CIVE 603

Structural Dynamics

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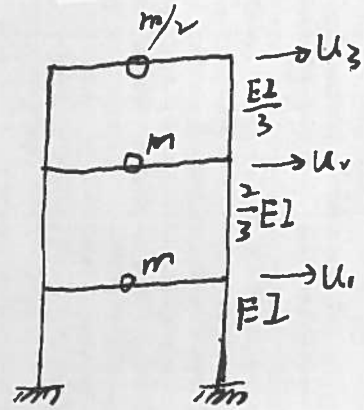
Problem 1:



We can easily get

$$[k] = \begin{pmatrix} \frac{5}{3} & -\frac{2}{3} & 0 \\ -\frac{2}{3} & 1 & -\frac{1}{3} \\ 0 & -\frac{1}{3} & \frac{1}{3} \end{pmatrix} \cdot \frac{24EI}{h^3}$$

$$[M] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{pmatrix} \cdot m$$

1.

$$[k - w_n^2 m] = \begin{pmatrix} \frac{5}{3}\frac{EI}{h^3} - w_n^2 m & -\frac{2}{3}\frac{EI}{h^3} & 0 \\ -\frac{2}{3}\frac{EI}{h^3} & \frac{EI}{h^3} - w_n^2 \cdot m & -\frac{1}{3}\frac{EI}{h^3} \\ 0 & -\frac{1}{3}\frac{EI}{h^3} & \frac{EI}{3h^3} - \frac{w_n^2 m}{2} \end{pmatrix} \cdot \frac{24EI}{h^3}$$

$$\det[k - w_n^2 m] = 0 \Rightarrow \begin{cases} w_{n1}^2 = 5.016\frac{EI}{mh^3} \\ w_{n2}^2 = 24\frac{EI}{mh^3} \\ w_{n3}^2 = 50.976\frac{EI}{mh^3} \end{cases} \Rightarrow \begin{cases} w_{n1} = 2.24\sqrt{\frac{EI}{mh^3}} \\ w_{n2} = 4.90\sqrt{\frac{EI}{mh^3}} \\ w_{n3} = 7.140\sqrt{\frac{EI}{mh^3}} \end{cases} \Rightarrow \begin{cases} T_1 = 2.80\sqrt{\frac{mh^3}{EI}} \\ T_2 = 1.28\sqrt{\frac{mh^3}{EI}} \\ T_3 = 0.88\sqrt{\frac{mh^3}{EI}} \end{cases}$$

For $w_{n1}^2 = 5.016 \cdot \frac{EI}{mh^3}$

$$\frac{24EI}{h^3}\begin{pmatrix} 4373/3000 & -\frac{2}{3} & 0 \\ -\frac{2}{3} & 791/1000 & -\frac{1}{3} \\ 0 & -\frac{1}{3} & 373/3000 \end{pmatrix} \cdot \begin{pmatrix} \phi_{11} \\ \phi_{21} \\ \phi_{31} \end{pmatrix} = 0$$

Assume $\phi_{31} = 1 \Rightarrow \phi_1 = \begin{pmatrix} 0.314 \\ 0.686 \\ 1 \end{pmatrix}$

For $w_{n2}^2 = 24 \cdot \frac{EI}{mh^3}$

$$\frac{24EI}{h^3}\begin{pmatrix} \frac{2}{3} & -\frac{1}{3} & 0 \\ -\frac{2}{3} & 0 & -\frac{1}{3} \\ 0 & -\frac{2}{3} & -\frac{2}{3} \end{pmatrix} \cdot \begin{pmatrix} \phi_{12} \\ \phi_{22} \\ \phi_{32} \end{pmatrix} = 0$$

Assume $\phi_{32} = 1 \Rightarrow \phi_2 = \begin{pmatrix} -0.5 \\ -0.5 \\ 1 \end{pmatrix}$

