CHAPTER 11
Properties of Stock Options

Practice Questions

11.1
The lower bound is

$$28-25e^{-0.08\times0.3333}=\$3.66$$

11.2
The lower bound is

$$15e^{-0.06\times0.08333}-12=\$2.93$$

11.3
Delaying exercise delays the payment of the strike price. This means that the option holder is able to earn interest on the strike price for a longer period of time. Delaying exercise also provides insurance against the stock price falling below the strike price by the expiration date. Assume that the option holder has an amount of cash K and that interest rates are zero. When the option is exercised early it is worth S_T at expiration. Delaying exercise means that it will be worth $\max(K, S_T)$ at expiration.

11.4
An American put when held in conjunction with the underlying stock provides insurance. It guarantees that the stock can be sold for the strike price, *K*. If the put is exercised early, the insurance ceases. However, the option holder receives the strike price immediately and is able to earn interest on it between the time of the early exercise and the expiration date.

11.5
An American call option can be exercised at any time. If it is exercised, its holder gets the intrinsic value. It follows that an American call option must be worth at least its intrinsic value. A European call option can be worth less than its intrinsic value. Consider, for example, the situation where a stock is expected to provide a very high dividend during the life of an option. The price of the stock will decline as a result of the dividend. Because the European option can be exercised only after the dividend has been paid, its value may be less than the intrinsic value today.

11.6
In this case, $c=1$, $T=0.25$, $S_0=19$, $K=20$, and $r=0.04$. From put–call parity

$$p=c+Ke^{-rT}-S_0$$

or

$$p=1+20e^{-0.04\times0.25}-19=1.80$$

so that the European put price is $1.80.

11.7
When early exercise is not possible, we can argue that two portfolios that are worth the same at time T must be worth the same at earlier times. When early exercise is possible, the

argument is no longer valid. Suppose that $P+S>C+Ke^{-rT}$. This situation does not lead to an arbitrage opportunity. If we buy the call, short the put, and short the stock, we cannot be sure of the result because we do not know when the put will be exercised.

11.8

The lower bound is

$$80-75e^{-0.1\times 0.5}=\$8.66$$

11.9

The lower bound is

$$65e^{-0.05\times 2/12}-58=\$6.46$$

11.10

The present value of the strike price is $60e^{-0.12\times 4/12}=\$57.65$. The present value of the dividend is $0.80e^{-0.12\times 1/12}=0.79$. Because

$$5<64-57.65-0.79$$

the condition in equation (11.8) is violated. An arbitrageur should buy the option and short the stock. This generates $64-5=\$59$. The arbitrageur invests \$0.79 of this at 12% for one month to pay the dividend of \$0.80 in one month. The remaining \$58.21 is invested for four months at 12%. Regardless of what happens a profit will materialize.

If the stock price declines below \$60 in four months, the arbitrageur loses the \$5 spent on the option but gains on the short position. The arbitrageur shorts when the stock price is \$64, has to pay dividends with a present value of \$0.79, and closes out the short position when the stock price is \$60 or less. Because \$57.65 is the present value of \$60, the short position generates at least $64-57.65-0.79=\$5.56$ in present value terms. The present value of the arbitrageur's gain is therefore at least $5.56-5.00=\$0.56$.

If the stock price is above \$60 at the expiration of the option, the option is exercised. The arbitrageur buys the stock for \$60 in four months and closes out the short position. The present value of the \$60 paid for the stock is \$57.65 and as before the dividend has a present value of \$0.79. The gain from the short position and the exercise of the option is therefore exactly $64-57.65-0.79=\$5.56$. The arbitrageur's gain in present value terms is exactly $5.56-5.00=\$0.56$.

11.11

In this case, the present value of the strike price is $50e^{-0.06\times 1/12}=49.75$. Because

$$2.5<49.75-47.00$$

the condition in equation (11.5) is violated. An arbitrageur should borrow \$49.50 at 6% for one month, buy the stock, and buy the put option. This generates a profit in all circumstances.

If the stock price is above \$50 in one month, the option expires worthless, but the stock can be sold for at least \$50. A sum of \$50 received in one month has a present value of \$49.75 today. The strategy therefore generates profit with a present value of at least \$0.25.

If the stock price is below \$50 in one month the put option is exercised and the stock owned is sold for exactly \$50 (or \$49.75 in present value terms). The trading strategy therefore generates a profit of exactly \$0.25 in present value terms.

