Further Result on Interval Observer Design for Discrete-Time Switched Systems and Application to Circuit Systems

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***Abstract*—This brief deals with the interval observer design for a class of discrete-time switched systems. In order to reduce the constraints of design conditions and improve the accuracy of estimation, the zonotope method is used to estimate the bounds of the system states. First, an H_∞ observer is constructed for discrete-time switched systems and sufficient conditions are derived to guarantee the existence of the H_∞ observer. Then, zonotope method is employed to estimate the states based on the designed observer. Finally, a numerical example in the background of booster converter is simulated to demonstrate the efficiency of the proposed approach.**

***Index Terms*—Interval observers, switched systems, zonotope method, H_∞ observers.**

I. Introduction

IN THE field of control theory, state estimation or observer design is an important topic [1], [2] and it can be applied in stabilization, optimal control, fault detection and so on. There always exist uncertainties in real systems, hence it is necessary to estimate the upper and lower bounds of the states [3], [4]. The investigation of interval observers has been paid much attention in recent years [5]–[8]. The most commonly used approach in mentioned works is the cooperative error system method. The main task is to design observer gains to make the corresponding error systems uniformly ultimately bounded and positive. Generally, it is not easy for the error systems to satisfy the uniformly ultimate boundedness and positivity simultaneously. Thus, the time-invariant coordinate change [9] and time-varying coordinate change [10] methods were proposed respectively. These methods were also proved to be useful for the interval observer design of specific systems, such as nonlinear systems [11], linear parameter varying (LPV) systems [12], singular systems [13], impulsive systems [14] and so on.

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As a class of important hybrid dynamical systems, switched systems have attracted increasing research interests in the control community. A switched system contains several subsystems which are activated by a switching rule. Up to now, many verification methods for the stability of switched systems were developed, such as traditional Lyapunov function method [15], multiple Lyapunov function method [16], and average dwell time (ADT) method [17]. To some extent, the interval observer design for switched systems is more complicated than that for the systems with single mode. Recently, the interval observer design problems of switched systems have been studied in the works [18]–[21]. In [18] and [19], the interval observers were designed for continuous time switched systems under the assumption that $A_i - L_iC_i$ are both Hurwitz and Metzler. The authors in [20] introduced a specified time-invariant coordinate transformation for each subsystem and improved the results of [18] and [19]. While in [21], discrete-time switched systems were considered and the interval observer framework was established under the time-varying transformation. As pointed out in [22], although the coordinate transformation is an effective approach, the transformation matrix and observer gains can not be synthesized to achieve the cooperativity as well as disturbance attenuation performance.

In the light of the above discussion, this brief addresses the problem of interval observer design for discrete-time switched systems by zonotope method. Some important works on the interval estimation by zonotope method can found in [23]–[26]. There are two main challenges: one is to design the H_∞ observers with optimal performance for discrete-time switched systems, and the other is to construct the framework of zonotope for switched systems and then give the upper and lower bounds of the system states. The contribution of this brief can be summarized as two aspects: (1) The error systems corresponding with H_∞ observers are not conformed to be cooperative, and the sufficient conditions are less restrictive. (2) Compared with the method presented in [21], it is verified that the method by using zonotope can provide more accurate bound of interval observer based on the simulation.

The remainder of this brief is organized as follows. Section II gives the problem formulation and some necessary preliminaries. Section III presents the main results of this brief including the interval observer design approach. An example is simulated in Section IV to illustrate the performance of the designed interval observers.

Notations: Throughout this brief, R^n is the n-dimensional Euclidean space, and $R^{n\times m}$ is the set of $n \times m$ real matrices. $||x||$ means the Euclidean norm of the vector x, and $x \le y$ implies that each component of x is less than the corresponding component of y. $\oplus$ stands for Minkowski sum, and $\odot$ is linear mapping. $A \prec 0$ means that A is a negative definite matrix, and I represents the identity matrix with appropriate dimensions. $\underline{\lambda}(P)$ and $\overline{\lambda}(P)$ are the minimum and maximum eigenvalues of matrix P.