For $w_{n3}^2 = 50.976 \frac{EI}{mh^3}$

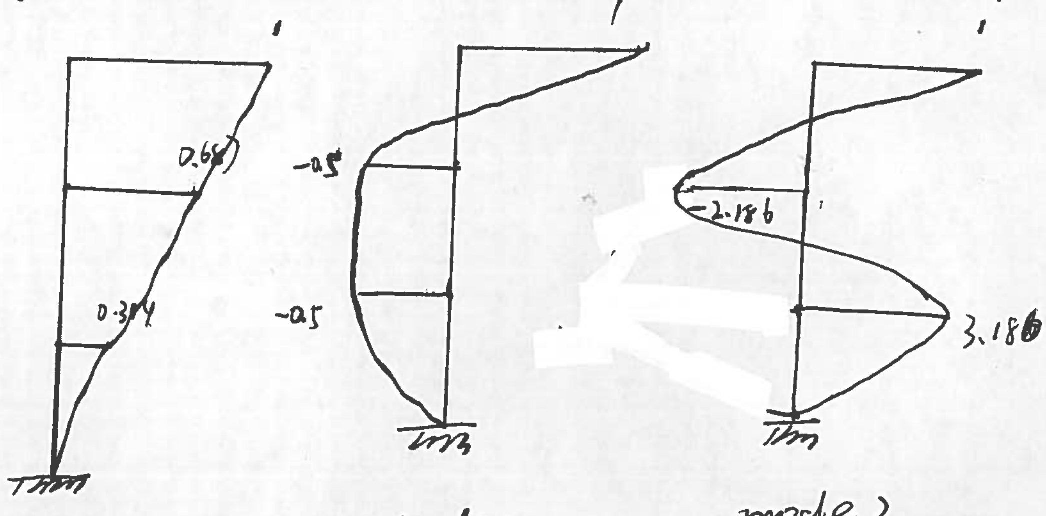
repeat the same process we get

$$\frac{24EI}{h^3}\begin{pmatrix} -0.4573 & -\frac{2}{3} & 0 \\ -\frac{2}{3} & -1.1240 & -\frac{1}{3} \\ 0 & -\frac{1}{3} & -0.7287 \end{pmatrix} \cdot \begin{pmatrix} \phi_{13} \\ \phi_{23} \\ \phi_{33} \end{pmatrix} = 0$$

Assume $\phi_{33} = 1 \Rightarrow \phi_3 = \begin{pmatrix} 3.186 \\ -2.186 \\ 1 \end{pmatrix}$

So the results are:

$\omega_n = 2.24\sqrt{\frac{EI}{mh^3}}$ $\qquad \omega_n = 4.90\sqrt{\frac{EI}{mh^3}}$ $\qquad \omega_n = 7.14\sqrt{\frac{EI}{mh^3}}$



mode 1 $\qquad$ mode 2 $\qquad$ mode 3

2. verify the orthogonality

The detail of calculation is shown in matlab.

$\phi_1^T \cdot k\phi_2 = 0 \qquad \phi_2^T \cdot m\phi_1 = 0$

$\phi_2^T \cdot k\phi_3 = 0 \qquad \phi_3^T \cdot m \cdot \phi_2 = 0$

$\phi_3^T \cdot k\phi_1 = 0 \qquad \phi_1^T \cdot m\phi_3 = 0$

So we can verify

$$\begin{cases} \phi_n^T \cdot k\phi_r = 0 \\ \phi_n^T \cdot m\phi_r = 0 \end{cases} \quad (r \neq n)$$

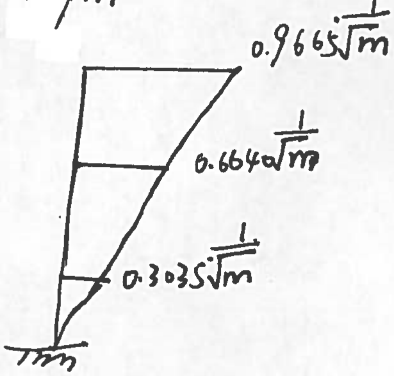
3. For mode 1 :

$$M_1 = \phi_1^T \cdot m \phi_1 = (0.314, 0.686, 1) \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} \cdot m \cdot \begin{pmatrix} 0.314 \\ 0.686 \\ 1 \end{pmatrix}$$

$$= 1.069 \cdot m$$

To make $M_1 = 1$ $\Rightarrow$ devide ϕ_1 by $\sqrt{1.069m}$

$$\phi_1 = \begin{pmatrix} 0.3035 \\ 0.6640 \\ 0.9665 \end{pmatrix} \cdot \frac{1}{\sqrt{m}}$$

