McGill University
Department of Civil Engineering and Applied Mechanics
CIVE 603 – Structural Dynamics
HM#3
Group assignment by

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Problem 1

Part A

First, use Duhamel's integral method:

$T_n = 1\,sec \quad w_n = \frac{2\pi}{T_n} = 2\pi\, rad/sec \quad t_d = 1.5\, sec$

$P(\tau) = -m\ddot{u}_g = -0.5g \sin(\frac{4}{3}\pi\tau)$

Thus $\quad u(t) = \frac{1}{m w_n}\int_0^t P(\tau)\sin[w_n(t-\tau)]\, d\tau$

$$= \frac{1}{1\times 2\pi}\int_0^t -0.5g\sin(\frac{4}{3}\pi\tau)\cdot \sin[w_n(t-\tau)]\, d\tau$$

$$= -\frac{g}{4\pi}\int_0^t \left[-\frac{1}{2}\cos(-\frac{2}{3}\pi\tau + 2\pi t) + \frac{1}{2}\cos(\frac{10}{3}\pi\tau - 2\pi t)\right] d\tau$$

$$= \frac{-g}{4\pi}\times\left[\frac{3}{4\pi}\sin(-\frac{2}{3}\pi\tau + 2\pi t) + \frac{3}{20\pi}\sin(\frac{10}{3}\pi\tau - 2\pi t)\right]\Big|_0^t$$

$$= \frac{-g}{4\pi}\times\left[\frac{9}{10\pi}\sin(\frac{4}{3}\pi t) - \frac{3}{5\pi}\sin(2\pi t)\right]$$

$$= -\frac{9g}{40\pi^2}\sin(\frac{4}{3}\pi t) - \frac{3g}{20\pi^2}\sin(2\pi t) \qquad (0 \le t < t_d)$$

During free vibration phase:

$$u(t) = u(t_d)\cos w_n(t-t_d) + \frac{(\dot{u}(t_d))}{w_n}\sin w_n(t-t_d)$$

$u(t_d) = 0 \qquad \dot{u}(t_d) = -\frac{3}{5}\frac{g}{\pi}$

Thus we get $u(t) = -\frac{3}{10}\frac{g}{\pi^2} \sin 2\pi(t-1.5) \quad (t \geq t_d)$

$$u(t) = \begin{cases} -\frac{9g}{40\pi^2} \sin(\frac{4}{3}\pi t) - \frac{3g}{20\pi^2} \sin(2\pi t) & (0 \leq t < t_d) \\ -\frac{3}{10}\frac{g}{\pi^2} \sin 2\pi(t-1.5) & (t \geq t_d) \end{cases}$$

In Figure 1, we could see two displacement response using central difference method and analytical solution seperately, which are exactly the same.

Part B

1. See the results in Figure 2 and Figure 3
2. See the ~~se~~ results in Figure 4 and Figure 5
3. See the results in Figure 6 and Figure 7
4. See the results in Figures 8 and Figure 9

For both the EI and Canoga earthquake, the absolute and ~~the~~ pseudo accelerations are ~~~~ almost same. For pseudo velocity it fits relative ~~pseudo~~ velocity better in shorter natural period, but the error is increasing with the natural period getting bigger.

Problem 2

Part A $\quad w = 100$ kips $\quad k = 4$ kips/in $\quad m = \frac{w}{g}$

$T_n = 2\pi\sqrt{\frac{m}{k}} = 2\pi \times \sqrt{\frac{100}{386\times 4}} = 1.599\,s$

Use the pseudo-acceleration spectrum

$A = 1.8 \times \frac{1}{1.599} \times 0.5 = 0.563\,g$

$D = A \div \left(\frac{2\pi}{T_n}\right)^2 = 14.08$ in

Thus base shear $V = k \cdot D = 14.08 \times 4 = 56.32$ kips

If $\xi = 2\%$, we would use damping modification factor B:

$$\frac{S_d(T_n, 2)}{S_d(T_n, 5)} = 1.4 - 0.248\ln(2) = 1.228$$

$S_d(T_n, 2) = 1.228 \times 14.08 = 17.29$ in

Base shear $= 17.29 \times 4 = 69.16$ kips

If $\xi = 20\%$

$$\frac{S_d(T_n, 20)}{S_d(T_n, 5)} = 1.4 - 0.248\ln(20) = 0.657$$

$S_d(T_n, 20) = 0.657 \times 14.08 = 9.25$ in

Base shear $= 9.25 \times 4 = 37$ kips

Pseudo-acceleration, pseudo-velocity and deformation spectrum can be seen in Figure 10, 11 and 12

