

第2章 电阻电路等效变换

等效变换的概念 Concept of Equivalence

串联与并联 Series and Parallel Connections

对称电路 Symmetric Circuits

电桥 Bridge Circuits

星-三角互换 Wye-Delta Transformation

电源变换 Source transformation

第2章 电阻电路等效变换

目标： 1.熟练应用支路电流分析法分析简单电路。
2.综合应用各种等效化简方法获得等效电路。

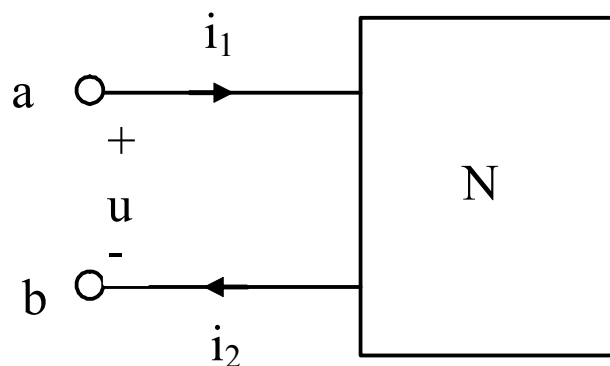
难点： 1.含受控源电路的等效化简。
2.综合应用多种等效变换方法化简复杂电路。

讲授学时： 4

讨论学时： 1

2.1 概述

一、二端电路及端口的概念



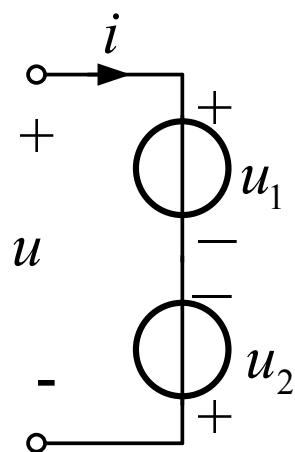
- N只有两个端子（a、b）与外部电路相连；进出两个端子的电流相同，即 $i_1=i_2=i$ ；
- N可由任意的元件组合而成；
- 两个端钮上的电压、电流分别称为端口电压和端口电流，它们之间的关系式 $u=f(i)$ 、 $i=f(u)$ 称为端口伏安关系。

三、等效变换的说明

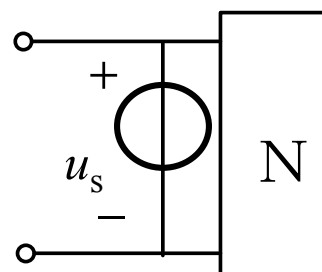
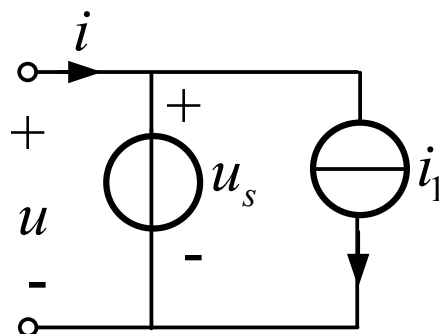
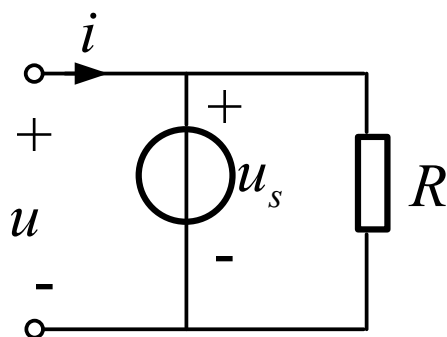
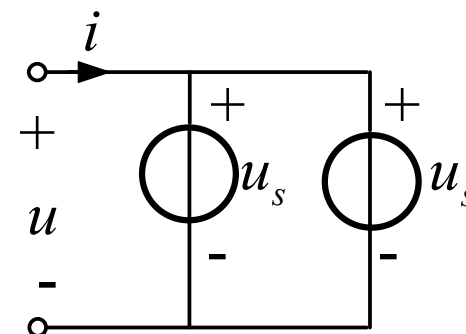
- 一个电路被它的等效电路替代后，未被等效的电路中的所有电压、电流不变。
- 两个内部结构不同的电路“等效” 等效的核心在于：两个电路对“任意”外电路的效果一致，而不是对某一特定的外电路等。
- 等效具有传递性。

2.2 串联与并联

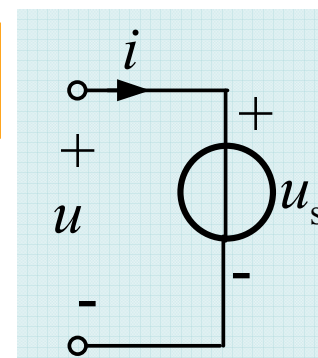
2.2.1 独立电压源串联与并联



$$u = u_1 - u_2 = u_s$$

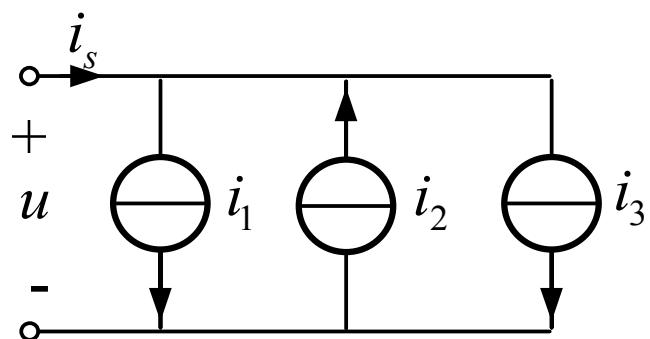


$$u = u_s$$

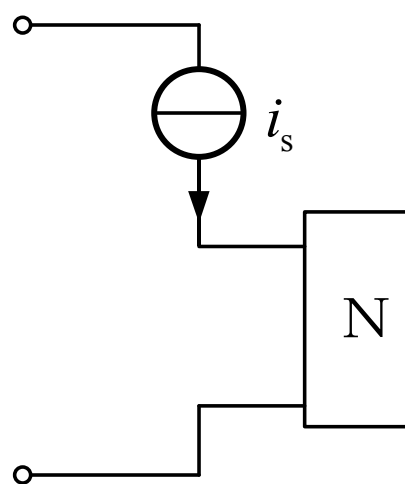
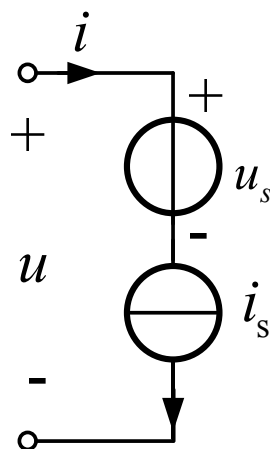
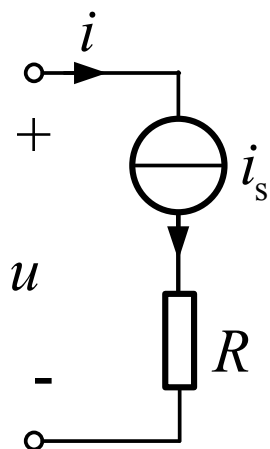
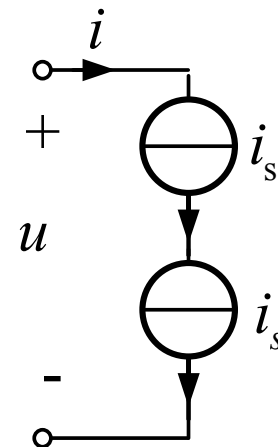


2.2 独立串联与并联

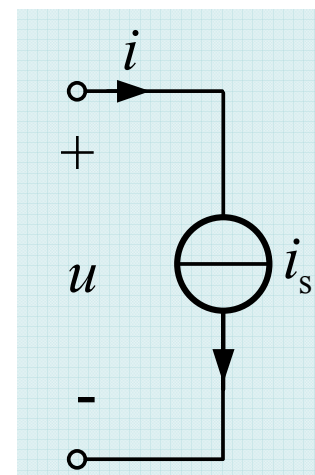
2.2.2 独立电流源串联与并联



$$i_s = i_1 - i_2 + i_3$$



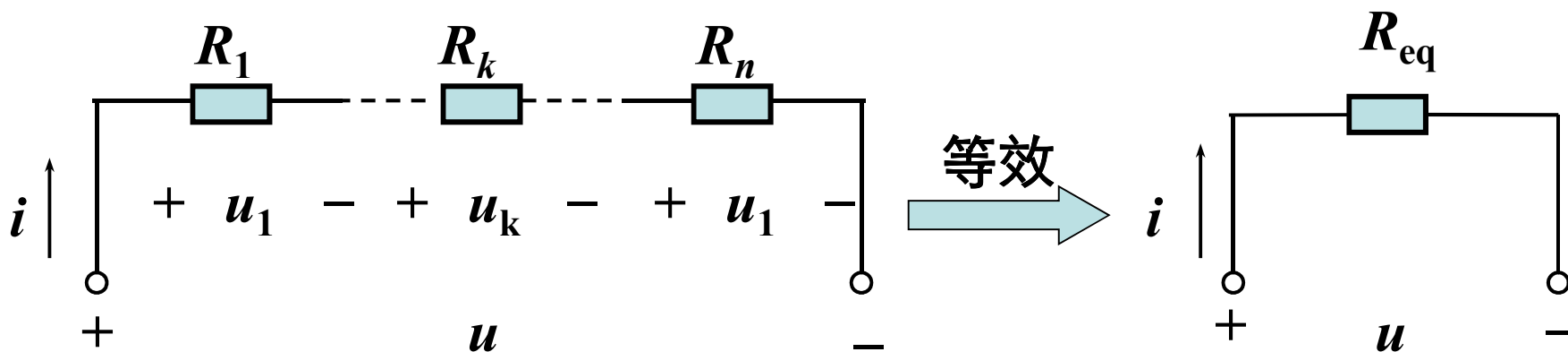
$$i = i_s$$



2.2.2 线性电阻元件的串联和并联

一 电阻元件的串联 (Series Connection of Resistors)

1. 等效电阻 R_{eq}



KVL
$$u = u_1 + u_2 + \dots + u_k + \dots + u_n$$
$$= (R_1 + R_2 + \dots + R_k + \dots + R_n) i = R_{eq} i$$

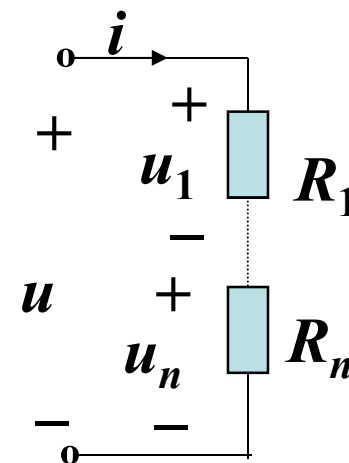
$$R_{eq} = (R_1 + R_2 + \dots + R_n) = \sum R_k$$

结论： 串联电路的总电阻等于各分电阻之和。

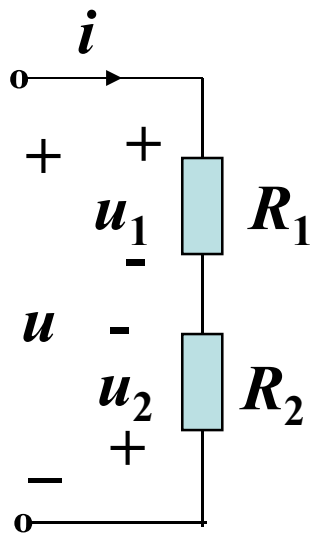
串联电阻上电压的分配

$$\text{由 } \frac{u_k}{u} = \frac{R_k i}{R_{\text{eq}} i} = \frac{R_k}{R_{\text{eq}}} = \frac{R_k}{\sum R_k}$$

即 电压与电阻成正比



例：两个电阻分压，如下图

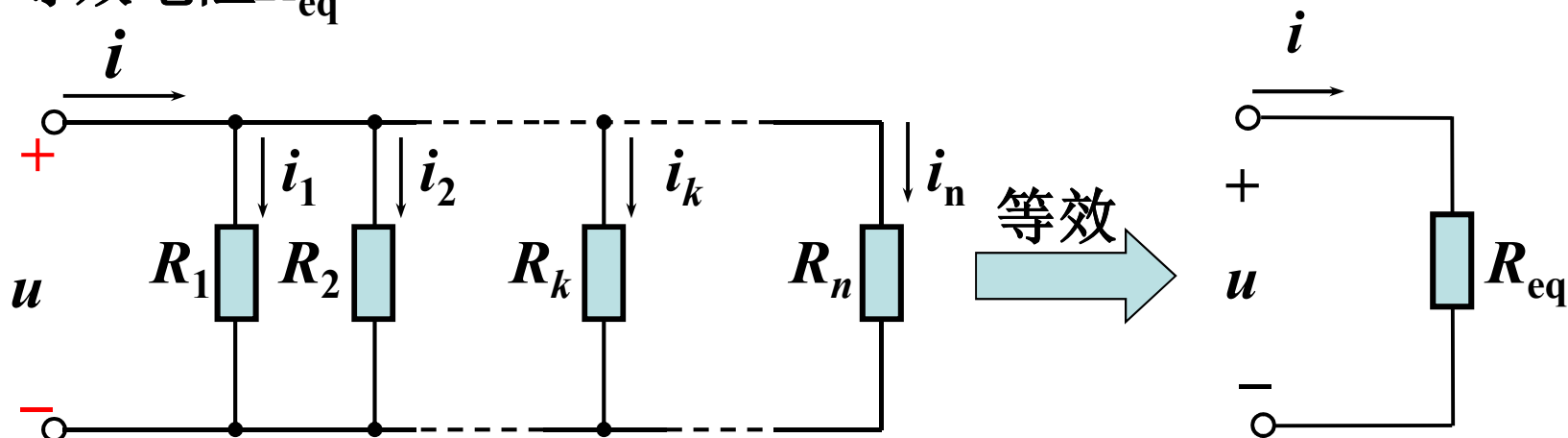


$$u_1 = \frac{R_1}{R_1 + R_2} u$$

$$u_2 = -\frac{R_2}{R_1 + R_2} u$$

二 电阻元件的并联 (Parallel Connection)

