第4章　电路定理

4.1叠加定理 Superposition Theorem

4.2替代定理 Substitution Theorem

4.3 戴维宁(诺顿)定理Thevenin(Norton) Theorem

4.4 最大功率传输定理Maximum Power Theorem

4.6 定理综合运用

第4章　电路定理

目标：

1.熟练应用叠加定理。

2.熟练应用戴维宁/诺顿定理。

3.熟练分析最大功率传输问题。

难点：

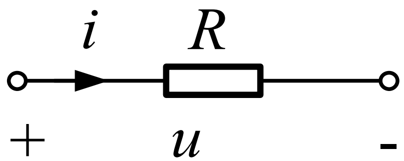
1.电路定理综合应用问题分析。

2.选择合适的分析方法。

讲授学时：4

4. 2 线性特性与线性电路

1.线性元件



$u = Ri$

If $i' = \mathrm{k}i$， then $u' = \mathrm{k}u$. **Homogeneity property 齐次性**

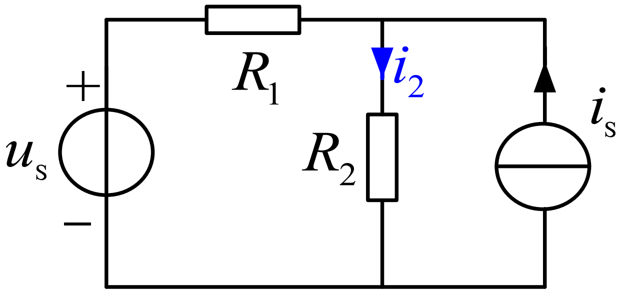
If $i = i_1 + i_2$， then $u = u_1 + u_2$. **Additivity property 可加性**

2. 线性电路

除独立电源外，电路中其他元件均为线性元件。

4.3 叠加定理 Superposition Theorem

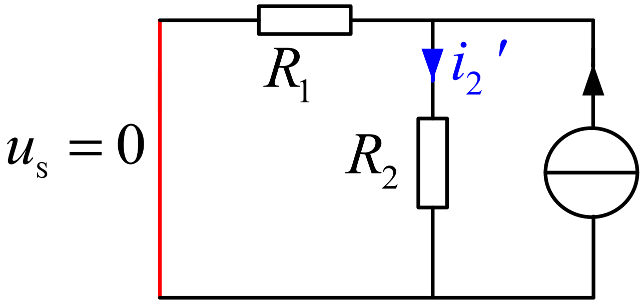
2. 线性电路



$$\left(\frac{1}{R_1}+\frac{1}{R_2}\right)R_2 i_2 = i_s + \frac{1}{R_1}u_s$$

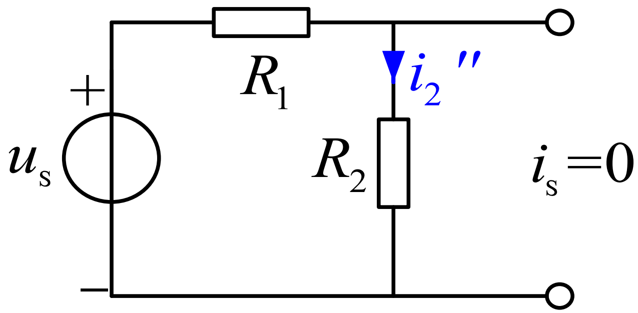
$$i_2 = \frac{R_1}{R_1+R_2}i_s + \frac{1}{R_1+R_2}u_s$$

电流源单独作用



$$i_2' = \frac{R_1}{R_1+R_2}i_s$$

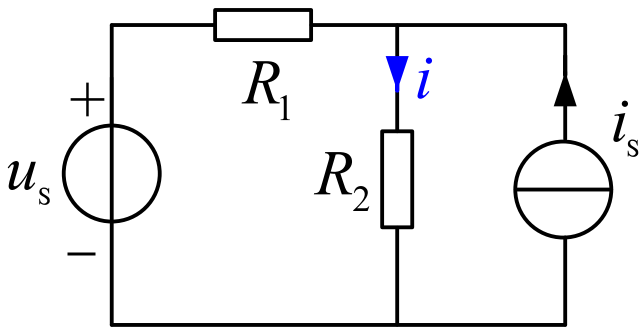
电压源单独作用



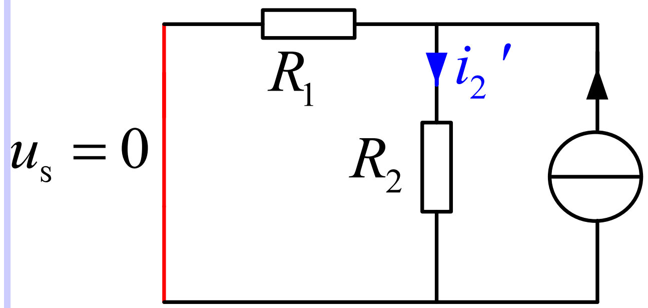
$$i_2'' = \frac{1}{R_1+R_2}u_s$$

4.3 叠加定理 Superposition Theorem

2. 线性电路 $i_2 = \dfrac{R_1}{R_1 + R_2} i_s + \dfrac{1}{R_1 + R_2} u_s$

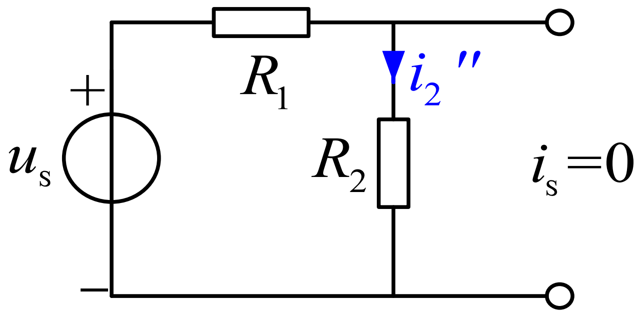


电流源单独作用



$$i_2' = \frac{R_1}{R_1 + R_2} i_s$$

电压源单独作用



$$i_2'' = \frac{1}{R_1 + R_2} u_s$$

3. 叠加定理

$\boxed{i_2 = i_2' + i_2''}$ 可加性

$\boxed{i_2' = k_1 i_s}$ $\boxed{i_2'' = k_2 u_s}$

齐次性（单电源作用）

$\boxed{i_2 = k_1 i_s + k_2 u_s}$

线性性（对功率不适用）

4.3 叠加定理 Superposition Theorem

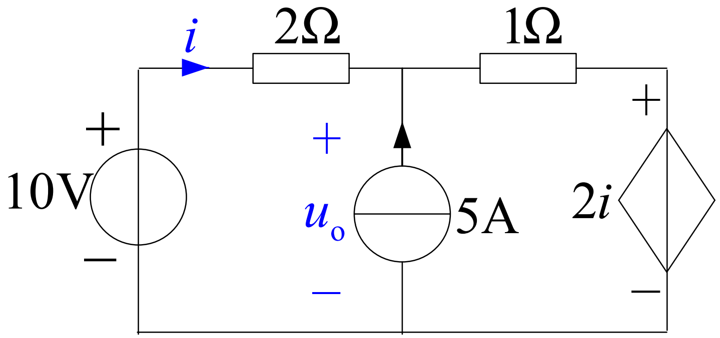
3. 叠加定理

线性电路中，多个独立电源共同激励下的响应（任意电流或电压），等于各独立电源单独（或分组）激励下的响应的代数和。

将多电源电路转化为单电源电路进行计算。

4. 定理应用Applications

【例 1】确定电压u_0电流i。



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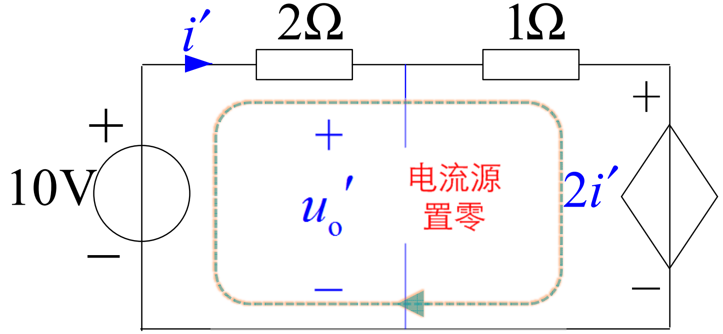
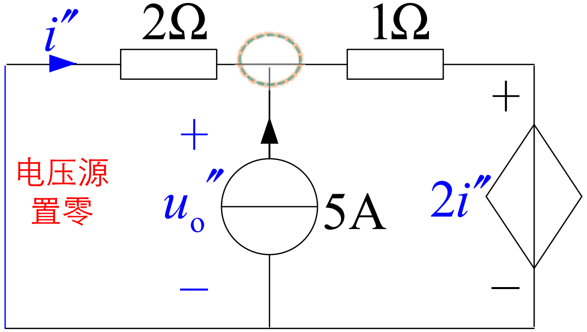


