Portfolio Allocation
Final Presentation

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11/03/2022

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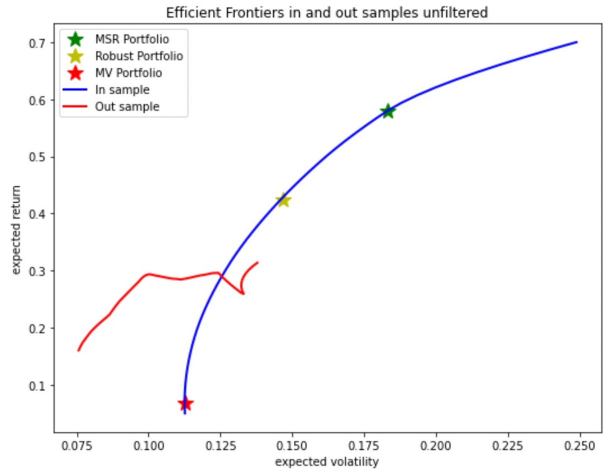
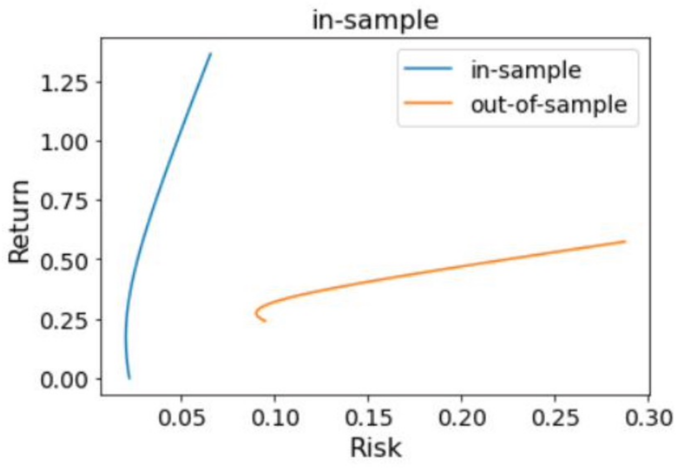
INTRODUCTION

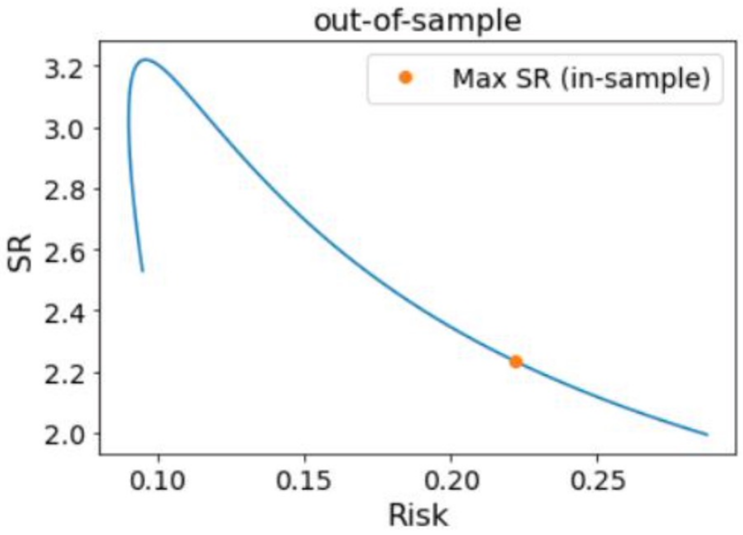
Overview of the dataset

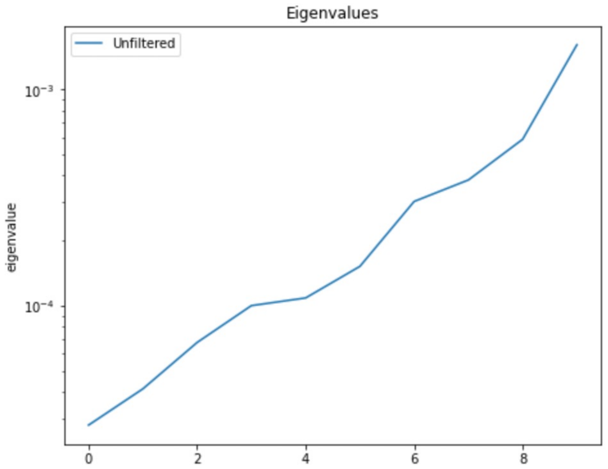
- In our report, we use the dataset of this course
- The initial version is log-return and we turn it back to the normal return
- For the in-sample we choose the length of 126 trading days
- For the out of sample we choose the length of 30 trading days
- After dropping the NA values, there are 468 equities left.

