McGill University

Department of Civil Engineering and Applied Mechanics

CIVE-609 – Risk Engineering

Assignment #3

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Instructor: Professor Luc E. Chouinard

TA: Masood Meidani (masood.meidani@mail.mcgill.ca)

Due: Thursday, March 22, 2018 11:30 am.

Problem #1

Consider the sequence of maximum daily temperatures, $\{X_i, i = 0, \pm1, \pm2 ...\}$. We observe X_i for i = i0 (today), and several previous days i_{0-1}, i_{0-2}, ... We want to use these observations, with values X_{i0}, X_{i0-1}, X_{i0-2}, ... to predict the future maximum daily temperatures X_{i0+1}, X_{i0+2}, ...

From a historical record we obtain the following statistics:

- mean value (assumed to be the same for all days of the month): m = 7°C
- Standard deviation (assumed to be the same for all days of the month): σ = 5°C
- correlation function ρ_{ij} (assumed to depend only on the time lag $|i-j|$):

$\|i-j\|$	ρ_{ij}	$\|i-j\|$	ρ_{ij}
0	1.00	6	0.16
1	0.95	7	0.08
2	0.81	8	0.04
3	0.63	9	0.02
4	0.44	$\geq$ 10	0.00
5	0.28		

Using conditional second-moment analysis one can obtain several important results on temperature prediction. For example, one can predict temperature n days ahead, X_{i0+n}, using only temperature today, X_{i0}, or both temperature today and temperature yesterday. If one wants to make predictions based on temperature today and temperature yesterday, then $X_1 = X_{i0+n}$ is a scalar and $X_2 = [X_{i0}, X_{i0-1}]^T$ is a vector.

Numerical results are shown in Figures 1 and 2. For the case when temperature today is X_{i0} = 15°C, Figure 1 shows $E[X_{i0+n}|X_{i0} = 15°C] - 7°C$ and $\{Var[X_{i0+n}|X_{i0}]\}^{1/2}$ as a function of the prediction time lag n. Notice that $E[X_{i0+n}|X_{i0} = 15°C] - 7°C$ is the amount by which the conditional mean (best predictor) deviates from the unconditional seasonal mean of 7°C and $\{Var[Xi0+n|Xi0]\}^{1/2}$ is the standard deviation of the prediction error. As one can see for $n \geq 10$, $\rho_n = 0$ and the best predictor is the seasonal mean. This is why, for $n \geq 10$, $E[X_{i0+n}|X_{i0} = 15°C] - 7°C = 0$ and $\{Var[X_{i0+n}|X_{i0}]\}^{1/2} = 5°C$.

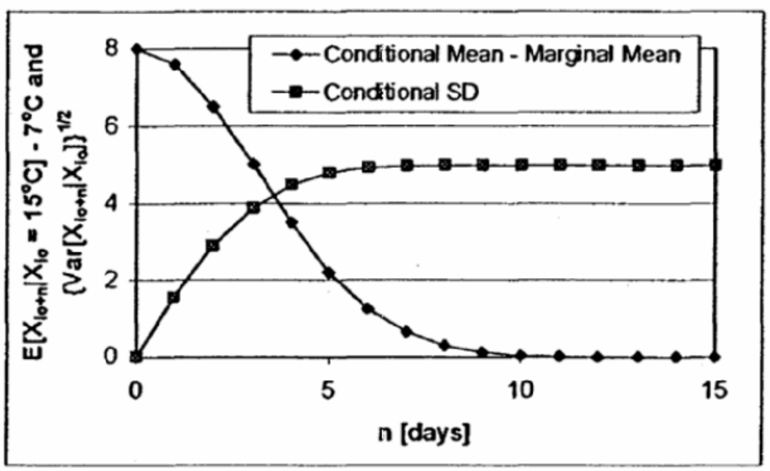


Figure 1: $E[X_{i_0+n}|X_{i_0} = 15°C] - 7°C$ and $\{Var[X_{i_0+n}|X_{i_0}]\}^{1/2}$ as a function of prediction lag in days, n

Figure 2 shows similar results when prediction is based on the observation of temperature today and yesterday. To exemplify, it was assumed that the observed temperature is 15°C for both days. While the trend of the conditional mean and conditional standard deviation are similar to those based only on temperature today, the values are not exactly the same. In particular, using information about temperature yesterday reduces the standard deviation of the prediction error (for example, the standard deviation for two-day lag prediction is about 3°C when one uses only temperature today and about 1.8°C when also temperature yesterday is used).

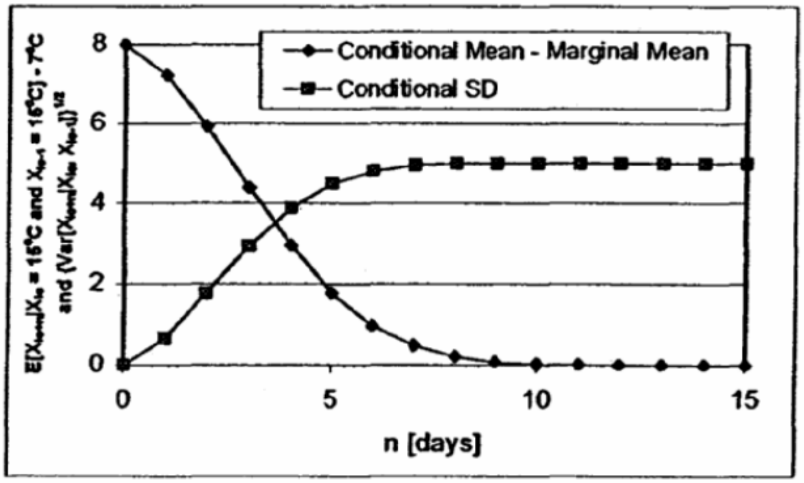


Figure 2: $E[X_{i_0+n}|X_{i_0} = 15°C$ and $X_{i_0-1} = 15°C] - 7°C$ and $\{Var[X_{i_0+n}|X_{i_0}, X_{i_0-1}]\}^{1/2}$ as functions of prediction lead in days, n

Question: Retain all the parameters of the example above, except for the correlations, which are now as follows.

