Mathematics

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Chapter 1

Pairs

1.1 Unordered, Ordered Pairs

Definition. Unordered Pair
$\{a, b\}$ is a set whose elements are exactly a and b.

Definition. Ordered Pair
Ordered pair of a and b is denoted by (a, b).

Definition. Coordinate
a is the first coordinate of the pair, b is the second coordinate.

Definition.
$(a, b) = \{\{a\}, \{a, b\}\}$.

Definition.
For two different sets α and β, $(a, b) = \{\{a, \alpha\}, \{b, \beta\}\}$.

Note. *An alternative definition of ordered pairs.*

Property 1.1.0.1.
Two ordered pairs are equal if and only if their first coordinates are equal and their second coordinates are equal.

Proof. Obviously. □

Property 1.1.0.2.
$(a,b) \neq (b,a)$ if $a \neq b$.

Proof. Obviously. □

Property 1.1.0.3.
If $a \neq b$, (a,b) has two elements, a singleton $\{a\}$ and an unordered pair $\{a,b\}$. If $a = b$, then $(a,a) = \{\{a\}\}$ has only one element.

Proof. Obviously. □

Note. *We find the first coordinate by looking at the element of $\{a\}$. The second coordinate is then the other element of $\{a,b\}$.*

Note. *Ordered pair should be defined in such a way that two ordered pairs are equal if and only if their first coordinates are equal and their second coordinates are equal.*

Note. *If $a \neq b$, (a,b) has two elements, a singleton $\{a\}$ and an unordered pair $\{a,b\}$. The first coordinate is the element of $\{a\}$; the second coordinate is the other element of $\{a,b\}$.*

Theorem 1.1.1.
$(a,b) = (a',b')$ if and only if $a = a'$ and $b = b'$.

Proof. Obviously. □

Definition. One-Tuples
$(a) = a$.

Definition. Ordered Triples
$(a,b,c) = ((a,b),c)$.

Definition. Ordered Quadruples
$(a,b,c,d) = (((a,b),c),d)$.

Property 1.1.1.1.
$(a,b) \in \mathcal{P}(\mathcal{P}(\{a,b\}))$. More generally, if $a \in A$ and $b \in A$, then $(a,b) \in \mathcal{P}(\mathcal{P}(\{a,b\}))$.

Proof. Obviously. □

Property 1.1.1.2.
$a,\ b \in \bigcup(a,b)$.

Proof. Obviously. □

Property 1.1.1.3.
(a,b), (a,b,c), and (a,b,c,d) exist for all a, b, c, and d.

Proof. Obviously. □

Property 1.1.1.4.
If $(a,b) = (b,a)$, then $a = b$.

Proof. Obviously. □

Property 1.1.1.5.
$(a,b,c) = (a',b',c')$ implies $a = a'$, $b = b'$, and $c = c'$. $(a,b,c,d) = (a',b',c',d')$ implies $a = a'$, $b = b'$, $c = c'$, and $d = d'$.

Proof. Obviously. □