The need to estimate the covariance









- The efficient frontier of the out sample is irregular and needs correction : in sample portfolios don't necessarily perform well in the future

- Lower eigenvalues are under-estimated while medium-higher are over-estimated due to some noise component

Hypothesis

- We consider the day 1000 as today
- We took a risk-free rate of 1%
- We don’t allow short selling
- We take only 10 assets in order to simplify the lecture of the result and reduce the time of computing
- Time metrics were calculated on a 30 or 90 day basis

Metrics

- The effective portfolio diversification：

$$N_{eff} = \frac{1}{||\mathbf{w}||_2^2}$$

It represents the effective number of stocks with a significant amount of money invested

- The number of stocks that account for q percent of the total amount of money invested：

$$N_q = \arg\min_l \Sigma_{i=1}^{l} |w_{\phi(i)}| \geq q||\mathbf{w}||_1$$

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PORTFOLIO OPTIMISATION

1. Introduction
3. Results
4. Comparison
5. Adding a risk-free asset
6. Conclusion

Building the efficient frontiers

- Portfolio optimisation is equivalent to a maximization/minimization problem. In general, it is written in the following form:

$$\begin{cases} \arg\min \mathbf{w}^T \cdot \Sigma \cdot \mathbf{w} & , \\ s.c \quad \mu^T \cdot \mathbf{w} \geq return\_level & . \end{cases}$$

- We can therefore plot a frontier on which any portfolio is considered efficient since it minimizes risk for a certain level of return

- We will see later how the choice of the covariance matrix and the minimization problem changes the efficient portfolios

The Minimum Variance (MV) Portfolio, and the Maximum Sharpe Ratio (MSR) Portfolio

- The MV Portfolio is the portfolio that minimizes the risk without a return constraint. Thus, the portfolio doesn't depend on returns.

$$\mathbf{w}_{MV} = \frac{\Sigma^{-1} \cdot \mathbf{1}}{\mathbf{1}^T \cdot \Sigma^{-1} \cdot \mathbf{1}}$$

- The MSR Portfolio is the portfolio that maximizes the return (in excess of the risk-free rate) per unit of risk. It is also the portfolio at the intersection of the efficient frontier and the capital market line.

$$\mathbf{w}_{MSR} = \frac{\Sigma^{-1} \cdot \mu}{\mathbf{1}^T \cdot \Sigma^{-1} \cdot \mu}$$

Robust optimisation

- Assuming we want to build a portfolio that is sensitive to error on returns and risk aversion of the investor, the optimisation problem can be written as:

$$\mathbf{w}_{rob} = \arg\max_{w} \mu^T \cdot \mathbf{w} - \kappa\sqrt{\mathbf{w}^T \cdot \Omega \cdot \mathbf{w}} - \frac{\lambda}{2}\mathbf{w}^T \cdot \Sigma \cdot \mathbf{w}$$

- Different choices of error aversion and covariance matrix on errors lead to different results

- In our case, we will choose a risk aversion of 13, an error aversion of 0.1 and a covariance matrix on errors of return equal to the covariance matrix on returns

- We will also consider a constraint to have a tracking error less thant 3% (benchmark is an equally weighted portfolio)

