CVIE 609 Risk Engineering

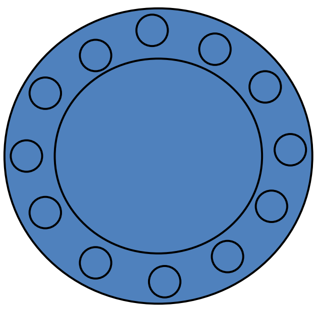
Assignment #2 :

Due March 1, 2018

PROBLEM 1

The figure below shows the plan view of a nuclear pressure vessel (the top lid).

In order for the lid to function properly, the total compression load P* provided by the 12 bolts must be at least 500 kips. The bolts are tightened by controlling the applied torque M. The relationship between the applied torque (M) an bolt tension P is



$$P = 0.02M \bullet \varepsilon$$

$$where$$

$$\varepsilon \sim N(m = 1, \sigma = 0.2)$$

The constant 0.02 is the calibration constant of the torque, and the random variable ε represents the uncertainty due to the lack of precision of the torqwue wrench,

a) Find the torque for which the total load exceeds P* with probability 0.95.
b) Find the torque for which the minimum bolt load exceeds P*/12 with probability 0.95.

PROBLEM 2

As an example, consider the time D it takes to commute in the morning from a suburb to downtown

For given traffic and weather conditions, respectively T and W, the distribution of D might be lognormal, with <u>mean value</u> $m_D(T,W)$ and <u>coefficient of variation</u> $V_D(T,W)$. The lognormal probability density function is usually written in terms of two other parameters, $m_{\ln D}$ and $\sigma^2_{\ln D}$, which are the mean value and variance of ln(D). This density has the form

$$f_D(d) = \frac{1}{d}\frac{1}{\sqrt{2\pi}\,\sigma_{\ln D}} e^{-(\ln d - m_{\ln D})^2/2\sigma^2_{\ln D}} \tag{3}$$

In turn, $m_{\ln D}$ and $\sigma^2_{\ln D}$ can be calculated from m_D and $\sigma^2_D = m^2_D V^2_D$ as

$$\begin{aligned} m_{\ln D} &= \ln(m^2_D) - \frac{1}{2}\ln\left(\sigma^2_D + m^2_D\right) \\ \sigma^2_{\ln D} &= -\ln(m^2_D) + \ln\left(\sigma^2_D + m^2_D\right) \end{aligned} \tag{4}$$

For a given suburb and a certain route, values of the distribution parameters $m_D(T,W)$ and $V_D(T,W)$ and probabilities of different (T,W) combinations might be as follows (m_D in minutes):

W	T	P[T∩W]	m_D (minutes)	V_D
good	light	0.24	25	0.05
good	normal	0.40	30	0.05
good	heavy	0.16	40	0.10
bad	light	0.02	30	0.07
bad	normal	0.08	40	0.10
bad	heavy	0.10	50	0.15
		$\Sigma = 1.00$		

(Eq. 2 : $f_D(d)=\sum_T \quad \sum_W (P(T \cap W) * f_{D|W,T}(d))$

Use Eq. 2 to find the marginal probability density function of D. Plot this density function. Is it a lognormal density? Using the fact that, for any given W and T, ln(D) has normal distribution with parameters in Eq. 4 and using tables of the normal distribution, find the unconditional probability that D > 60 minutes.

PROBLEM 3

(a) You are about to buy a used car, which has accumulated 50,000 miles without major problems. You are worried that something bad may soon be happening to it. The dealer reassures you that the car "is very safe, because statistics show that its lifetime distribution is uniform between 0 and 150,000 miles." In his sale pitch, the dealer adds: " This means that the car has the same chance of dying on you in the next 50,000 miles as it did during the first 50,000 miles after rolling out of the factory. Therefore, it is as good as new." Using Eq. 2, calculate the hazard function h(t) for the car when it was new (notice that here t has the units of miles). Compare h(t) for 0 < t < 50,000 miles and 50,000 < t < 100,000 miles. Do you agree with the dealer that, for the next 50,000 miles, the car is "as good as new?" Explain your reasoning.

(Eq. 2 h(t) = $\frac{f_T(t)}{1-F_T(t)}$)

(b) Now suppose that the dealer tells you that, for t > 50,000 miles, "cars like this have a lifetime distribution with probability density function $f_T(t) = t^{-a}$, for some $a > 1$." Calculate and plot the hazard function h(t) for a = 2 and t > 50,000 miles. Does this ***function increase or decrease with t? Is an older car of this type more or less reliable than a newer car?***

PROBLEM 4

The maximum yearly wind speed at a lcoation is described by the following distribution,

$$F_V(v) = e^{-(v/49.4)^{-6.5}}$$

where V is in mph.