网孔方程：$(2+1)i' = 10 - 2i'$

$$u_o' = 1 \times i' + 2i'$$



结点方程：

$$(\frac{1}{2} + \frac{1}{1})u_o'' = 5 + \frac{2i''}{1}$$

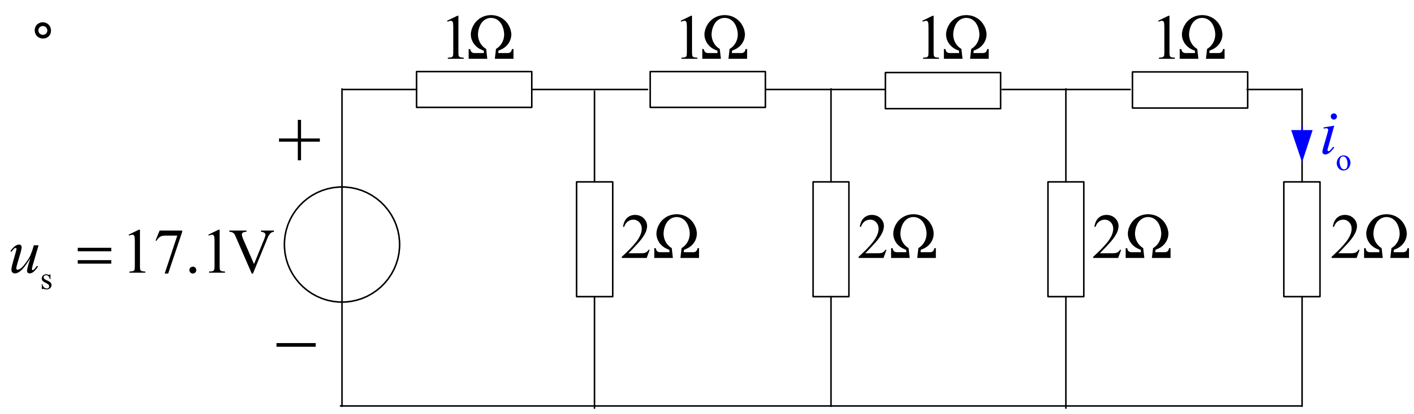
$$u_o'' = -2i''$$

$$u_o = u_o' + u_o'', \quad i = i' + i''$$

$$p_{2\Omega} = i^2 R \boxed{\neq i'^2 R + i''^2 R}$$

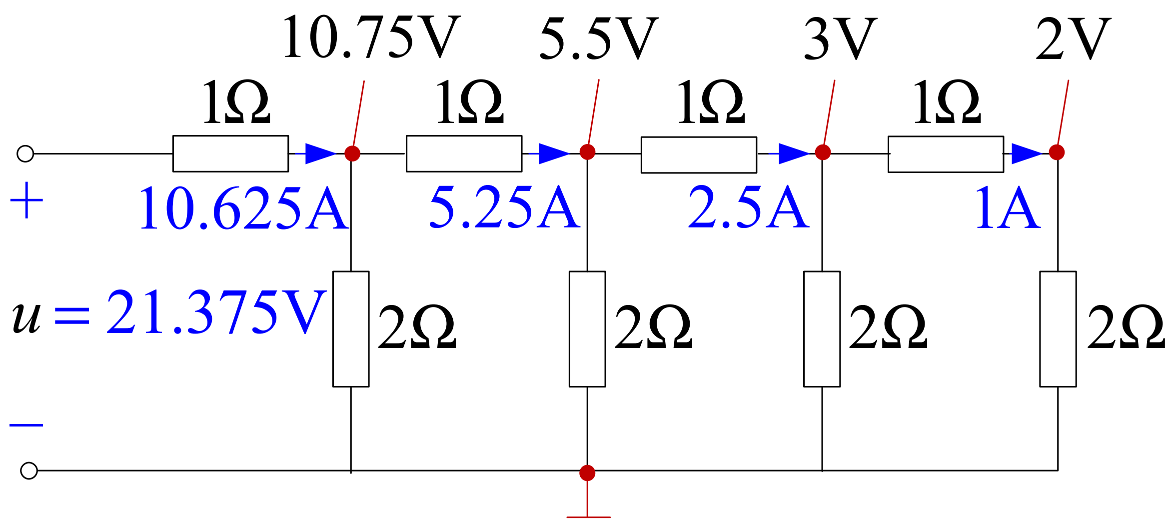
功率不符合叠加关系！

【例 2】确定 i_o 。



由此

$u_s = 21.375\text{V} \rightarrow i_o = 1\text{A}$



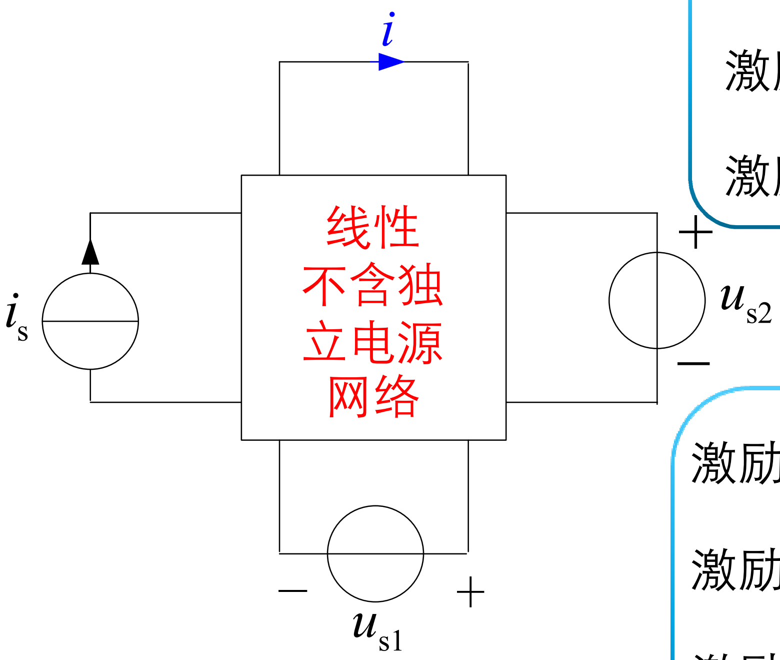
响应与激励的关系为

$$i_o = \frac{1}{21.375} u_s = \frac{8}{171} u_s$$

因此

$$u_s = 17.1\text{V} \rightarrow i_o = \frac{8}{171} \times 17.1 = 0.8\text{A}$$

【例 3】确定电流 i 。



已知条件

激励为 i_s 和 u_{s1} (u_{s2} 置零) → 响应 $i=2\text{A}$

激励为 i_s 和 u_{s2} (u_{s1} 置零) → 响应 $i=-0.5\text{A}$

激励为 i_s 、u_{s1} 和 u_{s2} → 响应 $i=1.2\text{A}$

↓ 确定

激励为 i_s 响应 $i=?$ → i'

激励为 u_{s1} 响应 $i=?$ → i''

激励为 u_{s2} 响应 $i=?$ → i'''

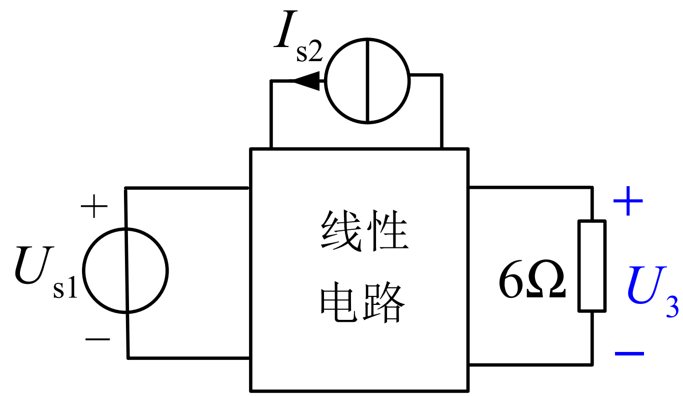
激励为 $0.5i_s$、$2u_{s1}$ 和 $3u_{s2}$ 响应 $i=?$ → i

$$\begin{cases} i'+i''=2 \\ i'+i'''=-0.5 \\ i'+i''+i'''=1.2 \end{cases}$$

解得 $i'=0.3$、$i''=1.7$、$i'''=-0.8$

$$i=0.5i'+2i''+3i'''=1.15\text{A}$$

【练习】一线性电路，U_{S1}=0V，I_{S2}=0A时，有U_3=3V；U_{S1}=1V，I_{S2}= -1A时，U_3=2V；U_{S1}= -4V，I_{S2}=1A时，U_3=1V。求当U_{S1}=1V，I_{S2}=2A时，U_3=?



解：由叠加定理

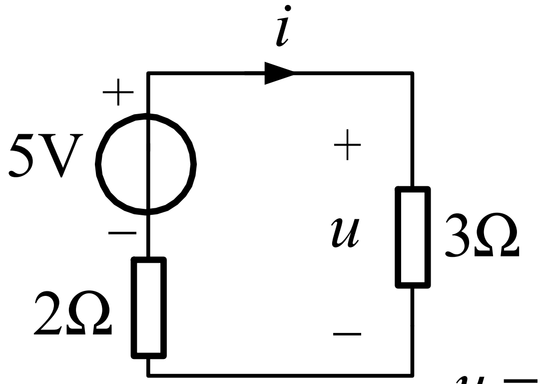
$$U_3 = U_3' + U_3'' + U_3'''$$
$$= k_1 U_{s1} + k_2 I_{s2} + k$$

- $k_1 U_{S1}$：U_{S1}单独激励产生的电压分量；
- $k_2 I_{S2}$：I_{S2}单独作用产生的电压分量；
- k：由电路内的独立源一起激励产生的电压分量；

$$\begin{cases} 3 = k_1 \times 0 + k_2 \times 0 + k \\ 2 = k_1 \times 1 + k_2 \times (-1) + k \\ 1 = k_1 \times (-4) + k_2 \times 1 + k \end{cases} \Rightarrow \begin{cases} k_1 = 1 \\ k_2 = 2 \\ k = 3 \end{cases} \Rightarrow U_3 = U_{S1} + 2I_{S2} + 3$$

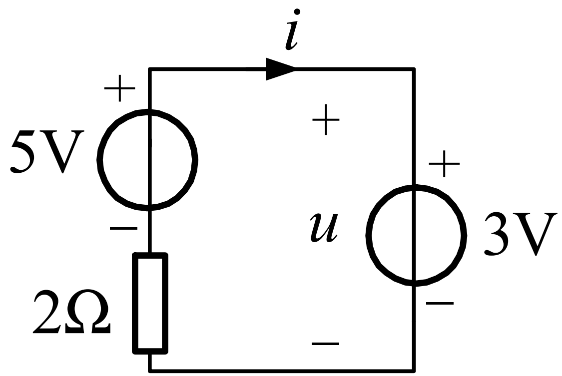
$$= 8\,\mathrm{V}$$

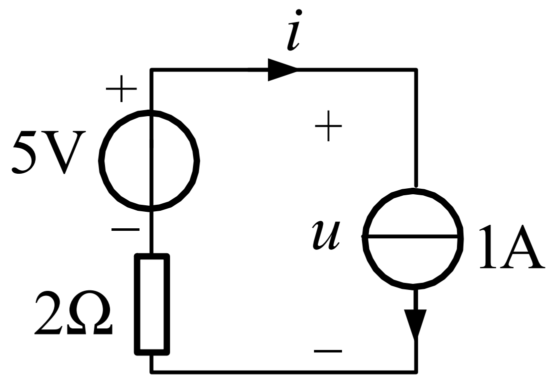
4.4 替代定理 (Substitution Theorem)



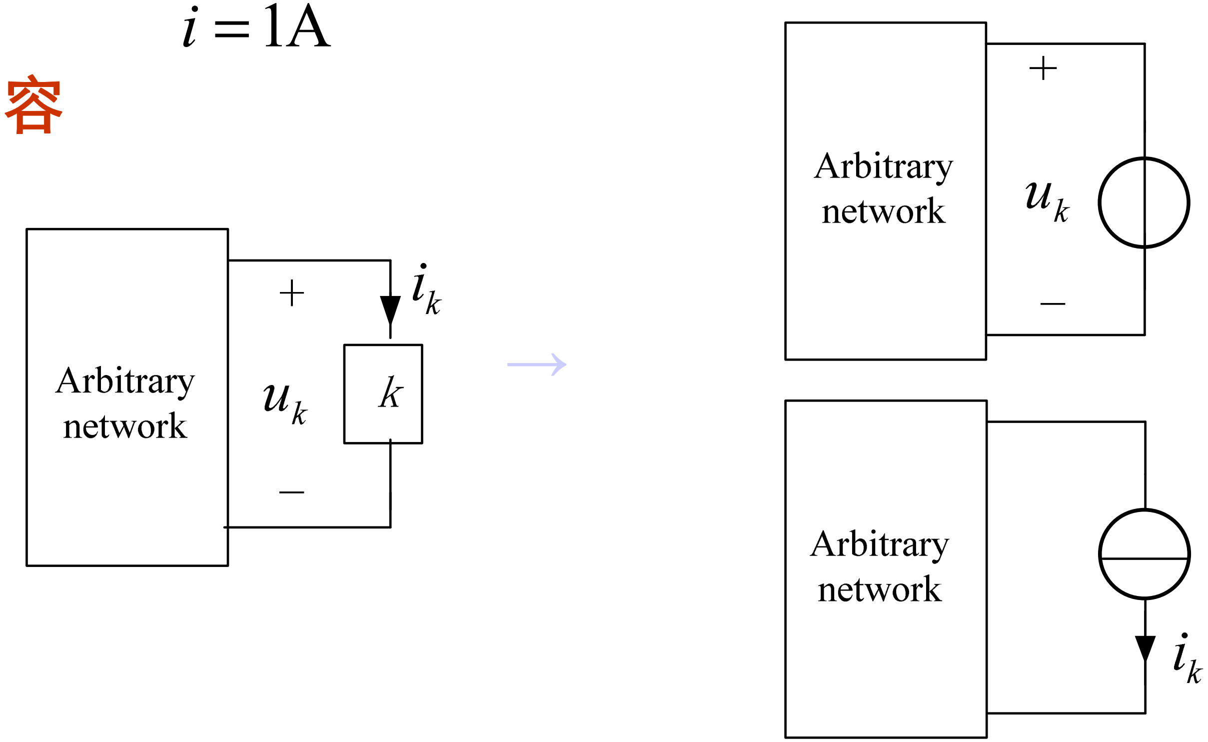
$u = 3\text{V}$

$i = 1\text{A}$





1.定理内容



4.4 替代定理 (Substitution Theorem)

1.定理内容:

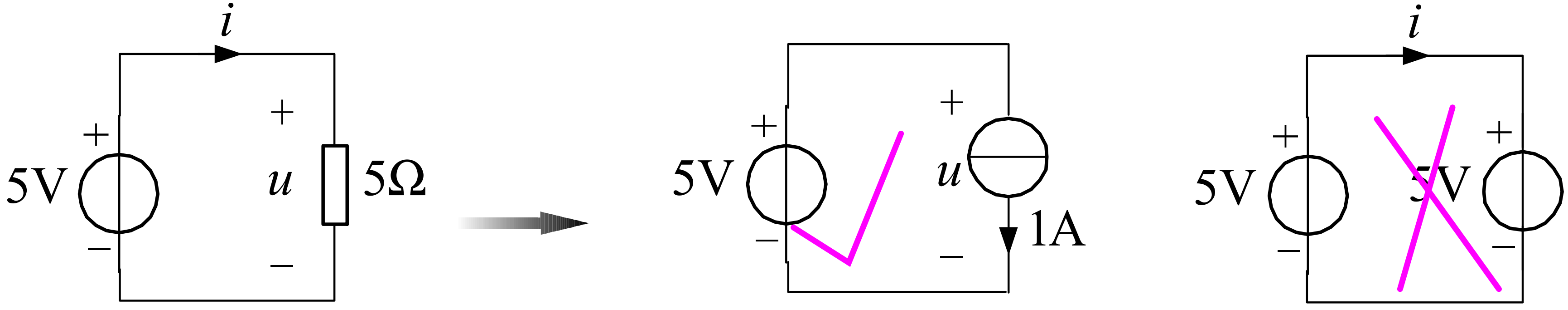
在任意一个电路中，若某支路k电压为u_k、电流为i_k，且该支路与其它支路不存在耦合，那么这条支路

- 可以用一个电压等于u_k的独立电压源替代；
- 或者用一个电流等于i_k的 独立电流源来替代；

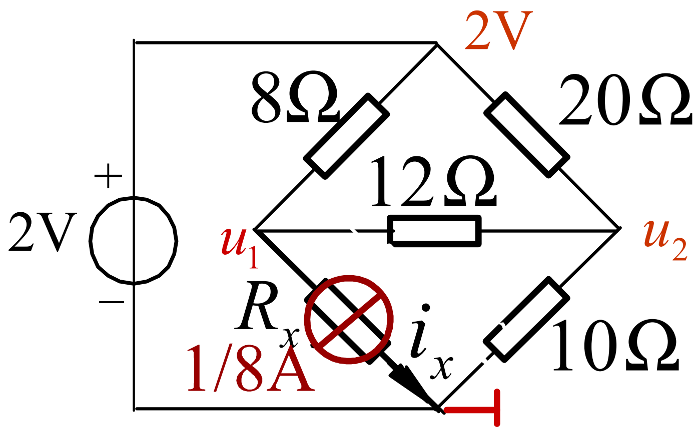
若替代后电路仍具有唯一解，则整个电路的各支路电压和电流保持不变。

2.定理应用 Applications

支路电压、电流具有唯一解



已知 i_x =1/8 A. 计算 R_x.



