McGill University
Department of Civil Engineering and Applied Mechanics
CIVE 603 – Structural Dynamics

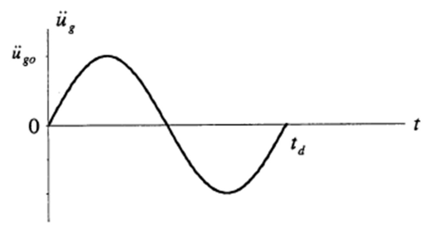
Instructor: Sarven Akcelyan, PhD
Macdonald Engineering Bld., Room 275, e-mail: sarven.akcelyan@mcgill.ca

Homework #3 (Total Points 100/100)
Group assignment: max. 2 students
Due Date: February 27th 2018 by 4:00pm
Suggested Solutions

Problem 1 (40 points)

Part A

Implement in a computer language of your choice the central difference method that was shown in class. Validate your solver with the analytical solution from the pulse shown in the figure below. Assume that T_n=1sec, ζ=0%, a unit seismic mass and that the pulse duration is t_d = 1.5sec. The peak ground acceleration of the pulse is $\ddot{u}_{go} = 0.5g$. Assume an integration time step Δt based on your judgment but you need to justify your selection.



Part B

By using the program that you developed in Part A determine the relative deformation response *u(t)* for $0 \le t \le 20$sec for a SDF system with natural period T_n = 2.0 sec to El Centro 1940 ground motion and the Canoga Park record from the 1994 Northridge earthquake for the following cases:

1. Damping ratio ζ=0%
2. Damping ratio ζ=5%
3. Develop the absolute acceleration, relative velocity and relative displacement response spectra for 5% damping ratio.
4. Compare the absolute acceleration and relative velocity response spectra with the equivalent pseudo acceleration and velocity response spectra for the two ground motions of interest.

Notes:

- Assume a unit seismic mass
- sampling rate of El Centro ground motion Δt = 0.02sec
- sampling rate of Canoga Park record Δt = 0.01sec

Solution

Part A

The algorithm is implemented in MATLAB based on the same principles that were shown during the lecture.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Full Sine Pulse Excitation for linear SDF systems (Homework 3, Problem 1
% Utilizing Central Difference Method
% Course: CIVE 603 - Structural Dynamics - Winter 2017
% Sarven Akcelyan, McGill University
% Created: Feb 20, 2017
%
%
%
%% Clear variables, close all figures/tables and clear the command line
clear all; close all; clc;

%% Define Input Parameters of SDF System
% units [kip,inch,sec]
Tn = 1.0;                          % Period of SDF system [sec]
wn = 2*pi/Tn;                      % circular frequency (calculated) [rads/sec]
m = 1.0;                           % Seismic mass of SDF system [kips-s^2/in]
k = m/(Tn/(2*pi))^2;               % lateral stiffness (calculated) [kips/in]
z = 0.00;                          % damping ratio of SDF system
c = z*(2*m*wn);                    % damping coefficient (calculated)
u0 = 0;                            % Initial Displacement [in]
v0 = 0;                            % Initial velocity [in/sec]
Tfree= 4*Tn;                       % Free vibration duration after forced vibration[sec]
g = 386.22;                        % acceleration of gravity [in/sec^2]

%% Full Sine Pulse
dtG = 0.02;
dtA = 0.02;
td = 1.5*Tn;
ao = 0.5;  %[g]
AGM = (ao*sin(2*pi*[dtA:dtA:td]/td))';
% Started from dtA to be synchronized with analytical solution
% because the solver adds 0 to the ground motion
%% Numerical Solution-Central Difference Method
[Time, u, v, acc, ab_acc, fs]=CentralDiff_Solver(m,Tn,z,dtG,dtA,AGM,g,Tfree);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

In order to validate the implementation of the algorithm in MATLAB we compare the numerical solution obtained from the algorithm with the closed form solution of an undamped SDF oscillator subjected to a pulse with t_d = 1.5sec.

Implementation of the analytical solution in MATLAB:

```
%% Analytical solution
t=[0:dtA:max(Time) max(Time)];
for i=1:length(t)
    if t(i)<=td
        u_a(i)=-0.5*g/k*(1/(1-(Tn/td)^2)*(sin(2*pi*t(i)/td)-Tn/td*sin(2*pi*t(i)/Tn)));
    else
        u_a(i)=-0.5*g/k*2*(Tn/td)*sin(pi*td/Tn)/((Tn/td)^2-1)*cos(2*pi*(t(i)/Tn-
1/2*td/Tn));
    end
end
```

因为需要带入一个地震加速的的vector，所以需要假设dt，在函数中，如果出现精度不够，在调整（用插值函数）

Central Difference Method as function

```
function [Time, u, v, acc, ab_acc, fs]=CentralDiff_Solver(m,Tn,z,dtG,dtA,AGM,g,Tfree)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This function performs Earthquake Response History Analysis for linear SDF systems
% By utilizing Central Difference Method
% Subroutine for Elastic_Response_Spectrum.m
% Course: CIVE 603 - Structural Dynamics - Winter 2017
% Instructor: Sarven Akcelyan, McGill University
% Created: Feb 19, 2017
%
% Input:
% m = mass of the SDF
% Tn = natural period of the SDF
% z = damping ratio (ex. 0.05)
% dtG= time step of the record [sec]
% dtA= given time step of the analysis [sec] (ex. dtG)
% AGM = time history of the external force (record)
% g = gravitational acceleration [other units are computed based on the unit of g]
% Tfree = duration of the free vibration after forced vibration (ex. 5*Tn sec)
%
% Output:
% Time = Time Vector [sec]
% u = relative deformation response history
% v = relative velocity response history
% acc = relative acceleration response history
% ab_acc = absolute acceleration response history
% fs = elastic resisting force history
%% Parameters of SDF System
wn = 2*pi/Tn;                    % circular frequency (calculated) [rads/sec]
k = m/(Tn/(2*pi))^2;             % lateral stiffness (calculated)
c = z*(2*m*wn);                  % damping coefficient (calculated)
u0 = 0;                          % Initial Displacement
v0 = 0;                          % Initial velocity
```