Consider now the problem of choosing the design wind speed v* for a building, such that the probability of non-exceeding v* during the design life of the building equals a target reliability value (R).

a) Plot the target reliability value as a function of number of years of service and design wind speed.

The following data on maximum yearly wind speed were recorded during the period 1944-1977.

year	*max speed (mph)*	*year*	*max speed (mph)*
1944	*57*	*1961*	*60*
1945	*65*	*1962*	*66*
1946	*62*	*1963*	*55*
1947	*58*	*1964*	*51*
1948	*64*	*1965*	*60*
1949	*65*	*1966*	*55*
1950	*59*	*1967*	*60*
1951	*65*	*1968*	*51*
1952	*59*	*1969*	*51*
1953	*60*	*1970*	*62*
1954	*64*	*1971*	*51*
1955	*65*	*1972*	*54*
1956	*73*	*1973*	*52*
1957	*60*	*1974*	*59*
1958	*67*	*1975*	*56*
1959	*50*	*1976*	*52*
1960	*74*	*1977*	*49*

(i) Calculate the sample mean and sample variance of this dataset.

(ii) Find the parameters of the Extreme Type I distribution (Gumbel) for which the theoretical mean and variance match the sample mean and sample variance.

(iii) Plot the target reliability value as a function of number of years of service and design wind speed.

PROBLEM 5

Define the maximum out of N identically distributed random variables as follows,

$$Y_2 = \max\{X_1, X_2, ..., X_N\}$$

Consider now the maximum of a Poisson number N of variables It is easy to obtain exact results for any given distribution F_X of the X variables and any given mean value λT of N.

(In other words, the number of events N occurring during the Period T is random and each event has a probability distribution function, so we are looking for the distribution of the maximum on N random events in the period of time T).

Let $F_N(y)$ be the distribution of Y_2 . Then $F_N(y)$ is the probability that none of the N events in T has intensity X greater than y. Events with this characteristic occur according to a Poisson process with reduced rate $\lambda_y = \lambda[1 - F_X(y)]$. Therefore, the probability that no such event occurs in T is $e^{-\lambda_y T} = e^{-\lambda T[1-F_X(y)]}$ and

$$F_N(y) = e^{-\lambda T[1-F_X(y)]}$$

For example, if X has exponential distribution $F_X(x) = 1 - e^{-x/m}$,

$$F_N(y) = e^{-\lambda T e^{-y/m}}, \qquad y \geq 0$$

(note: this is because the Poisson distribution is preserved under random selection).

Notice that, for $y = 0$, $F_N(0) = \exp\{-\lambda T\}$, which is the probability of no event in T. For $y > 0$, the form of the EX1 distribution has $\alpha = 1/m$ and $u = m\ln(\lambda T)$.

We conclude that, in the present case of exponentially distributed X variables, Y_2 has a distribution of mixed type (neither entirely discrete nor entirely continuous). The distribution has a probability mass $\exp\{-\lambda T\}$ at the origin and the rest of the distribution has the form of a "truncated EX1 distribution".

[Note that : EX1: $F(y) = e^{-e^{-\alpha(y-u)}}$, $-\infty < y < \infty$, $\alpha > 0$]

For the seismic design of structures, one needs to find the distribution of the maximum earthquake magnitude in T years in a given region. Potentially damaging earthquakes (say, earthquakes of magnitude greater than 5) occur with good approximation according to a Poisson process with rate $\lambda_{>5}$ and have independent and exponentially distributed magnitudes,

$$F_M(m) = 1 - e^{-m/m_o}, \qquad m \geq 5, \ m_o = 5.3$$

Notice that this is a shifted exponential distribution with 5 as minimum possible value and that m is used as a symbol for magnitude, not for mean value.

(a) Using results given above, find the distribution of the maximum magnitude in T years, as a function of T and the Poisson rate $\lambda_{>5}$;

(b) The distribution you obtained in Part (a) should depend on T and $\lambda_{>5}$ only through the product $\lambda_{>5}T$, which is the expected number of earthquakes of magnitude above 5 in T years. Plot the cumulative distribution of the maximum magnitude in T years for $(\lambda_{>5}T)$ = 0.1, 1, 10. Comment on the results.