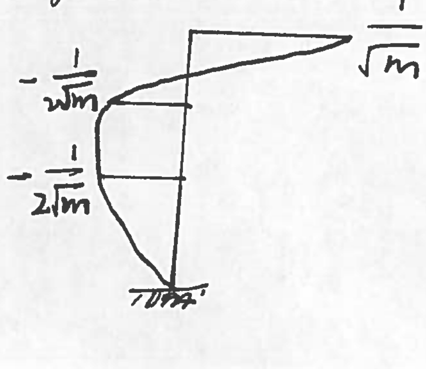


For mode 2 :

$$M_2 = \phi_2^T \cdot m \cdot \phi_2 = (-0.5, -0.5, 1) \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} m \cdot \begin{pmatrix} -0.5 \\ -0.5 \\ 1 \end{pmatrix} = m$$

To make $M_2 = 1$ devide ϕ_2 by $\sqrt{m}$

$$\phi_2 = \begin{pmatrix} -0.5 \\ -0.5 \\ 1 \end{pmatrix} \cdot \frac{1}{\sqrt{m}}$$

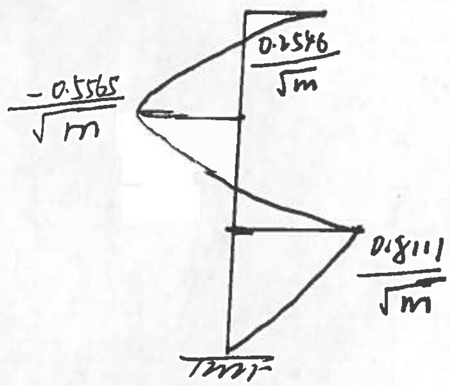


For mode 3:

$$M_3 = \phi_3^{T} \cdot m\, \phi_3 = (-3.186, -2.186, 1) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{pmatrix} m \begin{pmatrix} -3.186 \\ -2.186 \\ 1 \end{pmatrix} = 15.4292\,m$$

To make $M_3 = 1$ devide by $\sqrt{15.4292\,m}$

$$\phi_3 = \begin{pmatrix} 0.8111 \\ -0.5565 \\ 0.2546 \end{pmatrix} \frac{1}{\sqrt{m}}$$



Compare. The ~~prop~~ ratio of u_1, u_2, u_3 (displacement at mass point) is same in the modes and nomalized modes, but the values may be changed ~~at~~ by the normalization, but the made is multiples of the mode in the first question

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problem 2:

Condition a: $u_1 = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$:

$$q_1 = \frac{\phi_1^T m u_1}{\phi_1^T m \cdot \phi_1} = \frac{(0.314,\ 0.687,\ 1) \cdot \begin{bmatrix} 1, 0, 0 \\ 0, 1, 0 \\ 0, 0, 0.5 \end{bmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}}{(0.314,\ 0.687,\ 1) \cdot \begin{pmatrix} 1, 0, 0 \\ 0, 1, 0 \\ 0, 0, 0.5 \end{pmatrix} \cdot \begin{bmatrix} 0.316 \\ 0.687 \\ 1 \end{bmatrix}} = 2.9779$$

$$q_2 = \frac{\phi_2^T m u_1}{\phi_2^T m \phi_2} = 0$$

$$q_3 = \frac{\phi_3^T m u_1}{\phi_3^T \cdot m \phi_3} = 0.0204$$

and $\dot{q}_1(0) = 0$ $\quad \dot{q}_2(0) = 0$ $\quad \dot{q}_3(0) = 0$

$$u(t) = \sum_{n=1}^{N} \phi_n \left[q_n(0) \cos w_n t + \frac{\dot{q}_n(0)}{w_n} \sin w_n t \right]$$

$$= \begin{pmatrix} 0.314 \\ 0.687 \\ 1 \end{pmatrix} \cdot \left[2.9779 \cdot \cos\left(2.24 \sqrt{\frac{EI}{mh^3}}\, t\right)\right] + \begin{pmatrix} 3.186 \\ -2.186 \\ 1 \end{pmatrix} \left[0.0204 \cdot \cos\left(7.140 \sqrt{\frac{EI}{mh^3}}\, t\right)\right]$$