此处dA是先假设的数值

先跟dtG地面加速度的step比较，确定dtA分析的加速度step

```
%% Accuracy Check-GM, Interpolation in case dt is different than dtG
% Note: Central difference method is conditionally stable for dt < Tn/pi
dt=dtG;
while dt > Tn/10 % For accuracy in small periods (this is smaller than condition)
    dt = dt/2;
end
dt=min(dt,dtA); % final time step of the analysis [sec]
```

用除以2的方式保证了只需要精度提高的时候进行插值

保证dt小于 Tn/pi（为了保佑用10代替Pi）

```
if dt<dtG
    AGMN=interp1(0:dtG:dtG*(length(AGM)),[0;AGM],0:dt:dtG*(length(AGM)));
else
    AGMN=[0;AGM]';
end
```

此处是为了给地震加速度加上第一个位置是0，原来计算的地震加速度的向量用的dt是dtA，0.02，是given time step，如果现在用的dt比刚才计算用的dtA更小，则需要重新插值 获得新的地震加速度，保证每个step都有对应的地震加速度

```
%% External Force vector
Ag=[AGMN';zeros(round(Tfree/dt),1)];
p = -m*Ag*g;                     % External force vector

%% Solver - Central Difference Method)
% Step 1: Initial Calculations
p0 = p(1);                       % initial value of external force
a0 = (p0 - c*v0 - k*u0)/m;
u_1 = u0 - dt*v0 + dt^2/2*a0;
kh = m/dt^2 + c/(2*dt);
a = m/dt^2 - c/(2*dt);
b = k - 2*m/dt^2;
```

```
% Initialize Vectors to be used
Time(1) = 0.00;
acc(1) = a0;
v(1) = v0;
u(1) = u0;
ph(1) = p0 - a*u_1 - b*u0;
u(2) = ph(1)/kh;
% Step 2 Calculations for time step i
for i = 2:(length(p));
    ph(i) = p(i) - a*u(i-1) - b*u(i);
    u(i+1) = ph(i)/kh;
    v(i) = (u(i+1) - u(i-1))/(2*dt);
    acc(i) = (u(i+1) - 2*u(i) + u(i-1))/dt^2;
    Time(i) = Time(i-1) + dt;
end
u(end)=[];  % remove last extra u due to u(i+1);
% Absolute acceleration history = relative acceleration + ground acceleration
ab_acc = acc + Ag' * g;
fs = k*u; % fs = elastic resisting force history
fd = c*v; % fd = damping force history
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Figure 1 shows that there is nearly no difference between the closed-form solution (i.e., analytical solution) and the numerical solution that is obtained from the central difference method .

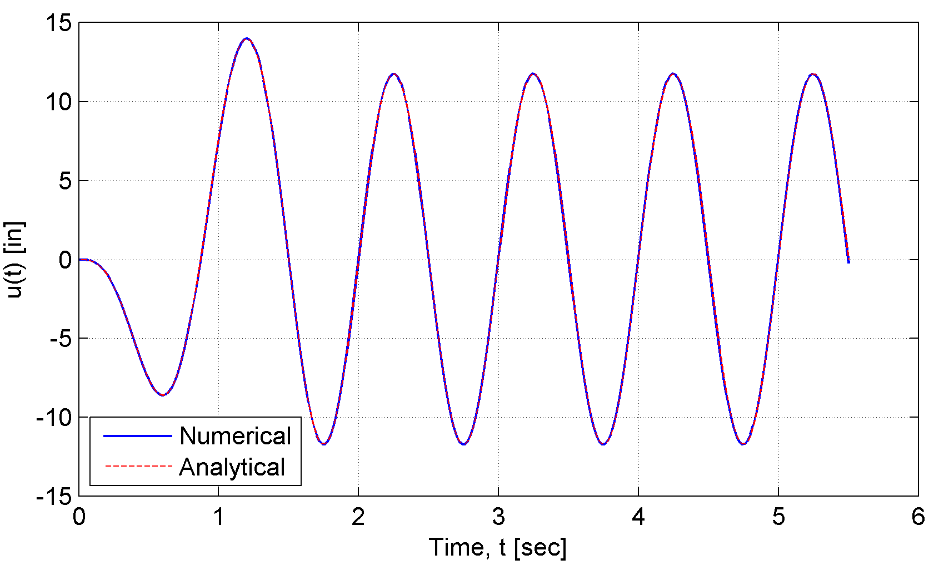
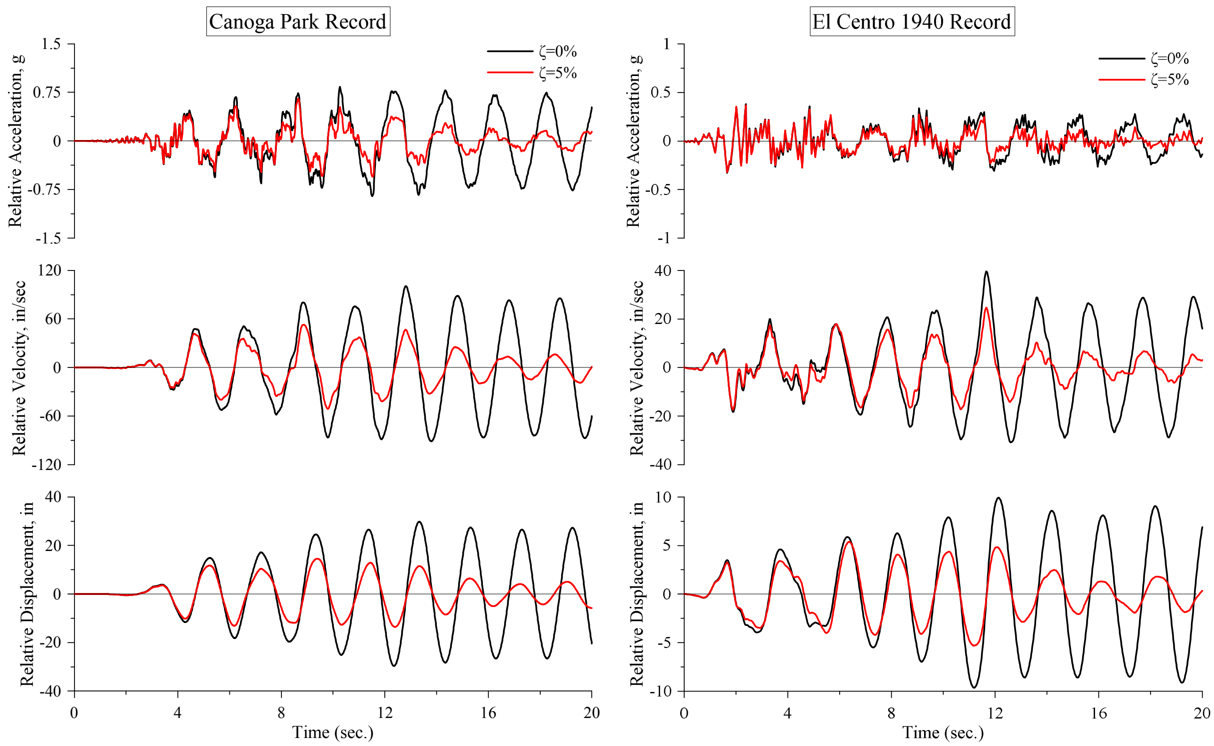


Figure 1. Comparison of analytical and numerical solutions for SDF with T_n = 1.0sec and ζ=0%.

Part B