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RESULTS

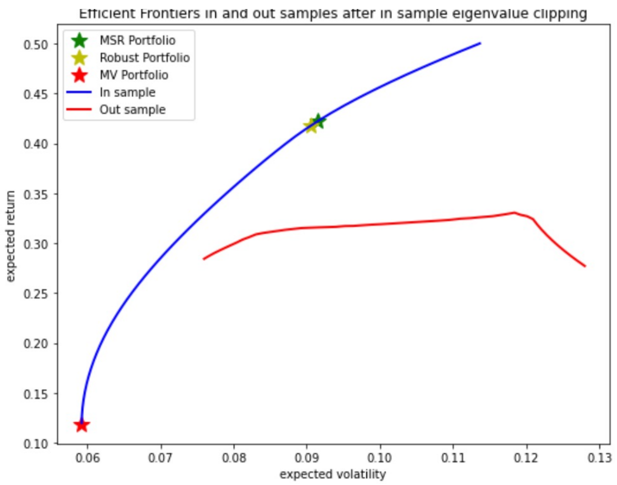
Eigenvalue clipping

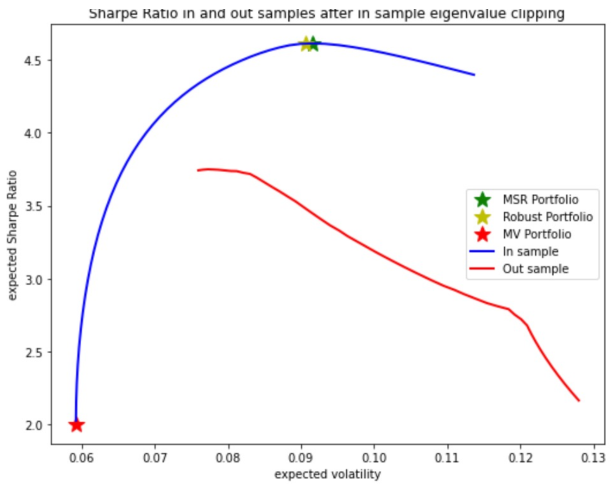
The eigenvalue clipping consists of averaging the directions on the bulk to reduce the sample-size error. It is done through the following steps:

1. Determine Λ and $\mathbf{V}$ such as Λ is diagonal and $\Sigma = \mathbf{V} \cdot \Lambda \cdot \mathbf{V}^T$,
2. Compute $\Lambda^{(c)}$ a diagonal matrix with:

$$\lambda_i^{(c)} = \frac{\sum_{j=1}^{N} \lambda_j \delta(\lambda_j < \lambda_{Max})}{\sum_{j=1}^{N} \delta(\lambda_j < \lambda_{Max})}$$

   where $(\lambda_j)_{j\in[1,N]}$ are the eigenvalues of Σ, and δ is the Heaviside function,
3. Compute $\mathbf{C}^{(t)} = \mathbf{V} \cdot \Lambda^{(c)} \cdot \mathbf{V}^T$,
4. For $i, j \in [1, N]$, compute $C_{i,j}^{(c)} = \frac{C_{i,j}^{(t)}}{C_{i,i}^{(t)} C_{j,j}^{(t)}}$,
5. Finally, compute the eigenclipped covariance matrix $\Sigma^{(c)}$ knowing that $\forall i, j \in [1, N], \Sigma_{i,j}^{(c)} = C_{i,j}^{(c)} \Sigma_{i,i} \Sigma_{j,j}$.