II. Problem Formulation and Preliminary

Consider the following switched system, which is described by:

$$\begin{cases} x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k) + E_{\sigma(k)}w(k), \\ y(k) = C_{\sigma(k)}x(k), \end{cases} \quad (1)$$

where $x(k) \in R^n$, $u(k) \in R^m$ and $y(k) \in R^q$ are the state, the control input, and the output, respectively. $w(k) \in R^r$ is the unknown but bounded disturbance, i.e., $-w \le w(k) \le w$, and w is a constant vector. $\sigma(k)$ is a piecewise constant function, which takes values in the finite set $S = \{1, 2, \ldots, N\}$. For any $\sigma(k) = i \in S$, $A_i \in R^{n\times n}$, $B_i \in R^{n\times m}$, $C_i \in R^{q\times n}$ and $E_i \in R^{n\times r}$ are determined matrices.

First of all, the definitions of interval observer, H_∞ observer and switched system are introduced.

Definition 1: The pair $\{\hat{x}^+(k), \hat{x}^-(k)\}$ is said to be an interval observer for (1) if it holds that $\forall\ k > 0$

$$\hat{x}^-(k) \le x(k) \le \hat{x}^+(k), \text{ with } \hat{x}^-(0) \le x(0) \le \hat{x}^+(0).$$

In this section, an observer for the system (1) is constructed:

$$\hat{x}(k+1) = A_{\sigma(k)}\hat{x}(k) + B_{\sigma(k)}u(k) + L_{\sigma(k)}(y - C_{\sigma(k)}\hat{x}(k)), \quad (2)$$

where the gain $L_{\sigma(k)} \in R^{n\times q}$ will be determined later. Denote that $e(k) = x(k) - \hat{x}(k)$, subtracting (2) from (1) results in the error system:

$$e(k+1) = (A_{\sigma(k)} - L_{\sigma(k)}C_{\sigma(k)})e(k) + E_{\sigma(k)}w(k). \quad (3)$$

Definition 2: Observer (2) is said to be an H_∞ observer of system (1), if

(1) $w(k) = 0$, the error system (3) is asymptotically stable;

(2) $w(k) \ne 0$, under zero-initial condition, the following inequality holds:

$$\sum_{k=0}^{\infty} e^T(k)e(k) \le \gamma \sum_{k=0}^{\infty} w^T(k)w(k),$$

where $\gamma > 0$ is the disturbance attenuation level.

Definition 3 [17]: Denote the switching number of $\sigma(k)$ on the interval $[k_1, k_2)$ by $N_\sigma(k_1, k_2)$. If

$$N_\sigma(k_1, k_2) \le N_0 + (k_2 - k_1)/\tau^*$$

holds for given $N_0 \ge 0$ and $\tau^* > 0$, then τ^* is called an ADT of the switching signal $\sigma(k)$. In this brief, we let $N_0 = 0$.

Since zonotope method is very important to the estimation of the system states, it is necessary to recall some definitions and properties of zonotope herein.

Definition 4 [23]: Let $p \in R^n$ be a given vector and $M \in R^{n\times s}$ be a given matrix, the set

$$Z = p \oplus MB^s = p + Mz, z \in B^s$$

is said to be a zonotope of order s, where $B = [-1, 1]$ is a unitary interval and B^s is a unitary box composed by s unitary intervals. For simplicity, the zonotope Z is denoted by $\langle p, M\rangle$ in the sequel.

Lemma 1 [24]: Given the zonotope $Z = \langle p, M\rangle$, where $p \in R^n$, $M \in R^{n\times s}$, the following properties hold:

(1) $\langle p_1, M_1\rangle \oplus \langle p_2, M_2\rangle = \langle p_1 + p_2, [M_1\ M_2]\rangle$,

(2) $L \odot \langle p, M\rangle = \langle Lp, LM\rangle$,

(3) $\langle p, M\rangle \subseteq \langle p, \bar{M}\rangle$,

where $\bar{M} \in R^{n\times n}$ is a diagonal matrix with the diagonal elements $\bar{M}_{i,i} = \sum_{j=1}^{s} |M_{i,j}|,\ i = 1, \ldots, n$.

