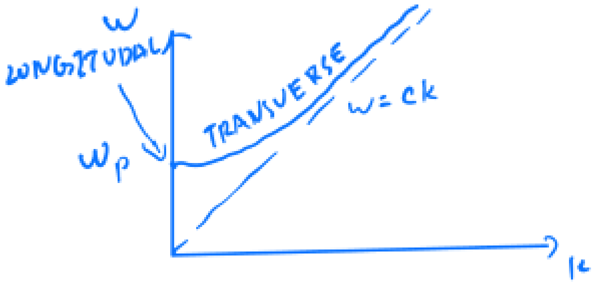
$\frac{\partial}{\partial t} \to -\omega^2 \mathbf{E}$

$\Rightarrow k^2\mathbf{E} - \underbrace{\mathbf{k}(\mathbf{k}\cdot\mathbf{E})}_{k^2 E_z \hat{\mathbf{z}}} = \frac{\omega^2 - \omega_p^2}{c^2}\mathbf{E}$

Transverse solutions: $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ we have $\omega^2 = \omega_p^2 + c^2k^2$

As for longitudinal solutions, we have $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, we have, hence, $\omega^2 = \omega_p^2$

We can therefore classify waves in the ω k graph



4.8 Dispersion

> Refractive index changes with frequency

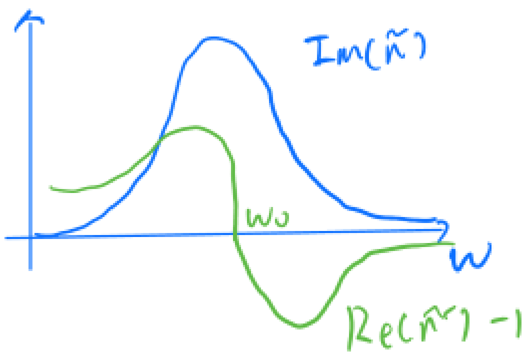
- classical theory of dispersion:

model electrons as a classical damped oscillator
$m\ddot{x} + m\gamma\dot{x} + m\omega_0^2 x = qE_0 e^{-i\omega t}$

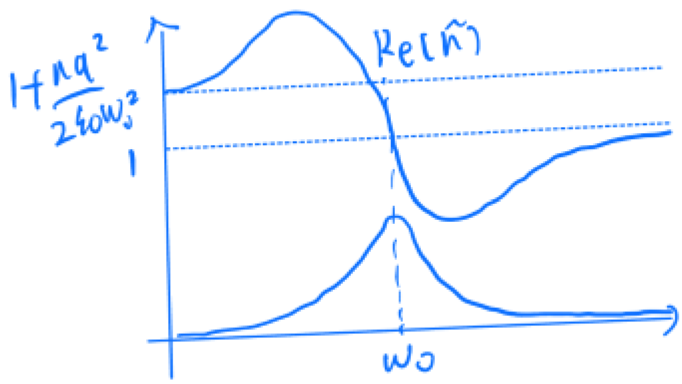
Assume that $x = x_0 e^{-i\omega t}$

$(-\omega^2 - i\omega\gamma + \omega_0^2)x_0 = \frac{qE_0}{m}$

$$\begin{aligned}
P = nqx_0 &= \frac{nq^2}{m} E_0 \frac{1}{\omega_0^2 - \omega^2 - i\omega\gamma} \\
&= \epsilon_0(\tilde{\epsilon}_r - 1)E_0 \\
\tilde{\epsilon}_r &= 1 + \frac{nq^2 E_0}{\epsilon_0 m} \frac{1}{\omega_0^2 - \omega - i\omega\gamma} \\
\tilde{n}^2 &= \tilde{\epsilon}_r \\
\text{For a gas, } \tilde{\epsilon} &= 1 + \text{small quantity} \\
\tilde{\epsilon}^{\frac{1}{2}} &= 1 + \frac{1}{2}\text{small quantities} + \cdots \\
Re(\tilde{n}) &= 1 + \frac{nq^2}{2\epsilon_0 m} \frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + \omega^2\gamma^2} \\
Im(\tilde{n}) &= \frac{nq^2}{2\epsilon_0 m} \frac{\gamma\omega}{(\omega_0^2 - \omega^2)^2 + \omega^2\gamma^2}
\end{aligned}$$



As $\omega \to 0$, $Re(\tilde{n}) \to 1 + \frac{nq^2}{2q_0\omega_0^2}$, $Im(\tilde{n}) \to 0$
As $\omega \to \infty$, $Re(\tilde{n}) \to 1$, we could get a better illustration, therefore, for the real part and imaginary part of $\tilde{n}$



We can also conclude that $Im(\tilde{n})$ corresponds to the absorption of light, and $Re(\tilde{n})$ corresponds to the refraction of light and generally increases with frequency

The sharp drop in the real part of $\tilde{n}$ in the real part of $\tilde{n}$ near ω_0 is called <u>anomalous dispersion</u>