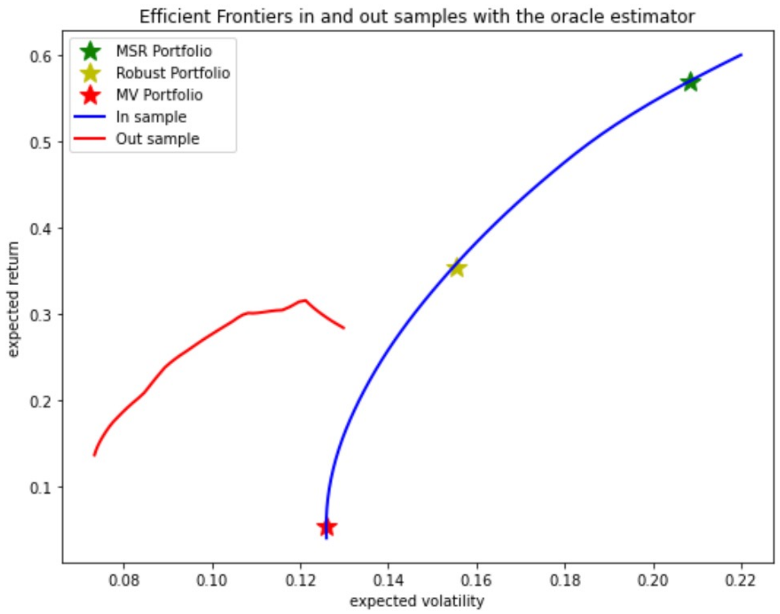


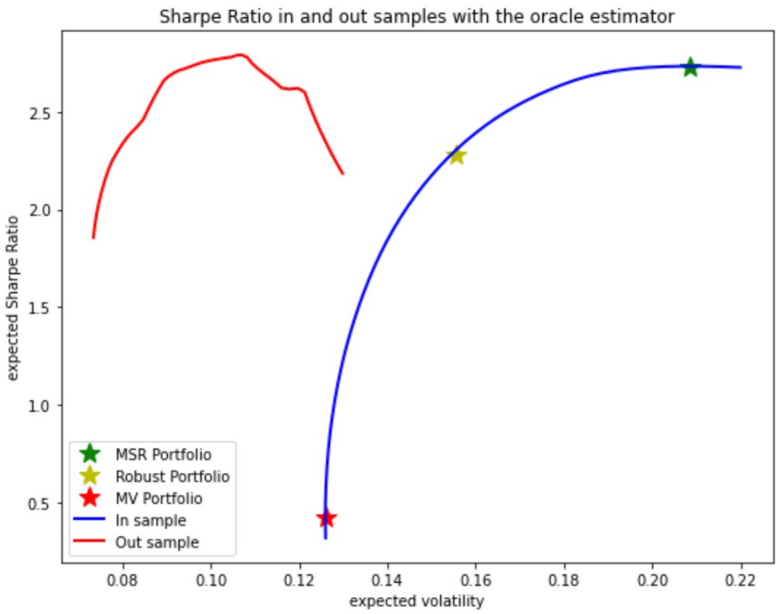
- The efficient frontier shrinks and moves to the right
- Portfolios computed have higher risk and less return for some
- The MV portfolio seems to have the best SR out sample

The oracle estimator

The oracle estimator is another way to clean the in sample covariance matrix. Assuming we know the out sample covariance matrix, we would like to adapt the in sample filtered covariance matrix Θ in order to minimize $d(\Sigma_{out}, \Theta)$. It is obtained with the following steps:

1. Determine Λ and $\mathbf{V}$ such as Λ is diagonal and $\Sigma = \mathbf{V} \cdot \Lambda \cdot \mathbf{V}^T$,
2. Compute $\mathbf{O} = diag((\mathbf{V}^T \cdot \Sigma_{out} \cdot \mathbf{V})_{diagonal},$
3. Compute $\Theta = \mathbf{V} \cdot \mathbf{O} \cdot \mathbf{V}^T$.

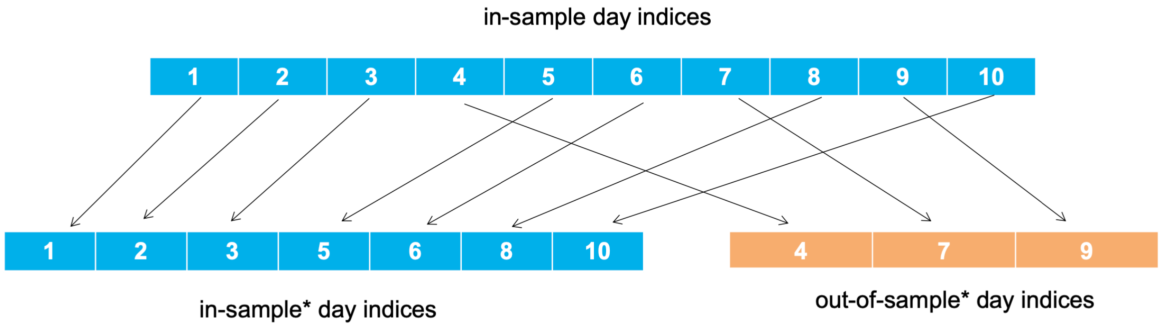




- In general, we don't know the out sample covariance matrix (solution : divide in sample into in sample* and out sample*)
- SR is globally higher
- Portfolios are less risky but also offer less return