Remark 1: $\bar{M}$ has the following form

$$\bar{M} = \begin{bmatrix} \sum_{j=1}^{s} |M_{1,j}| & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sum_{j=1}^{s} |M_{n,j}| \end{bmatrix}.$$

If $s > n$, then $\langle p, M\rangle \subseteq \langle p, \bar{M}\rangle$ is used for reducing the order of a zonotope.

Lemma 2 [23]: Consider the zonotope $Z = \langle p, M\rangle$, where $p \in R^n$, $M \in R^{n\times s}$, and the matrix $\widetilde{M}$ resulting from the reordering of the columns of matrix M in decreasing Euclidean norm, then the following holds

$$Z \subseteq \langle p, \hat{M}\rangle, \ (n \le q \le s),$$

where $\hat{M} = [\widetilde{M}_{q-n} \quad W] \in R^{n\times q}$, $\widetilde{M}_{q-n}$ is obtained from the first $q - n$ columns of matrix $\widetilde{M}$ and $W \in R^{n\times n}$ is a diagonal matrix with $W_{i,i} = \sum_{j=q-n+1}^{s} |\widetilde{M}_{i,j}|,\ i = 1, \ldots, n$.

III. Main Result

This section gives the sufficient conditions of the existence of the H_∞ observer (2), and then estimates the bounds of (1).

Theorem 1: Let $0 < \xi < 1$ and $\mu > 1$ be given constants. If there exist constant $\gamma > 0$ and matrices $P_i \in R^{n\times n} \succ 0$, $P_j \in R^{n\times n} \succ 0$, $\forall\ i, j \in S$, $i \ne j$ such that

$$\begin{bmatrix} \Lambda & * \\ E_i^T P_i(A_i - L_iC_i) & E_i^T P_i E_i - \gamma I \end{bmatrix} \prec 0, \quad (4)$$

$$P_i \prec \mu P_j, \quad (5)$$

where $\Lambda = (A_i - L_iC_i)^T P_i(A_i - L_iC_i) - \xi P_i + I$, and ADT satisfying

$$\frac{1}{\tau^*} + \frac{\ln \xi}{\ln \mu} < 0, \quad (6)$$

then (2) is an H_∞ observer of (1).

Proof: Let $\sigma(k_l) = i \in S$, and choose the Lyapunov function as follows:

$$V_i(k) = e^T(k)P_ie(k). \quad (7)$$

The proof is divided into two steps according to Definition 2.

(*i*) $w(k) = 0$. When $k \in [k_l, k_{l+1})$, along the error system (3), taking the forward difference of $V_i(k)$ results in

$$\begin{aligned}\Delta V_i(k) &= V_i(k+1) - V_i(k)\\ &= e^T(k)[(A_i - L_iC_i)^T P_i(A_i - L_iC_i) - P_i]e(k). \end{aligned}\tag{8}$$

The inequality (4) implies that $\Lambda \prec 0$. Substituting $\Lambda \prec 0$ into (8) yields

$$\begin{aligned}\Delta V_i(k) &\le (\xi - 1)e^T(k)P_ie(k) - e^T(k)e(k)\\ &\le (\xi - 1)e^T(k)P_ie(k) \le (\xi - 1)V_i(k),\end{aligned}\tag{9}$$