11.12

The early exercise of an American put is attractive when the interest earned on the strike price is greater than the insurance element lost. When interest rates increase, the value of the interest earned on the strike price increases making early exercise more attractive. When volatility decreases, the insurance element is less valuable. Again this makes early exercise more attractive.

11.13

Using the notation in the chapter, put−call parity [equation (11.10)] gives

$$c + Ke^{-rT} + D = p + S_0$$

or

$$p = c + Ke^{-rT} + D - S_0$$

In this case

$$p = 2 + 30e^{-0.1\times6/12} + (0.5e^{-0.1\times2/12} + 0.5e^{-0.1\times5/12}) - 29 = 2.51$$

In other words, the put price is \$2.51.

11.14

If the put price is \$3.00, it is too high relative to the call price. An arbitrageur should buy the call, short the put and short the stock. This generates $-2+3+29=\$30$ in cash which is invested at 10%. Regardless of what happens a profit with a present value of $3.00-2.51=\$0.49$ is locked in.

If the stock price is above \$30 in six months, the call option is exercised, and the put option expires worthless. The call option enables the stock to be bought for \$30, or $30e^{-0.10\times6/12}=\$28.54$ in present value terms. The dividends on the short position cost $0.5e^{-0.1\times2/12}+0.5e^{-0.1\times5/12}=\$0.97$ in present value terms so that there is a profit with a present value of $30-28.54-0.97=\$0.49$.

If the stock price is below \$30 in six months, the put option is exercised and the call option expires worthless. The short put option leads to the stock being bought for \$30, or $30e^{-0.10\times6/12}=\$28.54$ in present value terms. The dividends on the short position cost $0.5e^{-0.1\times2/12}+0.5e^{-0.1\times5/12}=\$0.97$ in present value terms so that there is a profit with a present value of $30-28.54-0.97=\$0.49$.

11.15

From equation (11.7)

$$S_0 - K \le C - P \le S_0 - Ke^{-rT}$$

In this case,

$$31 - 30 \le 4 - P \le 31 - 30e^{-0.08\times0.25}$$

or

$$1.00 \le 4.00 - P \le 1.59$$

or

$$2.41 \le P \le 3.00$$

Upper and lower bounds for the price of an American put are therefore \$2.41 and \$3.00.

11.16

If the American put price is greater than \$3.00, an arbitrageur can sell the American put, short the stock, and buy the American call. This realizes at least $3+31-4=\$30$ which can be invested at the risk-free interest rate. At some stage during the 3-month period either the American put or the American call will be exercised. The arbitrageur then pays \$30, receives the stock and closes out the short position. The cash flows to the arbitrageur are $+\$30$ at time zero and $-\$30$ at some future time. These cash flows have a positive present value.

11.17

As in the text, we use c and p to denote the European call and put option price, and C and P to denote the American call and put option prices. Because $P \geq p$, it follows from put–call parity that

$$P \geq c + Ke^{-rT} - S_0$$

and since $c = C$,

$$P \geq C + Ke^{-rT} - S_0$$

or

$$C - P \leq S_0 - Ke^{-rT}$$

For a further relationship between C and P, consider

Portfolio I: One European call option plus an amount of cash equal to K.

Portfolio J: One American put option plus one share.

Both options have the same exercise price and expiration date. Assume that the cash in portfolio I is invested at the risk-free interest rate. If the put option is not exercised early, portfolio J is worth

$$\max(S_T, K)$$

at time T. Portfolio I is worth

$$\max(S_T - K, 0) + Ke^{rT} = \max(S_T, K) - K + Ke^{rT}$$

at this time. Portfolio I is therefore worth more than portfolio J. Suppose next that the put option in portfolio J is exercised early, say, at time τ. This means that portfolio J is worth K at time τ. However, even if the call option were worthless, portfolio I would be worth $Ke^{r\tau}$ at time τ. It follows that portfolio I is worth at least as much as portfolio J in all circumstances. Hence

$$c + K \geq P + S_0$$

Since $c = C$,

$$C + K \geq P + S_0$$

or

$$C - P \geq S_0 - K$$

Combining this with the other inequality derived above for $C - P$, we obtain

$$S_0 - K \leq C - P \leq S_0 - Ke^{-rT}$$

11.18

As in the text, we use c and p to denote the European call and put option price, and C and P to denote the American call and put option prices. The present value of the dividends will be denoted by D. As shown in the answer to Problem 11.24, when there are no dividends

$$C - P \leq S_0 - Ke^{-rT}$$

Dividends reduce C and increase P. Hence this relationship must also be true when there are dividends.