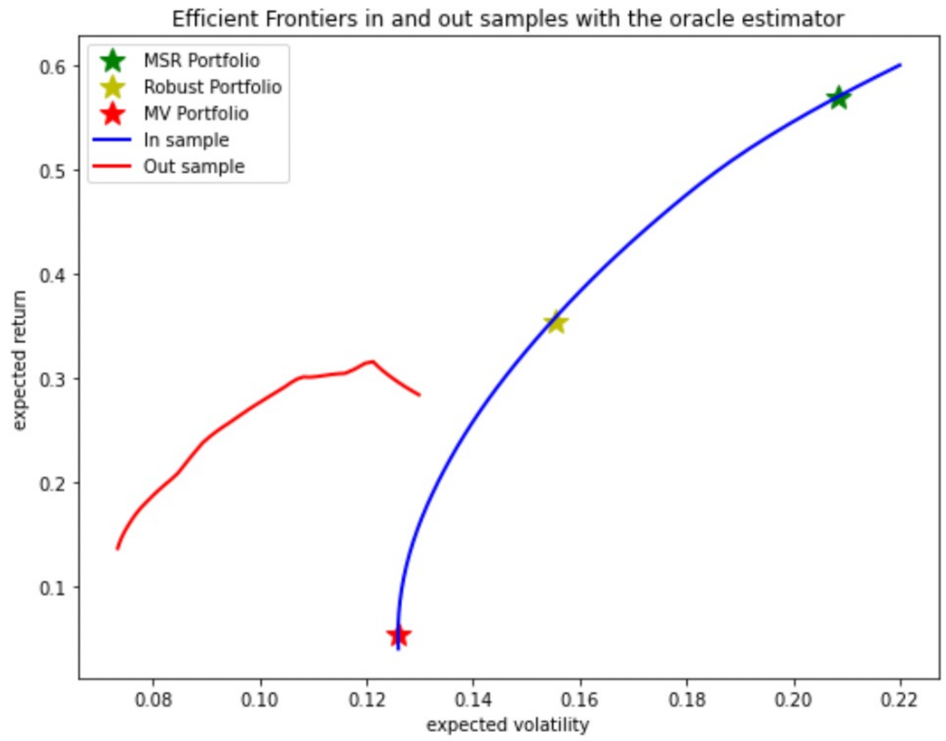
Cross validate eigenvalue shrinkage

The objective of this method is to find an alternative to zero eigenvalues in high dimension (T<N) or when the assets are highly correlated.



$$\Lambda_{CV_i}^{(t)} = (\mathbf{V}_{in_i^*}^T \cdot \Sigma_{out_i^*} \cdot \mathbf{V}_{in_i^*})_{diagonal}$$

$$\Sigma_{CV} = \mathbf{V}_{in} \cdot diag(\Lambda_{CV}^{(ISO)}) \cdot \mathbf{V}_{in}^T$$



The results are more cautious (less risk but less return)

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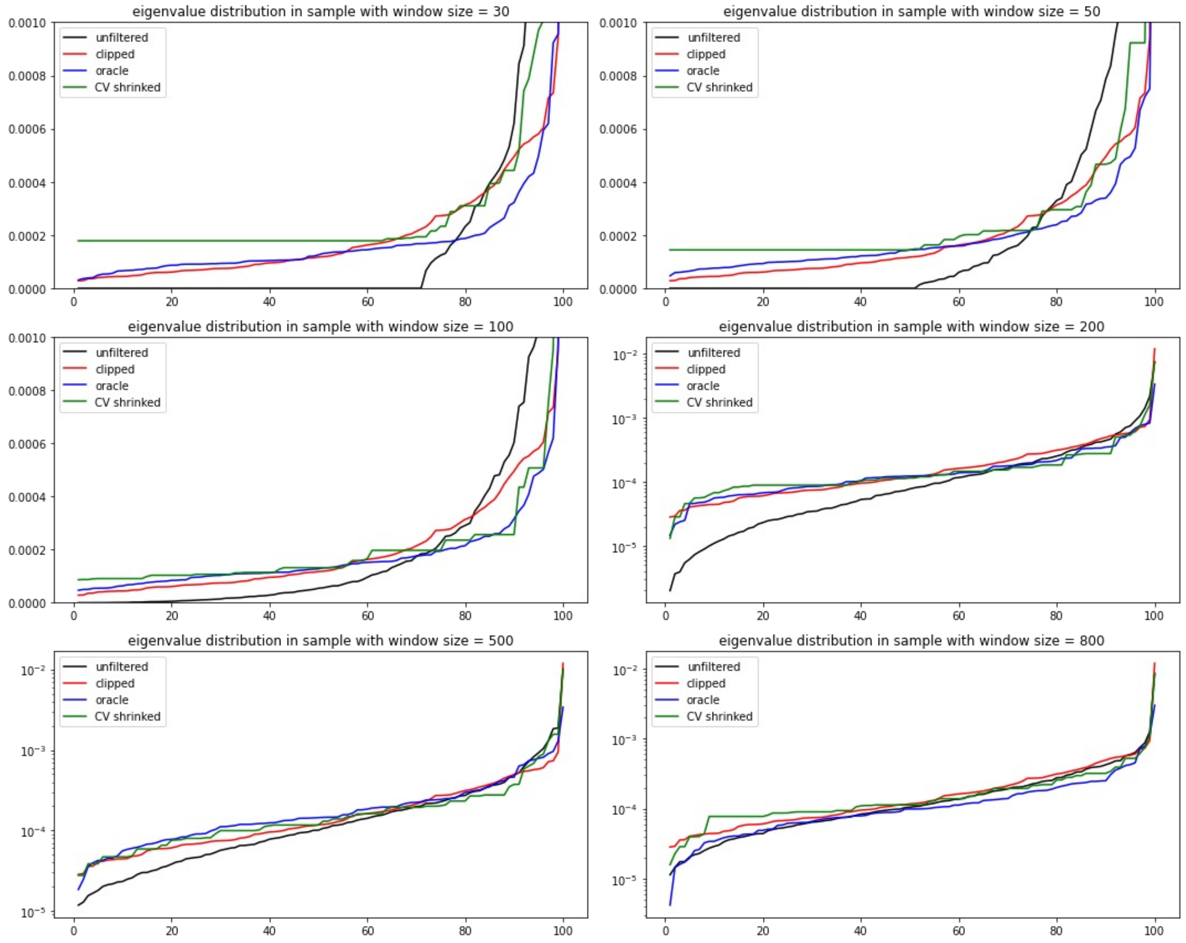
COMPARISON