hence

$$V_i(k+1) \le \xi V_i(k). \tag{10}$$

Considering the interval $[k_l, k)$, we obtain

$$V_i(k) \le \xi^{k-k_l} V_i(k_l). \tag{11}$$

Suppose that $\sigma(k_{l-1}) = j$. By using (5), inequality (11) becomes

$$V_i(k) \le \mu\xi^{k-k_l} V_j(k_l). \tag{12}$$

Repeating (11) and (12), one can get

$$V_i(k) \le \mu^{N_\sigma(0,k)} \xi^k V_{\sigma(0)}(0). \tag{13}$$

From Definition 3, since $N_\sigma(0, k) \le \frac{k}{\tau*}$, it deduces that

$$V_i(k) \le \mu^{\frac{k}{\tau*}} \xi^k V_{\sigma(0)}(0) \le \mu^{k(\frac{1}{\tau*}+\frac{\ln\xi}{\ln\mu})} V_{\sigma(0)}(0). \tag{14}$$

Due to $V_i(k) \ge \underline{\lambda}(P_i)\|e(k)\|^2$, $V_{\sigma(0)}(0) \le \overline{\lambda}(P_i)\|e(0)\|^2$, we have

$$\underline{\lambda}(P_i)\|e(k)\|^2 \le \mu^{k(\frac{1}{\tau*}+\frac{\ln\xi}{\ln\mu})}\overline{\lambda}(P_i)\|e(0)\|^2,$$

i.e.,

$$\|e(k)\| \le \sqrt{\frac{\mu^{k(\frac{1}{\tau*}+\frac{\ln\xi}{\ln\mu})}\overline{\lambda}(P_i)}{\underline{\lambda}(P_i)}}\|e(0)\|. \tag{15}$$

By (6) and the fact that $\mu > 1$, it follows from (15) that

$$\lim_{k\to\infty} \|e(k)\| = 0. \tag{16}$$

(*ii*) $w(k) \neq 0$. Let

$$\begin{aligned}J &= \sum_{k=0}^{\infty} e^T(k)e(k) - \gamma \sum_{k=0}^{\infty} w^T(k)w(k)\\ &= \sum_{k=0}^{\infty}[e^T(k)e(k) - \gamma w^T(k)w(k)].\end{aligned}\tag{17}$$

By using the zero-initial condition $e(0) = 0$, we get

$$\begin{aligned}\sum_{k=0}^{\infty}\Delta V_i(k) &= \lim_{k\to\infty}\Big\{V_i(k) - V_{\sigma(k-1)}(k-1)\\ &\quad + V_{\sigma(k-1)}(k-1) - V_{\sigma(k-2)}(k-2)\\ &\quad + \cdots + V_{\sigma(1)}(1) - V_{\sigma(0)}(0)\Big\}\\ &= \lim_{k\to\infty} V_i(k) \ge 0.\end{aligned}\tag{18}$$

Thus,

$$J \le \sum_{k=0}^{\infty}[e^T(k)e(k) - \gamma w^T(k)w(k) + \Delta V_i(k)]. \tag{19}$$

Denote that $S(k) = e^T(k)e(k) - \gamma w^T(k)w(k) + \Delta V_i(k)$. By (4), together with $0 < \xi < 1$, we obtain

$$\begin{aligned}S(k) &= e^T(k)[(A_i - L_iC_i)^T P_i(A_i - L_iC_i) - P_i + I]e(k)\\ &\quad + 2w^T(k)E_i^T P_i(A_i - L_iC_i)e(k)\\ &\quad + w^T(k)(E_i^T P_iE_i - \gamma I)w(k)\\ &\le (\xi - 1)e^T(k)P_ie(k) \le 0.\end{aligned}\tag{20}$$

It follows from (19) and (20) that

$$J \le \sum_{k=0}^{\infty} S(k) \le 0, \tag{21}$$

which implies

$$\sum_{k=0}^{\infty} e^T(k)e(k) - \gamma\sum_{k=0}^{\infty} w^T(k)w(k) \le 0, \tag{22}$$

i.e.,

$$\sum_{k=0}^{\infty} e^T(k)e(k) \le \gamma\sum_{k=0}^{\infty} w^T(k)w(k). \tag{23}$$

By (i) and (ii), the proof is completed. ■

The following theorem presents an optimal H_∞ observer for the system (1), and sufficient conditions can be formulated by the linear matrix inequality (LMI) forms.

