Mathematics

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August 2023

Contents

Chapter 1

Pairs

1.1 Unordered, Ordered Pairs

Notion. *Summary*
Sets define ordered and unordered pairs. Ordered pairs contain coordinates. Coordinates define equivalent.

Definition. Unordered Pair
$\{a, b\}$ is a set whose elements are exactly a and b.

Definition. Ordered Pair
Ordered pair of a and b is denoted by (a, b).

Definition. Coordinate
a is the first coordinate of the pair, b is the second coordinate.

Definition.
$(a, b) = \{\{a\}, \{a, b\}\}$.

Definition.
For two different sets α and β, $(a, b) = \{\{a, \alpha\}, \{b, \beta\}\}$.

Note. *An alternative definition of ordered pairs.*

Theorem 1.1.1.
$(a, b) = (a', b')$ if and only if $a = a'$ and $b = b'$.

Proof. Obviously. □

Proposition 1.1.1.1.
Two *ordered pairs* are *equal* if and only if their *first coordinates* are equal and their *second coordinates* are equal.

Proof. Obviously. □

Note. *Ordered pair should be defined in such a way that two ordered pairs are equal if and only if their first coordinates are equal and their second coordinates are equal.*

Proposition 1.1.1.2.
$(a, b) \neq (b, a)$ if $a \neq b$.

Proof. Obviously. □

Proposition 1.1.1.3.
If $a \neq b$, (a, b) has two elements, a *singleton* $\{a\}$ and an *unordered pair* $\{a, b\}$. We find the *first coordinate* by looking at the element of $\{a\}$. The *second coordinate* is then the other element of $\{a, b\}$. If $a = b$, then $(a, a) = \{\{a\}\}$ has only one element.

Proof. Obviously. □

Note. *If $a \neq b$, (a, b) has two elements, a singleton $\{a\}$ and an unordered pair $\{a, b\}$. The first coordinate is the element of $\{a\}$; the second coordinate is the other element of $\{a, b\}$.*

Definition. One-Tuples
$(a) = a$.

Definition. Ordered Triples
$(a, b, c) = ((a, b), c)$.

Definition. Ordered Quadruples
$(a, b, c, d) = (((a, b), c), d)$.

Property 1.1.1.
$(a, b) \in \mathcal{P}(\mathcal{P}(\{a, b\}))$. More generally, if $a \in A$ and $b \in A$, then $(a, b) \in \mathcal{P}(\mathcal{P}(\{a, b\}))$.

Proof. Obviously. □

Property 1.1.2.
$a, b \in \bigcup(a, b)$.

Proof. Obviously. □

Property 1.1.3.
(a, b), (a, b, c), and (a, b, c, d) exist for all a, b, c, and d.

Proof. Obviously. □

Property 1.1.4.
If $(a, b) = (b, a)$, then $a = b$.

Proof. Obviously. □

Property 1.1.5.
$(a, b, c) = (a', b', c')$ implies $a = a'$, $b = b'$, and $c = c'$. $(a, b, c, d) = (a', b', c', d')$ implies $a = a'$, $b = b'$, $c = c'$, and $d = d'$.

Proof. Obviously. □

1.2 Relation

Notion. *Summary*
Undering.

Definition. Binary Relation
A set A is a binary relation if all elements of R are ordered pairs, i.e., if for any $z \in R$ there exist x and y such that $z = (x, y)$.

Note. *Relations between objects of two sorts called binary relations.*

Note. *A binary relation is determined by specifying all ordered pairs of objects in that relation; it does not matter by what property the set of these ordered pairs is described.*

Definition.
Let A be a binary relation.

1. The set of all x which are in relation A with some y is called the domain of R and denoted by $\operatorname{dom} R$.

2. The set of all y such that, for some x, x is in relation R with y is called the range of A, denoted by $\operatorname{ran} R$.

3. The set $\operatorname{dom} R \cup \operatorname{ran} R$ is called the field of R and is denoted by $\operatorname{field} R$.

4. If $\operatorname{field} R \subseteq X$, we say that R is a relation in X or that R is a relation between elements of X.

Note. $\operatorname{dom} R$ is the set of all first coordinates of ordered pairs in R.

Note. $\operatorname{ran} R$ is the set of all second coordinates of ordered pairs in R.

Proposition 1.2.0.1.
Both $\operatorname{dom} R$ and $\operatorname{ran} R$ exist for any relation R.

Proof. Obviously. □