Now that we have validated the algorithm that was developed in Part A two earthquake excitations are used in order to compute the response of a SDF system with a period T_n = 2.0sec. The figures below illustrate the displacement response history of the above SDF system for ζ=0% and 5% damping ratios (i.e., questions 1 and 2):



(a) Canoga park record (b) El Centro record

Figure 2. Response history of relative acceleration, relative velocity and relative displacement of the SDF system with T_n=2.0sec and ζ=0% and 5% (images courtesy of Seong-Hoon Hwang).

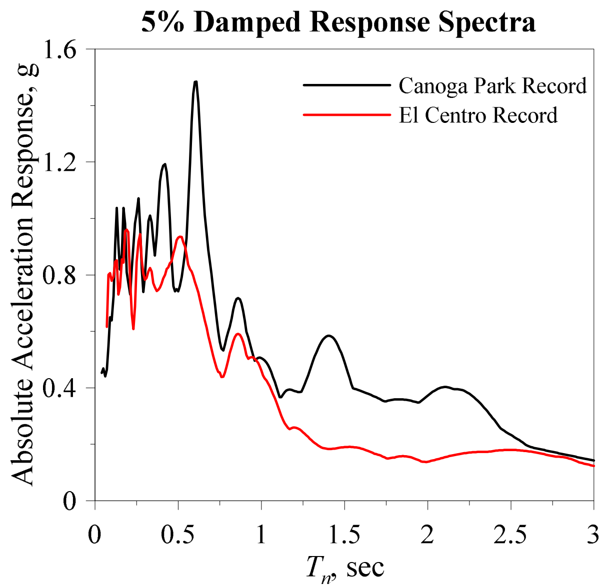
In order to compute the response spectra for the two ground motions that are given the same algorithm that was implemented in Part A is employed. However, for a range of SDF periods given the damping ratios of interest (ζ=0% and 5%) the following script is developed that compute the response spectra:

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This script computes and plots Elastic Response Spectrum
% Subroutine: Newmark_AvAcc_Solver.m  or CentralDiff_Solver.m
% Course: CIVE 603 - Structural Dynamics - Winter 2017
% Instructor: Sarven Akcelyan, McGill University
% Created: Feb 19, 2017
%
%
%
%% Clear variables, close all figures/tables and clear the command line
clear all; clc; close all
% Inputs
AGM = load('ElCentro.th');  % Load ground motion file
dtG = 0.02; % Ground motion sampling time step
dtA = dtG;  % Response history analysis time step
g=386.22;   % Gravitation Acceleration [in/sec^2]
Tns=0.01:0.01:3; % SDF Periods for response spectrum [sec]
Zetas=[0.0 0.05]; % damping ratio for response spectrum
m = 1; % assume unit mass
%% Running Response Histories for Tns and Zetas
for j=1:length(Zetas)
    for i=1:length(Tns)
        Tn=Tns(i);
        z=Zetas(j);
        Tfree=0*Tn;
        [Time, u, v, acc, ab_acc, fs] =CentralDiff_Solver(m,Tn,z,dtG,dtA,AGM,g,Tfree);
        wn = 2*pi/Tn;                     % circular frequency (calculated)
        RelD(i,j)=max(abs(u));            % relative displacement
        RelV(i,j)=max(abs(v));            % relative velocity
        RelA(i,j)=max(abs(acc));          % relative acceleration
        AbsA(i,j)=max(abs(ab_acc));       % absolute acceleration
        SpA(i,j)=RelD(i,j)*wn^2;          % pseudo - acceleration
        SpV(i,j)=RelD(i,j)*wn;            % pseudo - velocity
    end
end
```

