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Event-triggered interval observer design for a class of Euler-Lagrange systems with disturbances

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Abstract

This paper investigates the problem of interval estimation for a class of Euler-Lagrange systems with unknown disturbances. An event-triggered mechanism is applied in the design of interval observers, and unnecessary data communication burden is reduced. The sufficient conditions are derived by both the positive system theory and the Lyapunov stability theory. Moreover, the gains of observer are determined by solving a set of inequalities in the sufficient conditions. At last, the validity of the presented event-triggered interval observers is demonstrated by a numerical simulation.

Keywords

Euler-Lagrange systems, interval observers, event-triggered mechanism

Introduction

The Euler-Lagrange systems (ELSs) have a broad engineering background, for instance, humanoid robots, manipulators, and underwater robots. Over the last two decades, the research on ELSs, such as adaptive control (Patre et al., 2010a, 2010b), tracking control (Morabito et al., 2004), sliding mode control (Yang and Kim, 1999; Islam and Liu, 2010) and so on, has achieved considerable results. In addition, there were also many important works on distributed ELSs. The authors in Ren (2009) proposed and analysed distributed leaderless consensus algorithms for ELSs; meanwhile, the limitation of actuator saturation was also considered. Mei et al. (2011) studied a tracking problem for distributed multiple ELSs and presented a distributed protocol design algorithm. Cai and Huang (2015) investigated the consensus problem for uncertain ELSs and designed an adaptive distributed observer. In addition, many interesting works have been reported in Min et al. (2011), Tran et al. (2019) and Dao et al. (2021).

In applications, the states in the system are usually not available, and there also exist unknown but bounded disturbances. In recent years, the investigation on interval observers (IOs) has attracted tremendous attention. Compared with asymptotical observers, the bounds of the system states can be estimated by the IOs. Specially, the IOs can also estimate the systems with unknown disturbances (Gouzé et al., 2000). In general, there are two methods to design IOs, that is, the method of coordinate transformation and the set membership estimation method. By using the first method, Mazenc and Bernard (2011) presented the IOs framework for linear systems with time invariant, and Efimov et al. (2013) extended the results of Mazenc and Bernard (2011) to linear time varying systems, while Raïssi et al. (2010) proposed IOs design method for nonlinear systems. With the help of set membership estimation method, the problem of designing IOs for switched systems was studied in Huang et al. (2019). In addition, Huang et al. (2021) further investigated the functional IOs design for singular switched systems with uncertainty. Very recently, Zhang et al. (2022) addressed the problem of interval estimation for fractional order systems. Some studies on IOs are Mazenc et al. (2022), Wang et al. (2022), Zheng et al. (2016), Yin et al. (2022) and Huang et al. (2022b).

Actually, it is very important for a computer system to limit the sensor or actuator operation or communication to finite instances since the capability of the whole system is limited. To save computation resources, the concept of event-triggered mechanism (ETM) was proposed in Tabuada (2007). Then, it became a hot spot. In Eqtami et al. (2010), the authors designed an ETM for discrete-time systems, whereas a periodic event-triggered control design method for linear systems was presented in Heemels et al. (2012). In the context of event-triggered IO (ETIO), there are some new works such as Li et al. (2020), Huong et al. (2021) and Huang et al. (2022a). Li et al. (2020) designed an ETIO with an improved ETM for the systems under cyber attacks. The authors in Huong et al. (2021) studied a functional ETIO for linear systems. An event-triggered estimation approach is

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proposed for cyber–physical systems which contain unknown inputs in Huang et al. (2022a). As far as authors know, the ETIO design problem for ELSs has not been reported yet.

Based on the above discussion, an ETIO framework is formulated for ELSs in this paper. Both the positive system theory and Lyapunov stability theory are used to formulate sufficient conditions for the existence of ETIO. The contribution of this paper mainly lies in two aspects: (a) In this paper, the design of IO with ETM is applied to ELSs for the first time; (a) a nonlinear IO is presented to estimate the system states with the Lipschitz condition, and the bounds information of the states is recovered. The structure of the paper is as follows. The system model and preliminary knowledge are suggested in the 'Preliminaries and problem statement' section. In the 'Main results' section, a design method of the ETIO is given. A simulation of a two degrees of freedom (2-DOF) manipulator model in MATLAB is conducted to illustrate the effectiveness of the ETIO in the 'Numerical example' section. The 'Conclusion' section is the conclusion of this paper.

Notations

$x \geqslant 0$ means that the values of all components are greater than or equal to zero. For a matrix $Z \in \mathscr{R}^{m \times n}$, $Z \geqslant 0$ denotes that values of all elements in Z are non-negative, and $Z^+ = max\{Z, 0\}, Z^- = max\{-Z, 0\}$, and $Z^+ = Z^- + Z$. In addition, $Z \succ 0(\prec 0)$ means that Z is a positive (negative) definite matrix.