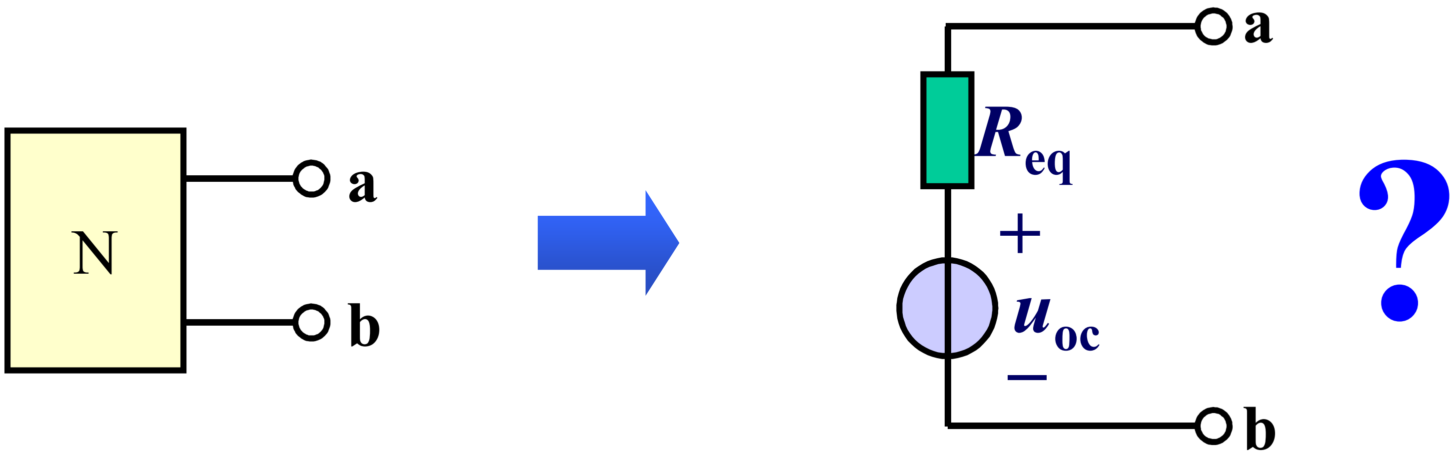
Nodal analysis:

$$\begin{cases} (\frac{1}{8}+\frac{1}{12})u_1-\frac{1}{12}u_2-\frac{1}{8}\times 2=-\frac{1}{8} \\ -\frac{1}{12}u_1(\frac{1}{20}+\frac{1}{12}+\frac{1}{10})u_2-\frac{1}{20}\times 2=0 \end{cases}$$

解方程得出： u_1=0.9V

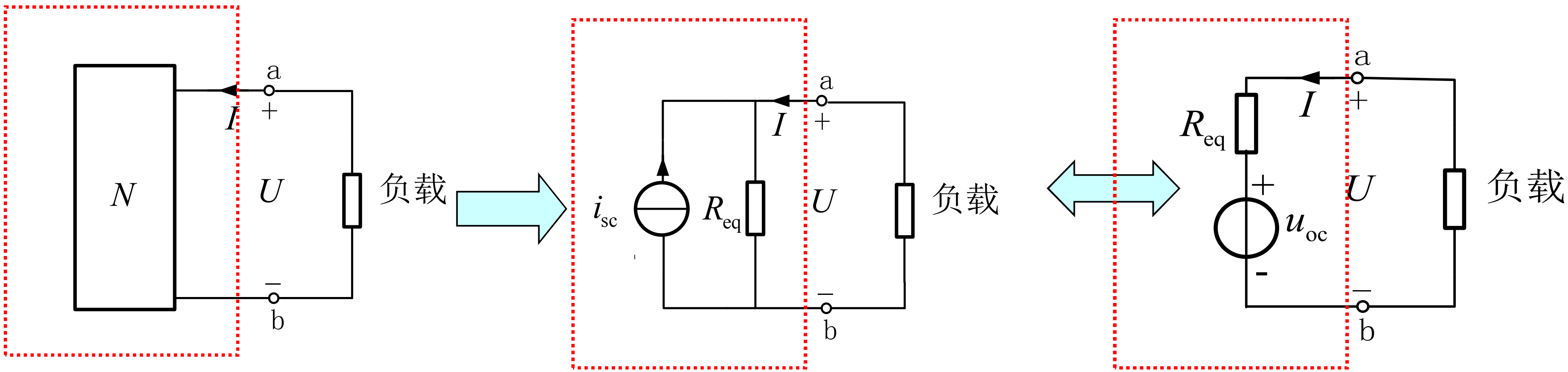
$$R_x=u_1\Big/\frac{1}{8}=0.9\times 8=7.2\Omega$$

4.5 戴维南定理与诺顿定理Thevenin-Norton Theorem



1 定理

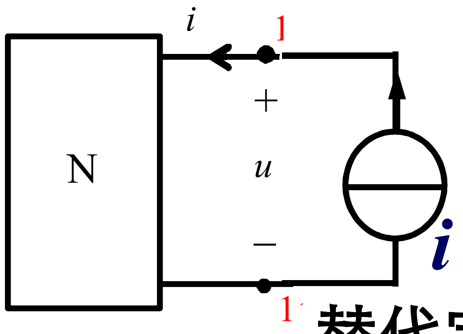
戴维南-诺顿定理：一个线性含有独立电源、线性电阻和线性受控源的一端口网络，对外电路来说，可用一个电压源和电阻串联等效，也可用一个电流源和电阻并联等效。



原始电路　　　诺顿等效电路　　　戴维南等效电路

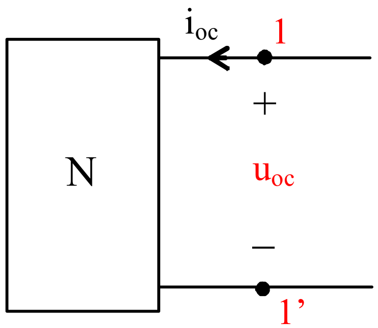
- u_{oc} 是端口的开路电压 （Open-circuit voltage）
- R_{eq}独立电源置零后的端口等效电阻（Equivalent resistance）
- i_{sc} 是端口的短路电流（Short-circuit current）

2 定理证明:



利用替代定理，将外部电路用电流源替代

替代定理



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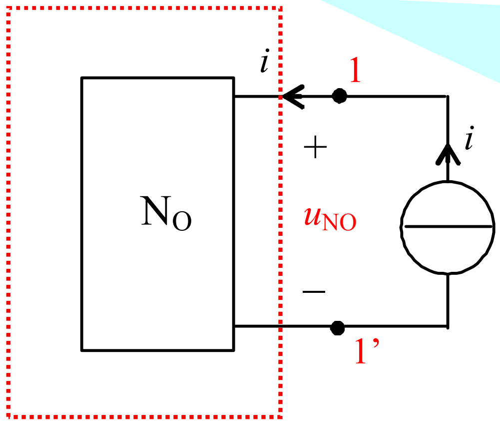


