McGill University
Department of Civil Engineering and Applied Mechanics
CIVE 603 – Structural Dynamics

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Homework #2 (Total Points 100/100)
Group Assignment: max. 2 students
Due Date, February 8th 2018 by 4:00pm

Problem 1 (10 Points)

The mass, m, stiffness k, and natural frequency ω_n of an undamped SDF system are unknown. These properties are to be determined by harmonic excitation tests. At an excitation frequency of 4*Hz*, the response of the SDF tends to increase without bound (i.e., a resonant condition). Next, a weight $\Delta w = 5lb$ is attached to the mass m and the resonance test is repeated. This time resonance occurs at $f = 3\ Hz$. Determine the mass and the stiffness of the SDF system.

Problem 2 (10 points)

In a forced vibration test under harmonic excitation it was noted that the amplitude of motion at resonance was exactly four times the amplitude at an excitation frequency 20% higher than the resonant frequency. Determine the damping ratio of the system.

Problem 3 (10 points)

A viscously damped system with weight W=100 lb has a static deflection of 0.5 in. due to its own weight. The mass of the system is given an initial displacement u_0 = 0.25 in. and released suddenly. After 3 cycles of free vibration, the amplitude of motion is measured to be 0.1 in. For this SDF system, determine:

a) the damping ratio, ζ
b) the damping coefficient, c
c) the frequency of damped vibration, ω_d
d) After free vibration, the SDF system is excited by a harmonic force with amplitude p_0=10 lb at its resonant frequency. Determine the steady state displacement amplitude at resonance.

Problem 4 (25 points)

The pulse force shown in the Figure 1 is applied to a SDF structure with a mass *m* and stiffness *k*. Write the equation of motion and derive the equation describing displacement response *u(t)* during the forced vibration phase (neglect damping)

a) By using classical ODE solution method.
b) By using Duhamel's Integral.
c) By using superposition method.

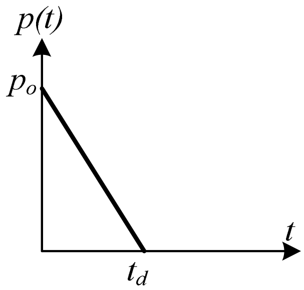

Figure 1

Problem 5 (25 points)

An undamped system is subjected to the triangular pulse shown in Figure 2 below.

(a) Show that the displacement response is given by the following set of equations,

$$\frac{u(t)}{(u_{st})_o} = \begin{cases} \dfrac{t}{t_d} - \dfrac{1}{2\pi}\left(\dfrac{T_n}{t_d}\right)\sin\left(\dfrac{2\pi t}{T_n}\right), & 0 \le t \le t_d \\ \cos\left(\dfrac{2\pi}{T_n}(t-t_d)\right) + \dfrac{1}{2\pi}\left(\dfrac{T_n}{t_d}\right)\sin\left(\dfrac{2\pi}{T_n}(t-t_d)\right) - \dfrac{1}{2\pi}\left(\dfrac{T_n}{t_d}\right)\sin\left(\dfrac{2\pi t}{T_n}\right), & t \ge t_d \end{cases}$$

plot the response for two values of t_d/T_n = ½ and 2.

(b) Derive the equations for the dynamic response factor R_d during (i) the forced vibration phase, and (ii) the free vibration phase.

(c) Plot R_d for the two phases against t_d/T_n. Also plot the shock spectrum.

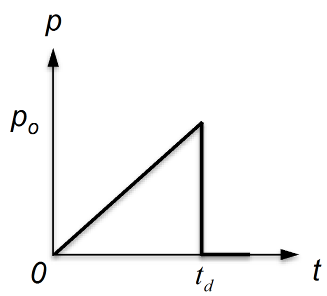

Figure 2

Problem 6 (20 points)

A free vibration is conducted on an empty elevated water tank. A cable attached to the tank applies a lateral horizontal force of 16.4kips and pulls the tank horizontally by 2-in. The cable is suddenly cut and the resulting free vibration is recorded. At the end of four complete cycles, the time is 2.0 sec and the amplitude is 1-in. From these data compute the following:

(a) damping ratio
(b) natural period of undamped vibration
(c) lateral stiffness of the water tank
(d) weight
(e) damping coefficient
(f) number of cycles required for the displacement amplitude to decrease to 0.2-in.

The 80-*ft*-high water tank is subjected to the force *p(t)* below. Determine the maximum base shear and bending moment at the base of the tower supporting the tank. Assume that the tank is full (weight = 100.03*kips*).

(g) If the tank is empty (weight = 20.03*kips*), calculate the maximum base shear and bending moment at the base of the tower supporting the tank.
(h) By comparing these results with those for the full tank, comment on the effect of mass on the response to impulsive forces. Explain the reason.

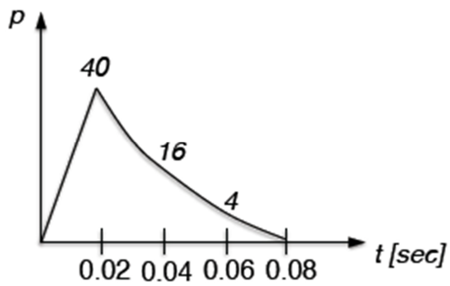

Figure 3