Preliminaries and problem statement

In general, a second-order ELS dynamics can be described as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = u + \tau_d \tag{1}$$

where $q \in \mathscr{R}^n$ and $u \in \mathscr{R}^n$ denote the state and the vector of input force, $M(q) \in \mathscr{R}^{n \times n}$, $C(q, \dot{q}) \in \mathscr{R}^{n \times n}$ and $G(q) \in \mathscr{R}^n$ mean mass matrix, Coriolis and centrifugal force and gravity. τ_d stands for lumped disturbances and uncertainties existing in the system. In what follows, we employ the following transformation for system (1)

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = h(x_1)u + f(x_1, x_2) + d \end{cases} \tag{2}$$

where $x_1 = q$, $x_2 = \dot{q}$, $h(x_1) = M^{-1}(x_1)$, $d = M^{-1}(x_1)\tau_d$, $f(x_1, x_2) = -M^{-1}(x_1)(C(x_1, x_2)x_2 + G(x_1))$. The joint displacements are measurable, under the above state reconstruction, we can get

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + F(x(t)) + Dd(t), \\ y(t) = Cx(t) \end{cases} \tag{3}$$

where

$$A = \begin{bmatrix} 0_{n \times n} & I_{n \times n} \\ 0_{n \times n} & 0_{n \times n} \end{bmatrix} \in \mathscr{R}^{2n \times 2n}, B = \begin{bmatrix} 0_{n \times n} \\ h(x_1) \end{bmatrix} \in \mathscr{R}^{2n \times n},$$
$$F(x) = \begin{bmatrix} 0_{n \times 1} \\ f(x_1, x_2) \end{bmatrix} \in \mathscr{R}^{2n},$$
$$D = \begin{bmatrix} 0_{n \times n} \\ I_{n \times n} \end{bmatrix} \in \mathscr{R}^{2n \times n}, C = [I_{n \times n} \; 0_{n \times n}] \in \mathscr{R}^{n \times 2n}$$

To save network resources, we proposed an ETM. Let us define $y_{\tau_p}(t)$ as the output under the ETM, and the observer can be received at the t moment. The trigger condition is formulated as

$$e_y^T(t)e_y(t) < \eta y_{\tau_p}^T(t) y_{\tau_p}(t) \tag{4}$$

where $e_y(t) = y_{\tau_p}(t) - y(t)$ and η is the triggering threshold. When equation (4) is violated, the ETM will transmit $y(t)$ to the observer.

Assumption 2.1. *The disturbances in system (1) are bounded, that is,*

$$d^- \leqslant d(t) \leqslant d^+ \tag{5}$$

where d^- and d^+ are constant vectors.

Assumption 2.2. *There is a Lipchitz constant $\alpha > 0$ such that $f(x_1, x_2)$ with respect to x_2 satisfying Lipchitz condition as follows*

$$\| f(x_1, x_2) - f(x_1, x_2 + \Delta) \| \leqslant \alpha \| \Delta \| \tag{6}$$

Lemma 2.1. *(Zhang et al., 2022) If $F(x)$ is a Lipschitz function which is globally differentiable, then there are two increasing Lipschitz functions $g_1(x)$ and $g_2(x)$ such that*

$$F(x) = g_1(x) - g_2(x) \tag{7}$$

Lemma 2.2. *(Zhang et al., 2022) For the function $F(x)$ in Lemma 2.1, there exists a global Lipschitz function $\bar{F}(x_a, x_b)$, such that*

$$\bar{F}(x, x) = F(x)$$

$$\frac{\partial \bar{F}}{\partial x_a} \geqslant 0, \text{ and } \frac{\partial \bar{F}}{\partial x_b} \leqslant 0$$

The above lemmas can help us get the boundaries of nonlinear function $F(x, x)$

$$\bar{F}(x^-, x^+) \leqslant F(x, x) \leqslant \bar{F}(x^+, x^-) \tag{8}$$

Lemma 2.3.*(Zhang et al., 2022) Consider the function $F(x)$ and its associate function $\bar{F}(x_a, x_b)$, if the Jacobian matrix of $F(x)$ is bounded, then*

$$\begin{cases} \bar{F}(x^+, x^-) - F(x, x) \leqslant F_1\bar{e} + F_2\underline{e}, \\ \bar{F}(x^-, x^+) - F(x, x) \leqslant F_3\bar{e} + F_4\underline{e} \end{cases} \tag{9}$$

where F_1, F_2, F_3 and F_4 are constant matrices.

Lemma 2.4. *(Efimov et al., 2013) For a constant matrix $Z \in \mathscr{R}^{n\times n}$, if there exists x satisfing $x^- \leqslant x \leqslant x^+$, then*

$$Z^+x^- - Z^-x^+ \leqslant Zx \leqslant Z^+x^+ - Z^-x^- \tag{10}$$

Lemma 2.5. *(Huang et al., 2019) Consider the system*

$$\begin{cases} \dot{x} = Rx + \mho(x), \\ x(0) = x_0 \geqslant 0 \end{cases} \tag{11}$$

where R is a known constant matrix, and the nonlinear function $\mho(x) > 0$. Then the system (11) is positive if R is a Metzler matrix.

Lemma 2.6. *(Huang et al., 2022a) Given two appropriate dimensions vectors W, Y and a positive constant γ, the following inequality holds*

$$W^TY + Y^TW \leqslant \frac{1}{\gamma}W^TPW + \gamma Y^TP^{-1}Y, \tag{12}$$

where $P \succ 0$.