$$= \begin{pmatrix} 0.935 \cdot \cos\left(2.24 \sqrt{\frac{EI}{mh^3}}\, t\right) + 0.065 \cdot \cos\left(7.140 \sqrt{\frac{EI}{mh^3}}\, t\right) \\ 2.046 \cdot \cos\left(2.24 \sqrt{\frac{EI}{mh^3}}\, t\right) - 0.044 \cos\left(7.140 \sqrt{\frac{EI}{mh^3}}\, t\right) \\ 2.978 \cdot \cos\left(2.24 \sqrt{\frac{EI}{mh^3}}\, t\right) + 0.020 \cos\left(7.140 \sqrt{\frac{EI}{mh^3}}\, t\right) \end{pmatrix}$$

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Condition b: $u_2 = \begin{pmatrix} -1 \\ 0.25 \\ 1 \end{pmatrix}$

$$q_1 = \frac{\phi_1^T \cdot m\, u_2}{\phi_1^T m\, \phi_1} = 0.3342$$

$$q_2 = \frac{\phi_2^T m\, u_2}{\phi_2^T m\, \phi_2} = 0.8750$$

$$q_3 = \frac{\phi_3^T \cdot m\, u_2}{\phi_3^T m\, \phi_3} = -0.2095 \qquad \dot{q}_1(0) = 0 \qquad \dot{q}_2(0) = 0 \qquad \dot{q}_3(0) = 0$$

$$u(t) = \sum_{n=1}^{3} \phi_n \left[q_n(0) \cdot \cos w_n t + \frac{\dot{q}_n(0)}{w_n} \sin w_n t \right]$$

$$= \begin{bmatrix} 0.314 \\ 0.687 \\ 1 \end{bmatrix} \left(0.3342 \cos\left(2.24 \cdot \sqrt{\frac{EI}{mh^3}}\, t\right)\right) + \begin{pmatrix} -0.5 \\ -0.5 \\ 1 \end{pmatrix} \left(0.8750 \cdot \cos\left(4.90 \sqrt{\frac{EI}{mh^3}}\, t\right)\right)$$

$$+ \begin{bmatrix} 3.186 \\ -2.186 \\ 1 \end{bmatrix} \left(-0.2095 \cdot \cos\left(7.140 \cdot \sqrt{\frac{EI}{mh^3}}\, t\right)\right)$$

$$= \begin{pmatrix} 0.105 \cos\left(2.24 \sqrt{\frac{EI}{mh^3}}\, t\right) - 0.438 \cdot \cos\left(4.90 \sqrt{\frac{EI}{mh^3}}\, t\right) - 0.667 \cdot \cos\left(7.140 \sqrt{\frac{EI}{mh^3}}\, t\right) \\ 0.230 \cdot \cos\left(2.24 \sqrt{\frac{EI}{mh^3}}\, t\right) - 0.438 \cos\left(4.90 \sqrt{\frac{EI}{mh^3}}\, t\right) + 0.458 \cdot \cos\left(7.140 \sqrt{\frac{EI}{mh^3}}\, t\right) \\ 0.334 \cdot \cos\left(2.24 \sqrt{\frac{EI}{mh^3}}\, t\right) + 0.875 \cos\left(4.90 \sqrt{\frac{EI}{mh^3}}\, t\right) - 0.209 \cdot \cos\left(7.140 \sqrt{\frac{EI}{mh^3}}\, t\right) \end{pmatrix}$$