1. 等效电阻 R_{eq}



$$\begin{aligned}\text{由KCL: } i &= i_1 + i_2 + \dots + i_k + i_n \\ &= u/R_1 + u/R_2 + \dots + u/R_n \\ &= u(1/R_1 + 1/R_2 + \dots + 1/R_n) = u/R_{eq}\end{aligned}$$

即

$$1/R_{eq} = 1/R_1 + 1/R_2 + \dots + 1/R_n$$

电导表示: $G_{eq} = G_1 + G_2 + \dots + G_k + \dots + G_n = \sum G_k = \sum 1/R_k$

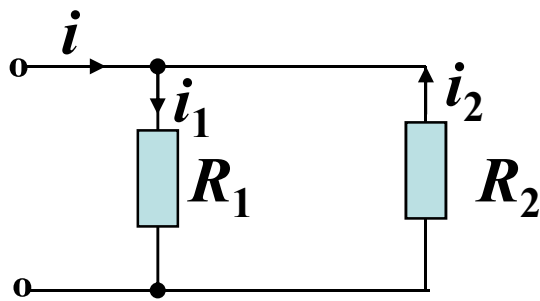
2. 并联电阻的电流分配

$$\text{由 } \frac{i_k}{i} = \frac{u / R_k}{u / R_{\text{eq}}} = \frac{G_k}{G_{\text{eq}}}$$

$$\Rightarrow i_k = \frac{G_k}{\sum G_k} i$$

即 电流分配与电导成正比

对于两电阻并联， 有



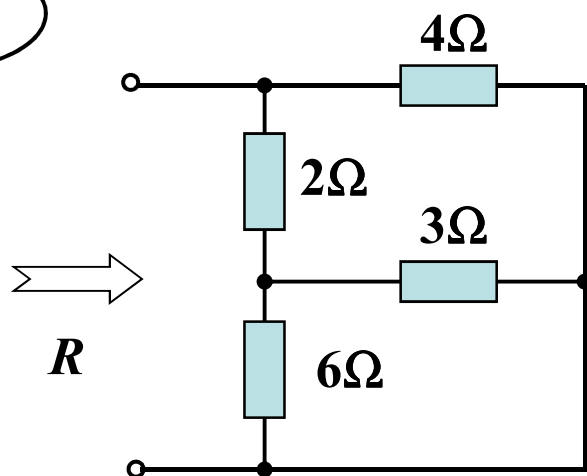
$$i_1 = \frac{1/R_1}{1/R_1 + 1/R_2} i = \frac{R_2}{R_1 + R_2} i$$

$$i_2 = -\frac{1/R_2}{1/R_1 + 1/R_2} i = -\frac{R_1}{R_1 + R_2} i$$

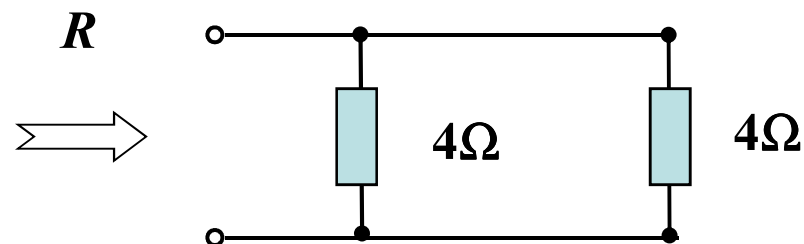
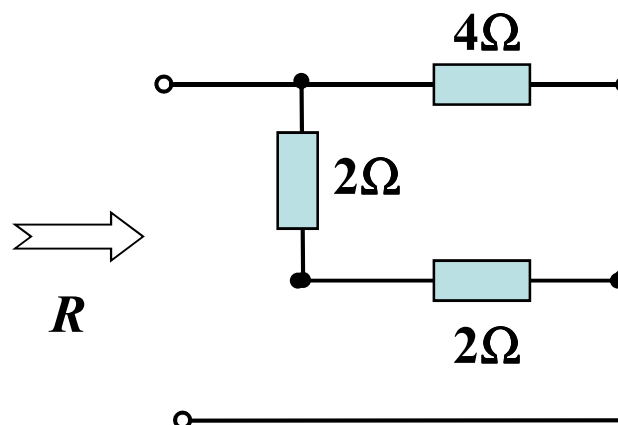
线性电阻元件的混联

要求：弄清楚串、并联的概念。交替运用串并联等效电阻计算指定端口的等效电阻。

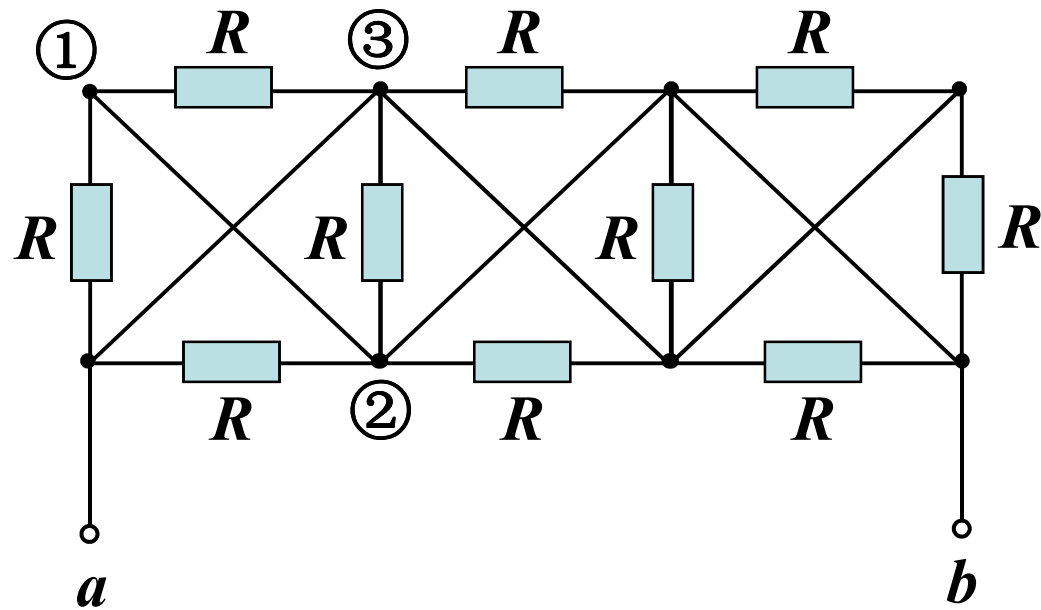
例1.



$$R = 2\Omega$$



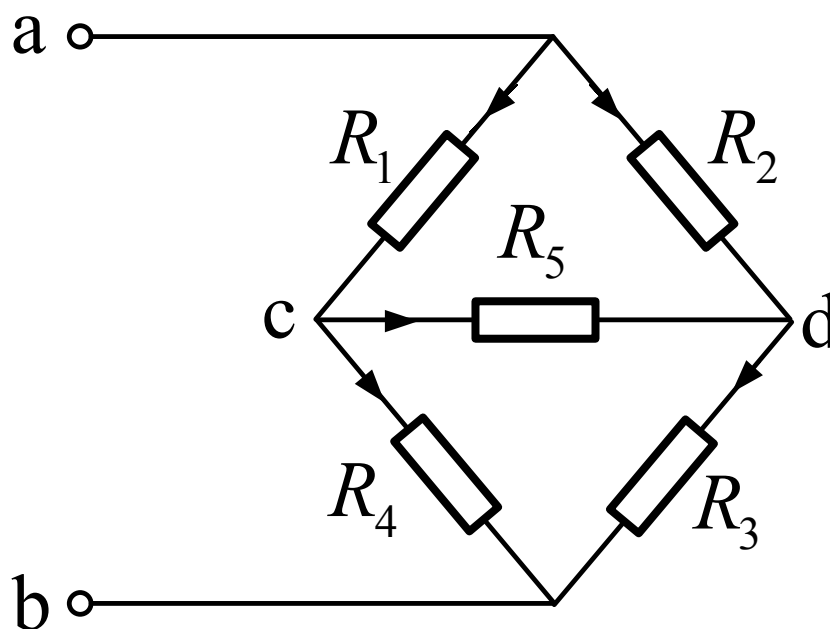
例2.



$$R_{ab} = 0.1R$$

2.3 星形与三角形电路等效变换

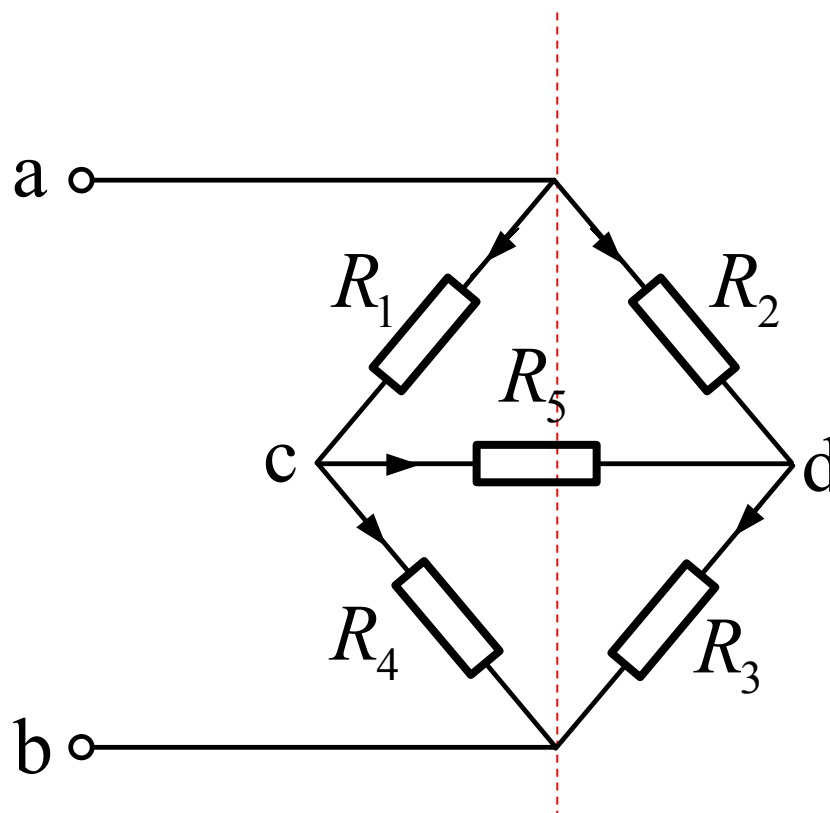
电桥电路



上图为电桥电路，电阻 R_1 、 R_2 、 R_3 、 R_4 称为电桥的“桥臂”， R_5 支路称为“桥”。

2.3.1 电路对称

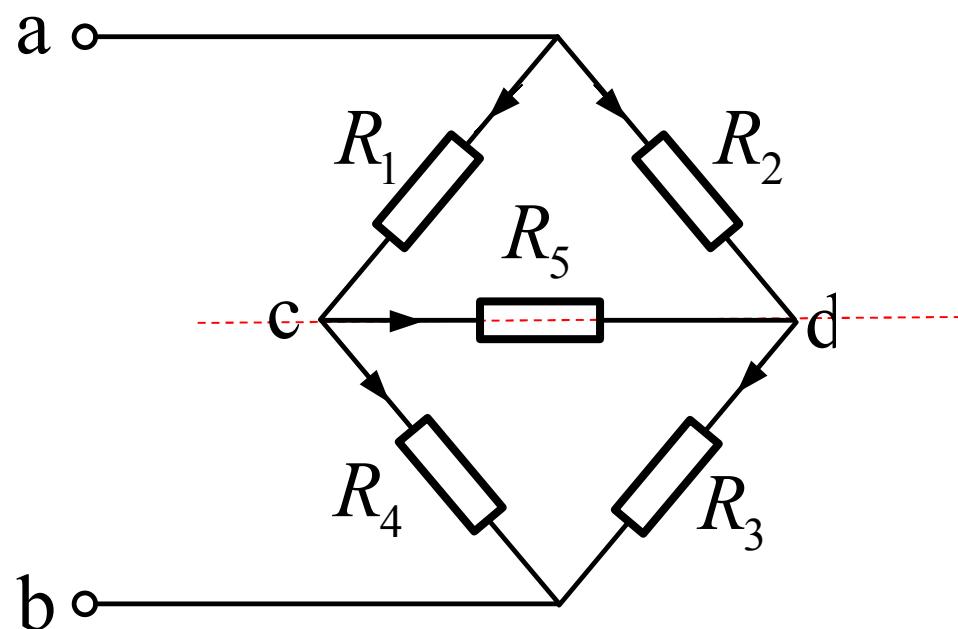
1、对称面通过端口：电阻 $R_1=R_2$ 、 $R_3=R_4$ ，则电路存在对称面，即左右对称。