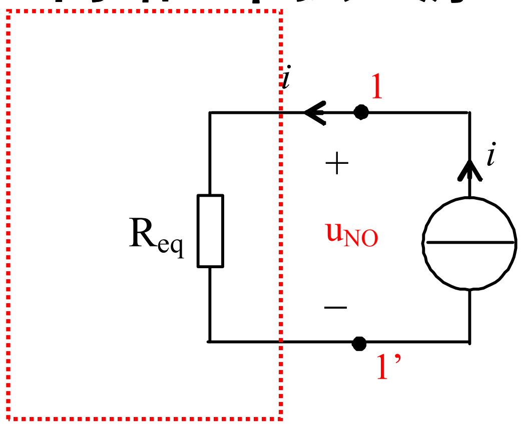
利用叠加定理，让电流源和N中电源分别单独作用。计算 ***u*** 值。

电流源 ***i*** 为零

网络 **N** 中独立源全部置零

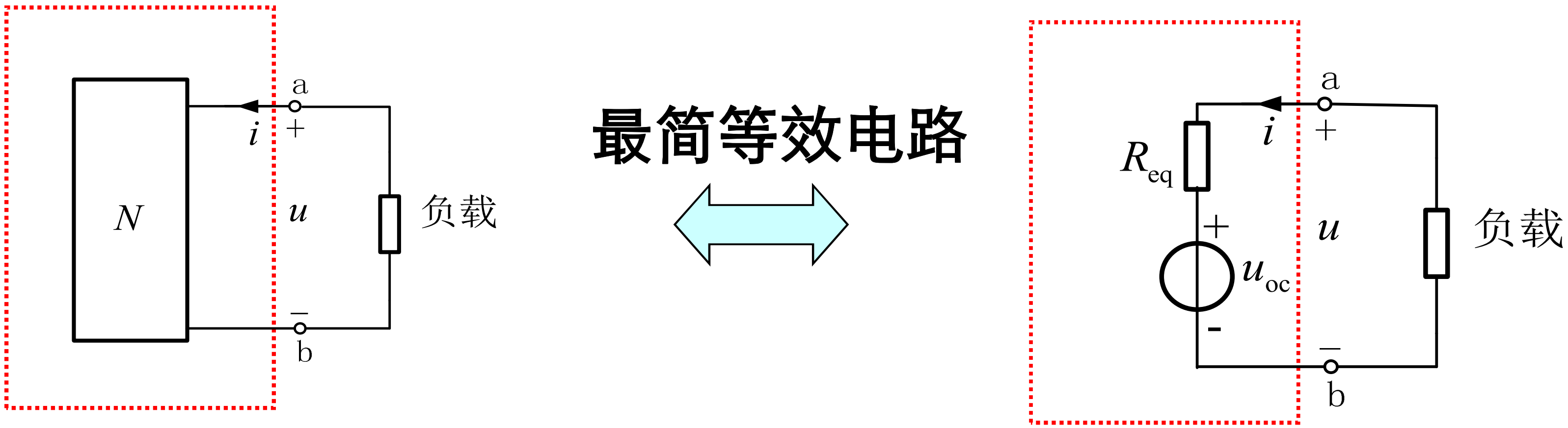
$$u = u' + u''$$

$$= u_{oc} + R_{eq} i$$



2 定理证明：

$$u = u_{oc} + R_{eq} i$$



结论：

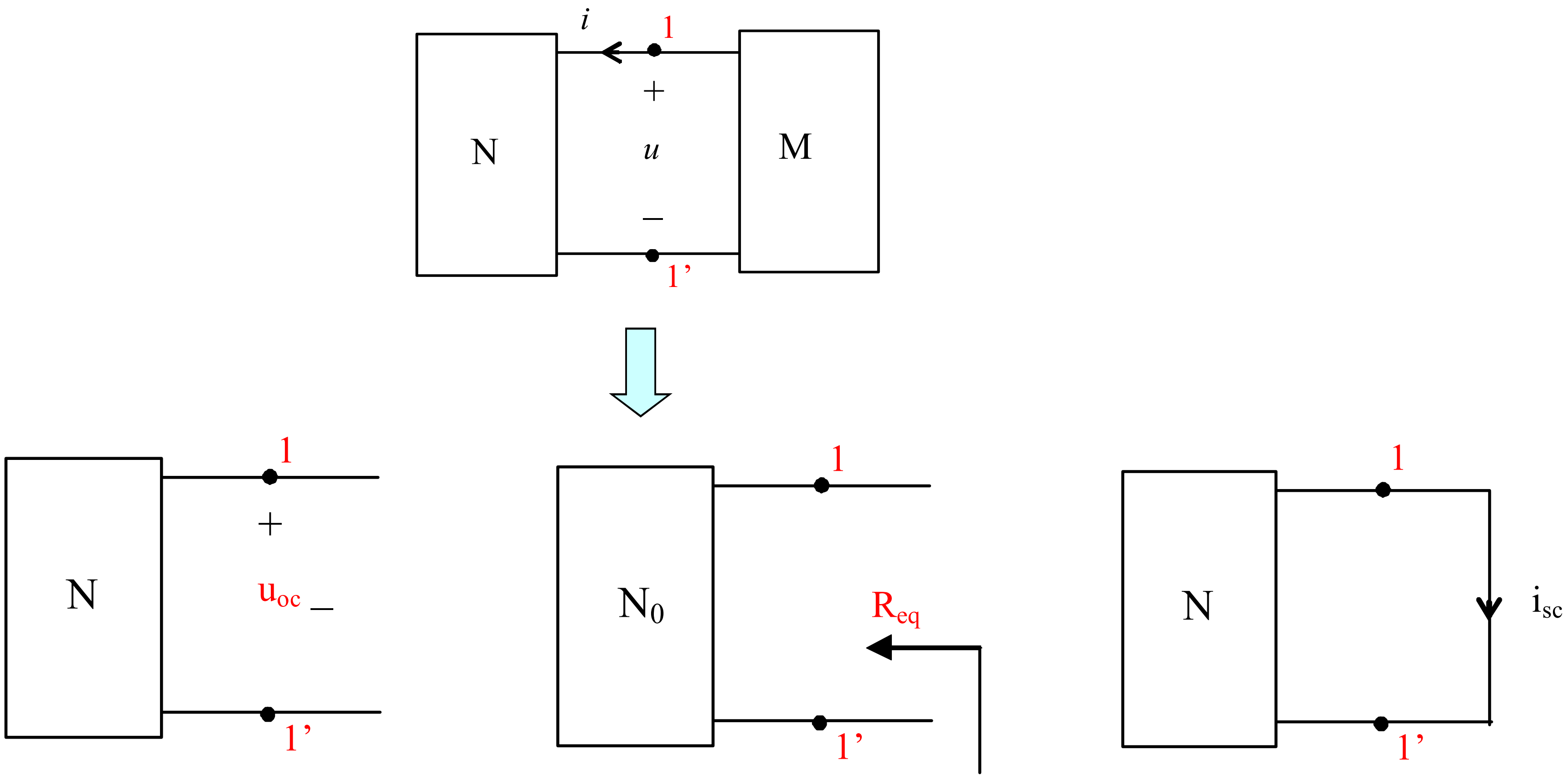
线性有源二端网络N，对外电路而言，可以用一个电压源和电阻元件串联组成的等效电路代替。

u_{oc} 是端口的开路电压；R_{eq}一端口中全部独立电源置零后的端口等效电阻。

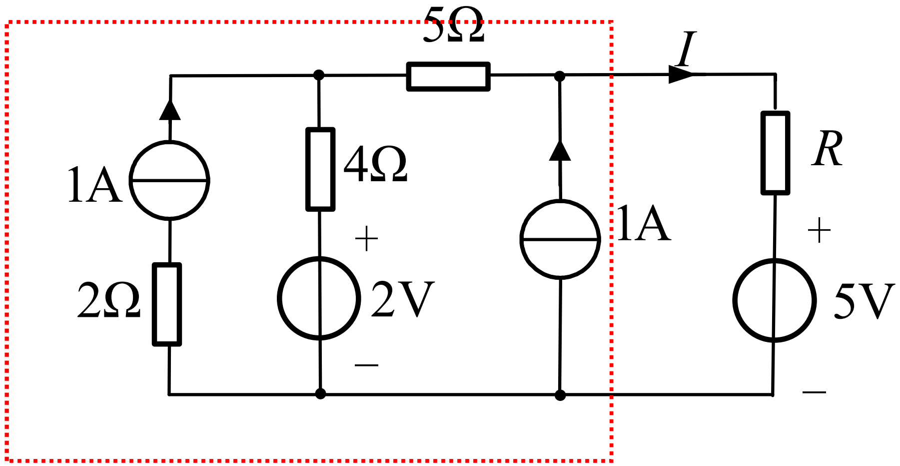
3.定理应用 Applications

确定戴维宁定理参数的方法:

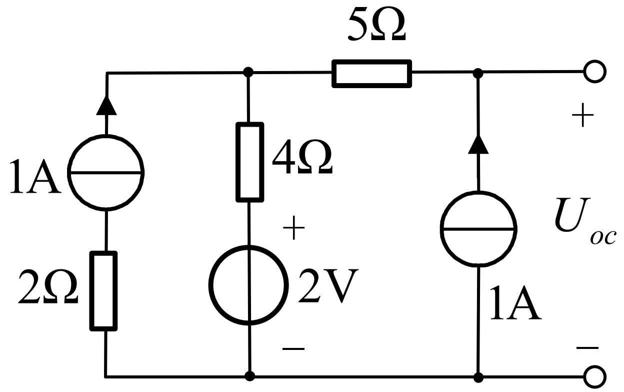
- 将待求支路移走,形成线性有源二端网络,求该网络的短路电流或开路电压或者入端电阻



【例 1】图示电路中，R为可调电阻。问：R为何值时，I=1A?

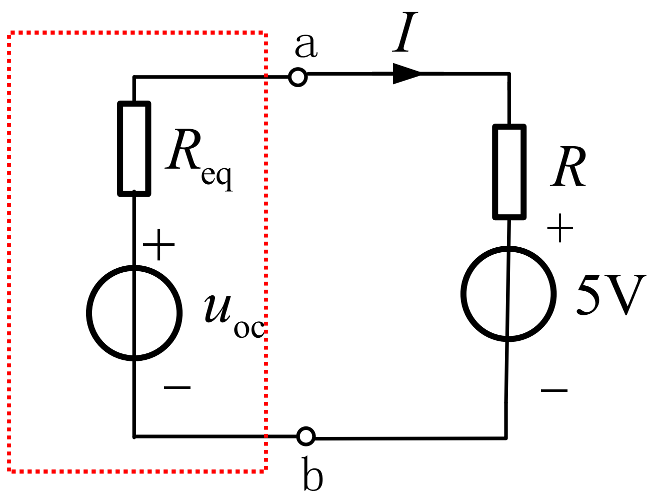


解：求开路电压u_{oc}，：



$$u_{oc} = 1\times5+4\times2+2=15\text{V}$$

求等效电阻R_{eq}：

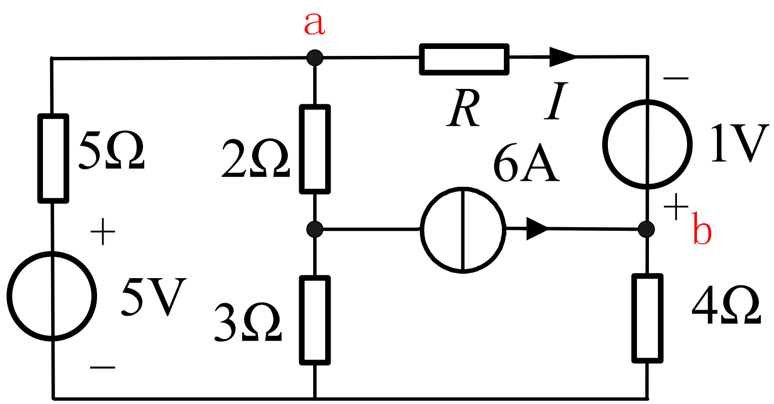


$$R_{eq} = 9\Omega$$

$$I = \frac{u_{oc}-5}{R+9} = 1$$

$$R = 1\Omega$$

【练习1】： 图示电路中，R为可调电阻。问：R为何值时，I=-1A?