Part B

From El Centro ground motion absolute acceleration response spectra, we get that $a = 71.56 \text{ in/sec}^2$

Thus base shear $V = F = ma = \frac{100}{386} \times 71.56 = 18.54$ kips

Lateral displacement: $D = \frac{F}{k} = 4.635$ in

From Canoga Park record ground motion absolute acceleration response spectra, we get that $a = 150.28 \text{ in/sec}^2$

Thus base shear $V = F = ma = \frac{100}{386} \times 150.28 = 38.93$ kips

Lateral displacement: $D = \frac{F}{k} = 9.733$ in

Part C

If $k = 8$ kips/in $T_n = 2\pi\sqrt{\frac{m}{k}} = 1.131$ s

$A = \cancel{1.8\times}\ 0.5\times 1.8 \times \frac{1}{1.131} = \cancel{1.5}\ 0.796\, g$

$D = 0.796 \times 386 \div \left(\frac{2\pi}{T_n}\right)^2 = 9.93$ in

base shear $V = kD = 79.44$ kips

By comparing these two systems, we would find that if we have larger stiffness, we will get smaller natural period, which will reduce the lateral deformation and increase the base shear.

Thus, stiffening the system will result in larger base shear and larger bending moment, so it is a disadvantage.

Problem 3

W 14×53 steel column $I_{xx} = 541\ in^4$ $I_{yy} = 57.7\ in^4$

$$m = \frac{200 \times 20 \times 30}{386} = 310.88\ lb-sec^2/in = 0.311\ kips-sec^2/in$$

1. For north-south excitation, from EL Centro spectrum

$$k = 4 \times \frac{12EI}{h^3} = \frac{4 \times 12 \times (29 \times 10^3) \times 541}{(12 \times 12)^3} = 252.202\ kips/in$$

$T_n = 2\pi\sqrt{\frac{m}{k}} = 0.2206\ s$ so from the spectrum

$a = 252.39\ in/sec^2$

Thus $F = ma = 78.49$ kips

Displacement $D = \frac{78.49}{252.202} = 0.311$ in

So the bending moment in each column:

$M = \frac{1}{4} \times 12 \times 78.49 = 235.47$ kips · ft

From Canoga Park record spectrum:

$a = 320.46$ in/sec^2 $F = ma = 99.66$ kips

$D = \frac{99.66}{252.202} = 0.395$ in

The bending moment in each column:

$M = \frac{1}{4} \times 12 \times 99.66 = 298.93$ kips · ft

2. For east-west excitation

Round hollow steel section HSS 3×0.12

$A = \pi[(1.5^2) - (1.5 - 0.12)^2] = 1.0857$ in^2

Although each frame has two cross-section, only the one in tension will provide lateral resistance; the one in compression will buckle at small axial force and will contribute little to the lateral stiffness.

$$k = \frac{2 \times 1.0857 \times 29 \times 10^3}{20 \times 12} \times \left(\frac{20}{\sqrt{12^2+20^2}}\right)^3$$

$$= 165.4356 \text{ kips/in}$$

From El Centro spectrum: $a = 358.90\ \cancel{ft}\ \text{in/sec}^2$

$F = ma = 111.62$ kips

the axial force in each tension brace:

$F_a \cdot \cos\theta \times 2 = F \qquad F_a = 65.08$ kips

From Canoga Park record: $a = 362.43 \text{ in/sec}^2$

$F = ma = 112.72$ kips $\qquad F_a = \frac{F}{2\cos\theta} = 65.73$ kips

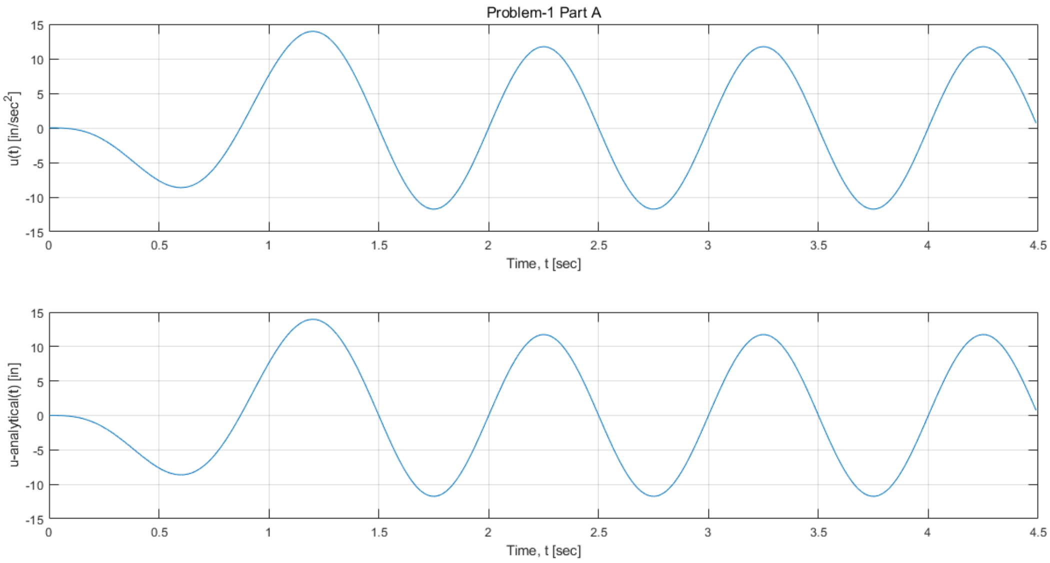

Figure 1 Problem 1 part A

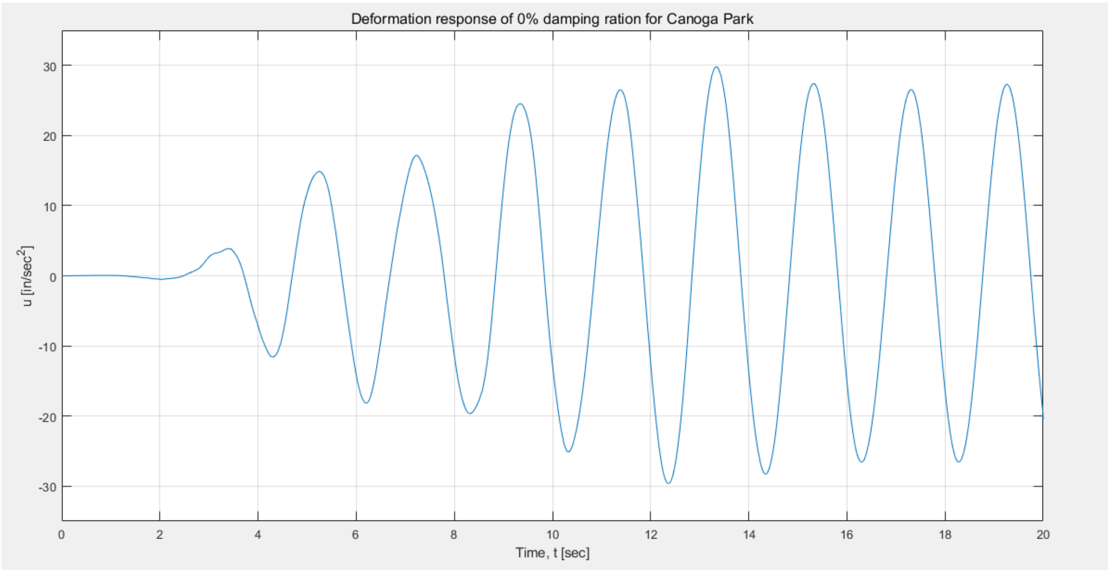

Figure 2 ζ =0% Canoga Park ground motion

Problem 1 part B 1.