Definition 2.1. *(Huang et al., 2022a) Under the initial condition $\delta(0) = 0$, the $\mathscr{K}$-class function δ is continuous and monotonically increasing. If function $\varepsilon(\cdot, t)$ is a $\mathscr{K}$*-class function with *$\forall t \geqslant 0$, and $\varepsilon(s, \cdot)$ is monotonically reducing, besides, $\varepsilon(s,t) \to 0$ as $t \to \infty$, then $\varepsilon : R^+ \times Z^+$* is *$\mathscr{K}\mathscr{L}$-class function.*

Definition 2.2. *(Huang et al., 2022a) For a system*

$$\dot{e}(t) = F(e(t), d(t)) \tag{13}$$

where F is a continuous mapping, $e(t) \in \mathscr{R}^n$ and $d(t) \in \mathscr{R}^n$ represent the error and disturbance, respectively. Given initial condition $e(0)$, then the system (13) is input-to-state stable (ISS) with external disturbance $d(t)$, if there exist a $\mathscr{K}$-class function δ and a $\mathscr{K}\mathscr{L}$-class function ε such that the following inequality holds

$$\| e(t) \| < \varepsilon(\| e(0) \|, t) + \delta(\| d[0,t] \|)$$

Main results

Design of the ETIO

Based on the 'Preliminaries and problem statement' section, we design the following ETIO for system (3)

$$\begin{cases} \dot{\hat{x}}^+(t) = A\hat{x}^+(t) + Bu(t) + \bar{F}(x^+(t), x^-(t)) + k^+ \\ \quad + L(y_{\tau_p} - y^+(t)) + \bar{h}_y(t), \\ \dot{\hat{x}}^-(t) = A\hat{x}^-(t) + Bu(t) + \bar{F}(x^-(t), x^+(t)) + k^- \\ \quad + L(y_{\tau_p} - y^-(t)) + \underline{h}_y(t) \end{cases} \tag{14}$$

where $\hat{x}^+(t)$ and $\hat{x}^-(t)$ are the estimated value of $x(t)$, and $k^+ = D^+d^+ - D^-d^-$, $k^- = D^+d^- - D^-d^+$. The gain L will be determined later. Moreover,

$$\begin{cases} \bar{h}_y(t) = L^-e_y^+ - L^+e_y^-, \\ \underline{h}_y(t) = L^-e_y^- - L^+e_y^+ \end{cases} \tag{15}$$

Theorem 3.1. *If $A - LC$ is Metzler, then*

$$\hat{x}^-(t) \leqslant x(t) \leqslant \hat{x}^+(t) \tag{16}$$

holds for $\forall t \geqslant 0$ under the condition $\hat{x}^-(0) \leqslant x(0) \leqslant \hat{x}^+(0)$.

Proof. Consider the event-triggered condition equation (4), and define $e_x^+ = \hat{x}^+ - x$ and $e_x^- = x - \hat{x}^-$, then the error systems are derived as below

$$\begin{cases} \dot{e}_x^+ = (A - LC)e_x^+ + \bar{F}(x^+, x^-) - F(x) + e_d^+ + Le_y + \bar{h}_y, \\ \dot{e}_x^- = (A - LC)e_x^- + F(x) - \bar{F}(x^-, x^+) + e_d^- - \underline{h}_y - Le_y \end{cases} \tag{17}$$

where $e_d^+ = k^+ - Dd$ and $e_d^- = Dd - k^-$.

According to Lemma 2.4, we have $e_d^+ \geqslant 0$ and $e_d^- \geqslant 0$; moreover

$$\begin{cases} Le_y + \bar{h}_y = Le_y - (L^+e_y^- - L^-e_y^+) \geqslant 0, \\ -\underline{h}_y - Le_y = (L^+e_y^+ - L^-e_y^-) - Le_y \geqslant 0 \end{cases} \tag{18}$$

If the initial conditions equation (16) are satisfied, and $A - LC$ is Metzler, then Lemma 2.5 can guarantee that system (17) is positive, that is

$$\hat{x}^-(t) \leqslant x(t) \leqslant \hat{x}^+(t), t \geqslant 0$$

Therefore, we have proved Theorem 3.1.

Transformation of coordinates

If $A - LC$ is not Metzler, we employ the coordinate transformation method to design the ETIO. An equivalent system is obtained by the transformation $w(t) = Mx(t)$; we define $\bar{A} = MAM^{-1}$, $\bar{B} = MB$, $\bar{C} = CM^{-1}$, $\bar{D} = MD$, then system (3) can be written as

$$\begin{cases} \dot{w}(t) = \bar{A}w(t) + \bar{B}u(t) + MF(M^{-1}w(t)) + \bar{D}d(t), \\ y = \bar{C}w(t) \end{cases} \tag{19}$$

To estimate the bounds of the nonlinear function $F(x)$, we define that

$$\begin{cases} \varphi(w^+, w^-) = \bar{F}(M_a^+w^+ - M_a^-w^-, M_a^+w^- - M_a^-w^+), \\ \varphi(w^-, w^+) = \bar{F}(M_a^+w^- - M_a^-w^+, M_a^+w^+ - M_a^-w^-) \end{cases} \tag{20}$$

where $w^+(t)$ and $w^-(t)$ represent the estimated value of $w(t)$, and $M_a = M^{-1}$.