Condition C: $u_3 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

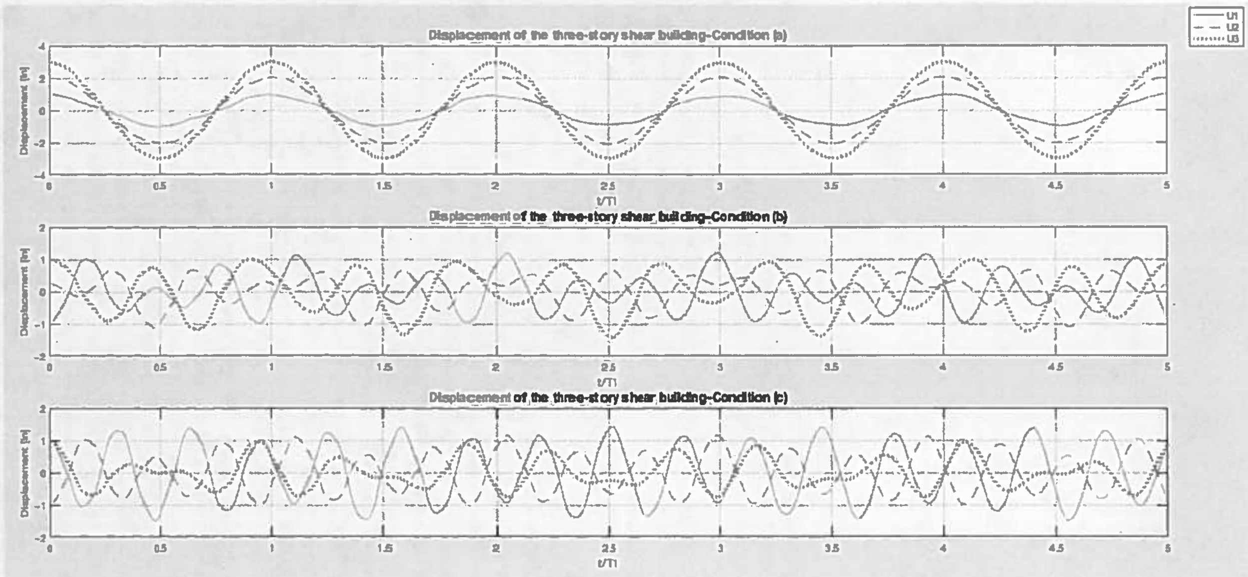
$$q_1 = \frac{\phi_1^T m u_3}{\phi_1^T m \phi_1} = 0.1186 \quad q_2 = \frac{\phi_2^T m u_3}{\phi_2^T m \phi_1} = 0.5 \quad q_3 = \frac{\phi_1^T m u_3}{\phi_1^T m \phi_1} = 0.3806$$

$$\dot{q}_1(0) = 0 \quad \dot{q}_2(0) = 0 \quad \dot{q}_3(0) = 0$$

$$u(t) = \sum_{n=1}^{3} \phi_n \left[q_n(0) \cdot \cos w_n t + \frac{\dot{q}_n(0)}{w_n} \sin w_n t \right]$$

$$= \begin{pmatrix} 0.314 \\ 0.687 \\ 1 \end{pmatrix} (0.1186 \cos w_1 t) + \begin{pmatrix} -0.5 \\ -0.5 \\ 1 \end{pmatrix} \cdot (0.5 \cos w_2 t) + \begin{pmatrix} 3.186 \\ -2.186 \\ 1 \end{pmatrix} \cdot (0.3806 \cos w$$

$$= \begin{pmatrix} 0.037 \cdot \cos(2.24 \sqrt{\frac{EI}{mh^3}} t) - 0.25 \cdot \cos(4.90 \sqrt{\frac{EI}{mh^3}} t) + 1.213 \cdot \cos(7.140 \sqrt{\frac{EI}{mh^3}} t) \\ 0.081 \cdot \cos(2.240 \sqrt{\frac{EI}{mh^3}} t) - 0.25 \cdot \cos(4.90 \sqrt{\frac{EI}{mh^3}} t) - 0.832 \cdot \cos(7.140 \sqrt{\frac{EI}{mh^3}} t) \\ 0.119 \cdot \cos(2.24 \sqrt{\frac{EI}{mh^3}} t) + 0.5 \cos(4.90 \sqrt{\frac{EI}{mh^3}} t) + 0.381 \cos(7.140 \sqrt{\frac{EI}{mh^3}} t) \end{pmatrix}$$