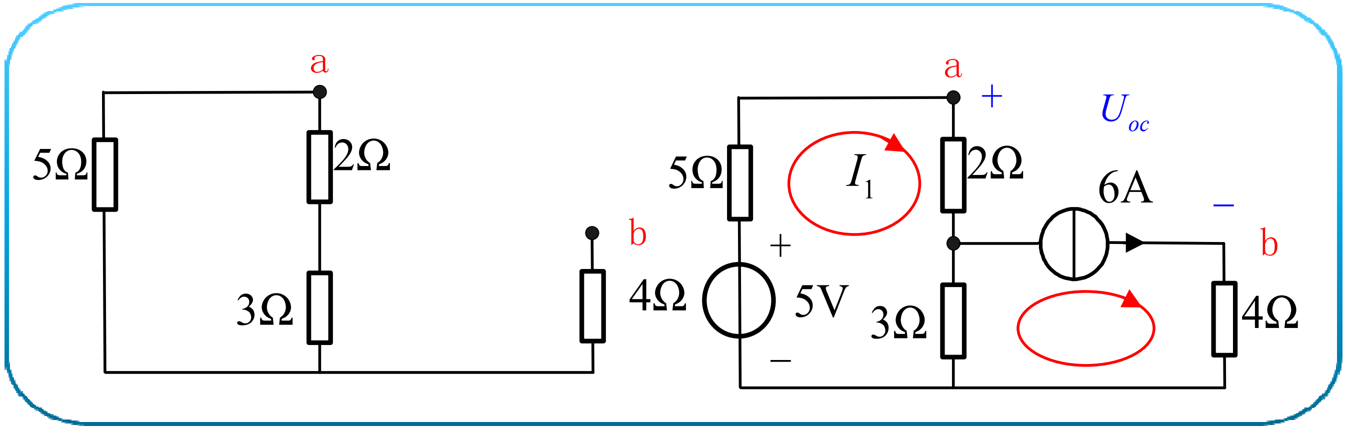


解：求开路电压u_{0C}，：

$$(5+2+3)I_1-3\times6=5$$

$$I_1=2.3\text{A}$$

$$u_{oc}=-5\times2.3+5-4\times6=-30.5\text{V}$$



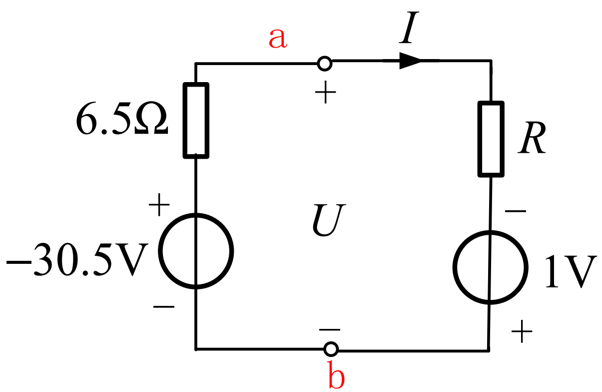
求等效电阻R_{eq}：

$$R_{eq}=6.5\Omega$$

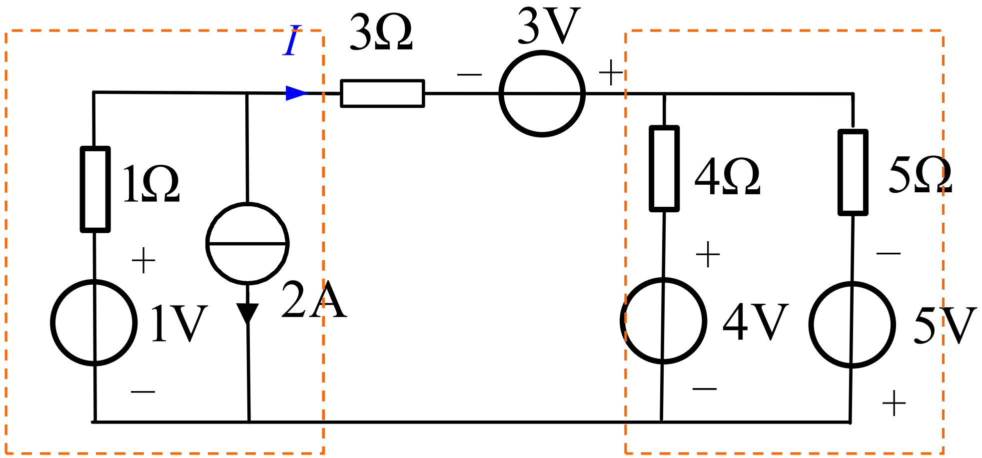
$$I_1=\frac{U_{oc}+1}{R_{eq}+R_1}$$

$$\therefore R_1=\frac{U_{oc}+1}{I_1}-R_{eq}$$

$$=23\Omega$$



【练习2】求电流I。



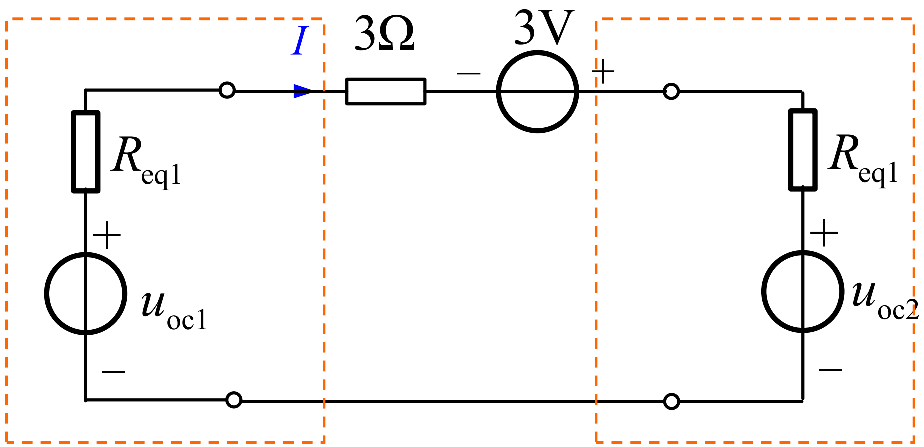
解：求开路电压与等效电阻

$$u_{oc1}=1-2\times1=-1\text{V}$$

$$R_{eq1}=1\Omega$$

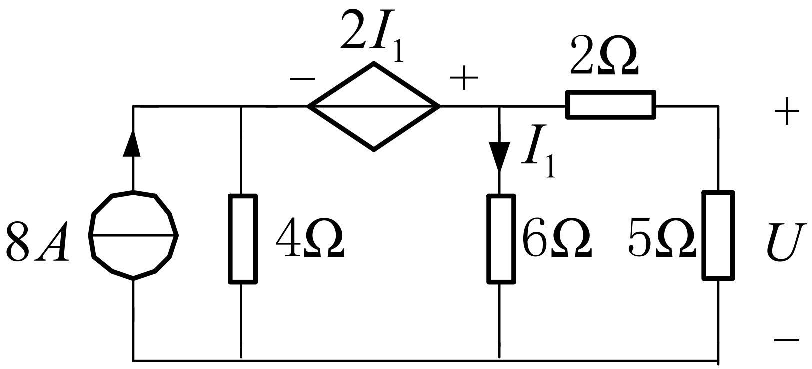
$$u_{oc2}=0\text{V}$$

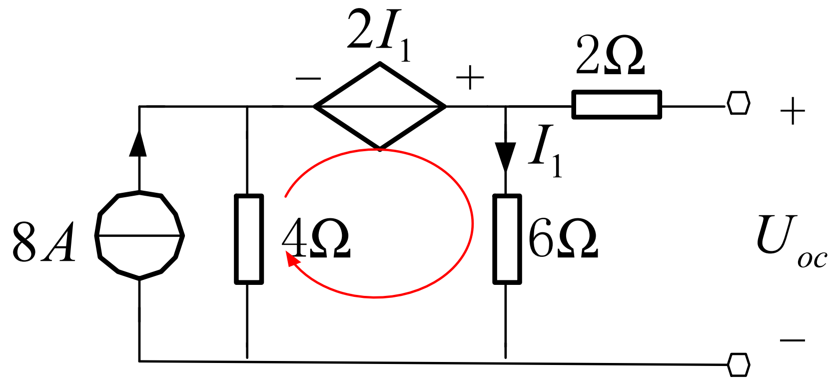
$$R_{eq2}=\frac{20}{9}\Omega$$



$$I=\frac{3+(-1)-0}{3+1+\dfrac{20}{9}}=\frac{9}{28}\text{A}$$

【例 2】.Find the voltage U.

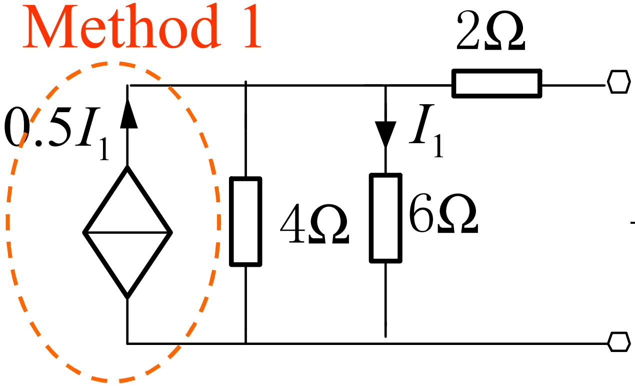


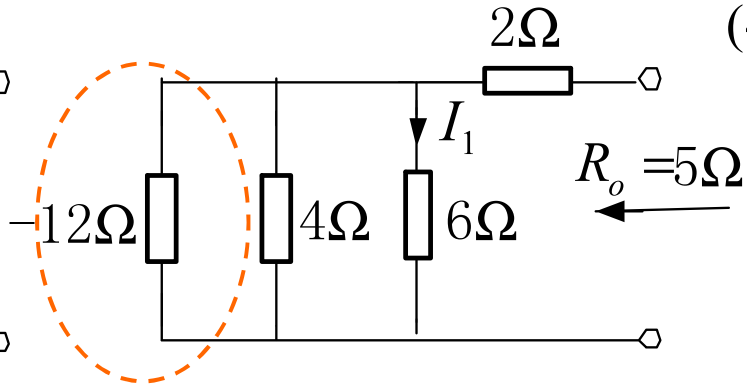


$$(4+6)I_1 - 8\times4 = 2I_1$$

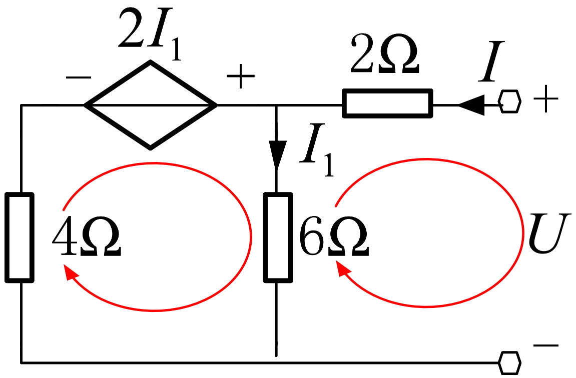
$$\Rightarrow I_1 = 4\text{A}$$

Method 1





Method 2

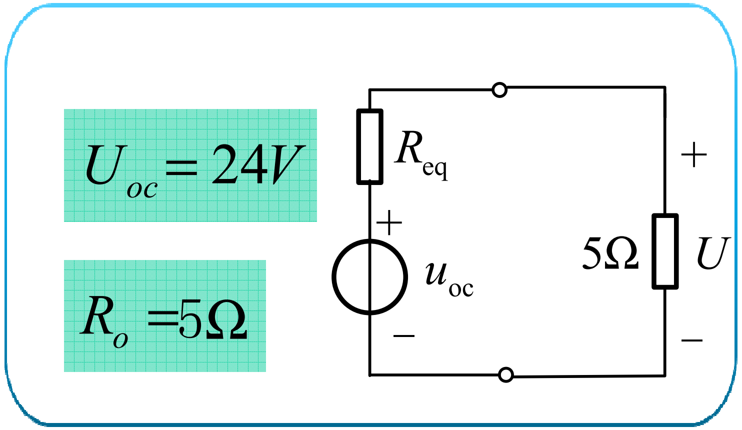


$$-6I_1 - 2I + U = 0$$

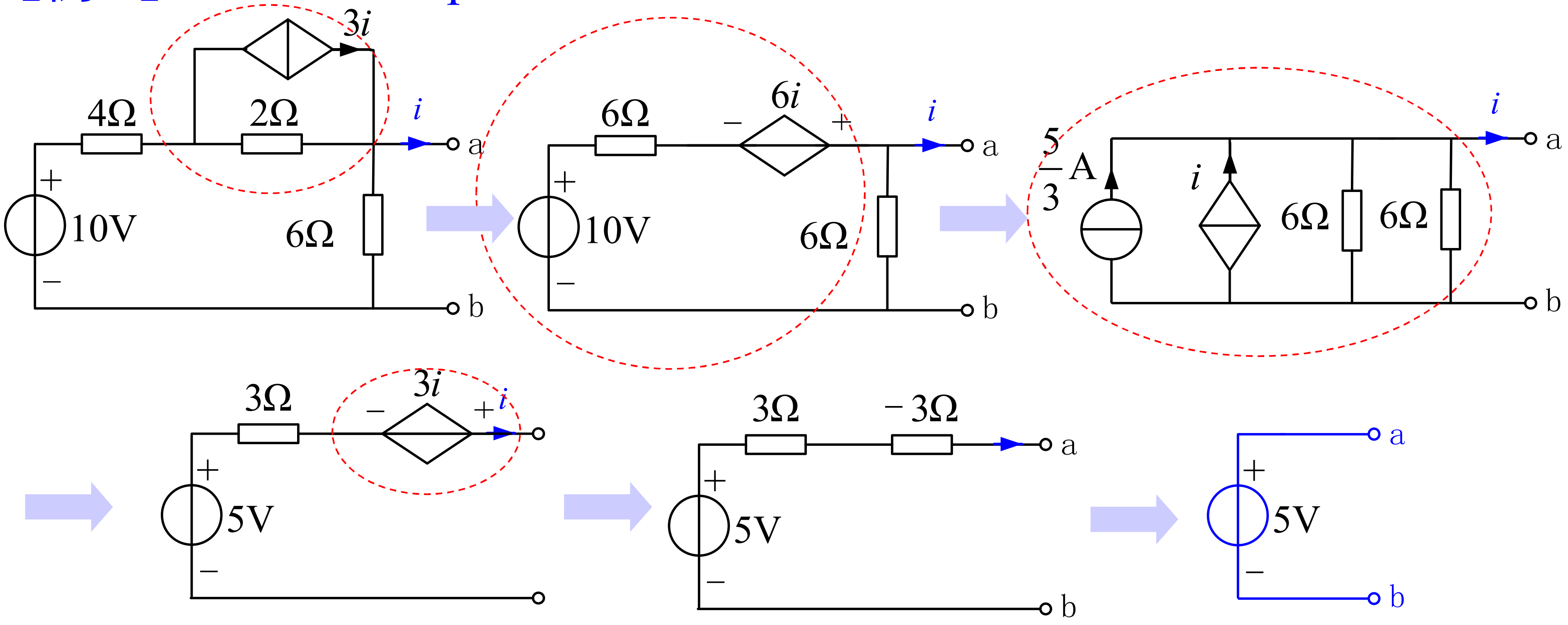
$$-2I_1 - 4(I - I_1) + 6I_1 = 0$$

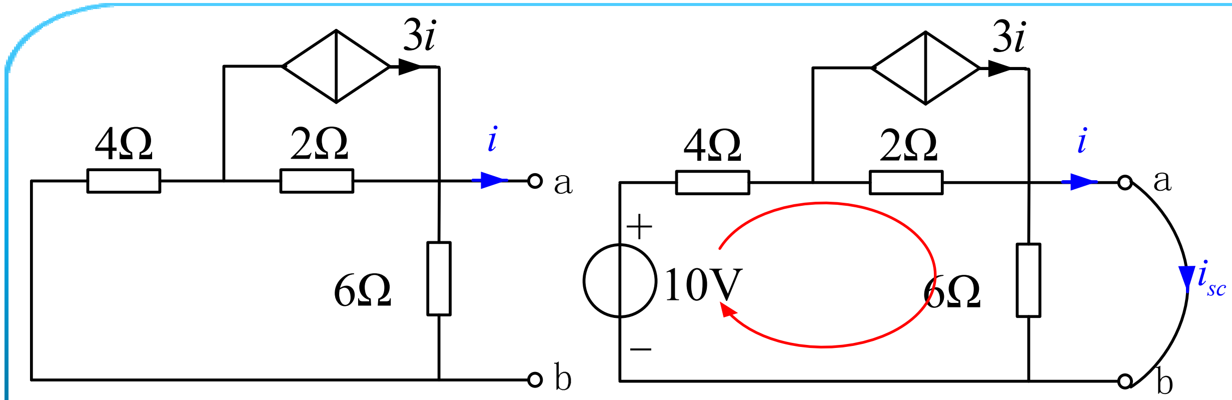
$$I_1 = 0.5I$$

$$R_o = \frac{U}{I} = 5\Omega$$



【例 3】. Find the equivalent circuit.