Under Lemma 2.3 and Lemma 2.4, the following inequality can be obtained

$$\begin{aligned} \bar{F}_w(w^-, w^+) = M^+\varphi(w^-, w^+) - M^-\varphi(w^+, w^-) \leqslant M\bar{F}(T^{-1}w) \\ \leqslant M^+\varphi(w^+, w^-) - M^-\varphi(w^-, w^+) = \bar{F}_w(w^+, w^-) \end{aligned}$$

An ETIO based on coordinate transformation is given as follows

$$\begin{cases} \dot{\hat{w}}^+(t) = \overline{A}\hat{w}^+(t) + \overline{B}u(t) + \overline{F}_w(w^+(t), w^-(t)) + \pi^+ \\ \quad + \overline{h}_y(t) + L(y_{\tau_p} - y^+(t)), \\ \dot{\hat{w}}^-(t) = \overline{A}\hat{w}^-(t) + \overline{B}u(t) + \overline{F}_w(w^-(t), w^+(t)) + \pi^- \\ \quad + \underline{h}_y(t) + L(y_{\tau_p} - y^-(t)) \end{cases} \tag{21}$$

and

$$\begin{cases} \pi^+ = \overline{D}^+ d^+ - \overline{D}^- d^-, \\ \pi^- = \overline{D}^+ d^- - \overline{D}^- d^+ \end{cases} \tag{22}$$

Remark 3.1. *Since the matrix T is nonsingular, we can also get the initial conditions after the coordinate transformation*

$$\hat{w}^-(0) \leqslant w(0) \leqslant \hat{w}^+(0) \tag{23}$$

where

$$\begin{cases} \hat{w}^+(0) = M^+\hat{x}^-(0) - M^-\hat{x}^+(0), \\ \hat{w}^-(0) = M^+\hat{x}^+(0) - M^-\hat{x}^-(0) \end{cases} \tag{24}$$

Denote that $e_w^+(t) = \hat{w}^+(t) - w(t)$ and $e_w^-(t) = w(t) - \hat{w}^-(t)$. Thus, the error dynamics is derived as

$$\begin{cases} \dot{e}_w^+(t) = (\overline{A} - L\overline{C})e_w^+ + \overline{F}_w(w^+, w^-) - M\overline{F}(M^{-1}w) \\ \quad + e_D^+ + Ly_{\tau_p}(t) + \overline{h}_y, \\ \dot{e}_w^-(t) = (\overline{A} - L\overline{C})e_w^- + M\overline{F}(M^{-1}w) - \overline{F}_w(w^-, w^+) \\ \quad + e_D^- + Ly_{\tau_p}(t) + \underline{h}_y \end{cases} \tag{25}$$

where $e_D^+ = \pi^+ - \overline{D}d$ and $e_D^- = \overline{D}d - \pi^-$.

Theorem 3.2. *If $\overline{A} - L\overline{C}$ is Metzler, then*

$$\hat{w}^-(t) \leqslant w(t) \leqslant \hat{w}^+(t) \tag{26}$$

holds $\forall t \geqslant 0$ under $\hat{w}^-(0) \leqslant w(0) \leqslant \hat{w}^+(0)$.

Proof. It can be obtained that the error system (25) is positive if $\overline{A} - L\overline{C}$ is Metzler, then by equations (21)–(23), Remark 3.1 and Lemma 2.5, we can get the conclusion of Theorem 3.2. The proof is similar to Theorem 3.1, so we omit the detailed proof here.

Next, we consider the case where $A - LC$ is not Metzler in the sequel, which relaxes the restriction of the proposed design approach.

Denote $\xi(t) = [e_w^{+T}\ e_w^{-T}\ w^T]^T$, according to equations (19) and (25), we have

$$\dot{\zeta}(t) = \quad G\zeta(t) + \hat{B}u + \hat{\Phi} + \hat{d} + \breve{L}\breve{e}_y + \breve{L}^+ e_{y1} + \breve{L}^- e_{y2} \tag{27}$$

where

$$G = \breve{A} - \breve{L}\breve{C}, \breve{A} = diag\{\overline{A}, \overline{A}, \overline{A}\},$$

$$\breve{L} = diag\{L, L, L\}, \breve{C} = diag\{\overline{C}, \overline{C}, 0\},$$

$$\breve{B} = \begin{bmatrix} 0 \\ 0 \\ \overline{B} \end{bmatrix}, \breve{\Phi} = \begin{bmatrix} \bar{F}_w(w^+, w^-) - M\bar{F}(M^{-1}w) \\ M\bar{F}(M^{-1}w) - \bar{F}_w(w^-, w^+) \\ M\bar{F}(M^{-1}w) \end{bmatrix}, \breve{d}$$
$$= \begin{bmatrix} \overline{D}^+ d^+ - \overline{D}^- d^- \\ \overline{D}^+ d^- - \overline{D}^- d^+ \\ \overline{D}d \end{bmatrix},$$
$$\breve{e}_y = \begin{bmatrix} e_y \\ -e_y \\ 0 \end{bmatrix}, e_{y1} = \begin{bmatrix} -e_y^- \\ e_y^+ \\ 0 \end{bmatrix}, e_{y2} = \begin{bmatrix} e_y^+ \\ -e_y^- \\ 0 \end{bmatrix}$$

Theorem 3.3. *Let λ and $\gamma_i (i = 1, 2, ..., 6)$ be positive constants, if Assumption 2.1 and Assumption 2.2 hold, and there is a positive definite and symmetric matrix P, such that*

$$\begin{bmatrix} \Sigma_1 P + \Sigma_2 & \sqrt{\frac{1}{\gamma_3}}\mathscr{A}^T & \sqrt{\frac{\eta}{\gamma_4}}\Theta \\ * & -P^{-1} & 0 \\ * & * & -P^{-1} \end{bmatrix} < 0 \tag{28}$$

where

$$\Sigma_1 = \gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + \gamma_6 + \lambda,$$
$$\Sigma_2 = G^T P + PG, \Theta = \hat{C}^T \breve{L}^T$$

$$\mathscr{A} = \begin{bmatrix} F_1 & F_2 & 0 \\ F_3 & F_4 & 0 \\ 0 & 0 & I \end{bmatrix}$$

Then system (25) is ISS, that is, the upper and lower errors are ultimately bounded.