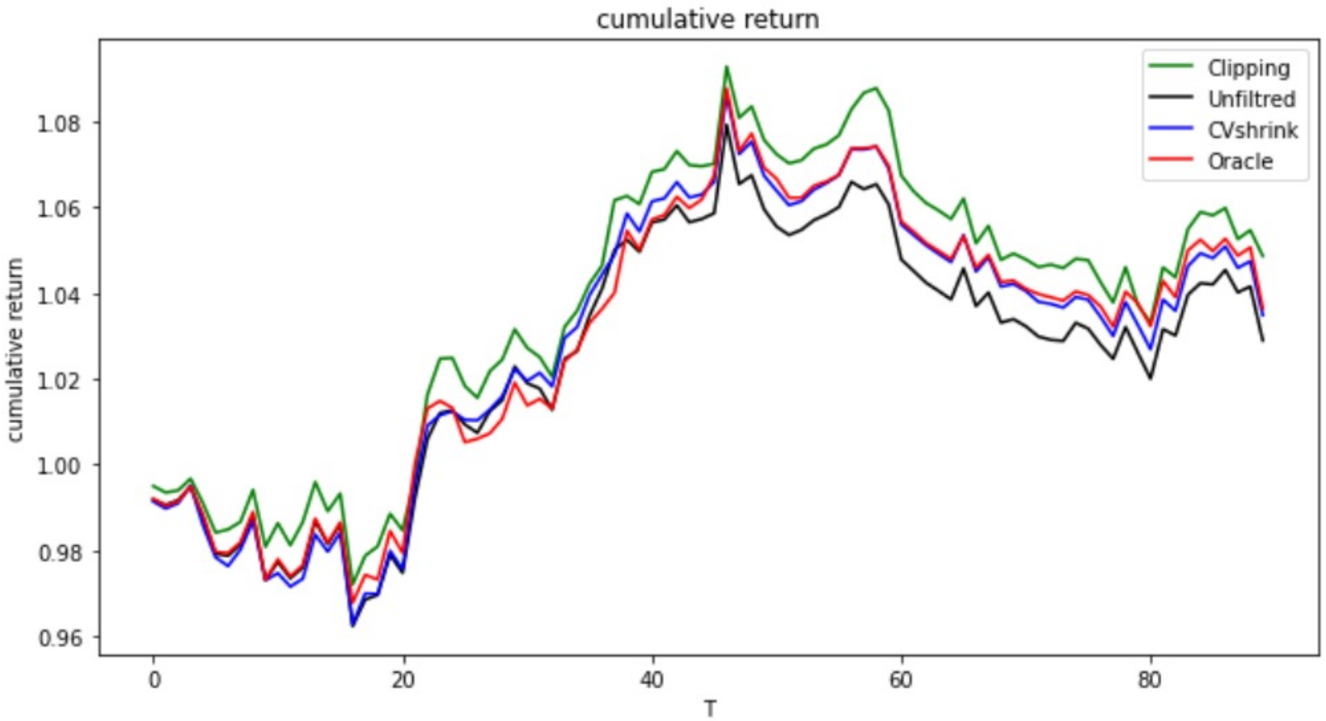
Eigenvalue distribution



- The effect of noise filtering and covariance cleaning is clearer with 100 assets
- In low regime, there are higher chances to have low eigenvalues (~0)
- These methods try to deal with sample size error

The cost of transaction, turnover and cumulative return

The cumulative return is $r_{t+1}^{cum} = c \cdot r_t^{cum} \sum_{i=1}^{N} (1 + r_i) w_i^{\tau+1}$ with $c = 1 - p \sum_{i=1}^{N} |w_i^{\tau} - w_i^{\tau+1}|$



The turnover enables to represent the number of transactions:

$$\Upsilon = \frac{\sum_{\tau=1}^{\Delta_\tau - 1} \sum_{i=1}^{N} |w_i^{\tau} - w_i^{\tau+1}|}{\Delta_\tau}$$

Cov	Unfiltered	CVshrink	Oracle	Clipping
Turnover	0,039	0,048	0,055	0,023

The different turnover explain partially the better performance of the clipping covariance.