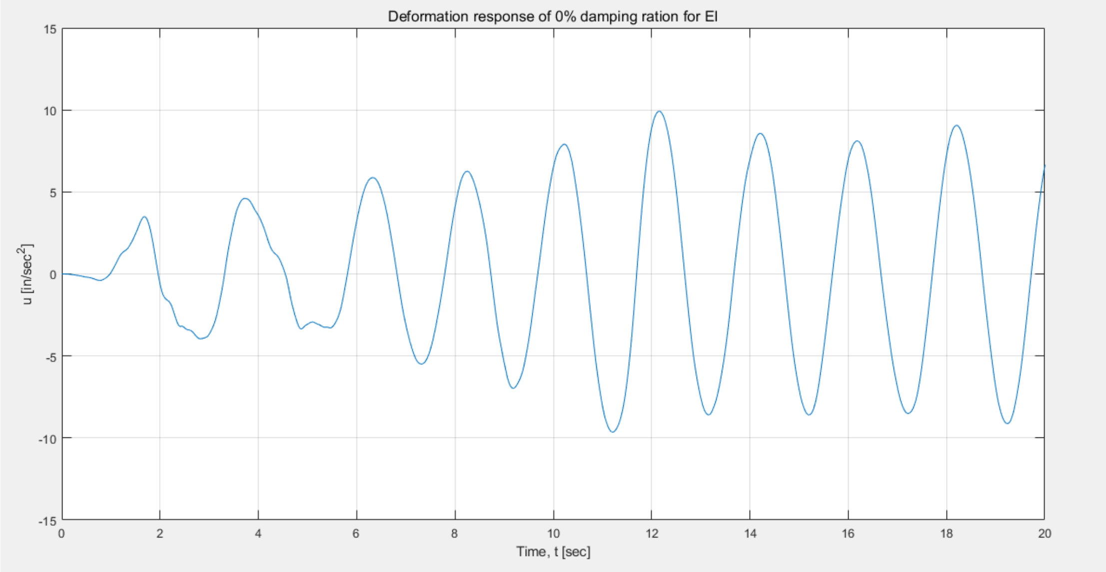

Figure 3 ζ =0% EI Centro ground motion

Problem 1 part B 1.

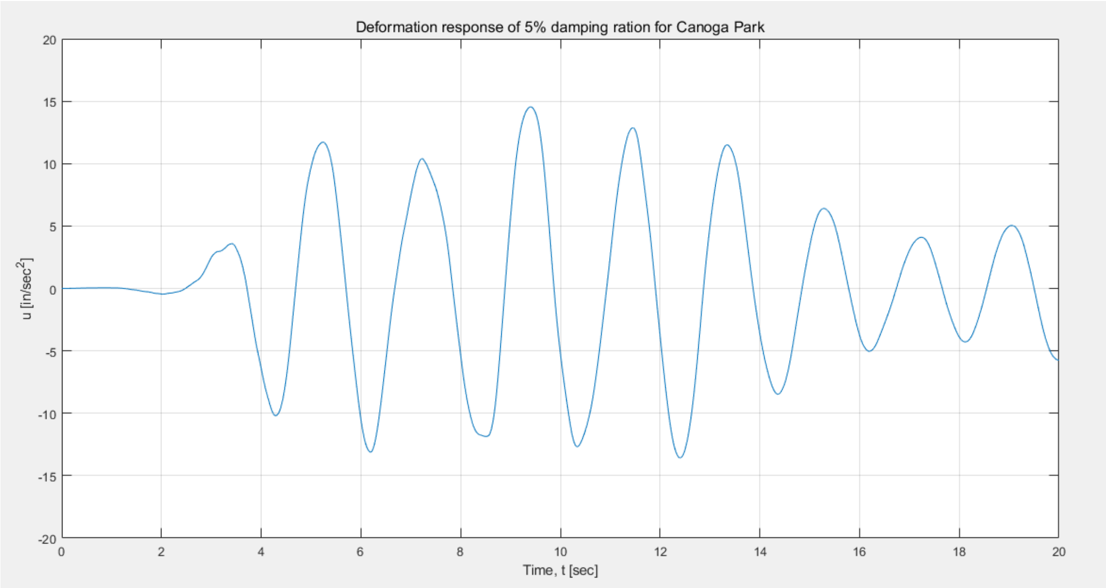

Figure 4 ζ =5% Canoga Park ground motion

Problem 1 part B 2.