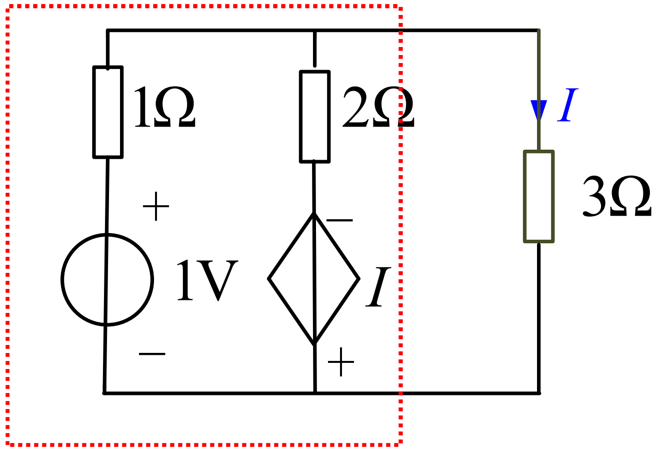
$$u_{oc} = 10 \times \frac{6}{4+2+6} = 5\text{V}$$

$$R_{eq} = 0\Omega$$

$$(4+2)i_{sc} - 2\times 3i_{sc} = 10$$

$$i_{sc} = \infty$$

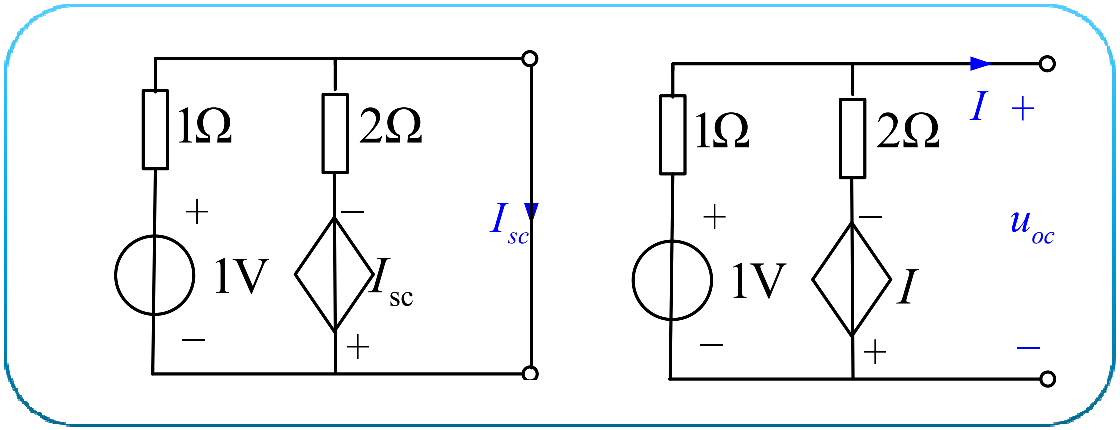
【练习3】求电流I。



解：求开路电压u_{oc}：

$$u_{oc}=1\times\frac{2}{1+2}=\frac{2}{3}\text{V}$$

求等效内阻（求短路电流）：

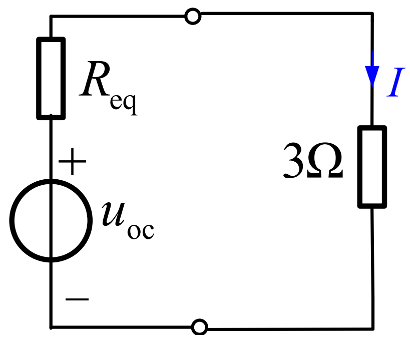


由KCL得：

$$I_{sc}-1+\frac{I_{sc}}{2}=0 \qquad I_{sc}=\frac{2}{3}\text{A}$$

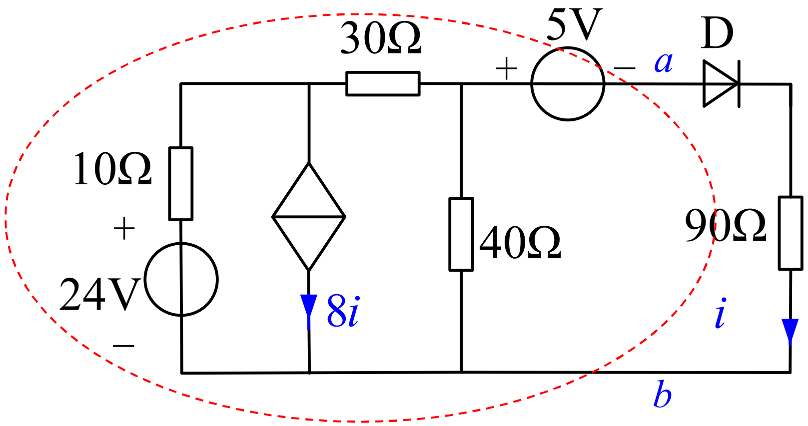
$$R_{eq}=\frac{U_{oc}}{I_{sc}}=1\Omega$$

求 I：

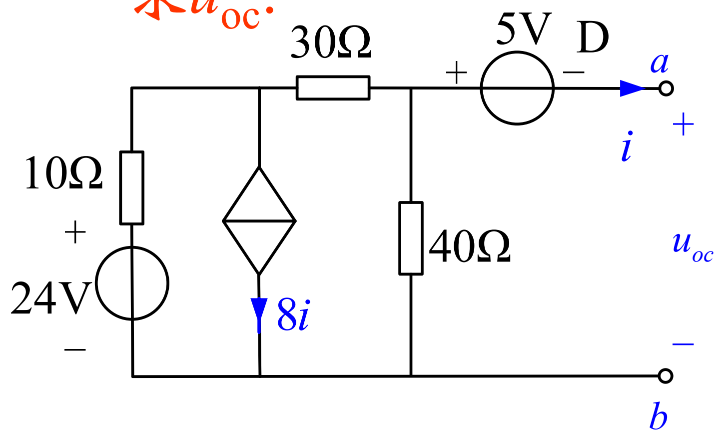


$$I=\frac{u_{oc}}{R_{eq}+3}=\frac{u_{oc}}{4}=\frac{1}{6}\text{A}$$

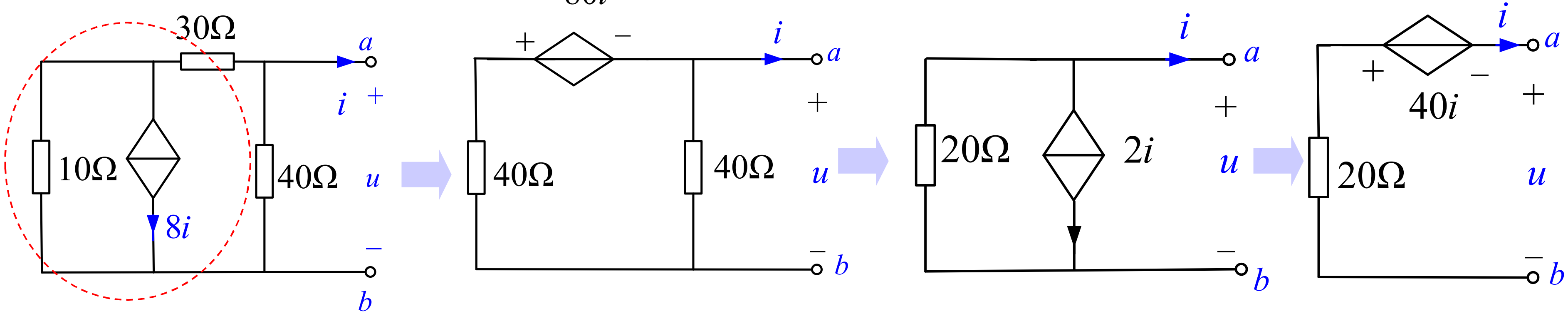
【练习4】. 二极管D状态？设D导通压降为0.7V，求i。(习题4-25)

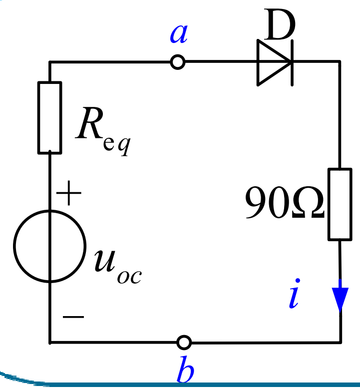


求u_{oc}:



求等效电阻



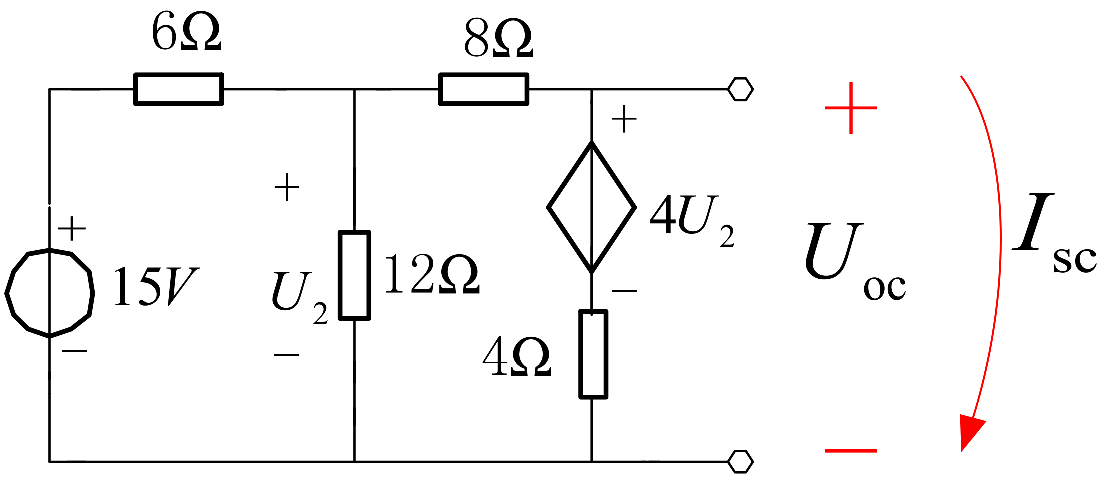


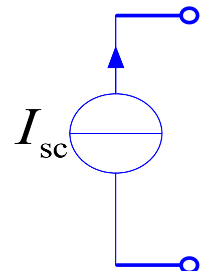
$$u_{oc}=\frac{40}{10+30+40}\times 24-5=7\text{V}$$

$$R_{eq}=20+40=60\Omega$$

$$i=\frac{7-0.7}{60+90}=0.042\text{A}$$

【课下练习】求等效电路。





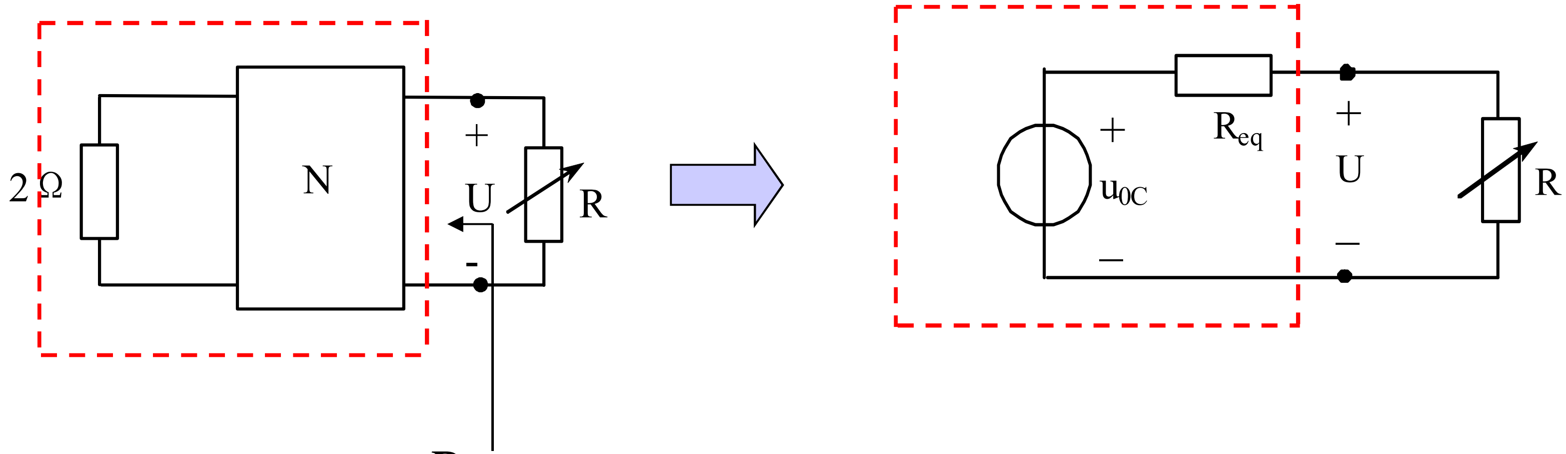
$$\left(\frac{1}{6}+\frac{1}{12}+\frac{1}{12}\right)U_2=\frac{15}{6}+\frac{4U_2}{12} \quad\Rightarrow\quad U_2=\infty，\ U_{oc}=\infty$$

$$\left(\frac{1}{6}+\frac{1}{12}+\frac{1}{8}\right)U_2=\frac{15}{6} \quad\Rightarrow\quad U_2=\frac{20}{3}\text{V} \quad\Rightarrow\quad I_{sc}=\frac{U_2}{8}+\frac{4U_2}{4}=\frac{15}{2}\text{A}$$