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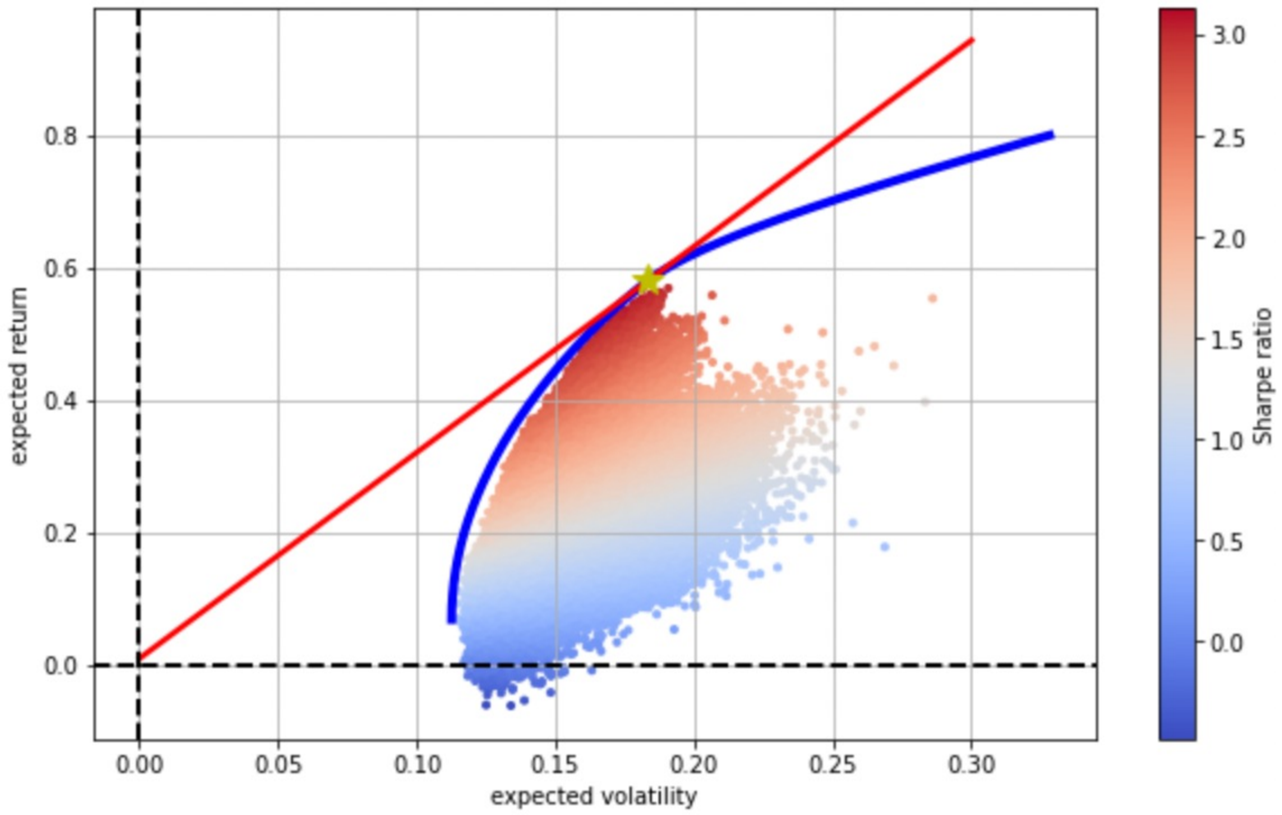
ADDING A RISK-FREE ASSET