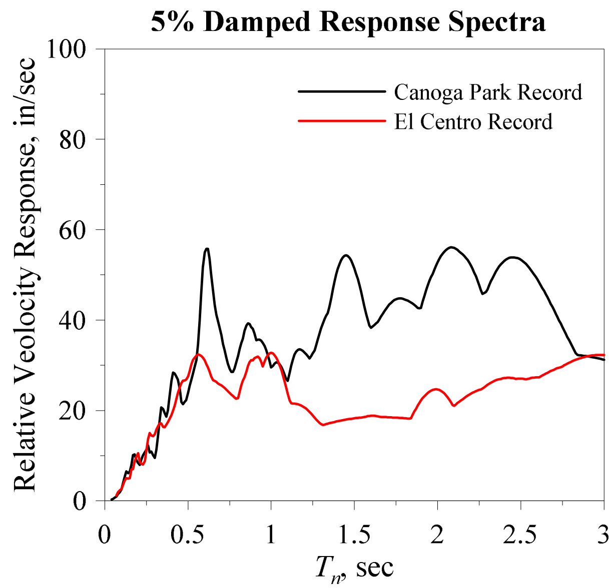
Tns()带入不同的周期分别求最大值

Figure 3 illustrates the response spectra for the two earthquakes for 5% damping ratio. The response spectra are plotted in the same figures for comparison purposes. Few observations:

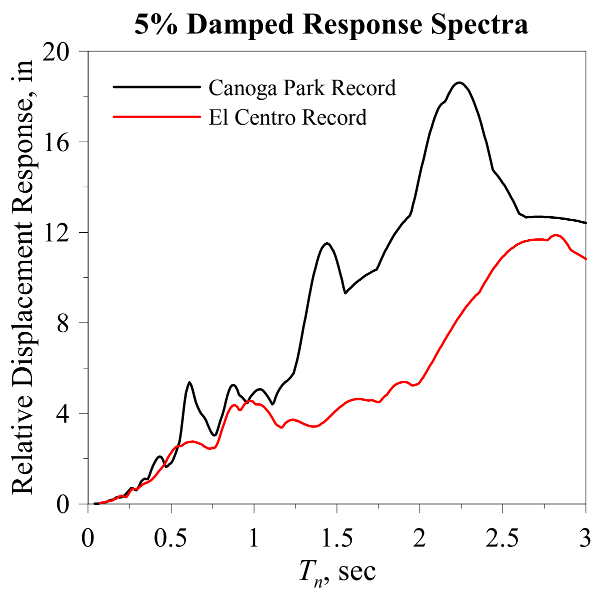
- From figure 3a note the high absolute acceleration demands that the Canoga Park record imposes to an SDF system in the range of periods 1 to 2.5sec compared to that of ElCentro.
- From figure 3c the displacement demands that the Canoga Park record imposes an SDF system in the range of periods 1 to 2.5sce is nearly double compared to ElCentro.



(a) Absolute acceleration response spectra



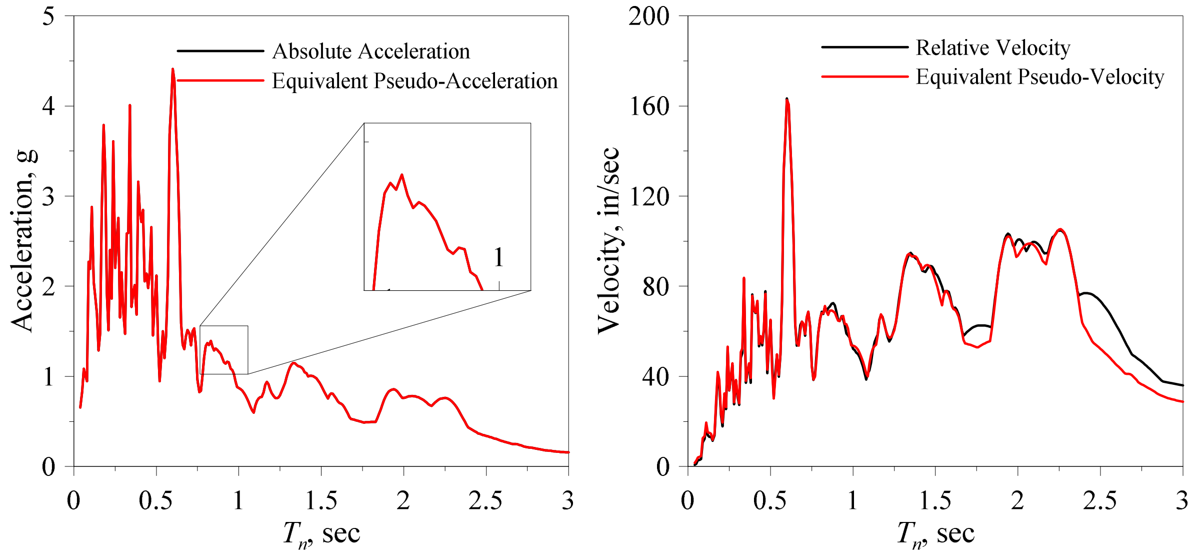
(b) Relative velocity response spectra.