3.定理应用 Applications

由网络端口伏安关系确定等效模型

【例 4】. N为含独立源的线性电阻网络，确定图中端口左侧的戴维南等效电路。已知当R=4Ω时，U=4V； R=12Ω时，U=6V。

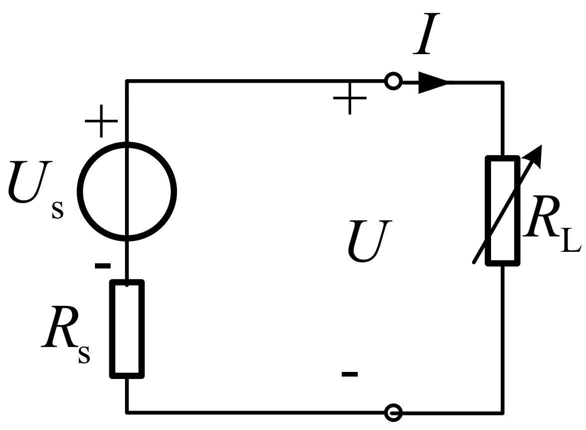


解:
$$\because U = \frac{R}{R + R_{eq}} u_{oc}$$

$$\Rightarrow \begin{cases} 4 = \dfrac{4}{4 + R_{\mathrm{eq}}} u_{oc} \\ 6 = \dfrac{12}{12 + R_{\mathrm{eq}}} u_{oc} \end{cases} \Rightarrow \begin{cases} u_{oc} = 8\mathrm{V} \\ R_{eq} = 4\Omega \end{cases}$$

4.6 最大功率传输定理Maximum Power Theorem

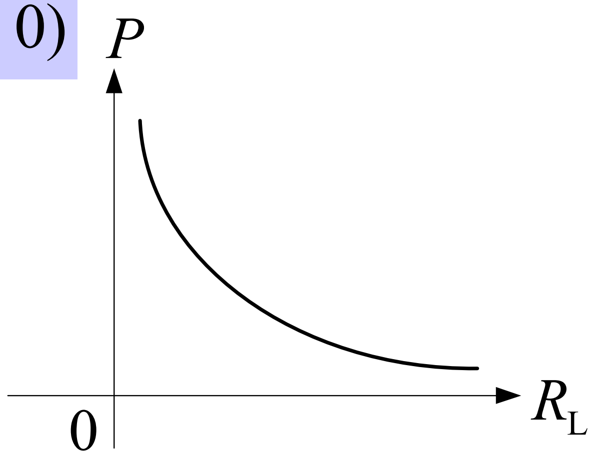
1.负载吸收功率的变化规律



$$P = R_L I^2 = R_L \times \left(\frac{U_s}{R_L + R_s}\right)^2$$

$R_s << R_L$ (或 $R_s \to 0$)

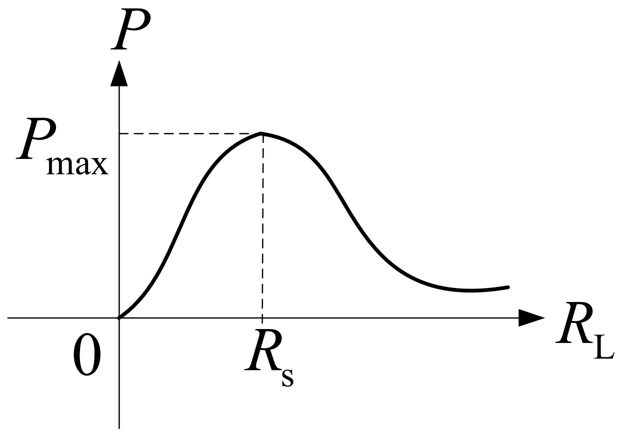
$$P \approx \frac{U_s^{\ 2}}{R_L}$$



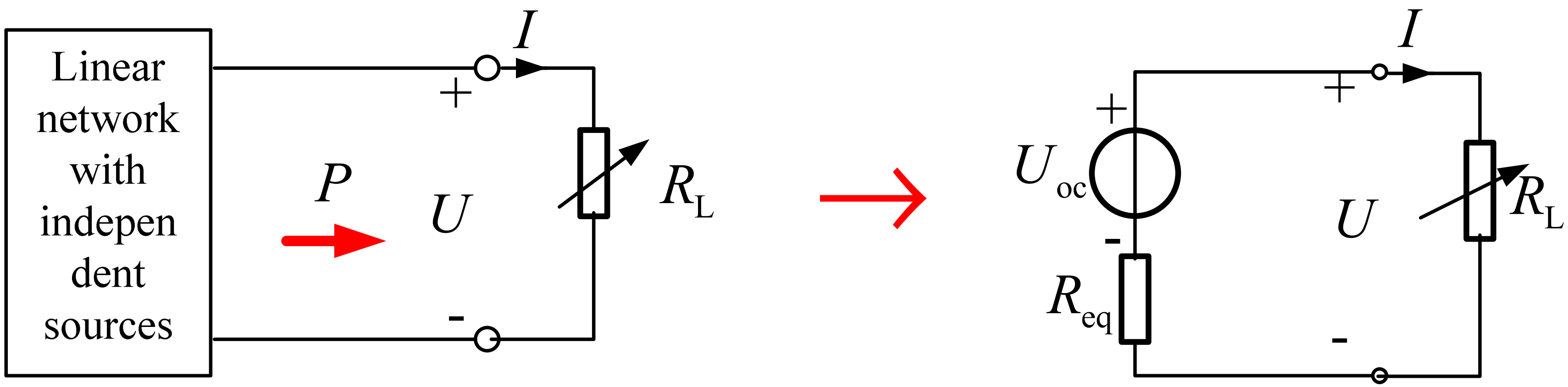
$R_s \neq 0$

$$P_{max} = ?$$

$$P_{max} = \frac{U_s^{\ 2}}{4R_s} \quad ?$$



4.6 最大功率传输定理Maximum Power Theorem



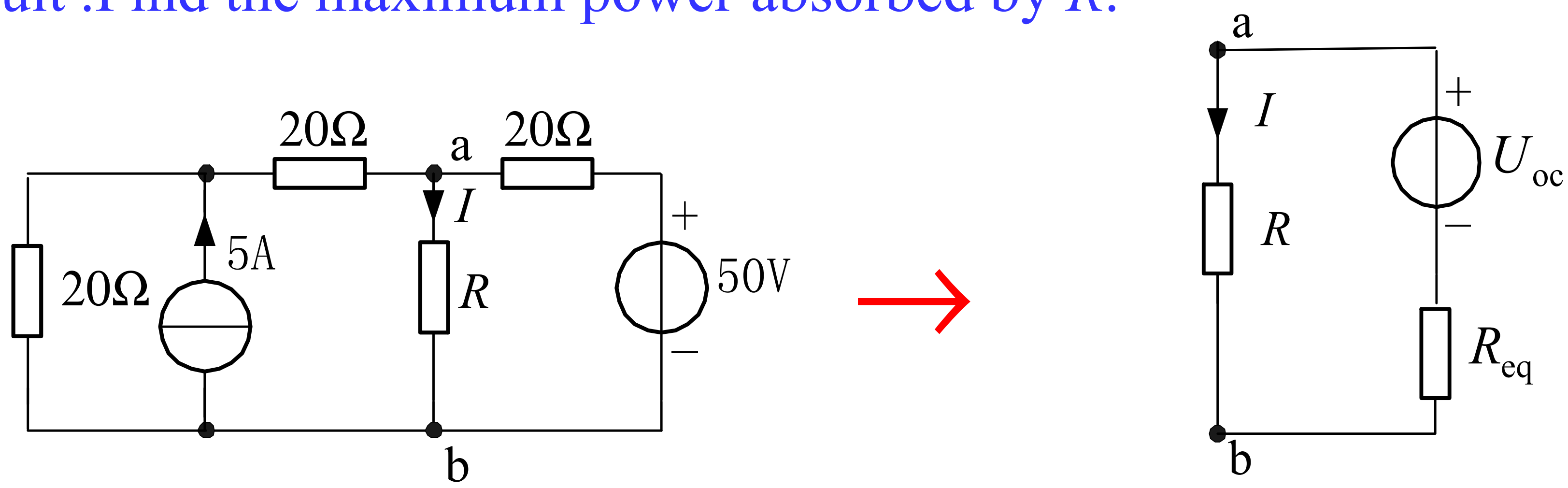
$$R_L = ? \Rightarrow P = \max = ?$$

$$\rightarrow P = R_L I^2 = R_L \times \left(\frac{U_{oc}}{R_L + R_{eq}}\right)^2$$

$$\rightarrow \frac{dP}{dR_L} = 0 \quad \rightarrow \quad R_L = R_{eq} \quad \rightarrow \quad P_{max} = \frac{U_{oc}^{\ 2}}{4R_{eq}}$$

讨论 ——目标：最大功率问题分析

【例 5】. Find the value of *R* for maximum power transfer in the circuit .Find the maximum power absorbed by *R*.

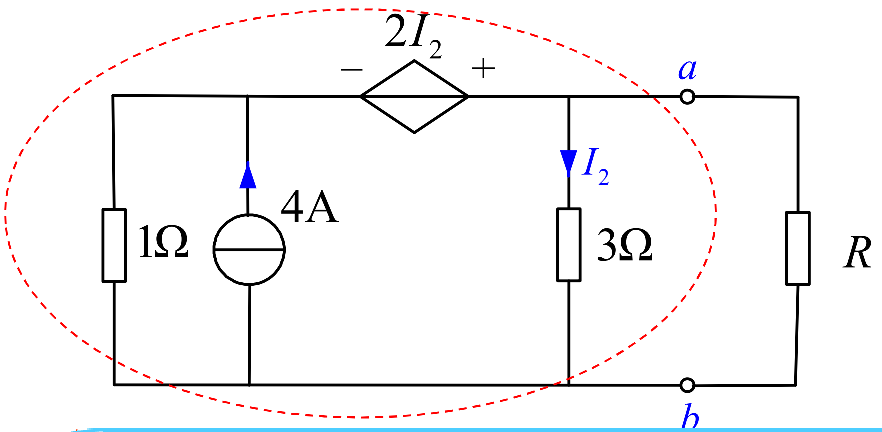


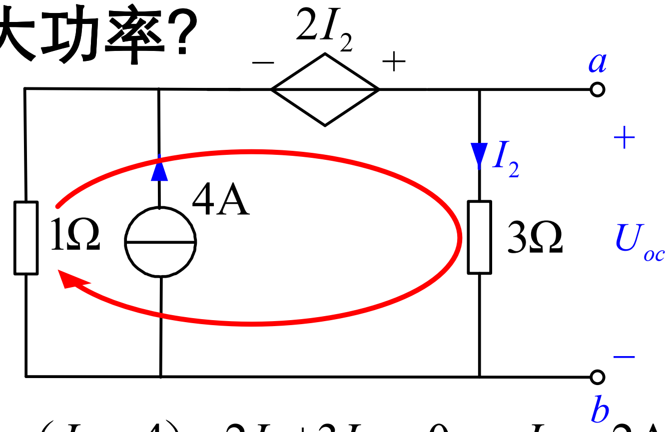
$$U_{oc} = \frac{20 \times 5}{20+40} \times 20 + \frac{40 \times 50}{20+40} = 66.7\text{V}$$