此时：对称面两侧有相同电流分布，**则垂直通过对称面的支路电流为0，可以断开该支路。** R_5 电流 $i_5=0$ ，可以断开该支路。

2.3.1 电路对称

2、对称面垂直于端口：电阻 $R_1=R_4$ 、 $R_2=R_3$ ，则电路存在对称面，即上下对称。

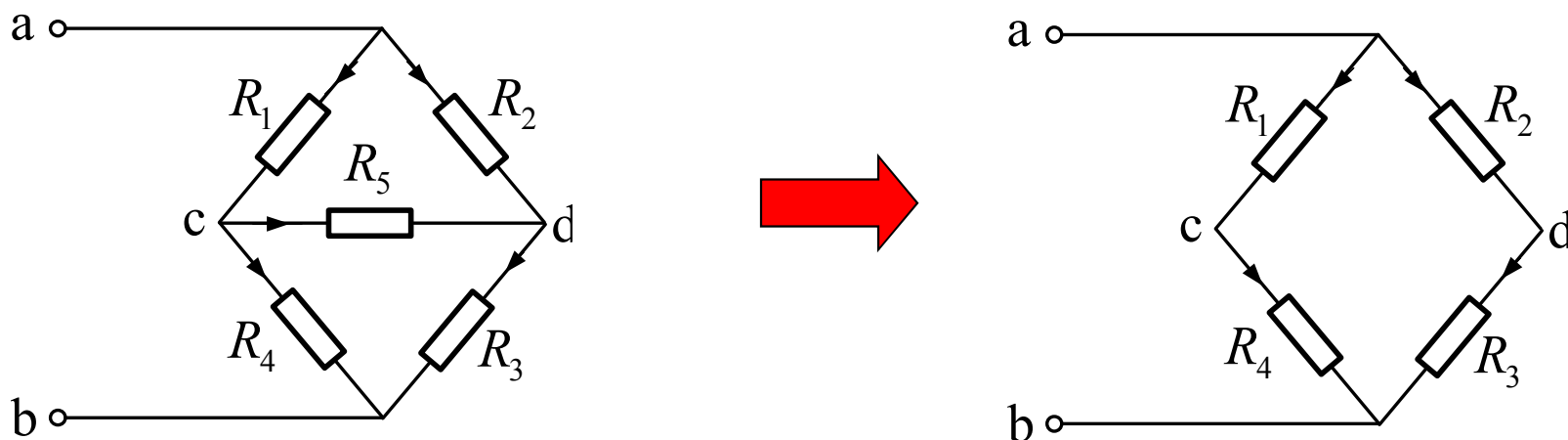


此时：位于对称面上结点ab电位相等，即 $u_{cd}=0$

可以短接结点或断开支路 R_5 。

2.3.2 电桥平衡电路

电桥平衡条件：当电路中的c、d两点为自然等电位点时，此电桥电路称为“平衡电桥电路”。



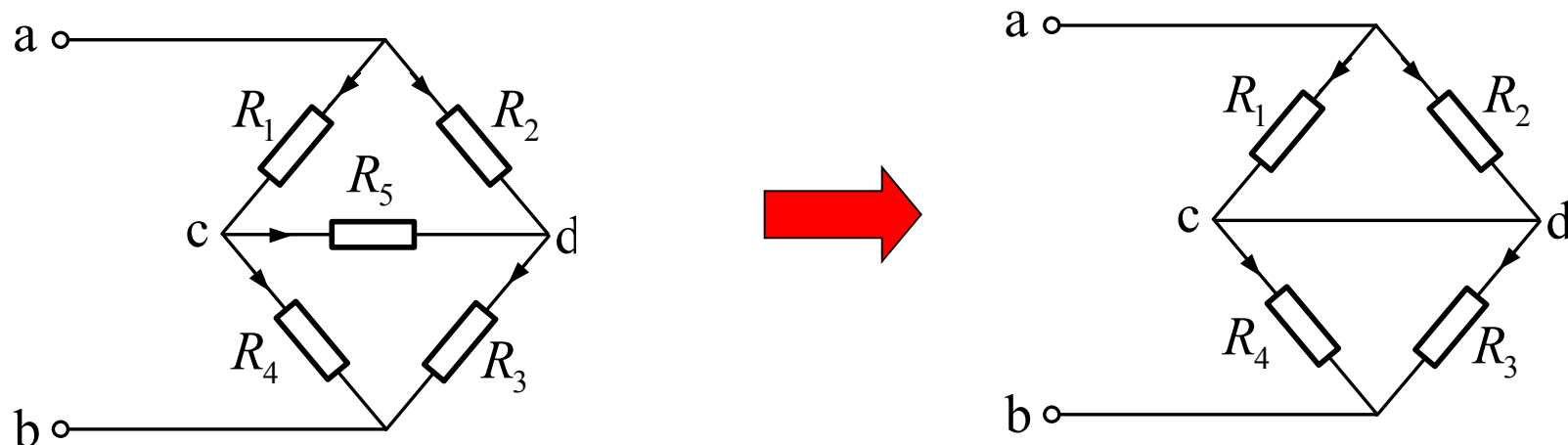
即 $u_{cd}=0$ 则 $i_5=u_{cd}/R_5=0$

电路中桥支路可以用开路代替，如右图所示：

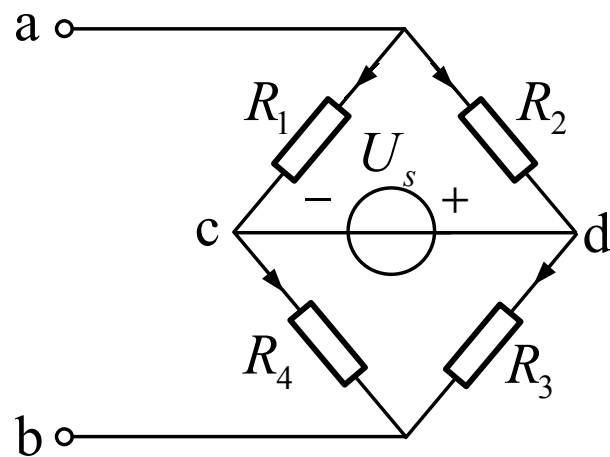
电桥平衡时，应满足的条件为：

$$u_{cd} = \frac{R_4}{R_1 + R_4} U_{ab} - \frac{R_3}{R_2 + R_3} U_{ab} = 0 \quad \Rightarrow \quad R_1 R_3 = R_2 R_4$$

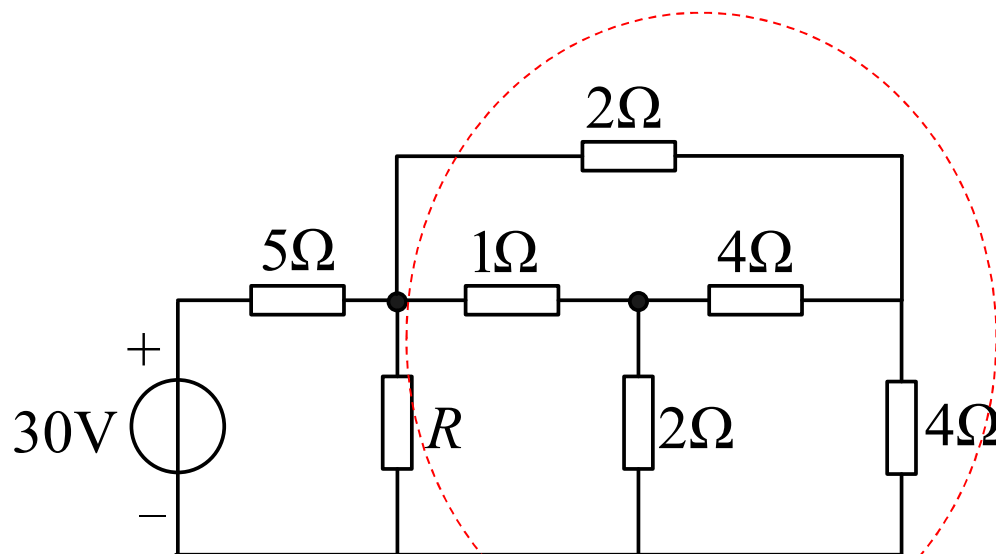
电路中桥臂可以用短路代替：



如果桥臂为“有源支路”，即使满足电桥平衡条件， c 、 d 两点也不是等电位点。

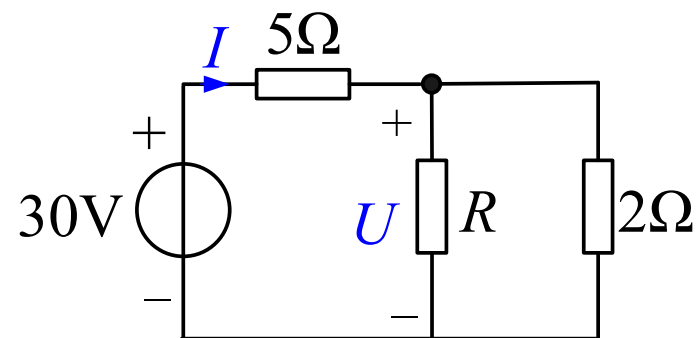
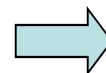
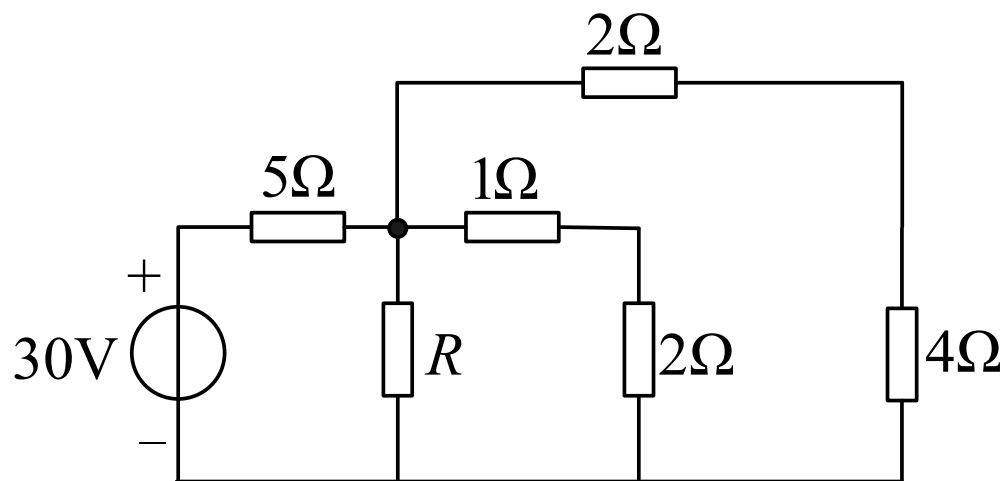


例3：电路消耗的总功率为150W，求R的阻值。

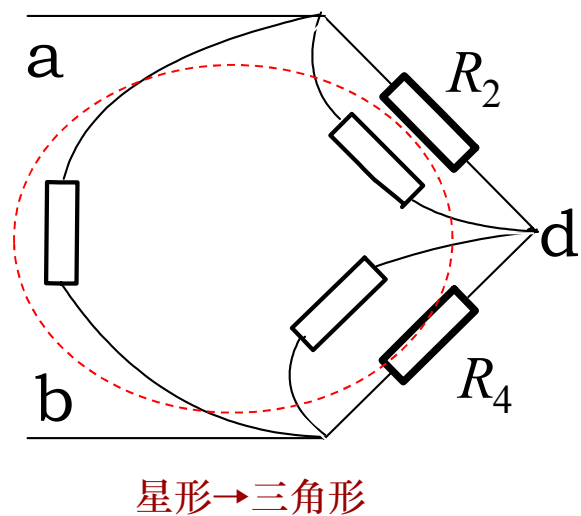
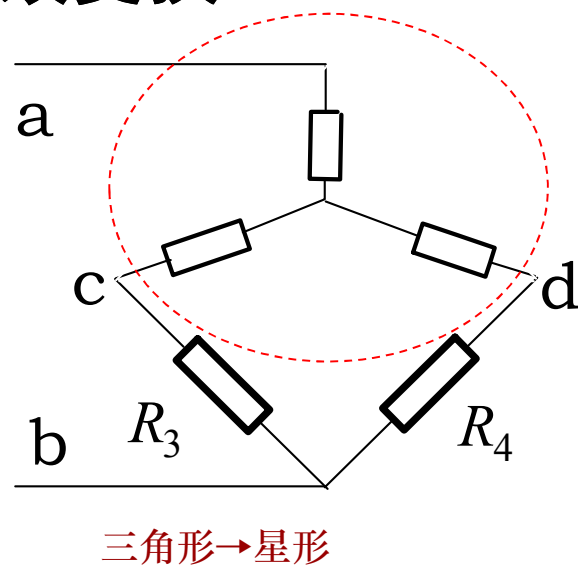
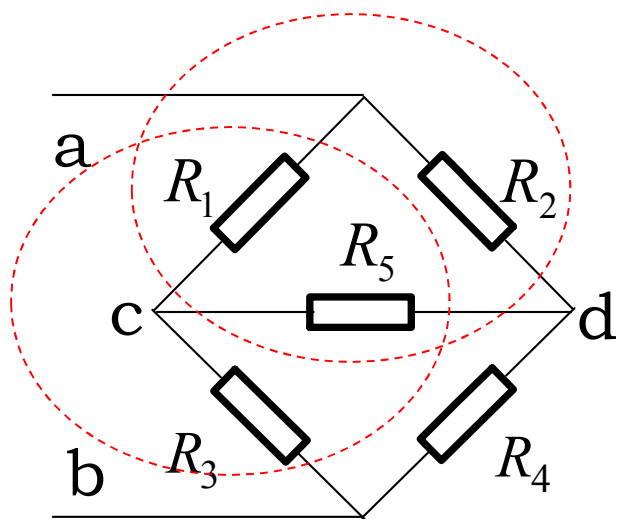