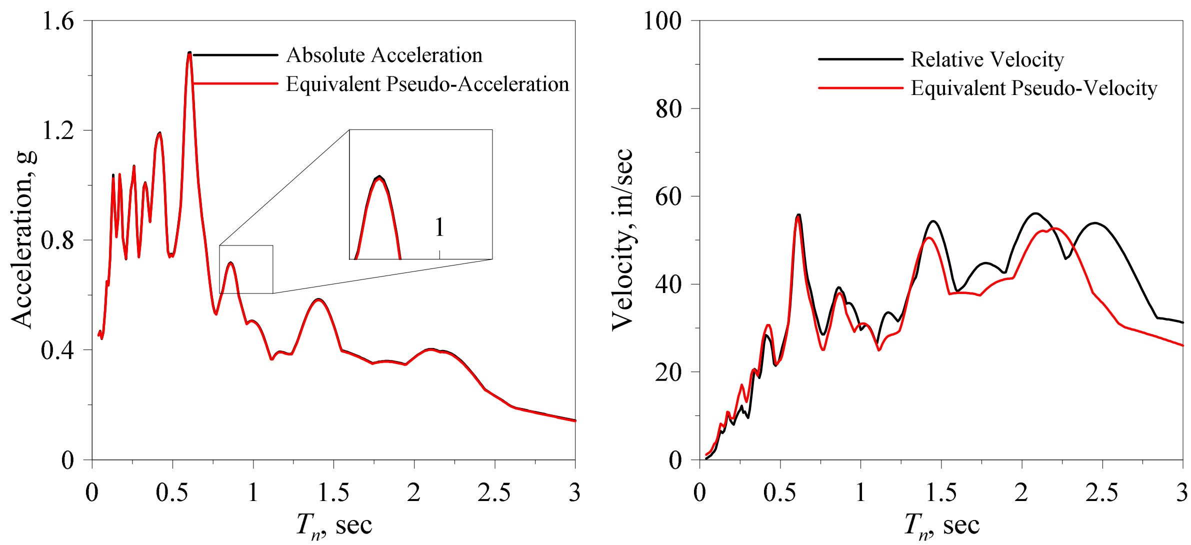
(c) Relative displacement response spectra

Figure 3. Response spectra for ζ=5% (images courtesy of Seong-Hoon Hwang).

The comparison of response spectra with the pseudo-spectra is shown in Figure 4. Note that in order to develop the pseudo-spectra ω_n changes over the range of spectra of interest. Note that the comparison between pseudo-spectra and real absolute acceleration and relative velocity spectra indicates that the two match relatively well for the entire range of periods shown in the graph.