Proof. Choosing the Lyapunov function $V(t) = \zeta^T(t)P\zeta(t)$, then we derive from $V(t)$ that

$$\begin{aligned} \dot{V}(t) = & [G\zeta(t) + \breve{\Phi} + \breve{d} + \breve{B}u + \breve{L}e_y + \breve{L}^+ e_{y1} + \breve{L}^- e_{y2}]^T P\zeta(t) \\ & + \zeta^T(t)P[G\zeta(t) + \breve{\Phi} + \breve{d} + \breve{B}u + \breve{L}e_y + \breve{L}^+ e_{y1} + \breve{L}^- e_{y2}] \\ = & \zeta^T(t)(G^T P + PG)\zeta(t) + \breve{\Phi}^T P\zeta(t) + \zeta^T(t)P\breve{\Phi} + \breve{d}^T P\zeta(t) + \zeta^T(t)P\breve{d} \\ & + (\breve{B}u)^T P\zeta(t) + \zeta^T(t)P\breve{B}u + (\breve{L}\breve{e}_y)^T P\zeta(t) + \zeta^T(t)P\breve{L}\breve{e}_y \\ & + (\breve{L}^+ e_{y1})^T P\zeta(t) + \zeta^T(t)P\breve{L}^+ e_{y1} + (\breve{L}^- e_{y2})^T P\zeta(t) + \zeta^T(t)P\breve{L}^- e_{y2} \end{aligned}$$

Define that

$$\begin{aligned} & M_1(t) = \zeta^T(t)P\breve{d}, M_2(t) = \zeta^T(t)P\breve{B}u, M_3(t) = \zeta^T(t)P\breve{\Phi}, \\ & M_4(t) = \zeta^T(t)P\breve{L}\breve{e}_y, M_5(t) = \zeta^T(t)P\breve{L}^+ e_{y1}, M_6(t) = \zeta^T(t)P\breve{L}^- e_{y2} \end{aligned} \tag{29}$$

It is derived from Lemma 2.6 that

$$\begin{aligned} & M_1(t) + M_1^T(t) \leqslant \gamma_1\zeta^T(t)P\zeta + \tfrac{1}{\gamma_1}\breve{d}^T P\breve{d}, M_2(t) \\ & + M_2^T(t) \leqslant \gamma_2\zeta^T(t)P\zeta + \tfrac{1}{\gamma_2}(\breve{B}u)^T P\breve{B}u, \\ & M_3(t) + M_3^T(t) \leqslant \gamma_3\zeta^T(t)P\zeta + \tfrac{1}{\gamma_3}(\breve{\Phi})^T P\breve{\Phi}, M_4(t) + M_4^T(t) \\ & \quad \leqslant \gamma_4\zeta^T(t)P\zeta + \tfrac{1}{\gamma_4}(\breve{L}\breve{e}_y)^T P\breve{L}\breve{e}_y, \\ & M_5(t) + M_5^T(t) \leqslant \gamma_5\zeta^T(t)P\zeta + \tfrac{1}{\gamma_5}(\breve{L}^+ e_{y1})^T P\breve{L}^+ e_{y1}, \\ & M_6(t) + M_6^T(t) \leqslant \gamma_6\zeta^T(t)P\zeta + \tfrac{1}{\gamma_6}(\breve{L}^- e_{y2})^T P\breve{L}^- e_{y2} \end{aligned} \tag{30}$$

Thus,

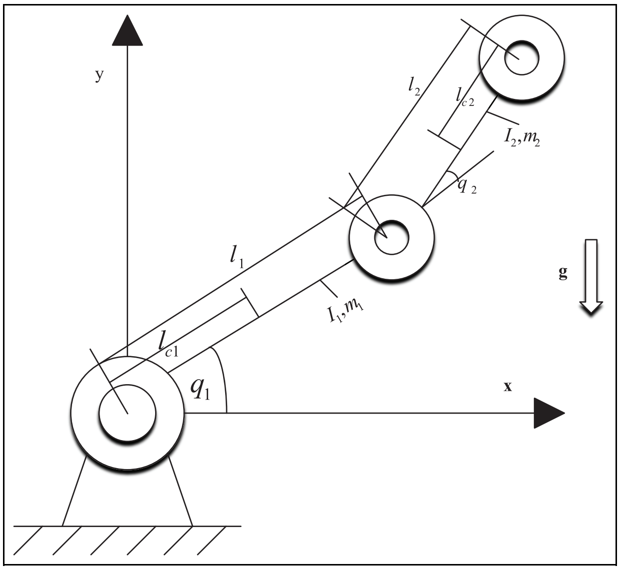

Figure 1. A 2-DOF robot manipulator model.