The Capital Market Line

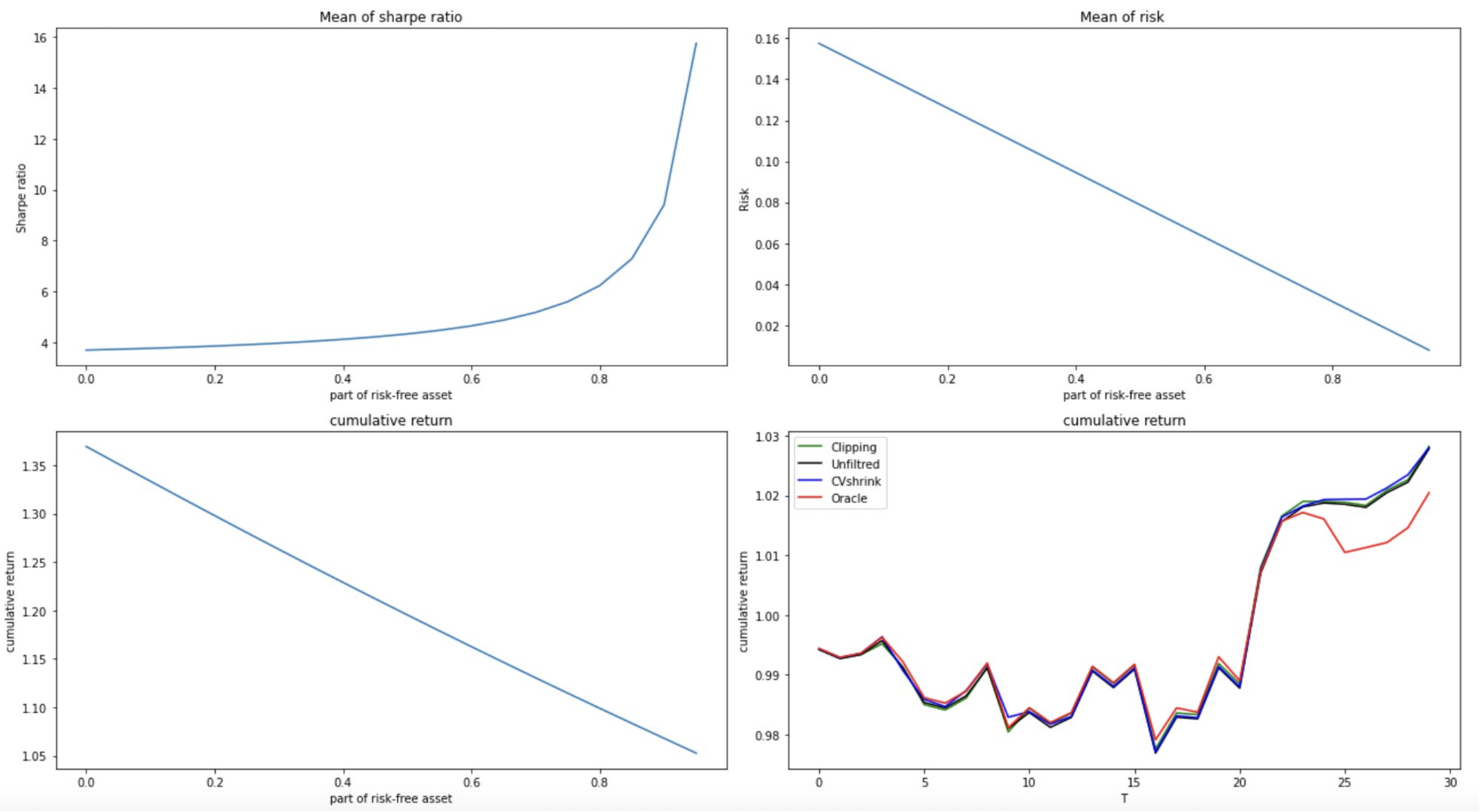
We have to find the intersection between the tangent of the efficient frontier and the risk-free asset.

$$R_p = r_f + \frac{R_t - r_f}{\sigma_T}\sigma_p$$

The advantage of this asset is to cross the efficient border and access to a low risk with a correct return.



The metrics with free-risk assets



The Sharpe ratio is not relevant if we build a portfolio with exclusively a risk free asset

The risk-free asset has a volatility of 0 and no correlation which explains the linearity of the mean of risk and of the cumulative return.

We observe on the cumulative return that the form of the efficient frontier reduce the sensitivity to the covariance.

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CONCLUSION

Key points to remember

- In general, portfolios don’t perform as well in out sample as they do in in sample
- We often need to work in low regime, which increases the risk of having 0-eigenvalues and sample size error
- An investor defines a criterion which translates in an optimisation problem
- Depending, on risk and error aversions, investor control the level of risk and error they can be exposed to
- Different covariance estimators have different results
- The oracle estimator assumes we know an out sample covariance matrix
- Adding a risk-free asset changes the efficient frontier and decreases the overall risk
- Depending on the regime and proportion of risk-free asset, covariance estimators may produce similar outputs

Thank you!