$$I = 5A$$

$$R = 2\Omega$$

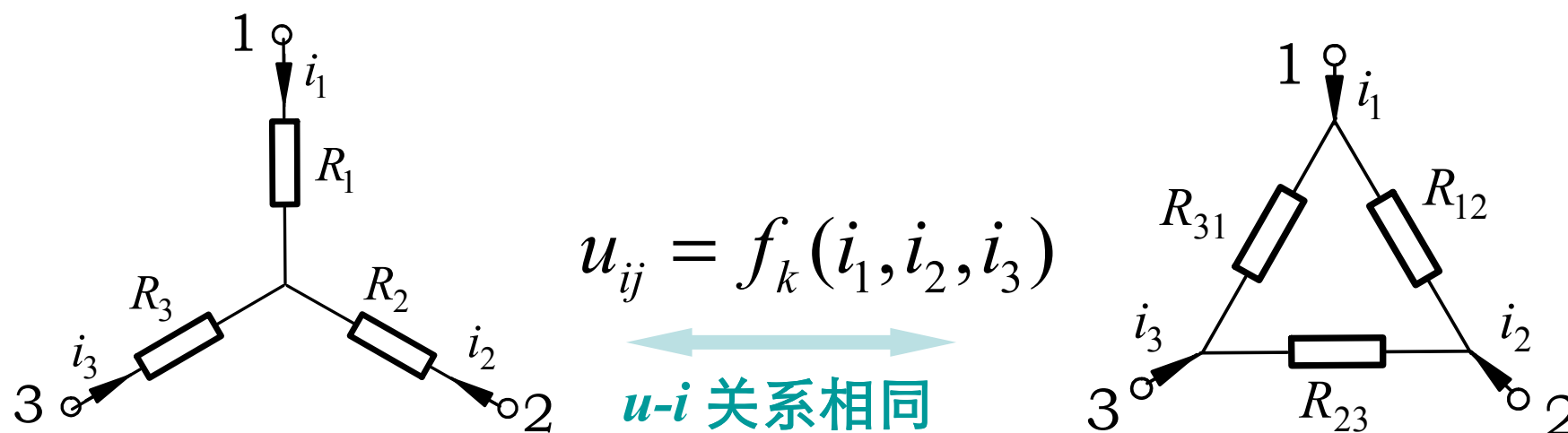


2.3.3 电阻电路的 Δ —Y等效变换



2.3.3 电阻电路的 Δ —Y等效变换

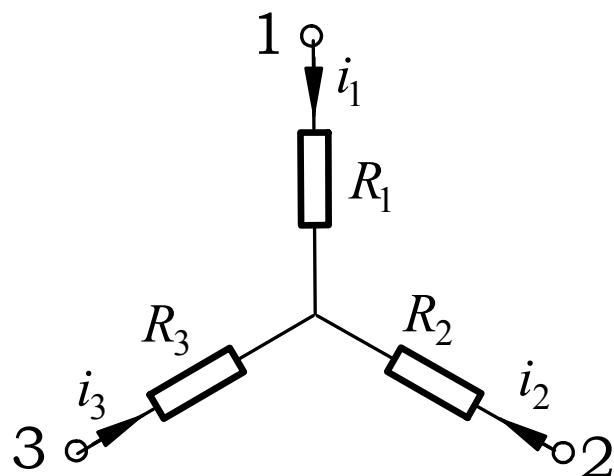
Y、 Δ 电路： 均有三条支路， 且有三个端钮与外部电路相连。



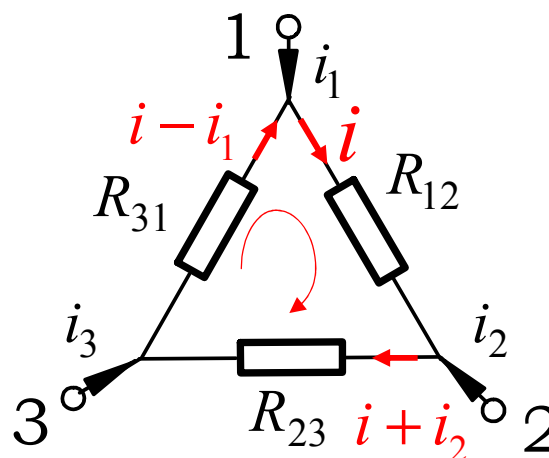
- 三个电阻的一端接在一个结点上，而它们的另一端分别接在三个不同的端钮上，这样的连接方式称为Y形（星形）电阻网络。
- 三个电阻的两端分别接在每两个端钮之间，使三个电阻本身构成回路这样的连接方式称为 Δ 形（三角形）电阻网络。

2.3.3 电阻电路的 Δ —Y等效变换

由KVL确定*i*:



$$u_{12} = R_1 i_1 - R_2 i_2$$



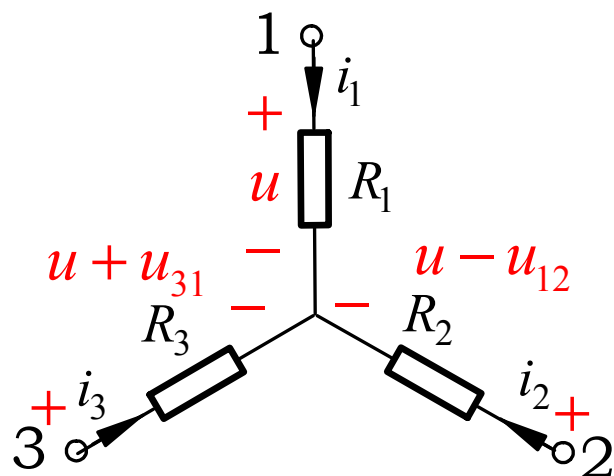
$$u_{12} = R_{12} i = \frac{R_{12} R_{31} i_1 - R_{12} R_{23} i_2}{R_{12} + R_{23} + R_{31}}$$

$$R_{12} i + R_{23} (i + i_2) + R_{31} (i - i_1) = 0$$

$$i = \frac{R_{31} i_1 - R_{23} i_2}{R_{12} + R_{23} + R_{31}}$$

$\Delta \rightarrow Y$	$R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}$	$R_2 = \frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}}$
		$R_3 = \frac{R_{23} R_{31}}{R_{12} + R_{23} + R_{31}}$

2.3.3 电阻电路的 Δ —Y等效变换

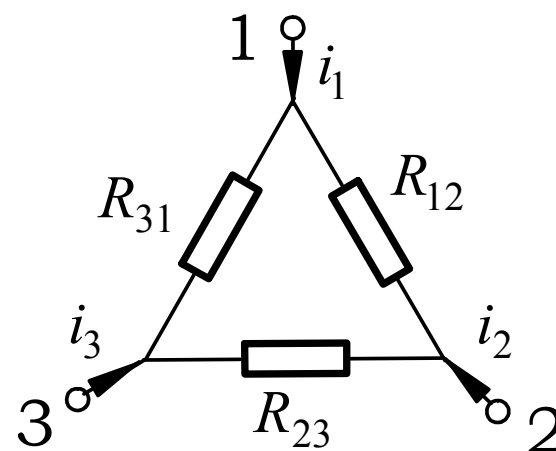


KCL确定 u :

$$G_1 u + G_2 (u - u_{12}) + G_3 (u + u_{31}) = 0$$

$$u = \frac{G_2 u_{12} - G_3 u_{31}}{G_1 + G_2 + G_3}$$

$$i_1 = G_1 u = \frac{G_1 G_2 u_{12} - G_1 G_3 u_{31}}{G_1 + G_2 + G_3}$$



$$i_1 = G_{12} u_{12} - G_{31} u_{31}$$

$Y \rightarrow \Delta$	$G_{12} = \frac{G_1 G_2}{G_1 + G_2 + G_3}$	$G_{31} = \frac{G_1 G_3}{G_1 + G_2 + G_3}$
		$G_{23} = \frac{G_2 G_3}{G_1 + G_2 + G_3}$

2.3.3 电阻电路的 Δ —Y等效变换

$\Delta \rightarrow Y$

$$R_1 = \frac{R_{12}R_{31}}{R_{12} + R_{23} + R_{31}}$$

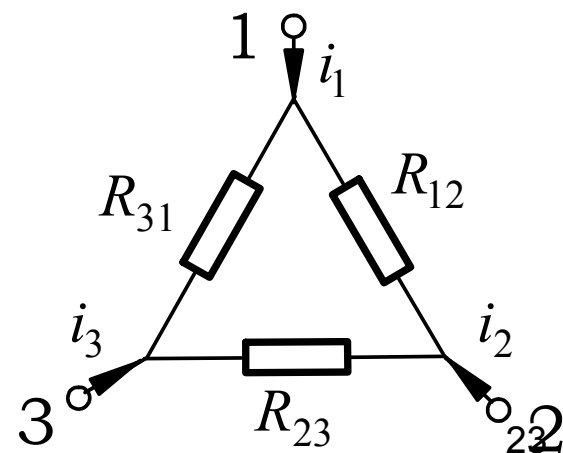
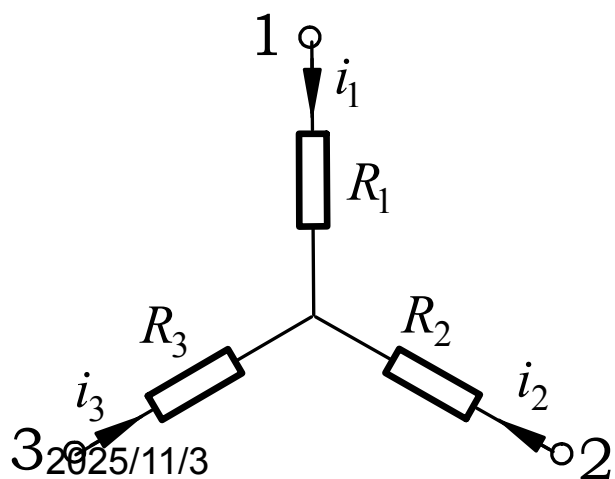
$Y \rightarrow \Delta$

$$G_{12} = \frac{G_1G_2}{G_1 + G_2 + G_3} \quad R_{12} = R_1 + R_2 + \frac{R_1R_2}{R_3}$$

规律：

Y的电阻 = $\frac{\Delta \text{相邻电阻之积}}{\Delta \text{电阻之和}}$

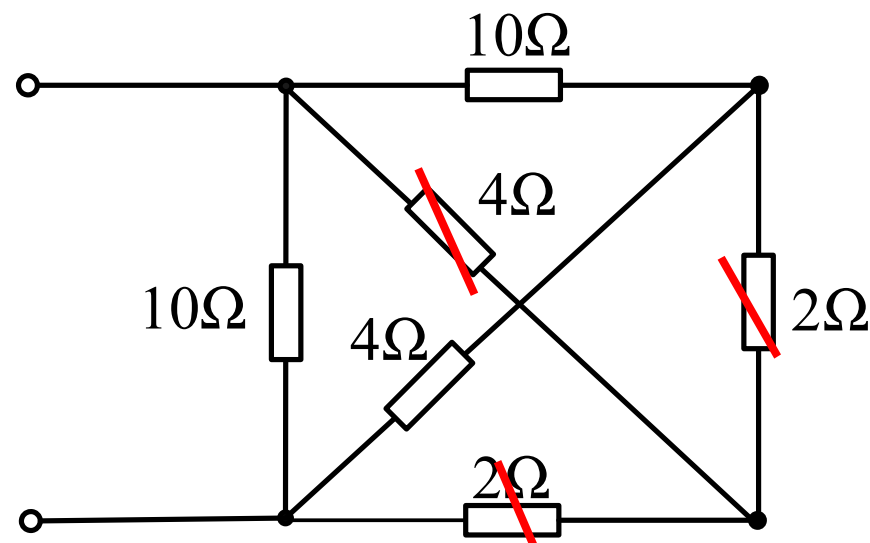
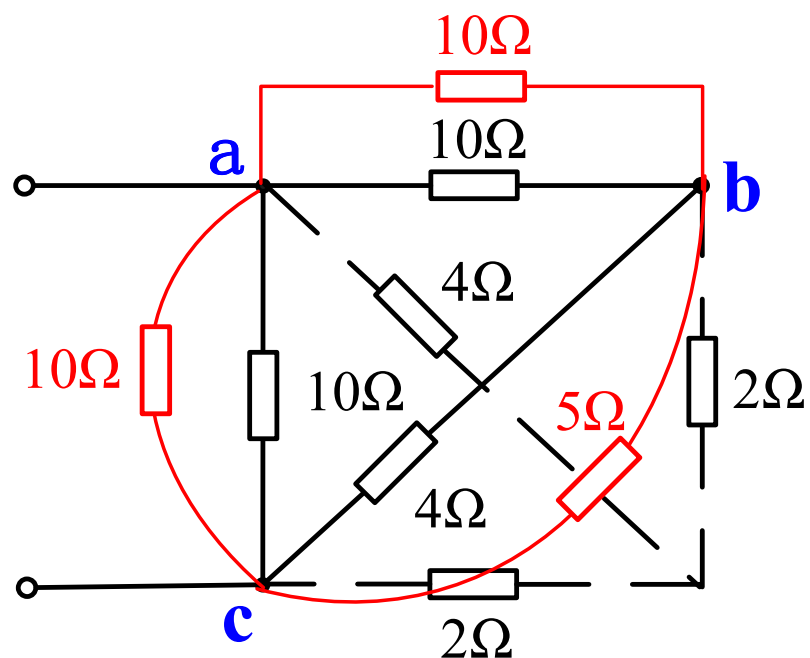
Δ 的电导 = $\frac{Y \text{相邻电导之积}}{Y \text{电导之和}}$