$$\dot{V}(t) \leqslant \zeta^T(t)[G^TP+PG+(\gamma_1+\gamma_2+\gamma_3+\gamma_4+\gamma_5+\gamma_6)P]\zeta(t) + \frac{1}{\gamma_1}\breve{d}^TP\breve{d} + \frac{1}{\gamma_2}(\breve{B}u)^TP\breve{B}u + \frac{1}{\gamma_3}\breve{\Phi}^TP\breve{\Phi} + \frac{1}{\gamma_4}(\breve{L}e_y)^TP\breve{L}e_y + \frac{1}{\gamma_5}(\breve{L}^+e_{y1})^TP\breve{L}^+e_{y1} + \frac{1}{\gamma_6}(\breve{L}^-e_{y2})^TP\breve{L}^-e_{y2} \tag{31}$$

Since equation (4) could be recast as

$$e_y^T(t)e_y(t) < \eta y^T(t)y(t) = \eta\zeta^T(t)\hat{C}^T\hat{C}\zeta(t) \tag{32}$$

then

$$\breve{e}_y{}^T(t)\breve{e}_y{}^T(t) < 2\eta\zeta^T(t)\hat{C}^T\hat{C}\zeta(t) \tag{33}$$

where $\hat{C} = diag\{0,0,\overline{C}\}$. Moreover, from Lemma 2.3, we can deduce that

$$\breve{\Phi}^TP\breve{\Phi} \leqslant \zeta^T(t)\mathscr{A}^TP\mathscr{A}\zeta \tag{34}$$

Substituting equations (33) and (34) into equation (31) and defining

$$\xi(t) = [e_{y1}^T \;\; e_{y2}^T \;\; u^T \;\; \breve{d}^T]^T$$

then

$$\dot{V}(t) \leqslant \zeta^T(t)[G^TP + PG + (\gamma_1+\gamma_2+\gamma_3+\gamma_4+\gamma_5+\gamma_6)P + \frac{2\eta}{\gamma_4}\hat{C}^T\breve{L}^TP\breve{L}\hat{C} + \frac{1}{\gamma_3}\mathscr{A}^TP\mathscr{A}]\zeta(t) + \Delta(t) \tag{35}$$

where

Table 1. System parameters.

Symbol	Estimated value
I_1	150×10^{-3}kgm^2
I_2	150×10^{-3}kgm^2
m_1	0.5 kg
m_2	1 kg
l_1	1 m
l_2	0.8 m
g	9.81 m/s^2

$$\Delta(t) = \tfrac{1}{\gamma_1}\breve{d}^TP\breve{d} + \tfrac{1}{\gamma_2}(\breve{B}u)^TP\breve{B}u + \tfrac{1}{\gamma_5}(\breve{L}^+e_{y1})^TP\breve{L}^+e_{y1} + \tfrac{1}{\gamma_6}(\breve{L}^-e_{y2})^TP\breve{L}^-e_{y2} \tag{36}$$

Therefore, we can obtain

$$\begin{aligned}\dot{V}(t) &\leqslant \zeta(t)^T(-\lambda P)\zeta(t) + \xi(t)^T\Lambda\xi(t) \\ &= -\lambda V(t) + \xi(t)^T\Lambda\xi(t)\end{aligned} \tag{37}$$

and

$$\Lambda = \begin{bmatrix} \sqrt{\frac{1}{\gamma_5}}\Theta_1 & 0 & 0 & 0 \\ * & \sqrt{\frac{1}{\gamma_6}}\Theta_2 & 0 & 0 \\ * & * & \sqrt{\frac{1}{\gamma_2}}\Theta_3 & 0 \\ * & * & * & \sqrt{\frac{1}{\gamma_1}}P \end{bmatrix} \tag{38}$$

$$\Theta_1 = (\breve{L}^+)^TP(\breve{L}^+), \Theta_2 = (\breve{L}^-)^TP(\breve{L}^-), \Theta_3 = \breve{B}^TP\breve{B}$$

where $\lambda > 0$, system (25) is ISS from Definition 2.2. Moreover, the upper and lower errors are ultimately bounded, which completes the proof.

Numerical example

A 2-DOF robot manipulator with revolute joints as shown in Figure 1 is applied to verify the validity of the presented ETIO in this section . Matrices in the manipulator dynamics Min et al. (2011) are given by

The model parameters are given in Table 1. Moreover, the disturbance $d(t)$, nonlinear function $f(x_1, x_2)$ and related parameters are chsoen as

$$d(t) = \begin{bmatrix} 0.5 + 0.5sin(t) \\ 0.5 - 0.5cos(t) \end{bmatrix}, f(x1, x2) = \begin{bmatrix} 0.5arctanx_1(t) \\ 0.5(1 - arctanx_2(t)) \end{bmatrix}$$