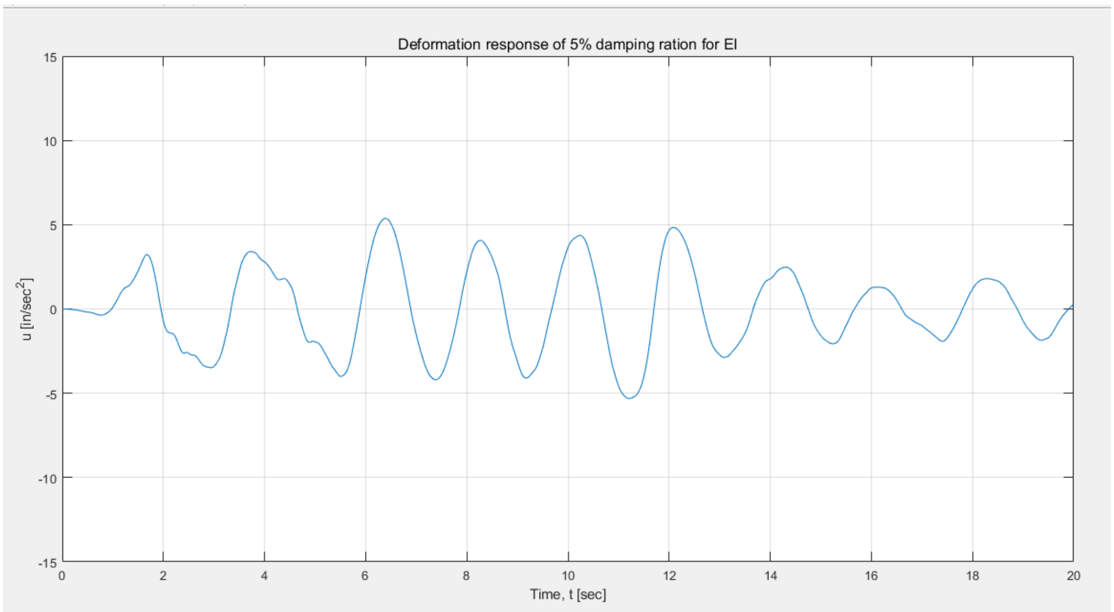

Figure 5 ζ =5% EI Centro ground motion

Problem 1 part B 2.

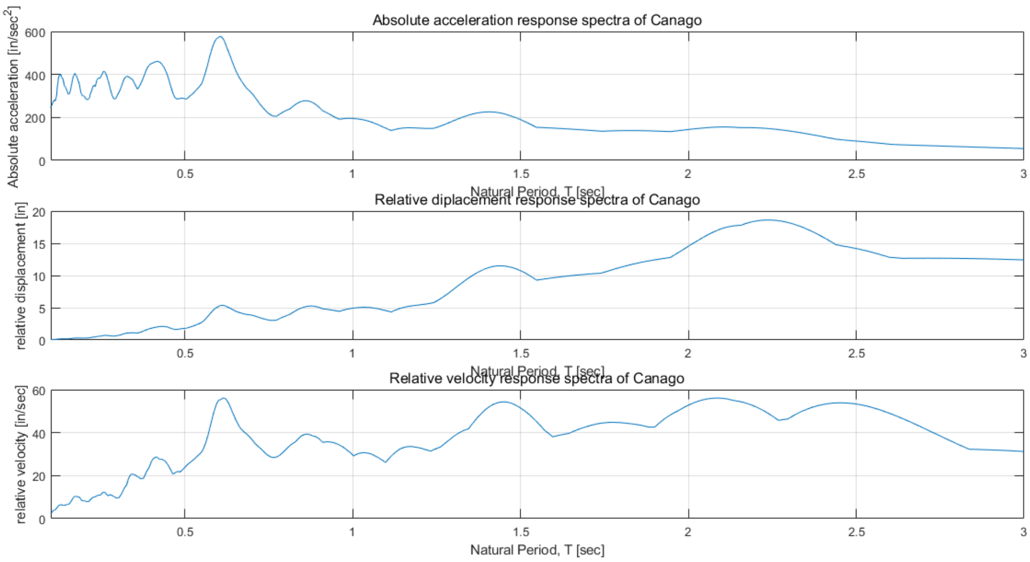

Figure 6 Canoga Park ground motion

Problem 1 part B 3.

