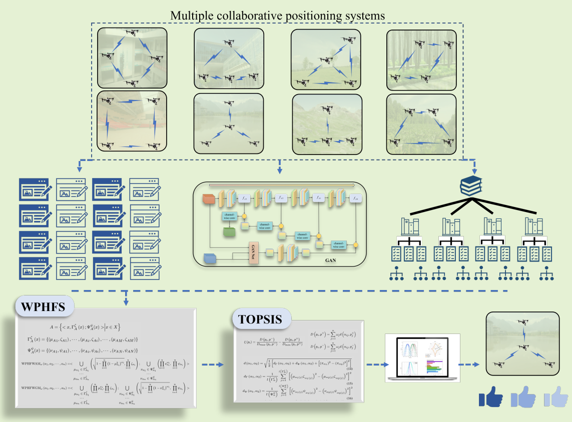
Evaluate and Make Decisions on Cooperative Positioning Systems in Weighted Pythagorean Hesitant Fuzzy Environment Combined with GAN

Cunle Zhang, Chengkai Tang*, *Senior Member, IEEE*, Haonan Wang, Baowang Lian and Lingling Zhang, *Member, IEEE*

***Abstract*—Current cooperative positioning schemes are complex and their performances vary, urgently necessitating evaluation. In view of the specific characteristics of Cooperative Positioning Systems (CPS) in terms of physical data, decision-making factors, and the professionalism of decisions, this study aims to construct a reasonable Multi-Criteria Cooperative Positioning Decision-Making Evaluation (MCCPDM) framework. The study applies Generative Adversarial Networks (GAN) to the decision-making process, proposes a weighted fusion of Pythagorean fuzzy sets and hesitant fuzzy sets, and provides a method for determining the weights of decision-making factors that combines objective physical entropy information with subjective multi-level judgments. An MCCPDE method integrating Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is developed under the weighted Pythagorean hesitant fuzzy environment. Finally, through experiments evaluating cooperative positioning schemes, the proposed framework is proven to be highly effective and practical in cooperative positioning evaluation issues, and the method presented in this paper surpasses the best baseline method by 2 orders of magnitude in terms of optimal ranking proximity.**



***Index Terms*—Multi-Criteria cooperative Positioning Decision-Making (MCCPDM), weighted Pythagorean hesitant fuzzy set (WPHFS),Generative Adversarial Networks (GAN),Technique for Order Preference by Similarity to Ideal Solution (TOPSIS),Entropy Weight Method (EWM), Analytic Hierarchy Process (AHP)**

Theorem 3.1.Let $\alpha_i =< \Gamma^{\zeta}_{\alpha_i}; \Psi^{\psi}_{\alpha_i} >$ $(i = 1, 2, 3)$ be a collection of WPHFNs, and $\lambda > 0$, then:

1) $\alpha_1 \bigoplus \alpha_2 = \alpha_2 \bigoplus \alpha_1$, $\alpha_1 \bigotimes \alpha_2 = \alpha_2 \bigotimes \alpha_1$;
2) $(\alpha_1 \bigoplus \alpha_2) \bigoplus \alpha_3 = \alpha_1 \bigoplus (\alpha_2 \bigoplus \alpha_3)$, $(\alpha_1 \bigotimes \alpha_2) \bigotimes \alpha_3 = \alpha_1 \bigotimes (\alpha_2 \bigotimes \alpha_3)$;
3) $(\alpha_1 \bigoplus \alpha_2) = \lambda\alpha_1 \bigoplus \lambda\alpha_2$, $(\alpha_1 \bigotimes \alpha_2)^{\lambda} = \alpha_1^{\lambda} \bigotimes \alpha_2^{\lambda}$;
4) $(\alpha_1 \bigoplus \alpha_2)^c = \alpha_1^c \bigotimes \alpha_2^c$, $(\alpha_1 \bigotimes \alpha_2)^c = \alpha_1^{\lambda c} \bigoplus \alpha_2^c$;
5) $(\alpha^c)^{\lambda} = (\lambda\alpha)^c$, $\lambda\alpha^c = \left(\alpha^{\lambda}\right)^c$.

Due to spatial constraints, we will only present the proof for the fifth item and omit the remaining ones.