Theorem 2: Let $0 < \xi < 1$ and $\mu > 1$ be given constants. If there exist constant $\gamma > 0$ and matrices $P_i \in R^{n\times n} \succ 0$, $P_j \in R^{n\times n} \succ 0$, $F_i \in R^{n\times q}$, $\forall\, i, j \in S$, $i \neq j$ such that

$$\begin{cases}\min\ \gamma\\ \textit{subject to}:\\ \begin{bmatrix} -\xi P_i + I & * & *\\ E_i^T P_iA_i - E_i^T F_iC_i & E_i^T P_iE_i - \gamma I & *\\ P_iA_i - F_iC_i & 0 & -P_i\end{bmatrix} \prec 0,\\ P_i \prec \mu P_j,\end{cases}\tag{24}$$

where the observer gain is determined by $L_i = P_i^{-1}F_i$ and ADT satisfying (6), then (2) is an optimal H_∞ observer for the system (1).

By using Shur complements and substituting $F_i = P_iL_i$ into (24), the proof can be completed. Due to the space limitation, the proof is omitted here.

Remark 2: Actually, the performance of the designed observer is determined by the ADT τ^* and disturbance attenuation level γ. Since the ADT $\tau^* > -\frac{\ln\mu}{\ln\xi}$ depends on μ and ξ, it is necessary to find suitable values of μ and ξ to minimize the ADT τ^*.

Denote that $w = [w_1\ w_2\ \cdots\ w_r]^T$. In view of $-w \le w(k) \le w$, $w(k)$ is described by the zonotope $\langle 0, W\rangle$, where $W \in R^{r\times r}$ has the following form:

$$W = \begin{bmatrix} w_1 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & w_r\end{bmatrix}.$$

Then, the state estimation of the system (1) based on the observer (2) is stated as the following theorem.

Theorem 3: Let $\sigma(k) = i \in S$, where $S = \{1, 2, \ldots, N\}$ is a finite set. If there exists an H_∞ observer (2) for the system (1), then the state of the system (1) can be described by

$$x(k) \in \langle \hat{x}(k), M_i(k) \rangle, \tag{25}$$

where $M_i(k)$ has the following recursive form:

$$M_i(k+1) = [(A_i - L_iC_i)\hat{M}_i(k) \quad E_iW]. \tag{26}$$

Proof: Since $x(k) = \hat{x}(k) + e(k)$, $x(k) \in \langle \hat{x}(k), M_i(k) \rangle$ is equivalent to $e(k) \in \Phi(k) = \langle 0, M_i(k) \rangle$. By Lemma 2, the following holds

$$e(k) \in \hat{\Phi}(k) = \langle 0, \hat{M}_i(k) \rangle. \tag{27}$$

It deduces from the error system (3) that

$$\begin{aligned} e(k+1) &\in \Phi(k+1) \\ &= (A_i - L_iC_i) \odot \langle 0, \hat{M}_i(k) \rangle \oplus E_i \odot \langle 0, W \rangle. \end{aligned} \tag{28}$$

In view of Lemma 1, we have

$$\begin{aligned} e(k+1) &\in \langle 0, (A_i - L_iC_i)\hat{M}_i(k) \rangle \oplus \langle 0, E_iW \rangle \\ &= \langle 0, [(A_i - L_iC_i)\hat{M}_i(k) \quad E_iW] \rangle, \end{aligned}$$

i.e.,

$$\Phi(k+1) = \langle 0, [(A_i - L_iC_i)\hat{M}_i(k) \quad E_iW] \rangle. \tag{29}$$