$$R_{eq} = 20 // (20+20) = 13.3\Omega$$

$$R = R_{eq} \qquad P_{max} = \frac{U_{oc}^2}{4R_{eq}} = 83.4\text{W}$$

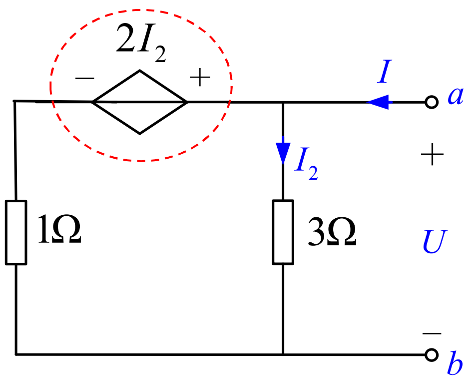
【例 6】. R可变，问R取何值时吸收最大功率？

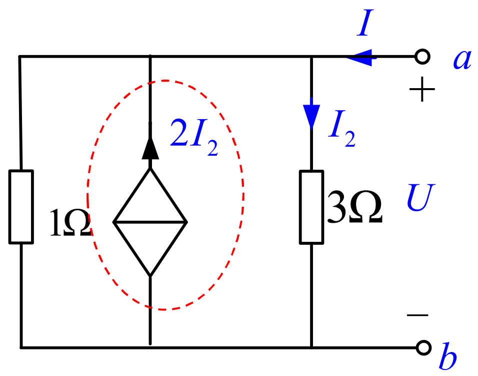


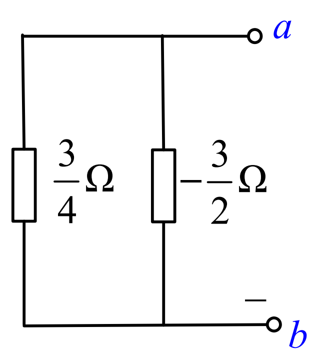


$$(I_2-4)-2I_2+3I_2=0 \qquad I_2=2\text{A}$$

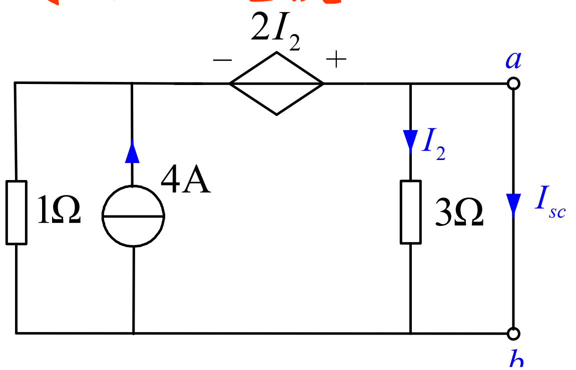
求等效电阻







求短路电流

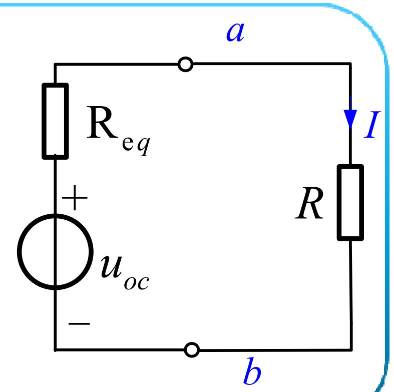


$$u_{\text{oc}}=6\text{V}$$

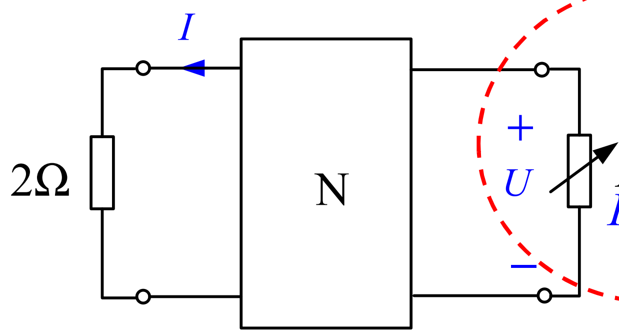
$$R_{\text{eq}}=\left(\frac{3}{4}//-\frac{3}{2}\right)=1.5\Omega$$

$$i_{\text{sc}}=4\text{A}$$

$$P=\frac{u_{oc}^{\ 2}}{4R_{eq}}=6\text{W}$$

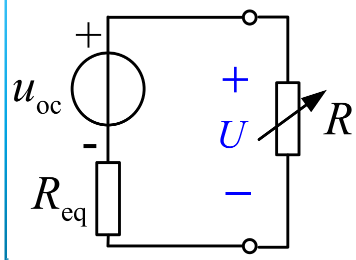


【例 7】. N为含独立源的线性电阻网络，已知当R=4Ω时，U=4V 、I=1.5A；R=12Ω时，U=6V、I=1.75A。求：R为何值时获得最大功率？R为何值时I=1.9A？



$U=\dfrac{8R}{R+4}$

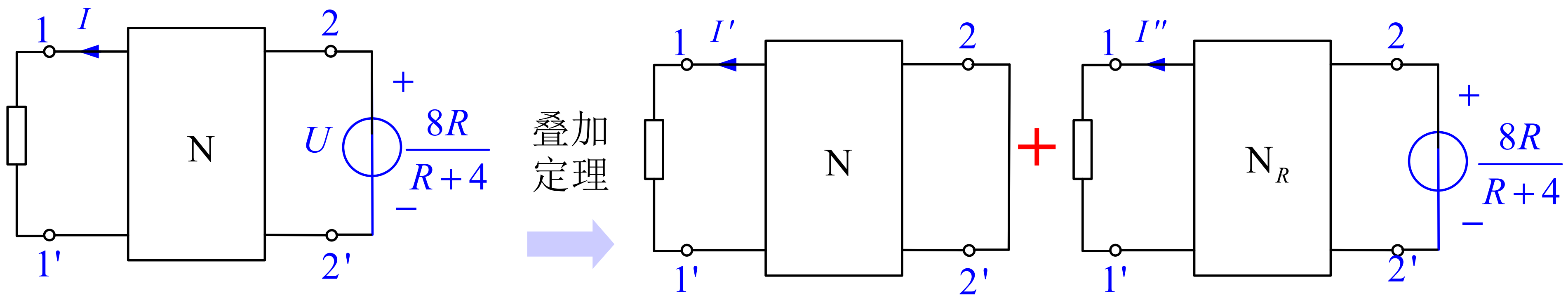
$I_R=\dfrac{8}{R+4}$



$\because U=\dfrac{R}{R+R_{eq}}u_{oc}$

$\Rightarrow\begin{cases}u_{oc}=8\text{V}\\ R_{eq}=4\Omega\end{cases}$ $P_{\max}=\dfrac{u_{oc}^2}{4R_{eq}}=4\text{W}$

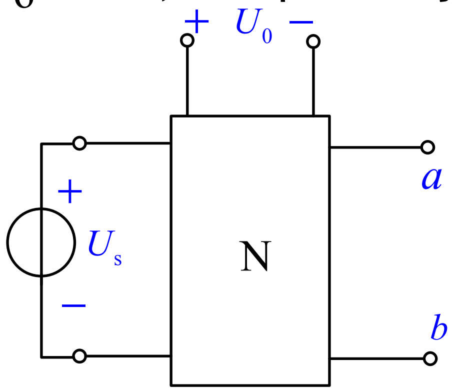
（2）应用替代和叠加定理

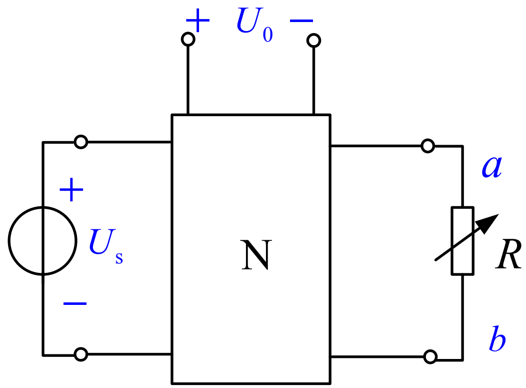


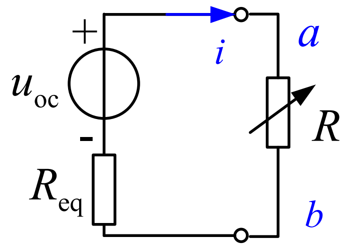
$\Rightarrow I=I'+I"\ \ =C\ +K\dfrac{8R}{R+4}$ $\Rightarrow\begin{cases}1.5=C+K\dfrac{8\times4}{4+4}\\ 1.75=C+K\dfrac{12\times4}{12+4}\end{cases}$ $\Rightarrow\begin{cases}C=1\\ K=0.125\end{cases}$

$I=1+0.125\dfrac{8R}{R+4}$ $\therefore 1.9=1+0.125\dfrac{8R}{R+4}$ $\therefore R=36\Omega$

【练习】. N为电阻网络，已知a、b开路时，U_o=6V；a、b短路时，U_o=8V， 当a、b 接可变电阻R，求 R获得最大功率时 U_o的值。

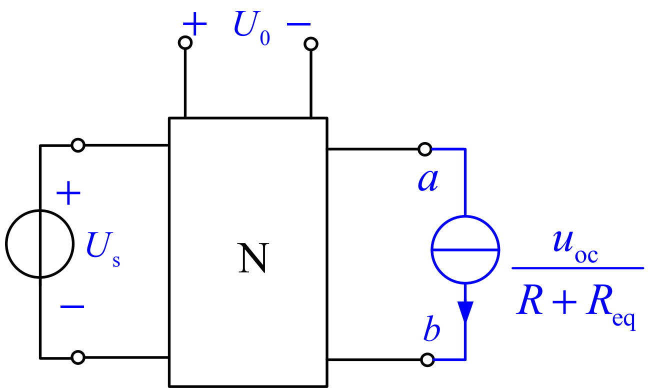




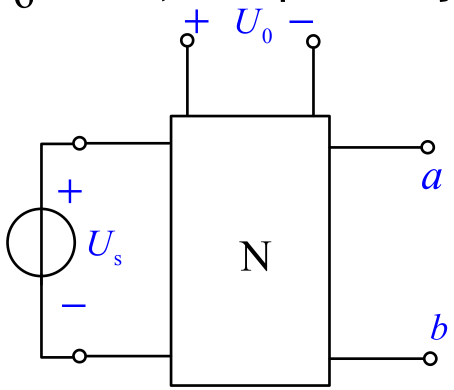


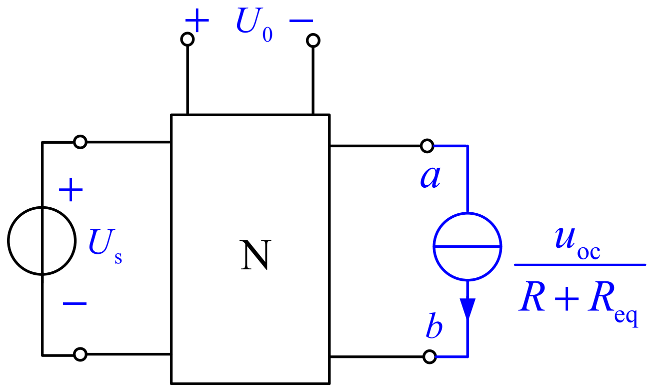
$$i = \frac{u_{oc}}{R + R_{eq}}$$

替代定理

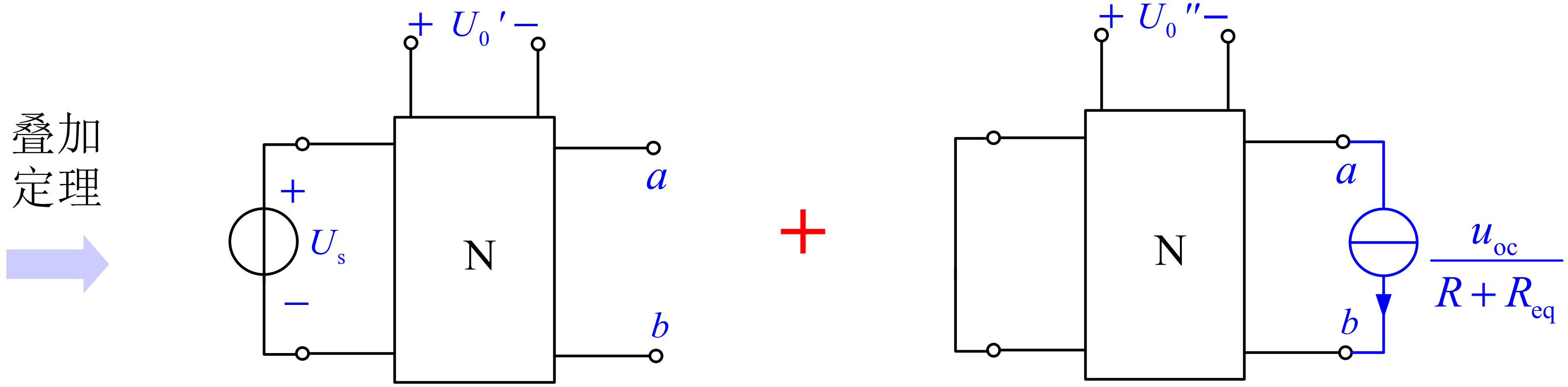


【练习】. N为电阻网络，已知a、b开路时，U_o=6V；a、b短路时，U_o=8V，当a、b 接可变电阻R，求 R获得最大功率时 U_o的值。





叠加定理



$$\Rightarrow U_0 = U_0{}' + U_0{}'' = C + K\frac{u_{oc}}{R+R_{eq}} \Rightarrow \begin{cases} U_0 = C + K\times 0 = 6 \\ U_0 = C + K\times\dfrac{u_{oc}}{R_{eq}} = 8 \end{cases} \Rightarrow \begin{cases} C = 6 \\ K\times\dfrac{u_{oc}}{R_{eq}} = 2 \end{cases}$$

R获得最大功率时 $\Rightarrow U_0 = C + K\times\dfrac{u_{oc}}{R_{eq}+R_{eq}} = 6 + \dfrac{1}{2}\times K\times\dfrac{u_{oc}}{R_{eq}} = 7\text{V}$