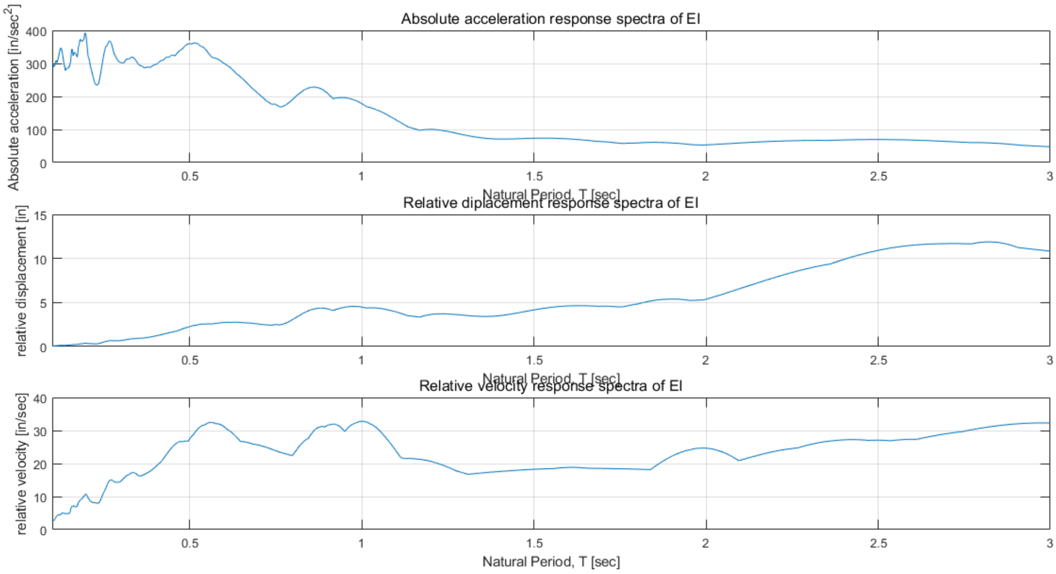

Figure 7 EI Centro ground motion

Problem 1 part B 3.

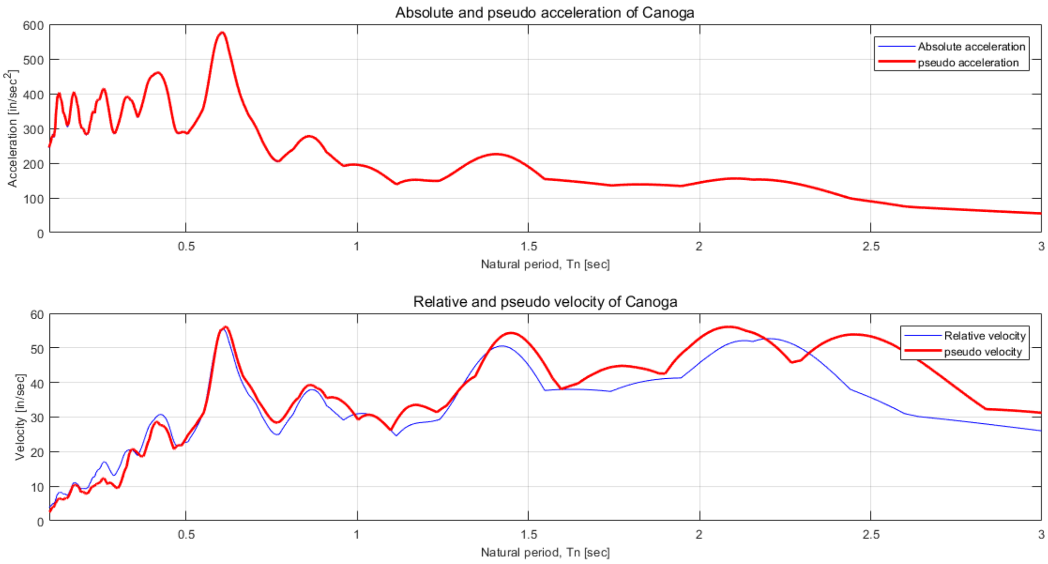

Figure 8 Canoga Park ground motion

Problem 1 part B 4.