By the fact that $\Phi(k+1) = \langle 0, M_i(k+1) \rangle$, it follows from (29) that

$$M_i(k+1) = [(A_i - L_iC_i)\hat{M}_i(k) \quad E_iW]. \tag{30}$$

Hence, we have completed the proof. ■

From the above theorems, an interval observer for the system (1) is designed as follows:

$$\begin{cases} (x^m)^+(k) = \hat{x}^m(k) + \sum_{j=1}^{s} |M_i^{m,j}(k)|, \\ (x^m)^-(k) = \hat{x}^m(k) - \sum_{j=1}^{s} |M_i^{m,j}(k)|, \end{cases} \tag{31}$$

where $m = 1, \ldots, n$, $j = 1, \ldots, s$, x^m is the m-th element of x, $\hat{x}^m$ is the m-th element of $\hat{x}$, and $M_i^{m,j}$ is the element in the m-th row and j-th column of the matrix M_i.

Remark 3: Different from [18] and [19], the derived condition that $A_i - L_iC_i$ need not be Metzler. In addition, compared with [20] and [21], we do not employ the coordinate change. Especially, in the discrete-time case [21], it is not easy to find a proper time-varying coordinate change.

Remark 4: The recent innovation [27] studies the optimal interval observer design for discrete-time linear switched systems. The method employed in [27] is the coordinate transformation combined with the theory of cooperative system. Specifically, by the coordinate transformation $z = R_qx$, $R_q(A_q - L_qC_q)R_q^{-1}$ is both nonnegative and Schur. As stated in this brief, we use two-step method: the first step is to design an optimal H_∞ observer and the second step is to use the zonotope method to estimate the bounds of the errors.

Remark 5: Generally, the zonotope method is not a limitation-free approach. There are two main limitations. Firstly, the bounds of the system states can be estimated by zonotope method provided that the H_∞ observer exists. Moreover, it is necessary to obtain the optimal H_∞ observer to make the bounds more compact. Secondly, as pointed out in [26], since the zonotope method is an iterative algorithm, the dimensionality reduction of $M_i(k)$ may enlarge the boundaries of state estimation.

Algorithm 1 Procedure to Design the Interval Observer by Zonotope Method

The procedure to design the presented interval observer is summarized by the following steps.

Step1: Solve the LMI problem in Theorem 2 by MATLAB;
Step2: Compute $L_i = P_i^{-1}F_i$ to obtain the observer gain;
Step3: Determine μ, ξ to estimate the ADT;
Step4: Calculate $\hat{M}_i(k)$ by Theorem 3;
Step5: Use (31) to estimate the bounds of the states.

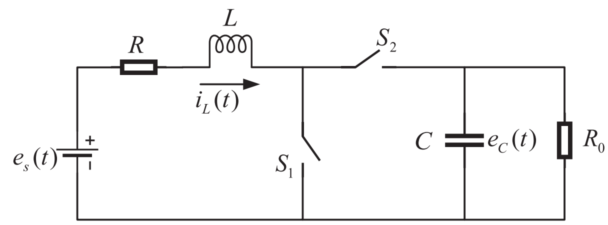

Fig. 1. A booster converter.

IV. Illustrative Example

Consider a booster converter (motivated by [28]), the block of which is presented in Fig. 1. The state space model can be described by:

$$\begin{cases} \dot{i}_L(t) = -\frac{R}{L}i_L(t) + (1 - S(t))\frac{1}{L}e_C(t) + \frac{1}{L}e_s(t), \\ \dot{e}_C(t) = (S(t) - 1)\frac{1}{C}i_L(t) - \frac{1}{R_0C}e_C(t), \end{cases} \tag{32}$$

where i_L is the current through the inductor, e_C is the capacitor voltage, and the value of switching signal $S(t)$ is 1 or 2. $S(t) = 1$ when S_1 switches on and S_2 switches off. $S(t) = 2$ when S_1 switches off and S_2 switches on.