例4：求输入端等效电阻。

➤ $\Delta \rightarrow Y$: 4

➤ $Y \rightarrow \Delta$: 2



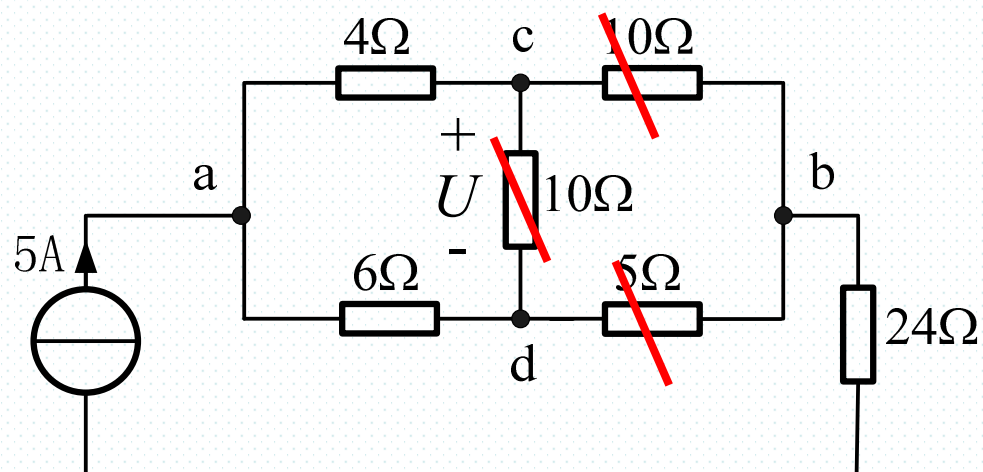
$$R_{ab} = 4 + 2 + \frac{4 \times 2}{2} = 10\Omega$$

$$R_{ac} = 4 + 2 + \frac{4 \times 2}{2} = 10\Omega$$

$$R_{bc} = 2 + 2 + \frac{2 \times 2}{4} = 5\Omega$$

$$R_{eq} = 10 // 10 // [10 // 10 + 4 // 5]$$

例5. 求 U .

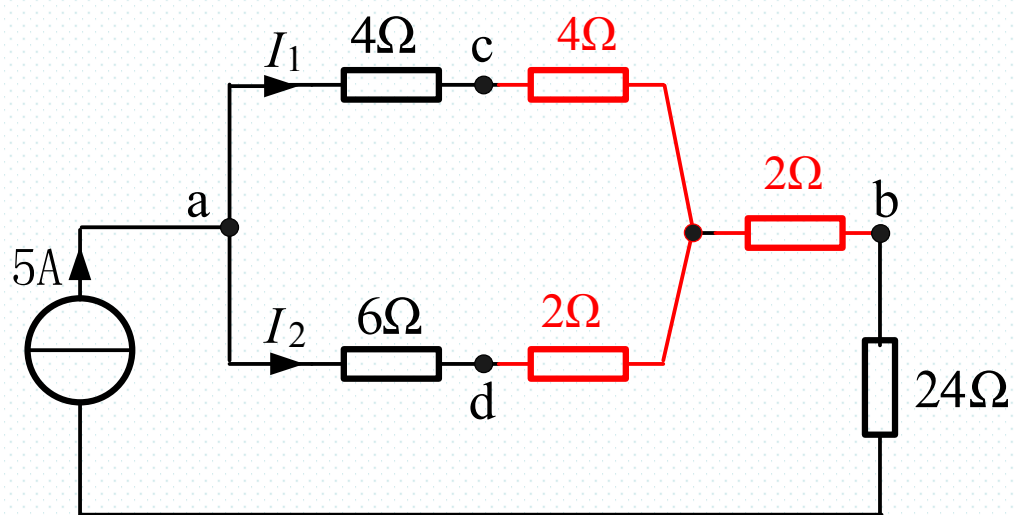


$$I_1 = I_2 = \frac{5}{2} \text{ A}$$

(Current division)

$$U = 4I_1 - 2I_2 = 5\text{V}$$

(KVL)



$$R_c = \frac{10 \times 10}{10 + 10 + 5} = 4\Omega$$

$$R_d = \frac{10 \times 5}{10 + 10 + 5} = 2\Omega$$

$$R_b = \frac{10 \times 5}{10 + 10 + 5} = 2\Omega$$

课下练习1：求入端等效电阻 R_{ab}

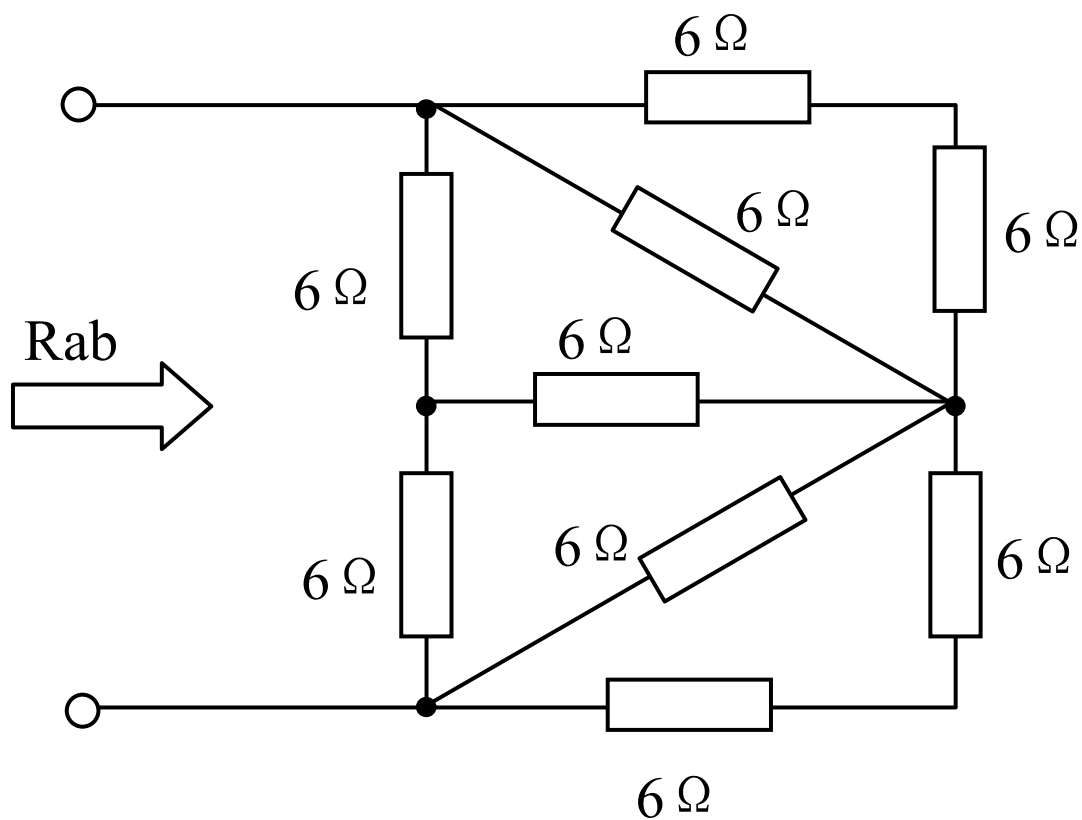
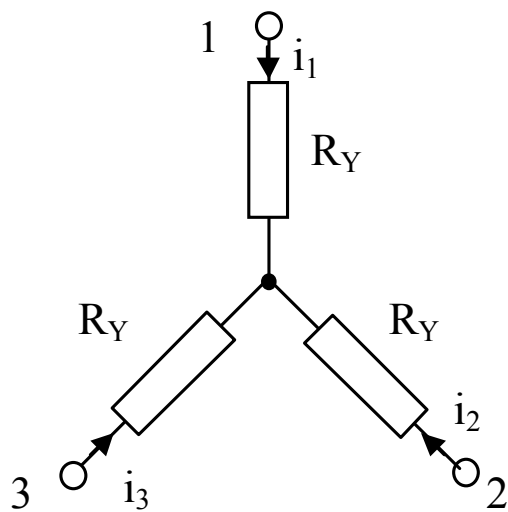


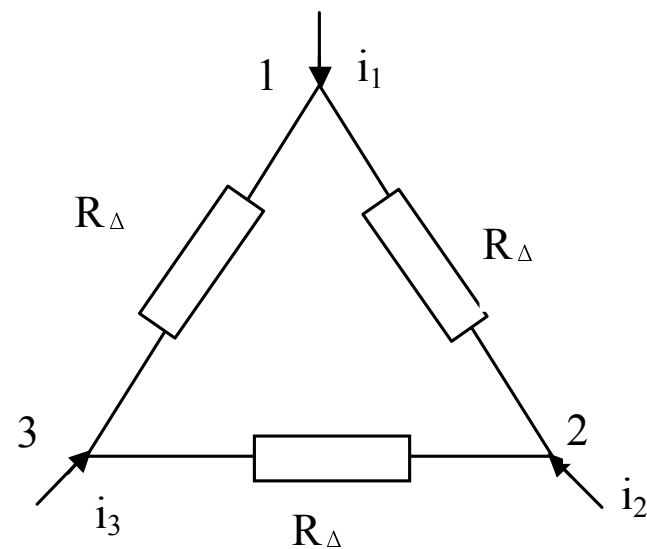
图 1

$$R=4.8\Omega$$

对称 Δ —Y联接电路的等效变换公式：



(a)



(b)

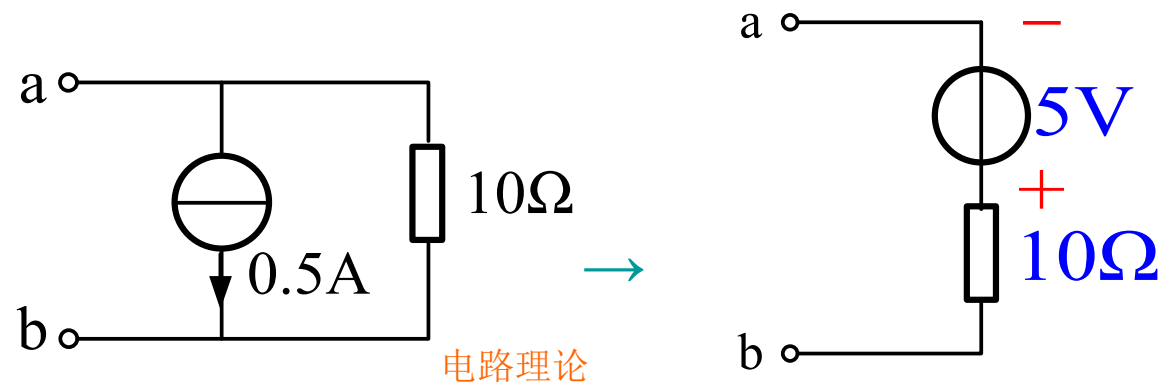
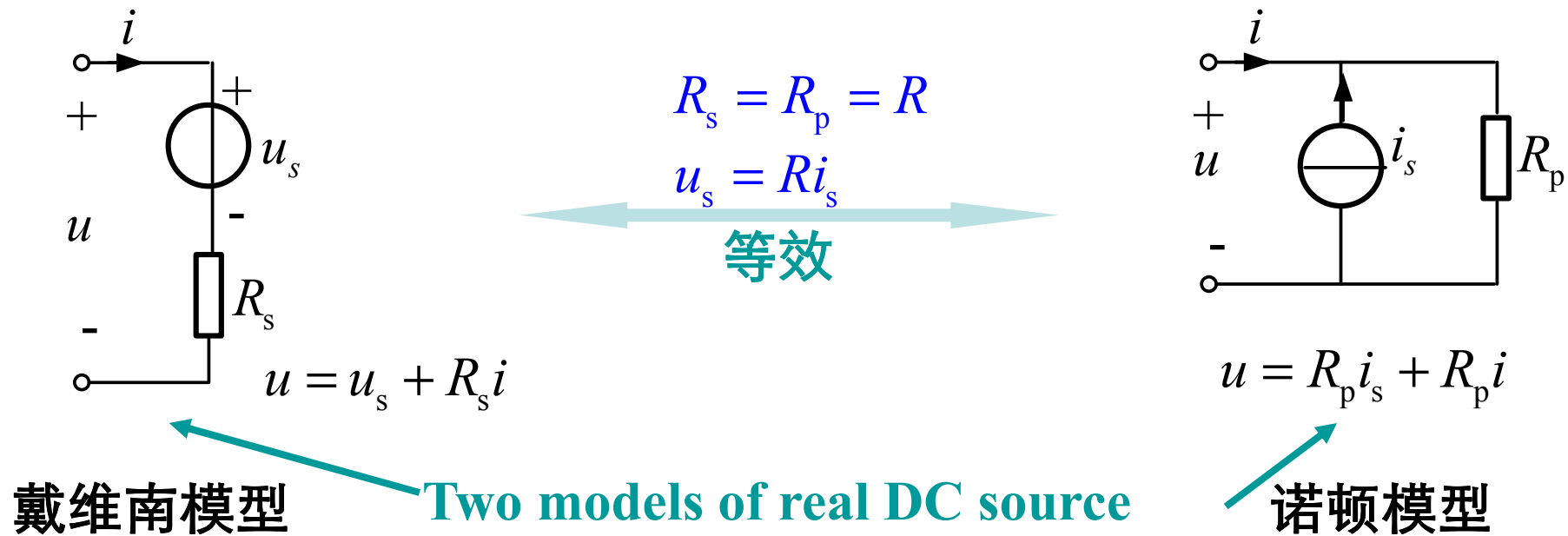
$$R_{\Delta} = R_Y + R_Y + R_Y \quad R_Y / R_Y = 3R_Y$$

