Mathematics

Zhang En-Yao

August 2023

Contents

Chapter 1

Pairs

1.1 Notions

Various general mathematical concepts, such as relations, functions, and orderings can be represented by ordered pairs.

Definition. Ordered Pair, Coordinate
Ordered pair of a and b is denoted by (a, b); a is the first coordinate of the pair, b is the second coordinate. $(a, b) = \{\{a\}, \{a, b\}\}$.

Property 1.1.1.
$(a, b) \neq (b, a)$ if $a \neq b$.

Proof. Obviously. □

Property 1.1.2.
If $a = b$, then $(a, a) = \{\{a\}\}$ has only one element.

Proof. Obviously. □

Note. *Ordered pair should be defined in such a way that two ordered pairs are equal if and only if their first coordinates are equal and their second coordinates are equal.*

Note. *If $a \neq b$, (a, b) has two elements, a singleton $\{a\}$ and an unordered pair $\{a, b\}$. The first coordinate is the element of $\{a\}$; the second coordinate is the other element of $\{a, b\}$.*

Theorem 1.1.3.
$(a, b) = (a', b')$ if and only if $a = a'$ and $b = b'$.

Proof. Obviously. □

Definition. One-Tuples
$(a) = a$.

Definition. Ordered Triples
$(a, b, c) = ((a, b), c)$.