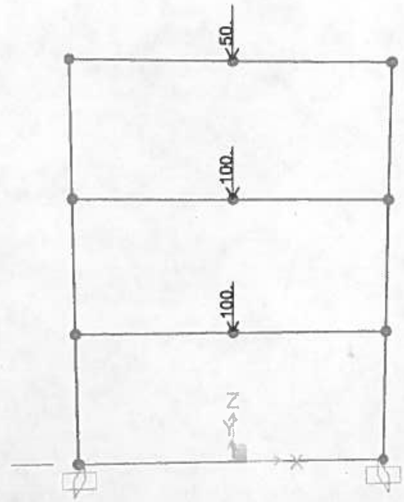


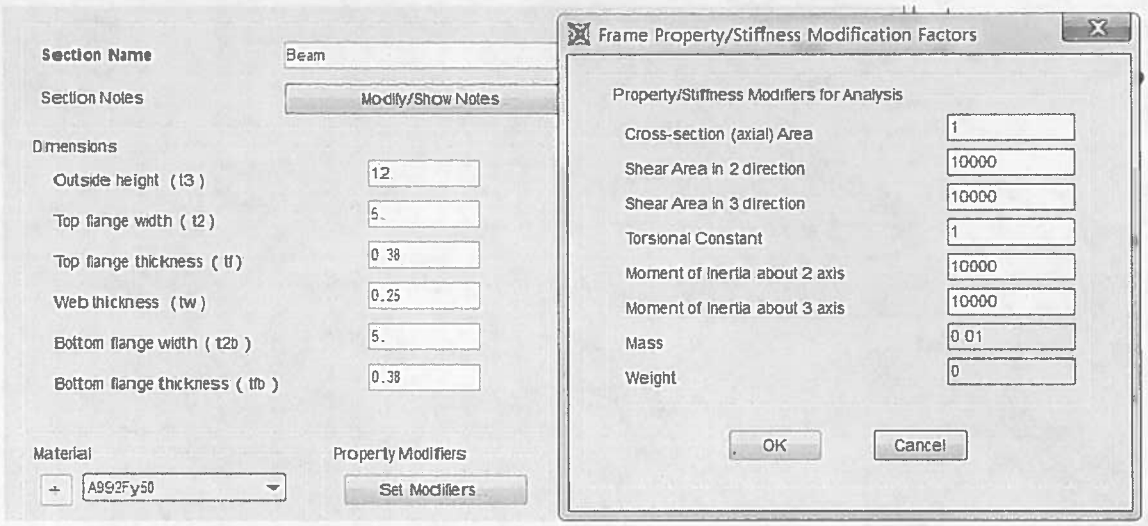
U1
U2
U3
Displacement of the three-story shear building-Condition (a)
Displacement [m]
t/T1
Displacement of the three-story shear building-Condition (b)
Displacement [m]
t/T1
Displacement of the three-story shear building-Condition (c)
Displacement [m]
t/T1

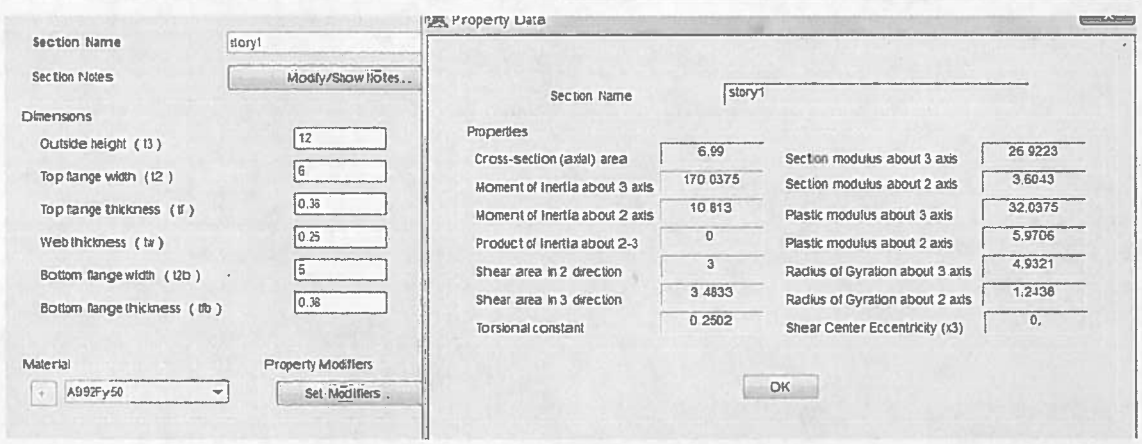
Problem 3:

The model and information are as shown.