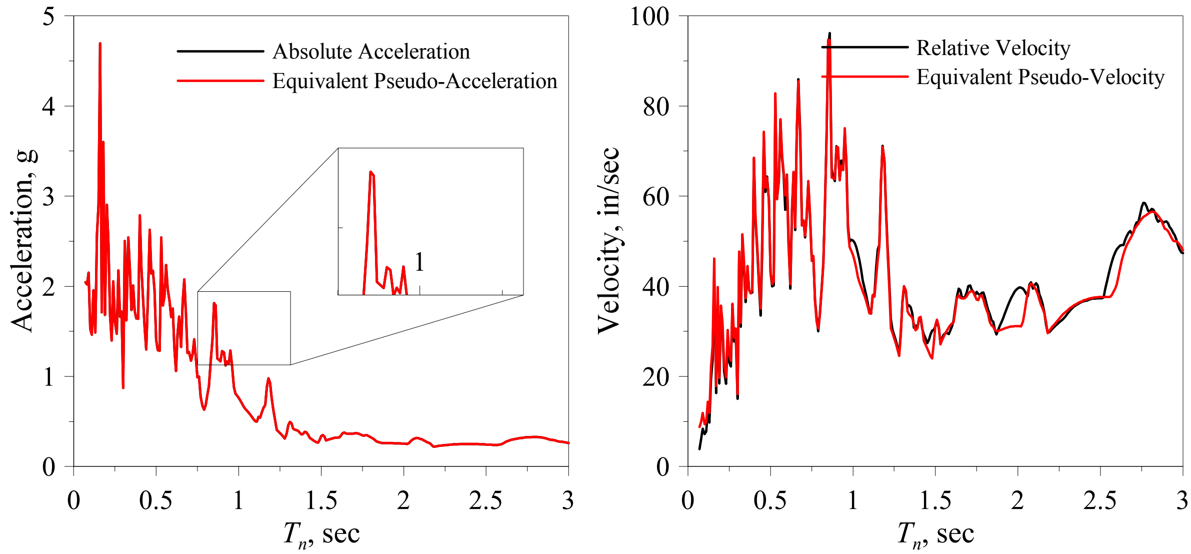


(a) ζ=0%

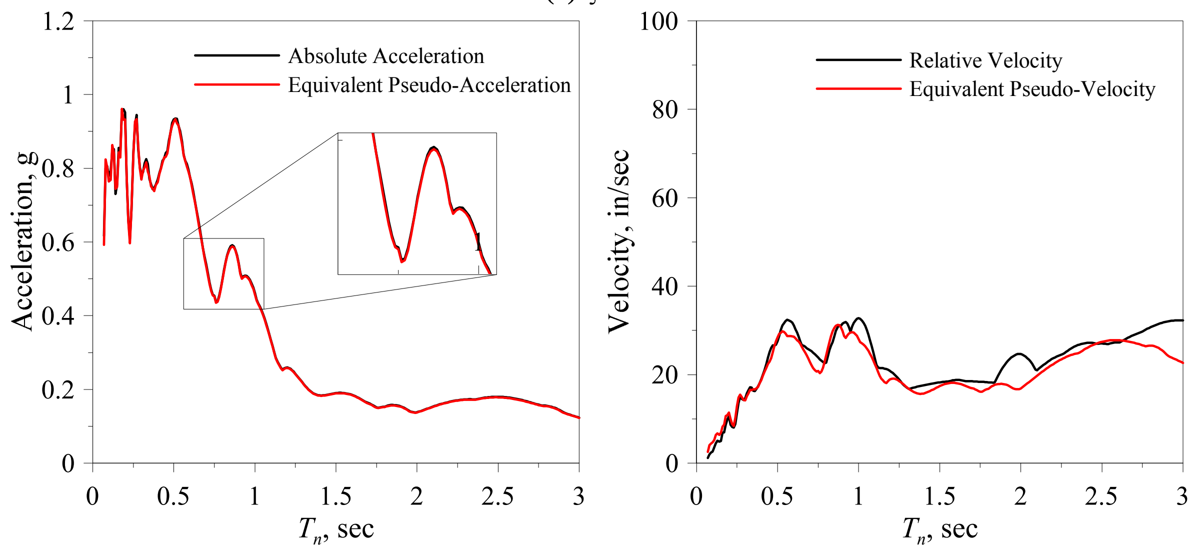


(b) ζ=5%

Figure 4. Comparison of the absolute acceleration and relative velocity response spectra with the equivalent pseudo acceleration and velocity response spectra for the Canoga Park record (images courtesy of Seong-Hoon Hwang)



(a) ζ=0%



(b) ζ=5%

Figure 5. Comparison of the absolute acceleration and relative velocity response spectra with the equivalent pseudo acceleration and velocity response spectra for the El Centro record (images courtesy of Seong-Hoon Hwang)

Problem 2 (30 points)

Part A

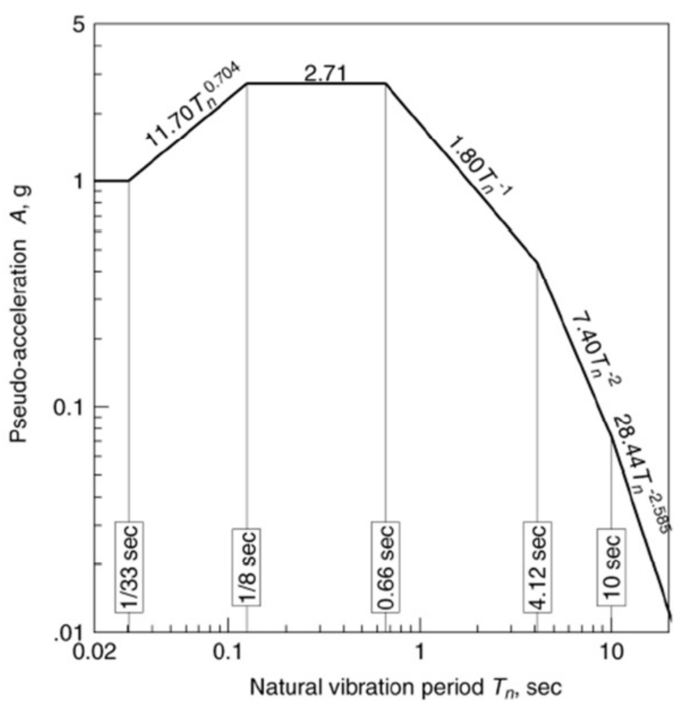
A full water tank is supported on an 80-ft-high cantilever tower. It is idealized as an SDF system with weight w = 100kips, lateral stiffness k = 4kips/in., and damping ratio ζ = 5%. The tower supporting the tank is to be designed for ground motion characterized by the design spectrum shown below scaled to 0.5g peak ground acceleration. Plot the pseudo-acceleration, pseudo-velocity and deformation spectrum up to T_n=12 sec. using linear (non-logarithmic) scale. Determine the design values of lateral deformation and base shear. Recalculate the design values of lateral deformation and base shear, assuming ζ = 2% and ζ = 20%. Utilize the Newmark-Hall formulations for damping modification factor, *B*, given in the lecture notes.

Part B

Determine the values of lateral deformation and base shear of the full water tank for the 5% absolute acceleration response spectra of the two ground motions that you used in Problem 1.

Part C

The deformation computed for the system in Part A seemed excessive to the structural designer, who decided to stiffen the tower by increasing its size. Determine the design values of deformation and base shear for the modified system if its lateral stiffness is 8 kips/in.; assume that the damping ratio is still 5%. Comment on how stiffening the system has affected the design requirements. What is the disadvantage of stiffening the system?



Solution

Part A

We first need to compute the cyclic frequency of the SDF system and its period.

$$T_n = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{100/386.22}{4}} = 1.60\,\text{sec}$$

The maximum absolute acceleration response spectra of the scaled intensity for T_n=1.60sec is given as follows:

$S_a(T_n\text{=1.60sec, 5\%}) = 0.5*1.80*(1.60)^{-1} = 0.5625g$

The displacement demand of this SDF system should be:

$$u_o = \frac{S_a\left(T_n, 5\%\right)}{\omega_n^2} = \frac{0.5625g}{\left(2\pi/1.60\right)^2} = 14.0625 - in.$$

The base shear of the water tank should be: $V_b = k^* u_o$ = 56.25kips.

Figure 6 shows the pseudo-acceleration, pseudo-velocity and relative displacement design spectra.

The displacement demand of this SDF system with 2% damping ratio should be:
Utilizing Newmark-Hall formulation for constant velocity region:

$$B = 1.4 - 0.248\ln(2) = 1.228$$

$$u_o = S_d\left(T_n, 2\%\right) = B \cdot S_d\left(T_n, 5\%\right) = 1.228 \cdot 14.06 = 17.27\text{in}$$

The base shear of the water tank should be: $V_b = k^* u_o$ = 69.12kips.

The displacement demand of this SDF system with 20% damping ratio should be:
Utilizing Newmark-Hall formulation for constant velocity region:

$$B = 1.4 - 0.248\ln(20) = 0.657$$

$$u_o = S_d\left(T_n, 20\%\right) = B \cdot S_d\left(T_n, 5\%\right) = 0.657 \cdot 14.06 = 9.24\text{in}$$

The base shear of the water tank should be: $V_b = k^* u_o$ = 36.98kips.