and

$$\gamma_i = i(i = 1,2,3...6), \lambda = 1, u(t) = 1.5$$

We choose the following coordinate transformation matrix

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

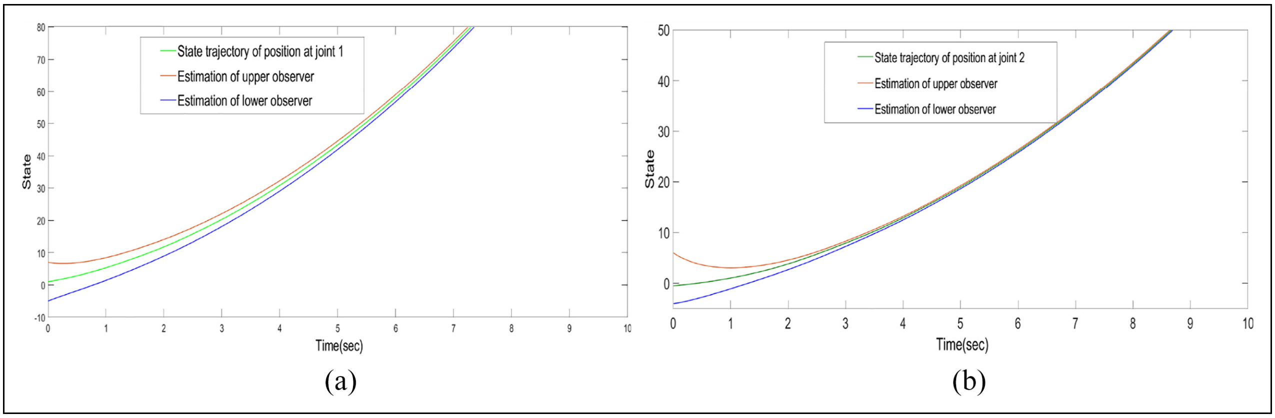

Figure 2. Evolutions of position of original system and ETIO at first joint and second joint: (a) is the position trajectory and interval estimates at the first joint and (b) is the position trajectory and interval estimates at the second joint.

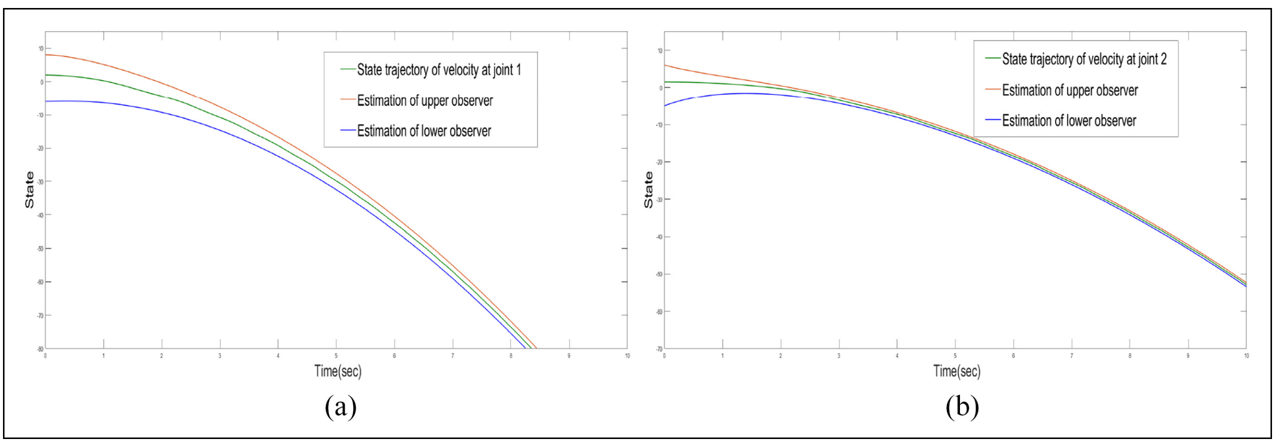

Figure 3. Evolutions of velocity of original system and ETIO at first joint and second joint: (a) is the velocity trajectory and interval estimates at the first joint and (b) is the velocity trajectory and interval estimates at the second joint.

then the weight matrix P is solved

$$P = \begin{bmatrix} 4.2691 & 0 & 0.2635 & 0 \\ 0 & 4.2691 & 0 & 0.2635 \\ 0.2635 & 0 & 0.0273 & 0 \\ 0 & 0.2635 & 0 & 0.0273 \end{bmatrix}$$

thus, L and Q can be obtained by Theorem 3.3

$$L = \begin{bmatrix} 3.214 & 0 \\ 0 & 3.258 \\ 2.151 & 0 \\ 0 & 2.121 \end{bmatrix}, Q = \begin{bmatrix} 14.2877 & 0 \\ 0 & 14.4676 \\ 0.9056 & 0 \\ 0 & 0.9164 \end{bmatrix}$$

The simulation results are displayed in the Figures 2–5. Figures 2 and 3 illustrate the trajectories of the original system and upper and lower observers, from which we can clearly see that the trajectories of the system are strictly surrounded by the states of upper and lower observers. Figures 4 and 5 draw the observed errors of position and velocity at each joint. We can see that the errors eventually converge to a bounded quantity and are greater than zero. The final oscillation convergence of Figure 5 is due to nonlinear effects in the system. Figure 6 depicts the moment with different η when $e_y^T(t)e_y(t) < \eta y_{\tau_p}^T(t)y_{\tau_p}(t)$ is violated; it is obvious that the ETM is effective. Different from Li et al. (2020), the weight matrix Q is not added to the ETM in this paper, but the simulation results are satisfactory, which means that the computation burden is smaller than that in Li et al. (2020). Compared with Yin et al. (2022), the ETM-based IO is considered in this paper, the network source is saved and the total design cost is reduced, which can be also seen from Figure 6. The results of simulation indicate that the performance of ETIO is sound. In summary, the proposed design method of ETIO for ELSs is effective and feasible.