或

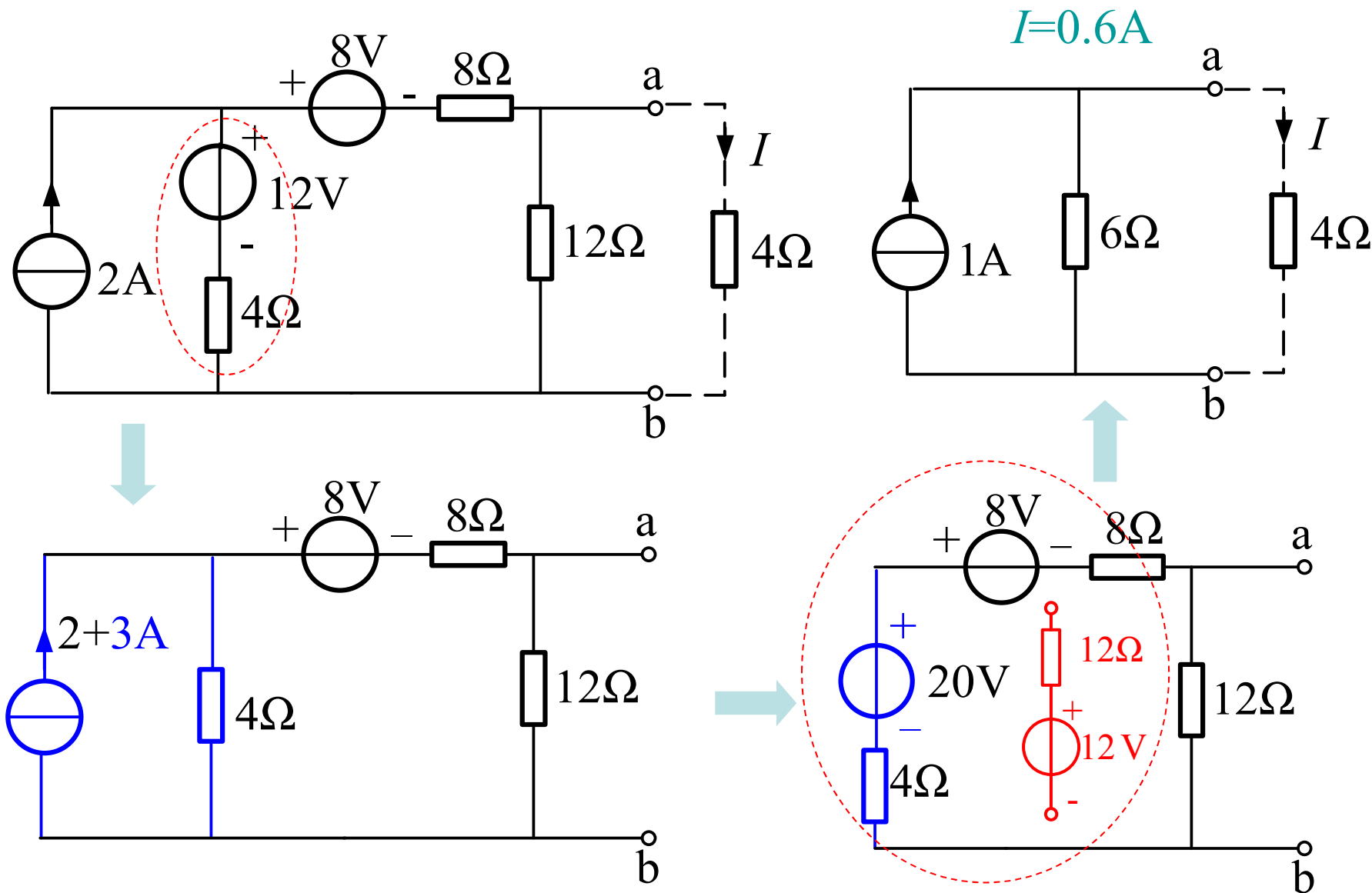
$$R_Y = R_{\Delta} / 3$$

2.4 电源变换Source transformation

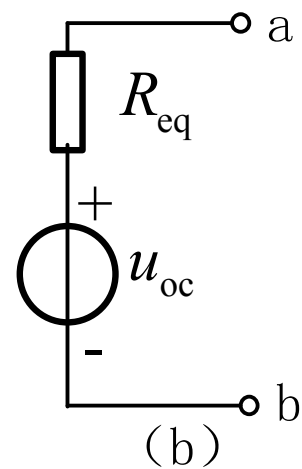
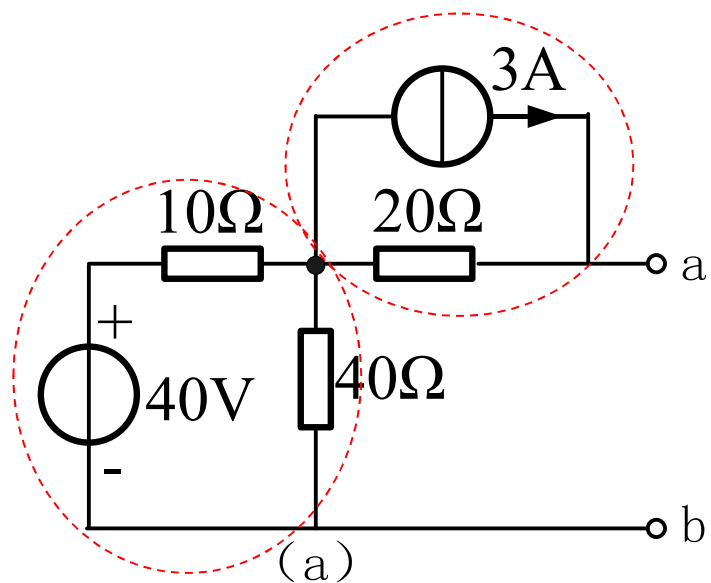
2.4.1 独立电源变换



例6.计算电流 I .



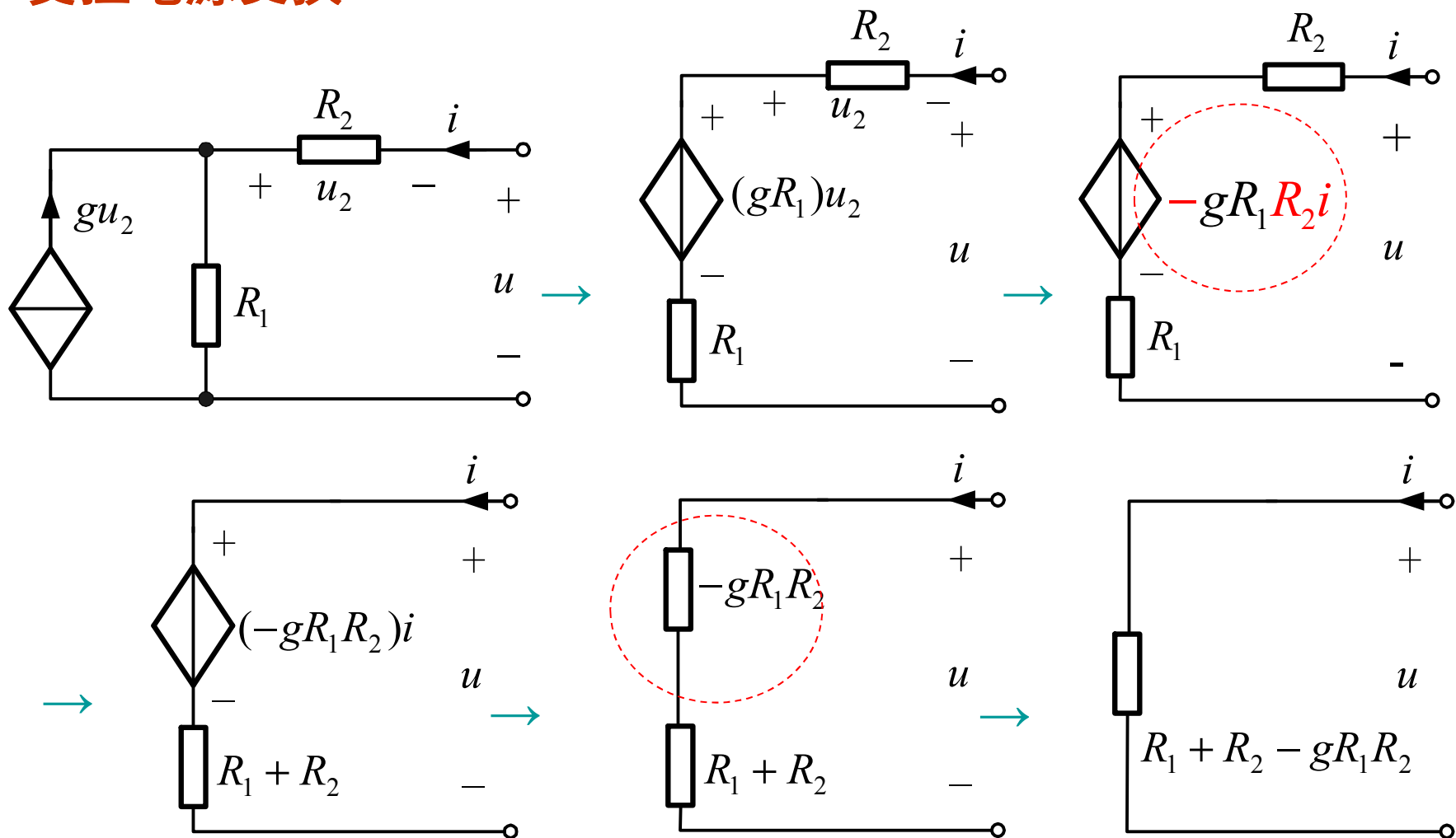
练习：求图 (a) 等效电路图 (b) 的参数



答案： 28Ω ， $92V$

2.4 电源变换Source transformation

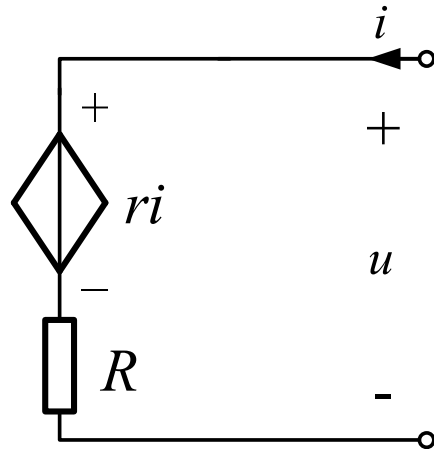
受控电源变换



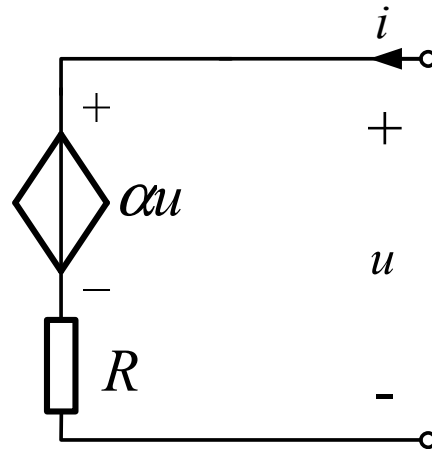
方法2:(1)图中 $u-i$ 关系: $u = R_2i + R_1(gu_2 + i) = (R_1 + R_2 - gR_1R_2)i$

2.4 电源变换Source transformation

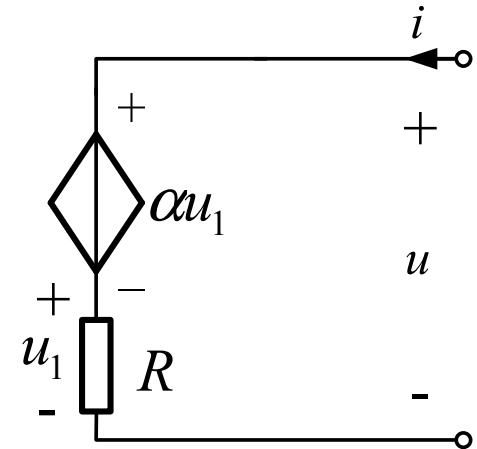
例7 计算等效电阻。



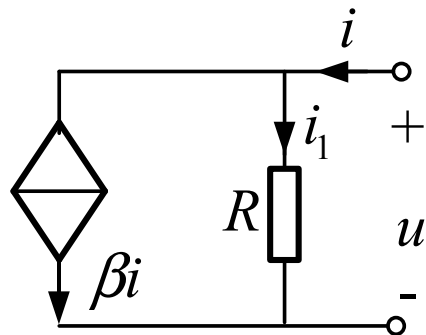
$$R_{\text{eq}} = R + r$$



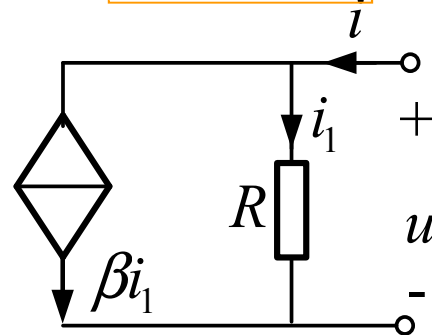
$$R_{\text{eq}} = \frac{R}{1 - \alpha}$$