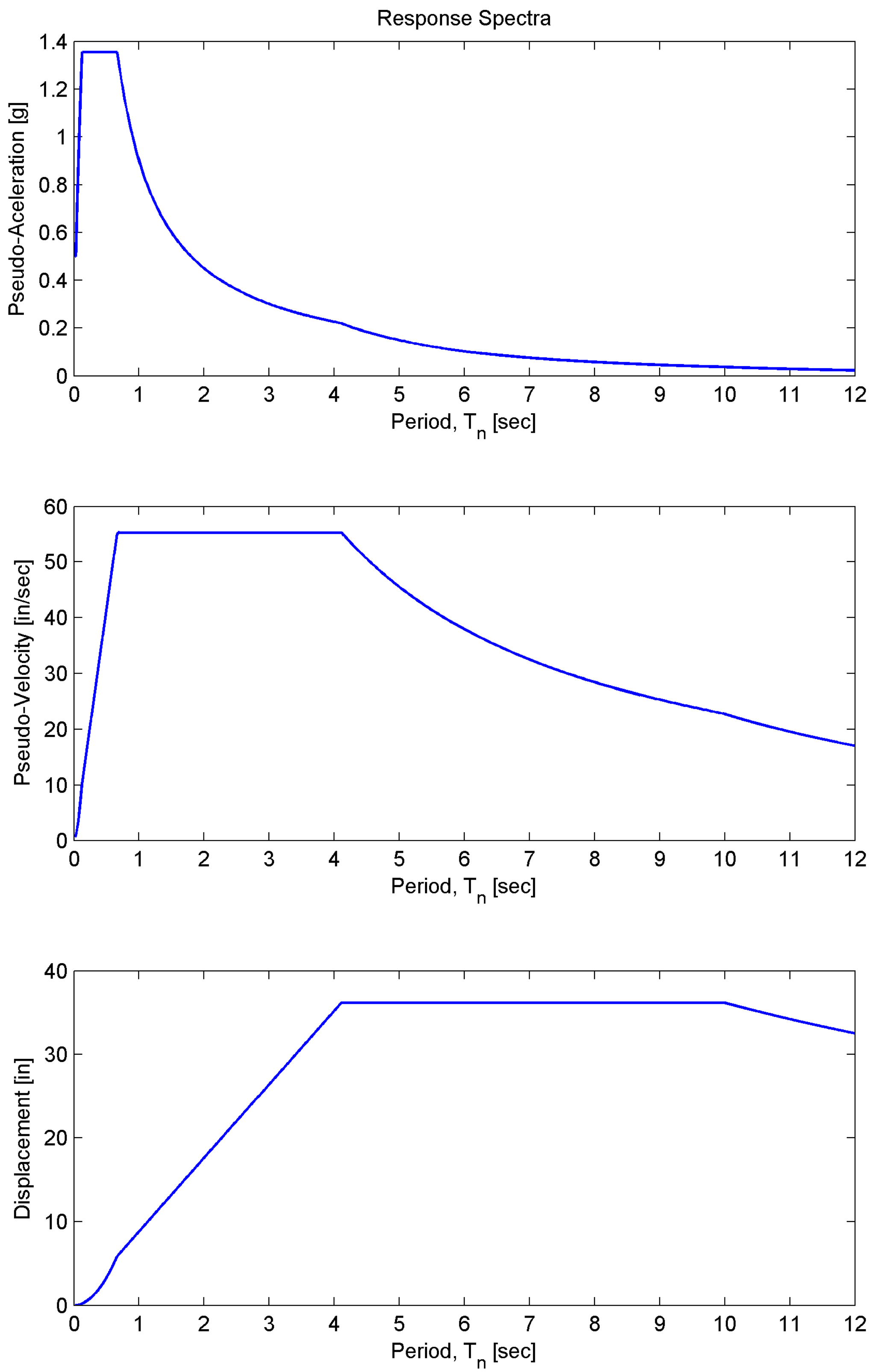
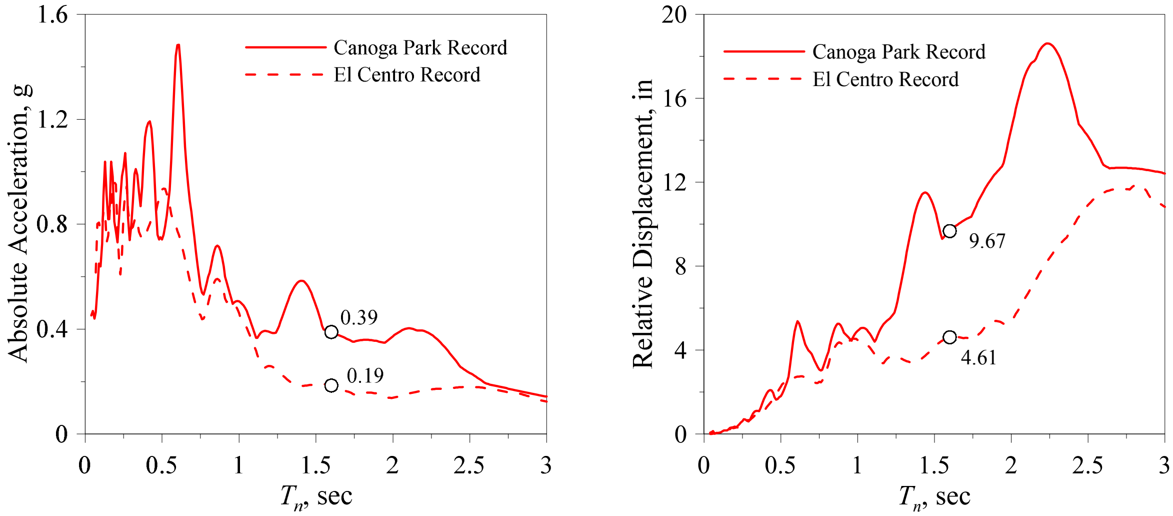


Figure 6. 5% pseudo-acceleration, pseudo-velocity and relative displacement design spectra

Part B

Figure 7 shows the absolute acceleration response spectra and relative displacement response spectra values for the two ground motions of interest at a period T_n = 1.60sec.