Conclusion

We focused on designing an ETIO for a class of ELSs containing unknown bounded disturbances in this paper. By using the information of input and output, the upper and

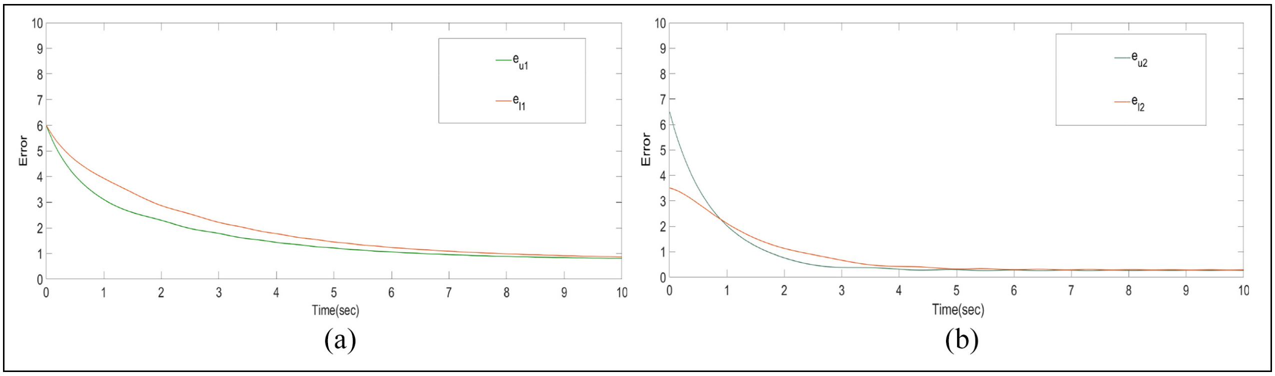

Figure 4. Observed errors of position at first joint and second joint: (a) is the upper and lower observed errors of position at the first joint and (b) is the upper and lower observed errors of position at the second joint.

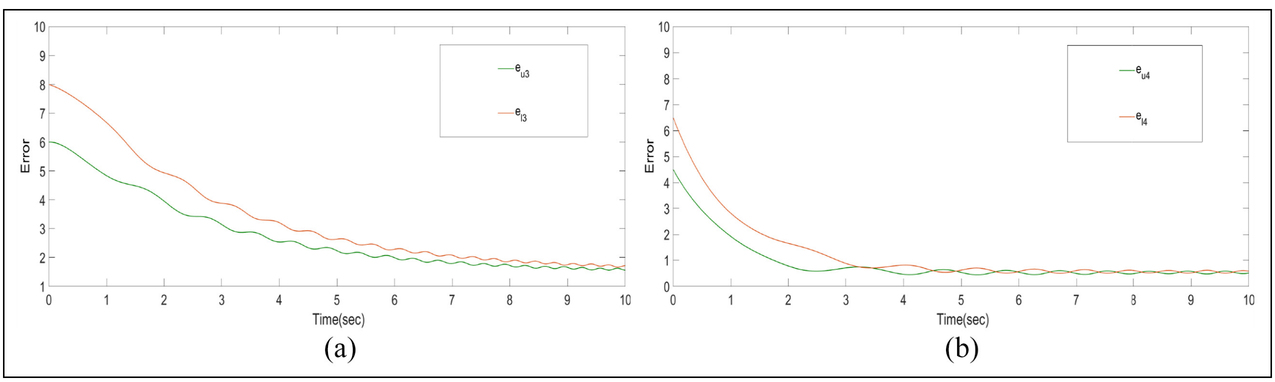

Figure 5. Observed errors of velocity at first joint and second joint: (a) is the upper and lower observed errors of velocity at the first joint and (b) is the upper and lower observed errors of velocity at the second joint.

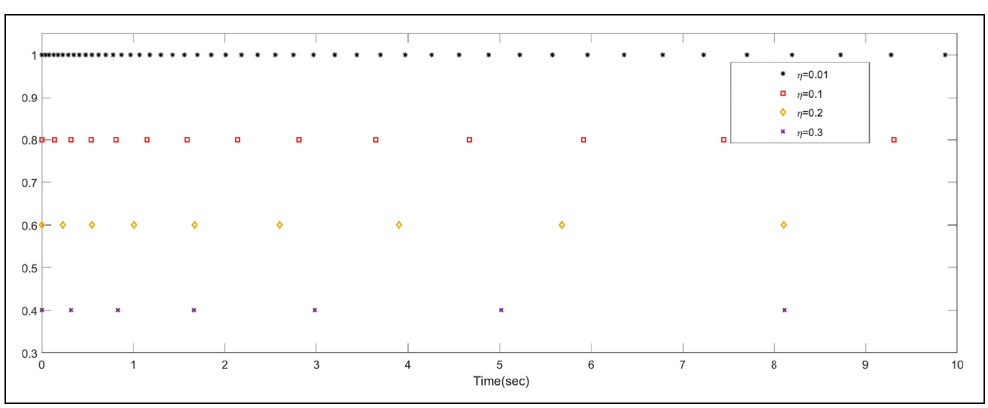

Figure 6. Event-triggered instants.

lower observers were designed for the system by the positive system method. We also present the coordinate transformation method to deal with the case where the matrix $A - LC$ is not Metzler. An ETIO is constructed under the proposed ETM, then the sufficient conditions for the gain of IOs are obtained, which also ensure the effectiveness of IOs by using ISS. The feasibility of the presented ETIO is verified by numerical simulation. In the future, we will try to increase robot experiments and consider the controller algorithm design based on ETIO.

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Data availability statement

All data analysed or used by the authors are available on request to the corresponding author.

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