Proof.Let $\alpha =< \Gamma^{\zeta}_{\alpha}; \Psi^{\psi}_{\alpha} >$ be a WPHFN, and $\lambda > 0$, then:

1) $(\alpha^c)^{\lambda} = \left(< \Psi^{\psi}_{\alpha}; \Gamma^{\zeta}_{\alpha} >\right)^{\lambda}$

$$=< \bigcup_{\nu_\alpha \in \Psi^{\psi}_{\alpha}} \left(\nu^{\lambda}_{\alpha},\ \psi_\alpha\right); \bigcup_{\mu_\alpha \in \Gamma^{\zeta}_{\alpha}} \left(\sqrt{1-\left(1-\mu^2_\alpha\right)^{\lambda}}, \zeta_\alpha\right) >$$
$$= (\lambda\alpha)^c.$$

2) $\lambda\alpha^c = \lambda < \Psi^{\psi}_{\alpha}; \Gamma^{\zeta}_{\alpha} >$

$$=< \bigcup_{\nu_\alpha \in \Psi^{\psi}_{\alpha}} \left(\sqrt{1-\left(1-\nu^2_\alpha\right)^{\lambda}},\ \psi_\alpha\right); \bigcup_{\mu_\alpha \in \Gamma^{\zeta}_{\alpha}} \left(\mu^{\lambda}_{\alpha}, \zeta_\alpha\right) >$$
$$= \left(\alpha^{\lambda}\right)^c.$$

Theorem 3.2.Let $\alpha_i =< \Gamma^{\zeta}_{\alpha_i}; \Psi^{\psi}_{\alpha_i} > (i = 1, 2, \cdots, n)$ be a collection of WPHFNs, while $\omega = (\omega_1, \omega_2, \cdots, \omega_n)$ corresponds to its weight vector, satisfying condition $\omega_i \in [0, 1]$, $i = 1, 2, \cdots, n$ and $\sum_{i=1}^{n} \omega_i = 1$, then:

$$\mathrm{WPHFWAM}_{\omega}(\alpha_1, \alpha_2, \cdots, \alpha_n) =< \bigcup_{\substack{\mu_{\alpha_1} \in \Gamma^{\zeta}_{\alpha_1} \\ \cdots \\ \mu_{\alpha_n} \in \Gamma^{\zeta}_{\alpha_n}}} \left(\sqrt{1 - \prod_{i=1}^{n} (1 - \mu^2_{\alpha_i})^{\omega_i}}, \prod_{i=1}^{n} \zeta_{\alpha_i} \right); \bigcup_{\substack{\nu_{\alpha_1} \in \Psi^{\psi}_{\alpha_1} \\ \cdots \\ \nu_{\alpha_n} \in \Psi^{\psi}_{\alpha_n}}} \left(\prod_{i=1}^{n} \nu^{\omega_i}_{\alpha_i}, \prod_{i=1}^{n} \psi_{\alpha_i} \right) > \quad (1)$$

$$\mathrm{WPHFWGM}_{\omega}(\alpha_1, \alpha_2, \cdots, \alpha_n) =< \bigcup_{\substack{\mu_{\alpha_1} \in \Gamma^{\zeta}_{\alpha_1} \\ \cdots \\ \mu_{\alpha_n} \in \Gamma^{\zeta}_{\alpha_n}}} \left(\prod_{i=1}^{n} \mu^{\omega_i}_{\alpha_i}, \prod_{i=1}^{n} \zeta_{\alpha_i} \right); \bigcup_{\substack{\nu_{\alpha_1} \in \Psi^{\psi}_{\alpha_1} \\ \cdots \\ \nu_{\alpha_n} \in \Psi^{\psi}_{\alpha_n}}} \left(\sqrt{1 - \prod_{i=1}^{n} (1 - \nu^2_{\alpha_i})^{\omega_i}}, \prod_{i=1}^{n} \psi_{\alpha_i} \right) > \quad (2)$$

Proof.When $n = 2$:

$\mathrm{WPHFWAM}_{\omega}(\alpha_1, \alpha_2) = \omega_1\alpha_1 \oplus \omega_2\alpha_2$

$$=< \bigcup_{\mu_{\alpha_1} \in \Gamma^{\zeta}_{\alpha_1}} \left(\sqrt{1 - (1 - \mu^2_{\alpha_1})^{\omega_1}}, \zeta_{\alpha_1} \right); \bigcup_{\nu_{\alpha_1} \in \Psi^{\psi}_{\alpha_1}} \left(\nu^{\omega_1}_{\alpha_1}, \psi_{\alpha_1} \right) >$$

$$\oplus < \bigcup_{\mu_{\alpha_2} \in \Gamma^{\zeta}_{\alpha_2}} \left(\sqrt{1 - (1 - \mu^2_{\alpha_2})^{\omega_2}}, \zeta_{\alpha_2} \right); \bigcup_{\nu_{\alpha_2} \in \Psi^{\psi}_{\alpha_2}} \left(\nu^{\omega_2}_{\alpha_2}, \psi_{\alpha_2} \right) >$$

$$=< \bigcup_{\substack{\mu_{\alpha_1} \in \Gamma^{\zeta}_{\alpha_1} \\ \mu_{\alpha_2} \in \Gamma^{\zeta}_{\alpha_2}}} \left(\sqrt{1 - \prod_{i=1}^{2} (1 - \mu^2_{\alpha_i})^{\omega_i}}, \prod_{i=1}^{2} \zeta_{\alpha_i} \right);$$

$$\bigcup_{\substack{\nu_{\alpha_1} \in \Psi^{\psi}_{\alpha_1} \\ \nu_{\alpha_2} \in \Psi^{\psi}_{\alpha_2}}} \left(\prod_{i=1}^{2} \nu^{\omega_i}_{\alpha_i}, \prod_{i=1}^{2} \psi_{\alpha_i} \right) >$$

