Mathematics

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August 2023

Contents

Chapter 1

Pairs

1.1 Unordered, Ordered Pairs

Notion. *Summary*

Sets define ordered and unordered pairs.
Ordered pairs contain coordinates.
Coordinates define equivalent.

Definition. Unordered Pair
$\{a, b\}$ is a set whose elements are exactly a and b.

Definition. Ordered Pair
Ordered pair of a and b is denoted by (a, b).

Definition. Coordinate
a is the first coordinate of the pair, b is the second coordinate.

Definition.
$(a, b) = \{\{a\}, \{a, b\}\}$.

Definition.
For two different sets α and β, $(a, b) = \{\{a, \alpha\}, \{b, \beta\}\}$.

Note. An alternative definition of ordered pairs.

Theorem 1.1.1.
$(a, b) = (a', b')$ if and only if $a = a'$ and $b = b'$.

Proof. Obviously. □

Proposition 1.1.1.1.
Two *ordered pairs* are *equal* if and only if their *first coordinates* are equal and their *second coordinates* are equal.

Proof. Obviously. □

Note. Ordered pair should be defined in such a way that two ordered pairs are equal if and only if their first coordinates are equal and their second coordinates are equal.

Proposition 1.1.1.2.
$(a, b) \neq (b, a)$ if $a \neq b$.

Proof. Obviously. □

Proposition 1.1.1.3.
If $a \neq b$, (a, b) has two elements, a *singleton* $\{a\}$ and an *unordered pair* $\{a, b\}$. We find the *first coordinate* by looking at the element of $\{a\}$. The *second coordinate* is then the other element of $\{a, b\}$. If $a = b$, then $(a, a) = \{\{a\}\}$ has only one element.

Proof. Obviously. □

Note. If $a \neq b$, (a, b) has two elements, a singleton $\{a\}$ and an unordered pair $\{a, b\}$. The first coordinate is the element of $\{a\}$; the second coordinate is the other element of $\{a, b\}$.

Definition. One-Tuples
$(a) = a$.

Definition. Ordered Triples
$(a, b, c) = ((a, b), c)$.

Definition. Ordered Quadruples
$(a, b, c, d) = (((a, b), c), d)$.

Property 1.1.1.
$(a, b) \in \mathcal{P}(\mathcal{P}(\{a, b\}))$. More generally, if $a \in A$ and $b \in A$, then $(a, b) \in \mathcal{P}(\mathcal{P}(\{a, b\}))$.

Proof. Obviously. □

Property 1.1.2.
$a,\ b \in \bigcup(a, b)$.

Proof. Obviously. □

Property 1.1.3.
(a, b), (a, b, c), and (a, b, c, d) exist for all a, b, c, and d.

Proof. Obviously. □

Property 1.1.4.
If $(a, b) = (b, a)$, then $a = b$.

Proof. Obviously. □

Property 1.1.5.
$(a, b, c) = (a', b', c')$ implies $a = a'$, $b = b'$, and $c = c'$. $(a, b, c, d) = (a', b', c', d')$ implies $a = a'$, $b = b'$, $c = c'$, and $d = d'$.

Proof. Obviously. □