

# Mathematics

Zhang En-Yao

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**Part I**

**Logic**



# Chapter 1

## Sentential logic

### 1.1 Arguments

#### Definition.

An *argument* can be defined as a **set** of **statements**, one of which is the **conclusion** (the thing **argue**-ed for) and the others are the **premises** (**reasons** for **accepting** the **conclusion**). The **sentences** in an **argument** must **express statements**—that is, say something that is either **true** or **false**. But even if every **sentence** in a **group** of **sentences** **expresses** a **statement**, the result is not necessarily an **argument**. The **statements** must be **related** to one another **statements** in the appropriate way. Something must be **argue**-ed for (the **conclusion**), and there must be **reasons** (the **premises**) for **accepting** the **conclusion**.

#### Property 1.1.1.

An **argument** may be embedded in other **arguments**.

*Proof.* Obviously

□

#### Property 1.1.2.

**Argument** is a kind of **reasoning**.

*Proof.* Obviously

□

#### Property 1.1.3.

**Argument** consists of **statements**.

*Proof.* Obviously

□

**Property 1.1.4.**

Not just any group of **sentences** makes an **argument**.

*Proof.* Obviously

□

**Property 1.1.5.**

A **statement** is best thought of as something that can be **expressed** using a **declarative sentence**.

*Proof.* Obviously

□

**Definition.**

**Statements** in an **argument** that give **reasons** for **accepting** a **statement** are said to be **premises** of an **argument**.

**Definition.**

The **argument** is called the **argument's conclusion**.

**Property 1.1.6.**

An **argument** may have any number of **premises** (even only one!)

*Proof.* Obviously

□

**Definition.**

To determine whether or not an **argument** is **valid**, one must ask whether there are any possible circumstances under which the **premises** could all be **true** and yet the **conclusion** be **false**. If not, the **argument** is **valid**.

**Property 1.1.7.**

The **truth** of the **premises** of a **valid argument** guarantees the **truth** of its **conclusion**.

*Proof.* Obviously

□

**Property 1.1.8.**

It is impossible for the **conclusion** of a **valid argument** to be **false** if all its **premises** are **true**.



*Proof.* Obviously

□

**Property 1.1.9.**

In a **valid argument**, if **premises** are both **true**, then the **conclusion** must be **true** also. But are both **premises** true? We have no way of knowing.

*Proof.* Obviously

□

**Definition.**

If it is possible for the **premises** to be **true** and the **conclusion** **false**, the **argument** is *invalid*.

**Property 1.1.10.**

An **invalid argument** is simply an **argument** that is not **valid**.

*Proof.* Obviously

□

**Property 1.1.11.**

**Deduction** and **induction** are commonly thought to be the cornerstones of good **reasoning**.

*Proof.* Obviously

□

**Definition.**

The fundamental logical property of a *deductively* **valid argument** is this: If all its **premises** are **true**, then its **conclusion** must be **true**. In other words, an **argument** is **valid** if it is impossible for all its **premises** to be **true** and yet its **conclusion** be **false**.

**Property 1.1.12.**

The **truth** of the **premises** of a **deductively valid argument** guarantees the **truth** of its **conclusion**.

*Proof.* Obviously

□

**Property 1.1.13.**

It is revealing to notice that in a typical case the information contained in the **conclusion** of a **deductively valid argument** is already "contained" in its **premises**. It is usually contained in the **premises** implicitly (along with other information not contained in the **conclusion**)

*Proof.* Obviously

□

**Definition.**

In addition to **deductive arguments**, there are also *inductive arguments*.

**Property 1.1.14.**

**Inductive arguments** differ from **deductively valid arguments** in having **conclusions** that go beyond what is contained in their **premises**.

*Proof.* Obviously

□

**Property 1.1.15.**

The crucial difference between **inductive strength** and **deductive validity** is that it is possible for the **premises** of a **strong inductive argument** to be **true** and yet the **conclusion** be **false**.

*Proof.* Obviously

□

**Property 1.1.16.**

The basic idea behind **inductive reasoning** is that of learning from **experience**. We notice **patterns**, **resemblances**, or other kinds of **regularity-s** in our **experiences**

*Proof.* Obviously

□

**Property 1.1.17.**

The great virtue of **inductive reasoning** is that it provides us with a way of **reasoning** to genuinely new beliefs, and not just to psychologically new ones that were implicit in what we already knew, as in the case of **valid deductions**. However, this benefit is purchased at the cost of an increase in the possibility of **error**.

*Proof.* Obviously

□

**Definition.**

Good **inductive arguments** are said to be **inductively strong**.

**Property 1.1.18.**

Whereas **true premises** in a **valid argument** guarantee the **truth** of the **conclusion**, **true premises** in a **strong inductive argument** make the **conclusion** likely or **probable**.

*Proof.* Obviously

□

**Property 1.1.19.**

Although an **inductively strong argument** does not guarantee that if its **premises** are **true**, then its **conclusion** also will be **true**, it does make its **conclusion** more **probable**.

*Proof.* Obviously

□

**Definition.**

The expression "**probable**-ity **argument**" is sometimes applied to **inductive arguments**.

**Definition.**

Argument **form** is, informally, the logical structure of an argument.

**Property 1.1.20.**

A **strong induction** may contain all **true premises** and yet have a **false conclusion**.

*Proof.* Obviously

□

**Property 1.1.21.**

Unlike **validity**, **inductive strength** comes in **degrees**. It makes no sense to speak of one **deductive argument** as being "more **valid**" than another. All **deductive arguments** are either **valid** or **invalid**. But it does make sense to describe one **argument** as being **inductively stronger** than another.

*Proof.* Obviously

□

**Property 1.1.22.**

Nevertheless, it is clear that if the **premises** are **true**, then the **conclusion** must be **true**. We know this from the **form** of the **argument** and not because of its **content**; its **form** makes this **argument** a **valid argument form**.

*Proof.* Obviously

□

**Property 1.1.23.**

Any argument having the same (**valid**) **form** is **deductively valid**. Of course, there are many other **valid argument forms**.

*Proof.* Obviously

□

**Property 1.1.24.**

**Logic** is concerned primarily with **argument forms**, and only secondarily with **arguments**, for all **arguments** that have a **valid argument form** are **valid**. The **form**, not the **content**, of the preceding **arguments** makes it impossible for them to have all **true premises** and a **false conclusion**.

*Proof.* Obviously

□

**Property 1.1.25.**

In general, **arguments** have several **forms**; any **argument** with at least one **valid form** is **valid**.

*Proof.* Obviously

□