$\|i-j\|$	ρ_{ij}	$\|i-j\|$	ρ_{ij}
0	1.0	6	0.4
1	0.9	7	0.3
2	0.8	8	0.2
3	0.7	9	0.1
4	0.6	≥ 10	0.0
5	0.5		

Produce plots analogous to those in Figures 1 and 2. Compare your results with those in Figures 1 and 2, giving qualitative explanations for the differences.

Problem #2

Pier foundations of bridges over water can be undermined by local scour. The best-fit scour model for bridge piers proposed by Johnson (1992) gives the scour depth X measured from the average channel bed to the bottom of the scour hole as

$$X = 2.02Y(b/Y)^{0.98}F_r^{0.21}W^{-0.24},$$

where Y is the depth of flow just upstream of the pier, F_r is the upstream Froude number ($F_r = V/(gY)^{1/2}$, V and g denote the approach flow velocity and acceleration due to gravity, respectively), W is sediment gradation (equal to $d_{84\%}/d_{50\%}$, the ratio between the 84% quantile to the median sediment diameter), and b is the pier width. All these quantities are measured in metric units. Using the Manning formula to compute the velocity for a wide rectangular channel cross section,

$$V = (1/n)S^{1/2}Y^{2/3},$$

where n is the roughness coefficient and S is the slope. Hence, the Froude number is

$$F_r = \frac{V}{(gY)^{1/2}} = S^{1/2}Y^{1/6}n^{-1}g^{-1/2},$$

thus,

$$X = 2.02Y(b/Y)^{0.98}(S^{1/2}Y^{1/6}n^{-1}g^{-1/2})^{0.21}W^{-0.24},$$

which, after substituting 9.81 m/s^2 for g, can be written as

$$X = 1.59b^{0.980}Y^{0.055}S^{0.105}n^{-0.210}W^{-0.240}.$$

The estimation of Y, S, n, and W is affected by uncertainties. We propose to model all these quantities as random variables. One can thus determine the probability distribution of X by simulation if the probability distributions of Y, S, n, and W are known. For a pier width of 2.5 m, suppose that sediment gradation $W \sim$ lognormal $(4, 1.6^2)$, the slope $S \sim N(0.002, 0.0004^2)$, the depth $Y \sim N(4.75\,\text{m}, 1.2^2\,\text{m}^2)$, and the roughness coefficient $n \sim$ uniform $(0.02, 0.04)$. Also, one can reasonably assume that Y, S, n, and W are independent of each other.

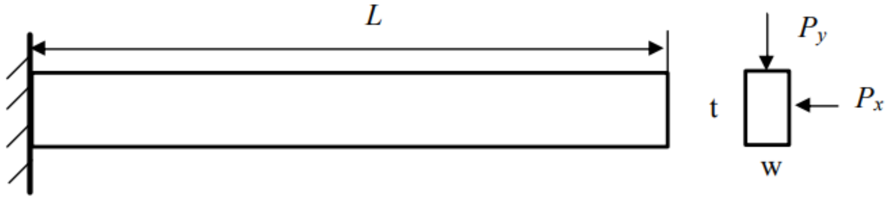
a) Use FOSM analysis to determine the mean and the variance of X

b) Use Monte Carlo simulation to determine the mean and variance of X\

c) Repeat a and b when S and Y have a correlation of 0.5.

Problem #3

A cantilever beam is illustrated below.



One of the failure modes is that the tip displacement exceeds the allowable value, D_0. The performance function is the difference between D_0 and the tip displacement and is given by

$$g = D_0 - \frac{4L^3}{Ewt}\sqrt{\left(\frac{P_y}{t^2}\right)^2 + \left(\frac{P_x}{w^2}\right)^2}$$

where D_0 = 3" , E = 30 * 10^6 psi is the modulus of elasticity, L =100" is the length, w and t are width and height of the cross section, respectively, and w = 2" and t = 4" . P_x and P_y are external forces which follow normal distributions, and $P_x \sim N$ (500,100) lb and $P_y \sim N$ (1000,100) lb . The probability of failure is defined as the probability of the allowable displacement less than the tip displacement, i.e.

$$p_f = P\left\{ g = D_0 - \frac{4L^3}{Ewt}\sqrt{\left(\frac{P_y}{t^2}\right)^2 + \left(\frac{P_x}{w^2}\right)^2} \le 0 \right\}$$

a) Find the probability of the failure using Monte Carlo simulation (10, 100, 1000 simulations).
b) Draw the failure path in a graph where P_x is horizontal and P_y is vertical axis. Show safe region and failure region in the graph.
c) Draw the *cdf* and *pdf* curve of the performance function (g) with 1000 simulations.