(1)







	OutputCase	StepType Text	StepNum Unitless	Period Sec	Frequency Cyc/sec	CircFreq rad/sec	Eigenvalue rad2/sec2
▶	MODAL	Mode	1	0.522942	1.9122568...	12.015063...	144.36176...
	MODAL	Mode	2	0.24631	4.0599163...	25.509206...	650.71964...
	MODAL	Mode	3	0.163235	6.1261526...	38.491752...	1481.6149...

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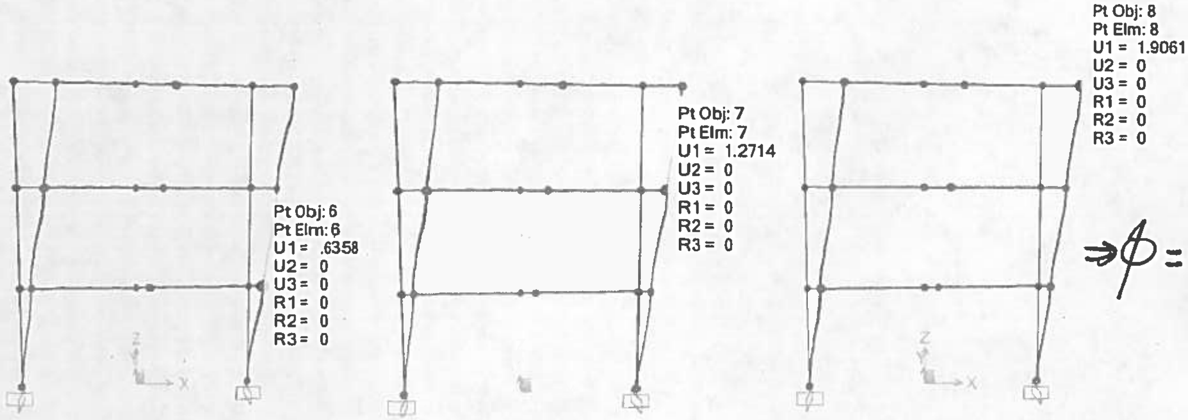
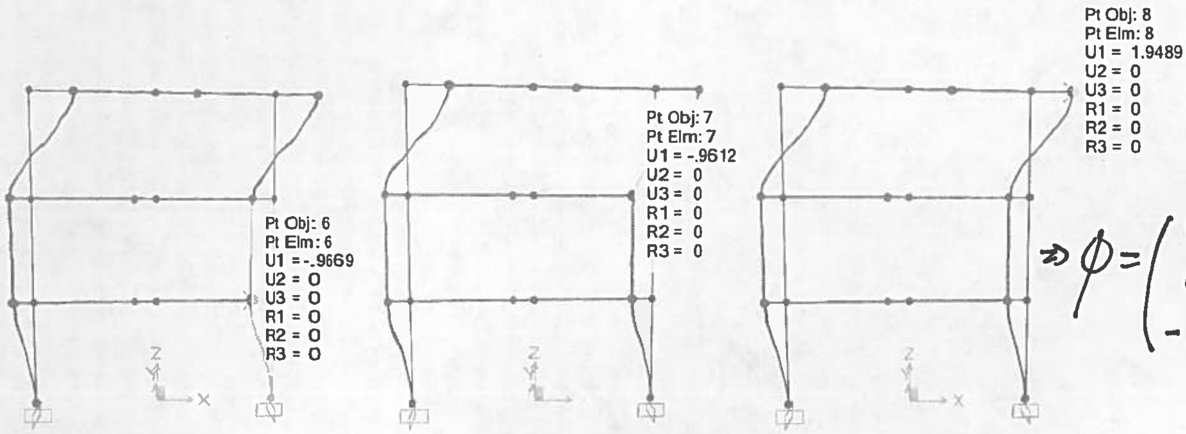
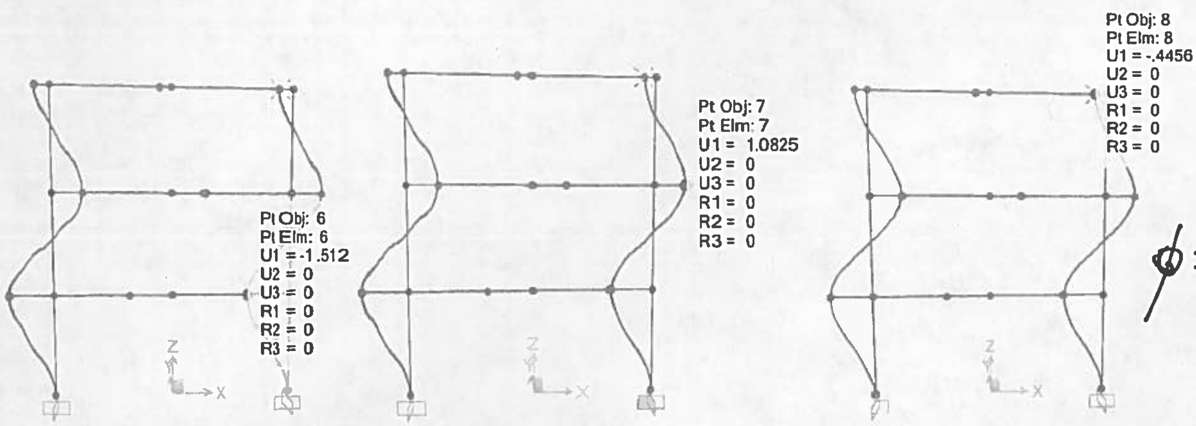
Almost Same as example in problem