(a) 5% absolute acceleration response spectra (b) 5% relative displacement response spectra

Figure 7. 5% absolute acceleration and relative displacement response spectra of the two ground motions (images courtesy of Seong-Hoon Hwang)

	Lateral deformation (in.)	Base shear (kips)
Canoga Park Record	9.67	38.68
El Centro Record	4.61	18.44

Part C

Because the lateral stiffness of the cantilever tank changes then its period T_n also changes.

$$T_n = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{100/386.22}{8}} = 1.13\text{sec}$$

The maximum absolute acceleration response spectra of the scaled intensity for T_n=1.13sec is given as follows:

$S_a(T_n$=1.13sec, 5%) = 0.5*1.80*$(1.13)^{-1}$ = 0.796g

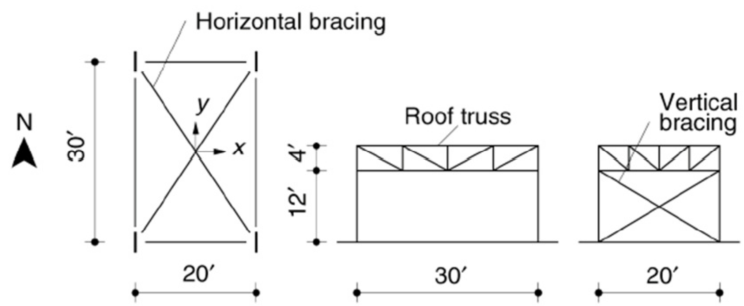
The displacement demand of this SDF system should be:

$$u_o = \frac{S_a(T_n, 5\%)}{\omega_n^2} = \frac{0.796g}{(2\pi/1.13)^2} = 9.95 - in.$$

The base shear of the water tank should be: $V_b = m^*$ $S_a(T_n$=1.13sec, 5%) = 79.6kips.

Problem 3 (30 points)

The small single story industrial building, 20 x 30 *ft* in plan, is shown in the figure below with moment frames in the north-south direction and braced frames in the east-west direction. The weight of the structure can be idealized as 200 lb/ft^2 lumped at the roof level. The horizontal cross bracing is at the bottom chord of the roof trusses. All steel columns are W14x53 sections. The modulus of elasticity of the steel material is E=29,000*ksi*. The vertical cross bracing made of round hollow steel sections HSS 3 x 0.12 (i.e., diameter d = 3-in., and HSS thickness, t = 0.12in.).



Determine the peak response of the single-story industrial building to ground motion characterized by the two absolute acceleration response spectra given in Problem 1 for 5% damping ratio.

1. For north-south excitation determine the lateral displacement of the roof and the bending moments in the columns.
2. For east-west excitation determine the lateral displacement of the roof and the axial force in each brace.

Solution

The weight of the structure shown in the figure above can be calculated as follows:

m = 200*20*30/386.22/1000 = 0.310 kips-s^2/in. This mass is moving in both loading directions.

Column section: W14x53: $I_x = 541in^2$, $I_y = 57.7in^2$.

North-South Direction (Moment frame):

Assume that the roof truss acts as a rigid diaphragm; therefore, the lateral system of the structure in this direction:

$$K = 4\frac{12EI_x}{L^3} = 4\frac{12\cdot 29000\cdot 541}{144^3} = 252.2 kips/in$$

The predominant period of the structure T_n in the North-South direction is as follows:

$$T_n = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.310}{252.2}} = 0.22\sec$$

East-West Direction (Braced Frame):

Assume tension-only brace (i.e., the compressive resistance is neglected in this case)
Assume that the roof truss acts as a rigid diaphragm; therefore, the lateral system of the structure in this direction:

L_b = 23.3ft, $\cos\theta = 20 \big/ \sqrt{12^2 + 20^2} = 0.8575$, A_b = 1.02in^2.

$$k_{EW} = 2\frac{EA_b}{L_b}\cos^2\theta = 2\frac{29000 \cdot 1.02}{23.3 \cdot 12}\left(0.8575\right)^2 = 155 kips/in$$

$$k_{EW,c} = 4\frac{12EI_y}{L^3} = 4\frac{12 \cdot 29000 \cdot 57.7}{144^3} = 26.9 kips/in$$

Note that you can ignore the column flexural stiffness in this case for simplicity. The lateral stiffness of the braced frame is dominated by the axial stiffness of the braces.

The predominant period of the structure T_n in the East-West direction is as follows:

$$T_n = 2\pi\sqrt{\frac{m}{k_{EW}}} = 2\pi\sqrt{\frac{0.310}{155}} = 0.28\sec$$

A summary of the results in then presented for questions 1 and 2:

	N-S		E-W	
	Canoga Park Record	El Centro Record	Canoga Park Record	El Centro Record
T_n, sec	0.22		0.28	
Lateral Displacement (in)	0.389	0.227	0.698	0.661
Bending Moment (columns) (kips-in)	1766	1233	--	--
Axial Force (each brace) (kips)	--	--	63.3	59.9

Column Bending Moment = $6EI_x/L^2 * u$
Brace Axial Force = $EA_b/L_b * (u * \cos\theta)$