Denote that $x(t) = [i_L(t) \quad e_C(t)]^T$, $u(t) = e_s(t)$ and $y(t) = e_C(t)$. Substituting the standard parameters $R_0 = 0.2\Omega$, $L = 2H$, $C = 4F$ and $R = 5\Omega$ into the above model, and using the discretization algorithm, we have the corresponding discrete-time system (1) where

$$A_1 = \begin{bmatrix} -0.5 & 0 \\ 0 & -0.25 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.5 & -0.5 \\ 0.25 & -0.25 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}, \quad C_1 = C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

In order to verify the effectiveness of the proposed method, we also add the disturbance term $w(k)$ to the circuit system, and the disturbance matrices are chosen as

$$E_1 = \begin{bmatrix} -0.15 \\ 0.22 \end{bmatrix}, \quad E_2 = \begin{bmatrix} -0.24 \\ 0.13 \end{bmatrix}.$$

By solving the conditions in Theorem 2, we can get

Case	Parameters	Results
1	$\mu = 2.28$, $\xi = 0.33$	$L_1 = \begin{bmatrix} 0.2202 \\ -0.2512 \end{bmatrix}$, $L_2 = \begin{bmatrix} -0.1369 \\ -0.4213 \end{bmatrix}$, $\gamma = 3.27$, $\tau^* > 0.7434$
2	$\mu = 1.89$, $\xi = 0.41$	$L_1 = \begin{bmatrix} 0.1747 \\ -0.2458 \end{bmatrix}$, $L_2 = \begin{bmatrix} -0.2384 \\ -0.3823 \end{bmatrix}$, $\gamma = 5.62$, $\tau^* > 0.714$

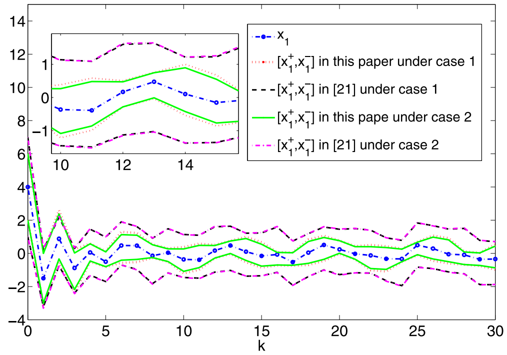

Fig. 2. Comparison of the results in [21] and this brief.

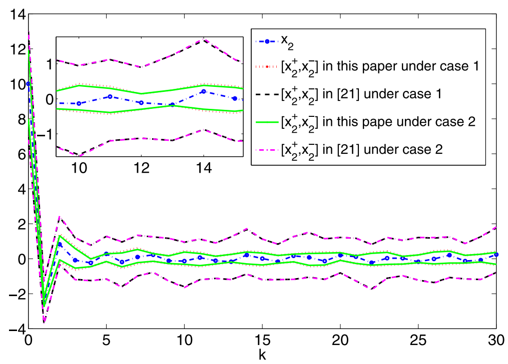

Fig. 3. Comparison of the results in [21] and this brief.

Then, we use (31) to estimate the bounds of the states. When $\mu = 2.28$ and $\xi = 0.33$, the dotted lines represent the bounds of the states obtained by Algorithm 1 and the dashed ones represent that obtained by the method in [21]. While $\mu = 1.89$ and $\xi = 0.41$, the solid lines and dash-dotted lines are used to describe the interval estimations obtained by the methods in this brief and [21], respectively. According to Fig. 2 and Fig. 3, under two sets of different values of μ and ξ, the estimation accuracy by zonotope method is better than the method in [21].

V. Conclusion

This brief considers the interval observer design method for uncertain discrete-time switched systems. The framework of the interval observer is established based on the H_∞ theory and zonotope approach. First, the observer gain is obtained by solving LMIs, and the robust observer is designed. The derived conditions do not require the cooperativity of the error system when we design the H_∞ observer. Then, the upper and lower bounds of the system states are estimated by zonotope method. Thus, the coordinate transformation is not employed in this brief. Furthermore, the method improves the performance of the interval observer. Finally, we use a practical example in the background of booster converter to show the effectiveness and advantages of our results over the existing works.

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