$$\Rightarrow \phi = \begin{pmatrix} 0.6358 \\ 1.2714 \\ 1.9061 \end{pmatrix}$$



$$\Rightarrow \phi = \begin{pmatrix} 0.9669 \\ 0.9612 \\ -1.9489 \end{pmatrix}$$



$$\phi = \begin{pmatrix} 1.512 \\ -1.0825 \\ 0.4456 \end{pmatrix}$$

The modes are almost as same as the results in the problem.

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(2)

$$m = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \cdot \frac{100}{386}$$

$$K = \frac{k}{9} \begin{bmatrix} 16 & -7 & 0 \\ -7 & 10 & -3 \\ 0 & -3 & 3 \end{bmatrix} = \frac{168}{9} \begin{bmatrix} 16 & -7 & 0 \\ -7 & 10 & -3 \\ 0 & -3 & 3 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} 1/w_i & w_i \\ 1/w_j & w_j \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \xi_i \\ \xi_j \end{bmatrix}$$

$$\Rightarrow \frac{1}{2} \begin{bmatrix} \frac{1}{12.01} & 12.01 \\ \frac{1}{38.90} & 38.90 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 0.05 \\ 0.05 \end{bmatrix} \Rightarrow \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} 0.9177 \\ 0.001964 \end{pmatrix}$$

$$C = a_0 m + a_1 K = \begin{bmatrix} 0.824 & -0.257 & 0 \\ -0.2575 & 0.604 & -0.100 \\ 0 & -0.100 & 0.229 \end{bmatrix}$$

$$\xi_2 = \frac{a_0}{2} \cdot \frac{1}{w_{n2}} + \frac{a_1}{2} w_{n2}$$

$$= \left(0.9177 \cdot \frac{1}{25.47} + 0.0020 \times 25.47\right) \times \frac{1}{2}$$

$$= 4.3\%$$

$\cancel{M = \Phi^T \cdot m}$

$$\Phi = \begin{bmatrix} 0.6375 & 0.9827 & 1.5778 \\ 1.2750 & 0.9829 & -1.1270 \\ 1.9125 & -1.9642 & 0.4508 \end{bmatrix}$$

$$M = \Phi^T \cdot m \cdot \Phi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{kips} \cdot s^2 \cdot in^{-1}$$

$$K = \Phi^T \cdot k \cdot \Phi = \begin{bmatrix} 144.14 & 0 & 0 \\ 0 & 648.62 & 0 \\ 0 & 0 & 1513.59 \end{bmatrix} \cdot \text{kip}/in$$

$$C = \Phi^T c \cdot \Phi = \begin{bmatrix} 1.20 & 0 & 0 \\ 0 & 2.19 & 0 \\ 0 & 0 & 3.89 \end{bmatrix}$$

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