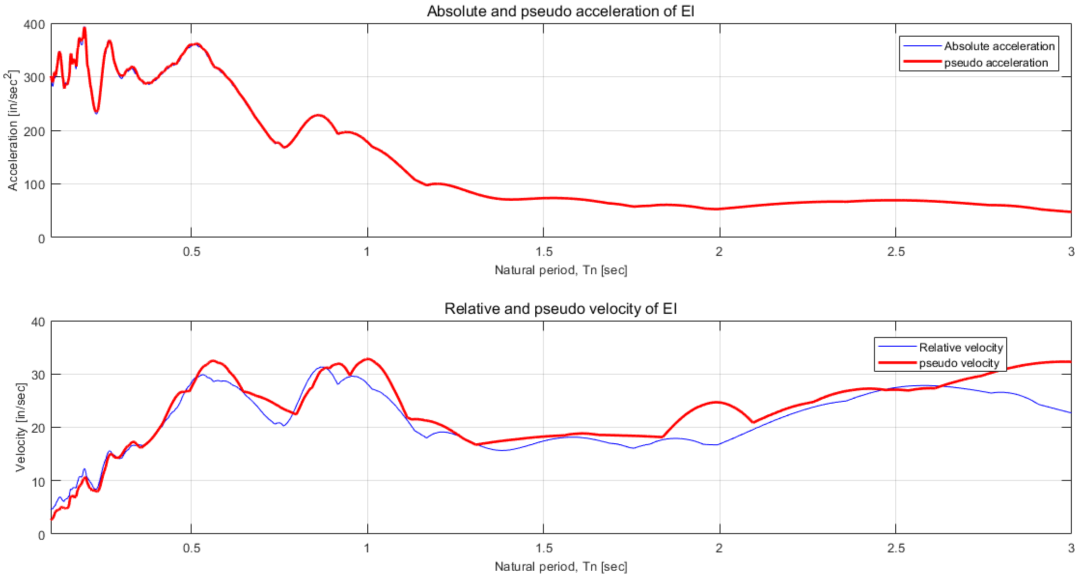

Figure 9 EI Centro ground motion

Problem 1 part B 4.