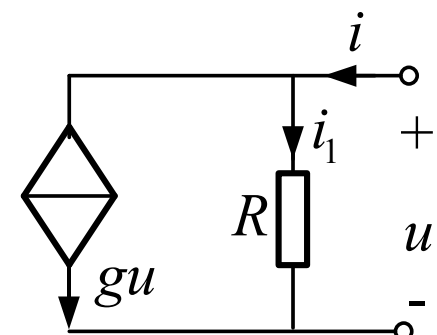
$$R_{\text{eq}} = (1 + \alpha)R$$



$$R_{\text{eq}} = (1 - \beta)R$$



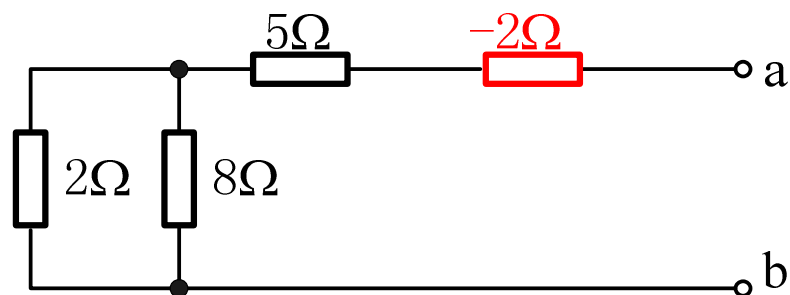
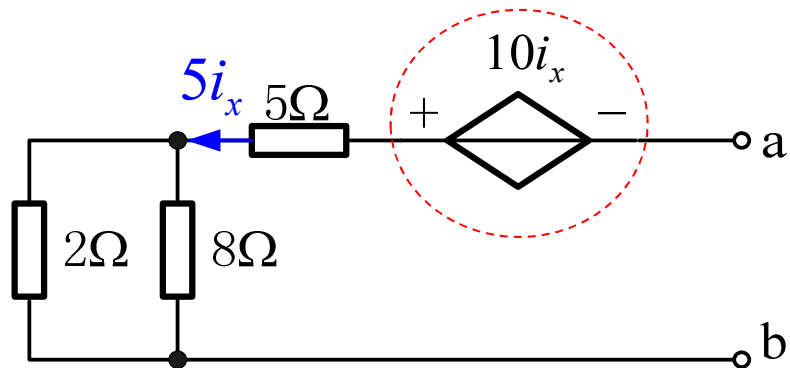
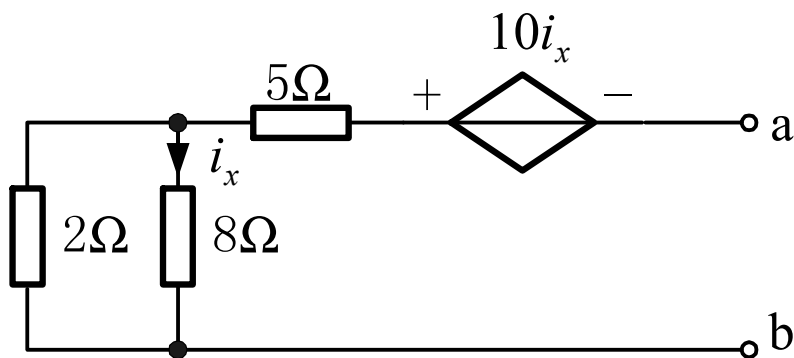
$$R_{\text{eq}} = \frac{R}{1 + \beta}$$



$$R_{\text{eq}} = \frac{R}{1 + gR}$$

2.4 电源变换Source transformation

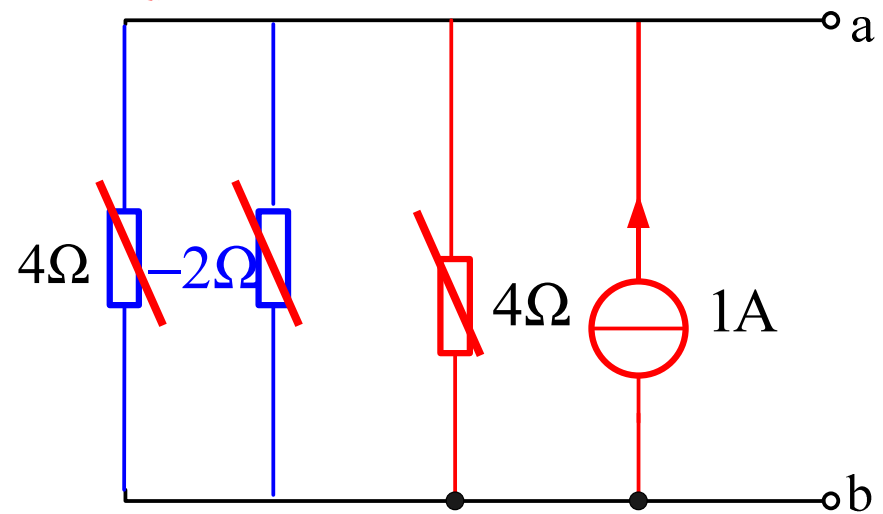
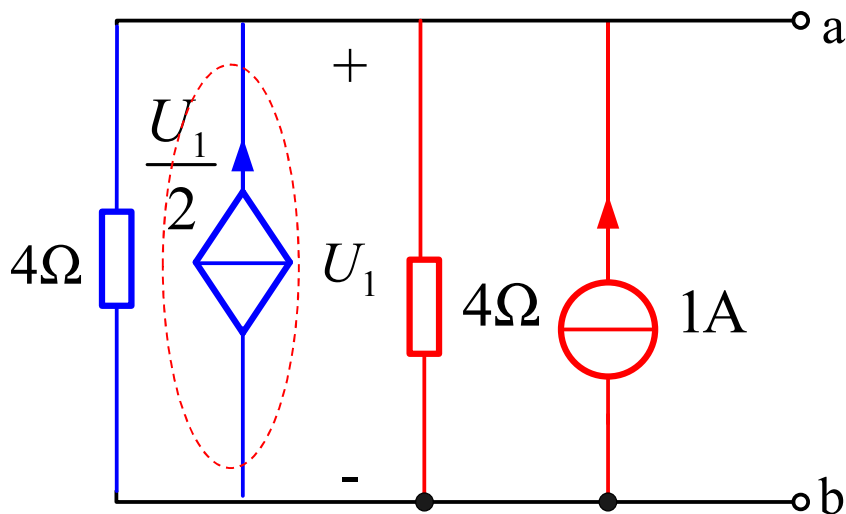
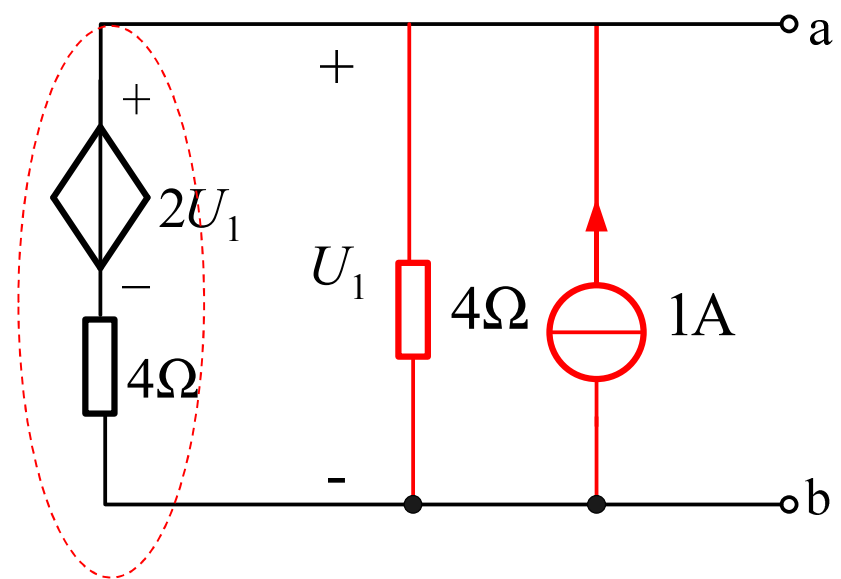
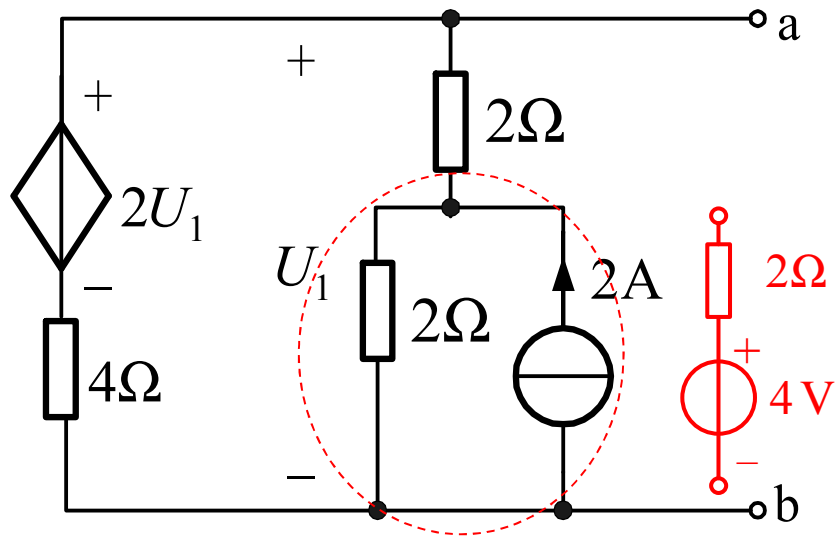
例8.计算端口等效电阻。



$$R_{ab} = 4.6\Omega$$

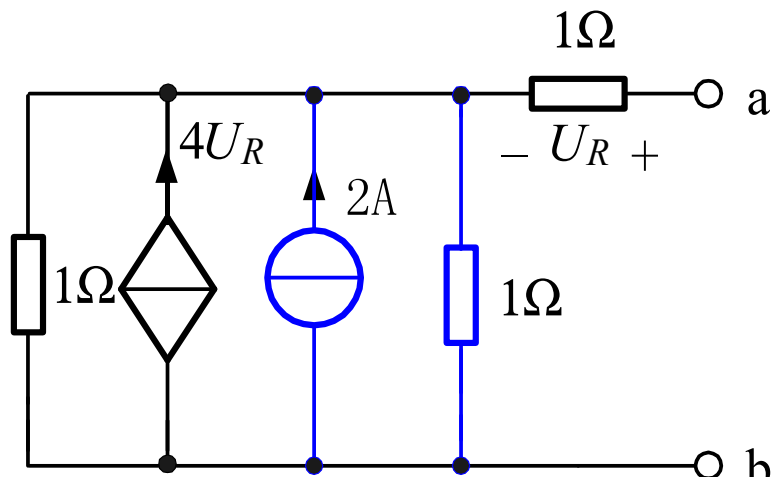
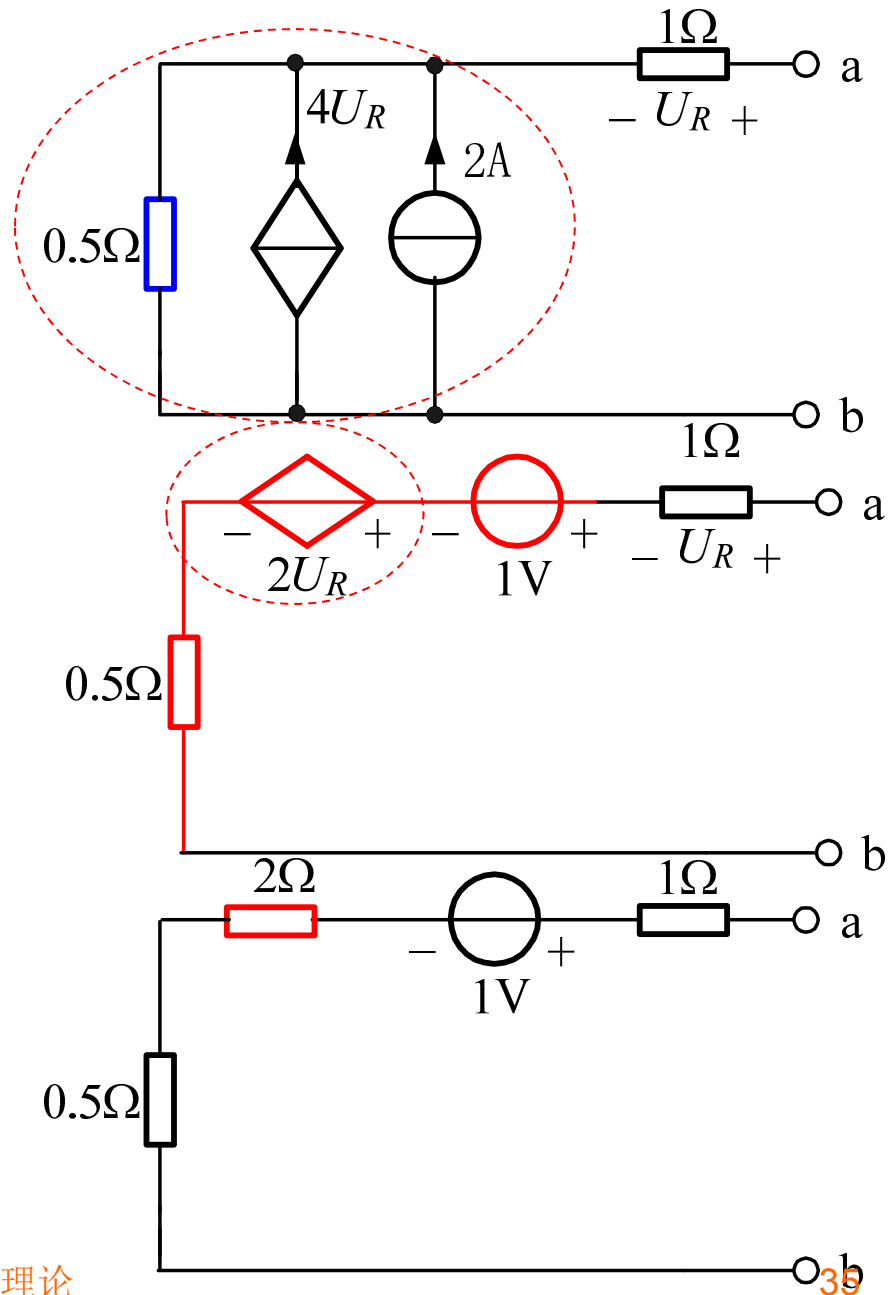
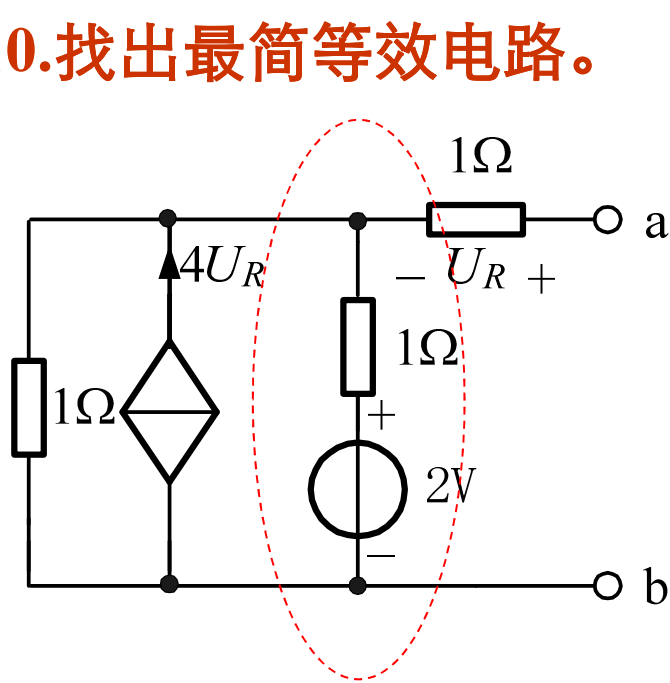
2.4 电源变换Source transformation