Let's assume

$\mathrm{WPHFWAM}_{\omega}(\alpha_1, \alpha_2, \cdots, \alpha_k) = \omega_1\alpha_1 \oplus \cdots \oplus \omega_k\alpha_k$

$$=< \bigcup_{\substack{\mu_{\alpha_1} \in \Gamma^{\zeta}_{\alpha_1} \\ \cdots \\ \mu_{\alpha_k} \in \Gamma^{\zeta}_{\alpha_k}}} \left(\sqrt{1 - \prod_{i=1}^{k} (1 - \mu^2_{\alpha_i})^{\omega_i}}, \prod_{i=1}^{k} \zeta_{\alpha_i} \right);$$

$$\bigcup_{\substack{\nu_{\alpha_1} \in \Psi^{\psi}_{\alpha_1} \\ \cdots \\ \nu_{\alpha_k} \in \Psi^{\psi}_{\alpha_k}}} \left(\prod_{i=1}^{k} \nu^{\omega_i}_{\alpha_i}, \prod_{i=1}^{k} \psi_{\alpha_i} \right) >$$

holds true when $n = k$.

Consequently, we can derive

$\mathrm{WPHFWAM}_{\omega}(\alpha_1, \alpha_2, \cdots, \alpha_k, \alpha_{k+1}) = \omega_1\alpha_1 \oplus \cdots \oplus \omega_k\alpha_k \oplus \omega_{k+1}\alpha_{k+1} = (\omega_1\alpha_1 \oplus \cdots \oplus \omega_k\alpha_k) \oplus \omega_{k+1}\alpha_{k+1}$

$$=< \bigcup_{\substack{\mu_{\alpha_1} \in \Gamma^{\zeta}_{\alpha_1} \\ \cdots \\ \mu_{\alpha_k} \in \Gamma^{\zeta}_{\alpha_k}}} \left(\sqrt{1 - \prod_{i=1}^{k} (1 - \mu^2_{\alpha_i})^{\omega_i}}, \prod_{i=1}^{k} \zeta_{\alpha_i} \right); \bigcup_{\substack{\nu_{\alpha_1} \in \Psi^{\psi}_{\alpha_1} \\ \cdots \\ \nu_{\alpha_k} \in \Psi^{\psi}_{\alpha_k}}} \left(\prod_{i=1}^{k} \nu^{\omega_i}_{\alpha_i}, \prod_{i=1}^{k} \psi_{\alpha_i} \right) >$$

$$\oplus < \bigcup_{\mu_{\alpha_{k+1}} \in \Gamma^{\zeta}_{\alpha_{k+1}}} \left(\sqrt{1 - \left(1 - \mu^2_{\alpha_{k+1}}\right)^{\omega_{k+1}}}, \zeta_{\alpha_{k+1}} \right); \bigcup_{\nu_{\alpha_{k+1}} \in \Psi^{\psi}_{\alpha_{k+1}}} \left(\nu^{\omega_{k+1}}_{\alpha_{k+1}}, \psi_{\alpha_{k+1}} \right) >$$

$$=< \bigcup_{\substack{\mu_{\alpha_1} \in \Gamma^{\zeta}_{\alpha_1} \\ \cdots \\ \mu_{\alpha_{k+1}} \in \Gamma^{\zeta}_{\alpha_{k+1}}}} \left(\sqrt{1 - \prod_{i=1}^{k+1} (1 - \mu^2_{\alpha_i})^{\omega_i}}, \prod_{i=1}^{k+1} \zeta_{\alpha_i} \right); \bigcup_{\substack{\nu_{\alpha_1} \in \Psi^{\psi}_{\alpha_1} \\ \cdots \\ \nu_{\alpha_{k+1}} \in \Psi^{\psi}_{\alpha_{k+1}}}} \left(\prod_{i=1}^{k+1} \nu^{\omega_i}_{\alpha_i}, \prod_{i=1}^{k+1} \psi_{\alpha_i} \right) >$$

For WPHFWGM,the same process applies. By employing mathematical induction, we can establish the validity of **Theorem 3.2**.