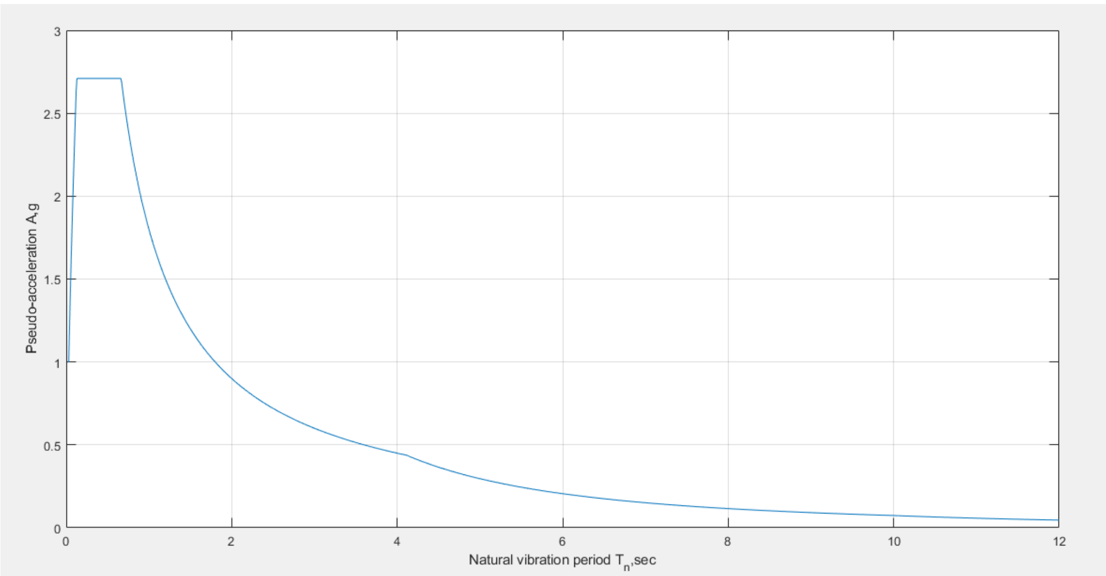

Figure 10 5 % damped pseudo-acceleration spectrum

Problem 2 part A

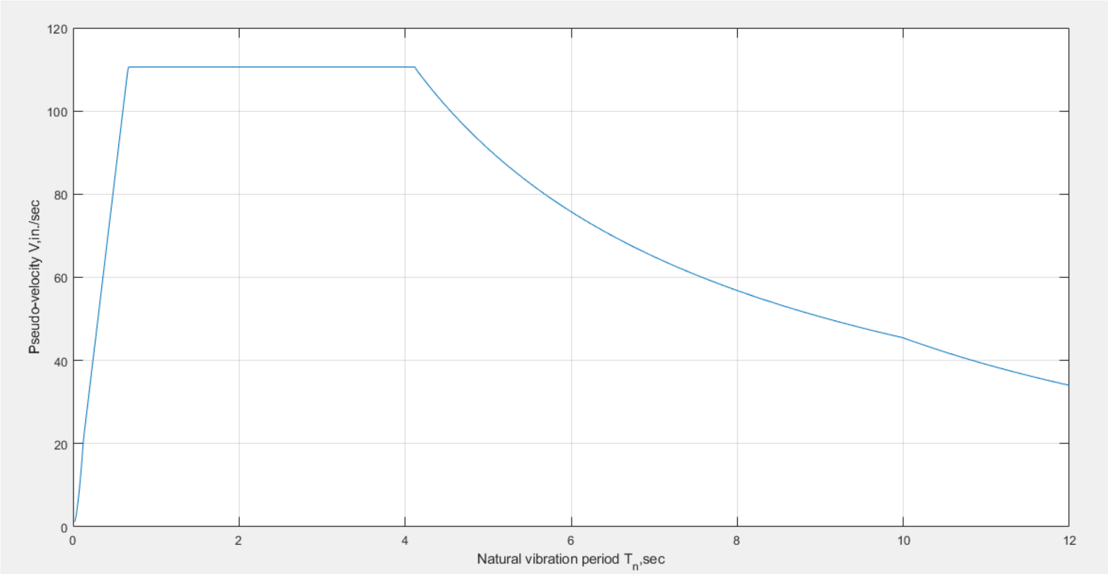

Figure 11　5 % damped pseudo-acceleration spectrum

Problem 2 part A

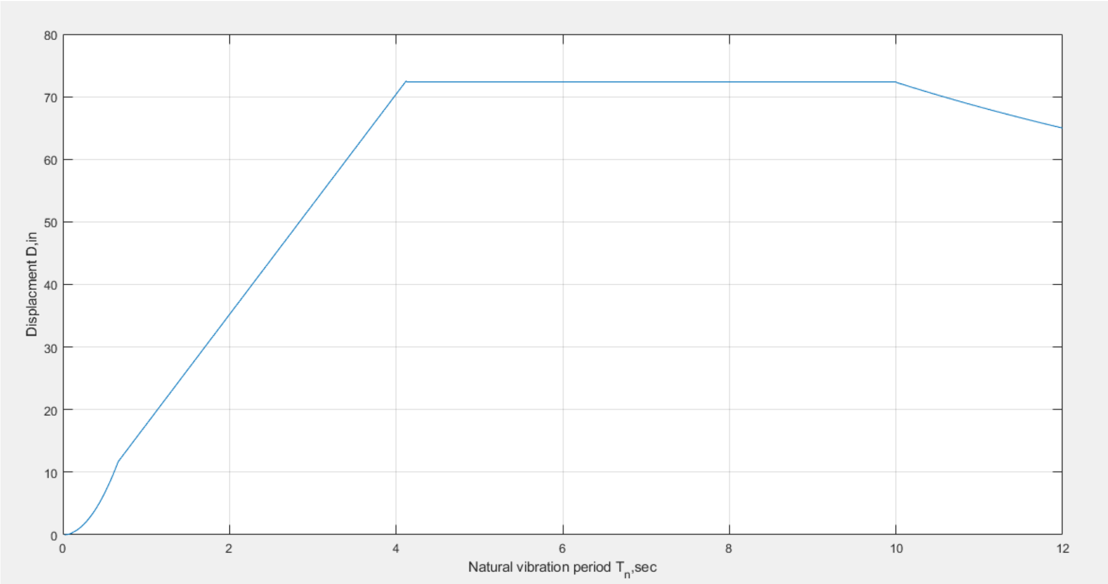

Figure 12　5 % damped displacement spectrum

Problem 2 part