例9.找出最简等效电路。

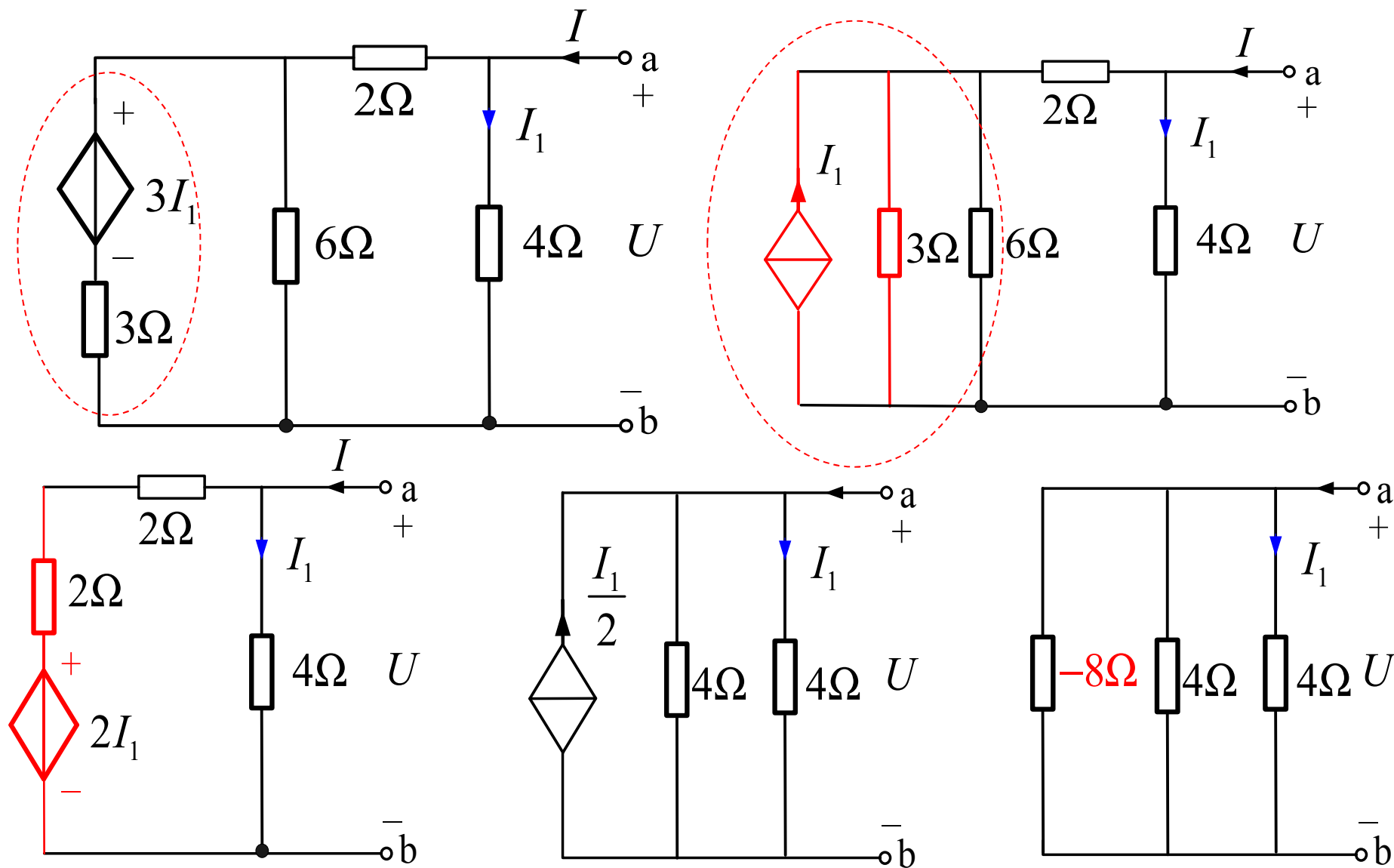


2.4 电源变换Source transformation

例10.找出最简等效电路。



练习.计算端口等效电阻。



$$R_{eq} = 4 // 4 // (-8) = \frac{8}{3} \Omega$$

计划学时：3学时；课后学习9学时

作业：

2-12, 2-14 /串并联

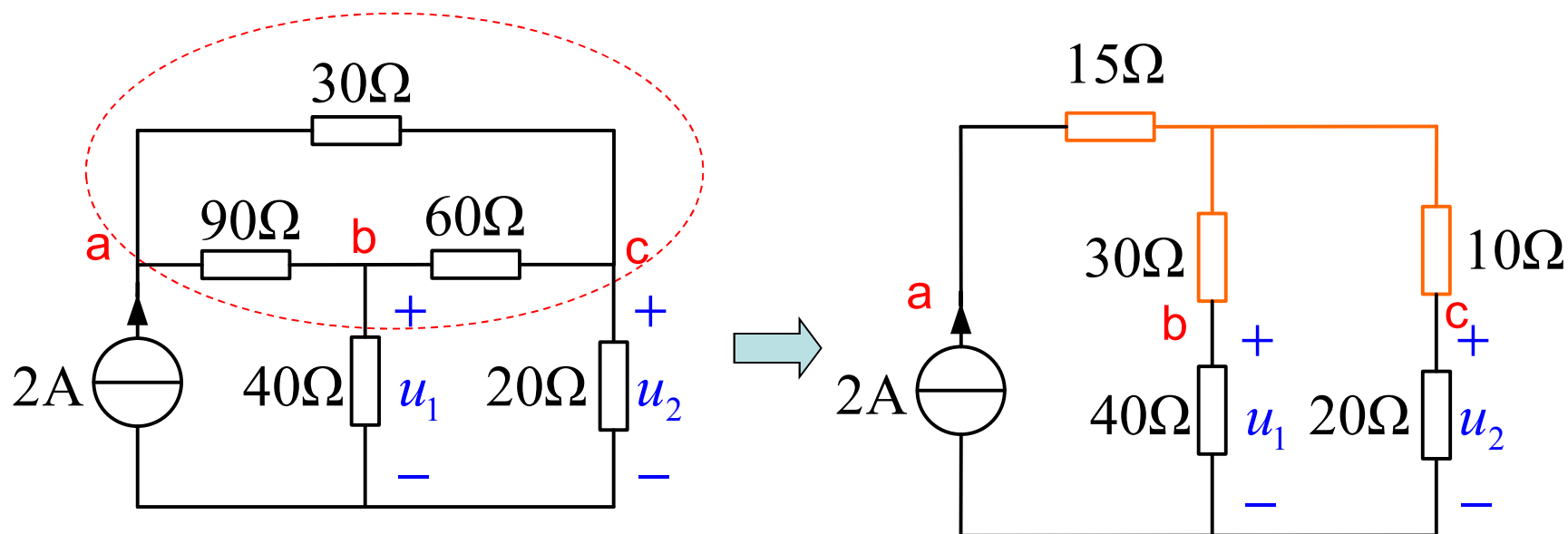
2-16, 2-20, 2-24 平衡电桥 星三角变换

2-26 /独立电源变换

2-32 / 受控电源变换

2-36 /综合分析

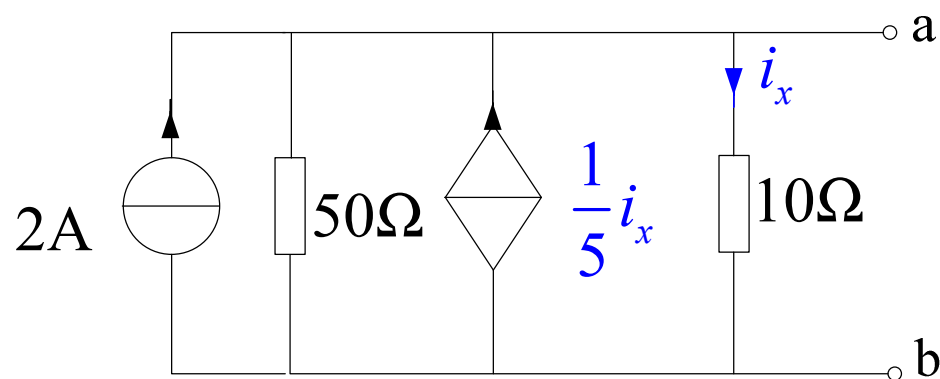
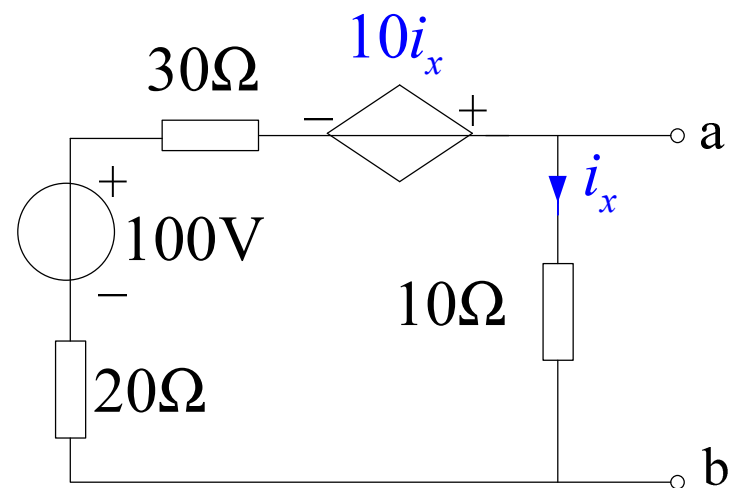
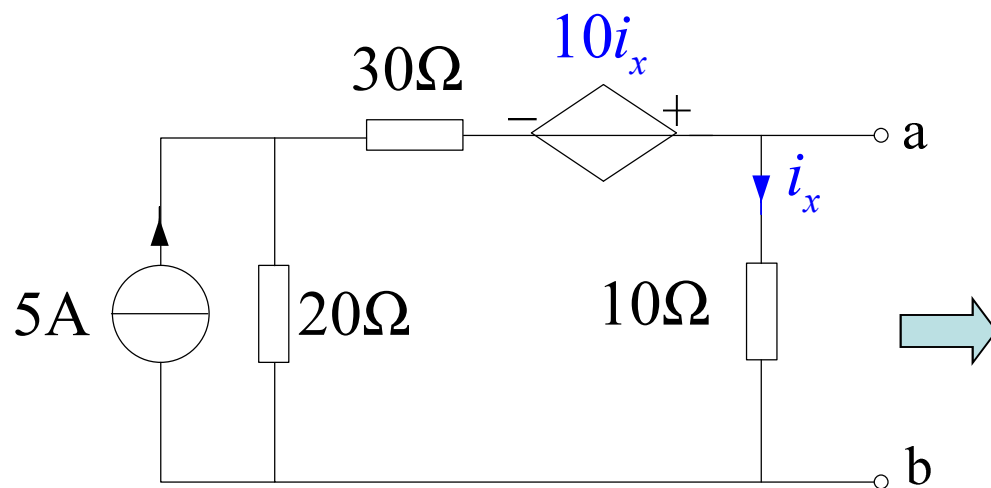
2-24：确定电路中电压 u_1 、 u_2 。



$$u_1 = \frac{30}{70 + 30} \times 2 \times 40 = 24\text{V}$$

$$u_2 = \frac{70}{70 + 30} \times 2 \times 20 = 28\text{V}$$

2-32：确定最简单等效电路。



$$R = 50 // (-50) // 10 = 10\Omega$$