For a further relationship between C and P, consider

Portfolio I: One European call option plus an amount of cash equal to $D + K$.

Portfolio J: One American put option plus one share.

Both options have the same exercise price and expiration date. Assume that the cash in portfolio I is invested at the risk-free interest rate. If the put option is not exercised early, portfolio J is worth

$$\max(S_T, K) + De^{rT}$$

at time T. Portfolio I is worth

$$\max(S_T - K, 0) + (D+K)e^{rT} = \max(S_T, K) + De^{rT} + Ke^{rT} - K$$

at this time. Portfolio I is therefore worth more than portfolio J. Suppose next that the put option in portfolio J is exercised early, say, at time τ. This means that portfolio J is worth at most $K + De^{r\tau}$ at time τ. However, even if the call option were worthless, portfolio I would be worth $(D+K)e^{r\tau}$ at time τ. It follows that portfolio I is worth more than portfolio J in all circumstances. Hence

$$c + D + K \geq P + S_0$$

Because $C \geq c$

$$C - P \geq S_0 - D - K$$

11.19

An employee stock option may be exercised early because the employee needs cash or because he or she is uncertain about the company's future prospects. Regular call options can be sold in the market in either of these two situations, but employee stock options cannot be sold. In theory, an employee can short the company's stock as an alternative to exercising. In practice, this is not usually encouraged and may even be illegal for senior managers. These points are discussed further in Chapter 16.

11.20

The graphs can be produced from the first worksheet in DerivaGem. Select Equity as the Underlying Type. Select Black−Scholes−European as the Option Type. Input stock price as 50, volatility as 30%, risk-free rate as 5%, time to exercise as 1 year, and exercise price as 50. Leave the dividend table blank because we are assuming no dividends. Select the button corresponding to call. Do not select the implied volatility button. Hit the *Enter* key and click on calculate. DerivaGem will show the price of the option as 7.15562248. Move to the Graph Results on the right hand side of the worksheet. Enter Option Price for the vertical axis and Asset price for the horizontal axis. Choose the minimum strike price value as 10 (software will not accept 0) and the maximum strike price value as 100. Hit *Enter* and click on *Draw Graph*. This will produce Figure 11.1a. Figures 11.1c, 11.1e, 11.2a, and 11.2c can be produced similarly by changing the horizontal axis. By selecting put instead of call and recalculating the rest of the figures can be produced. You are encouraged to experiment with this worksheet. Try different parameter values and different types of options.

11.21

a) The put−call parity result still holds. The arguments are unchanged.
b) Deep-in-the-money American calls might be exercised early because option holder will prefer to pay the strike price earlier.
c) Deep-in the-money American puts should not be exercised early because the holder would rather delay receiving the strike price.

11.22

Because no dividends are paid, the call can be regarded as a European call. Put−call parity can be used to create a European put from the call. A European put plus the stock equals a European call plus the present value of the strike price when both the call and the put have the same strike price and maturity date. A European put can be created by buying the call, shorting the stock, and keeping an amount of cash that when invested at the risk-free rate will grow to be sufficient to exercise the call. If the stock price is above the strike price, the call is exercised and the short position is closed out for no net payoff. If the stock price is below the strike price, the call is not exercised and the short position is closed out for a gain equal to the put payoff.

11.23

From put−call parity

$$20 + 120e^{-r\times 1} = 5 + 130$$

Solving this

$$e^{-r} = 115/120$$

so that $r = -\ln(115/120) = 0.0426$ or 4.26%

11.24

If the call is worth \$3, put−call parity shows that the put should be worth

$$3+20e^{-0.10\times 3/12}+e^{-0.1\times 1/12}-19=4.50$$

This is greater than \$3. The put is therefore undervalued relative to the call. The correct arbitrage strategy is to buy the put, buy the stock, and short the call. This costs \$19. If the stock price in three months is greater than \$20, the call is exercised. If it is less than \$20, the put is exercised. In either case, the arbitrageur sells the stock for \$20 and collects the \$1 dividend in one month. The present value of the gain to the arbitrageur is

$$-3-19+3+20e^{-0.10\times 3/12}+e^{-0.1\times 1/12}=1.50$$

11.25

Consider a portfolio that is long one option with strike price K_1, long one option with strike price K_3, and short two options with strike price K_2. The value of the portfolio can be worked out in four different situations:

$S_T \le K_1$: Portfolio Value $=0$

$K_1 < S_T \le K_2$: Portfolio Value $=S_T - K_1$

$K_2 < S_T \le K_3$: Portfolio Value $=S_T - K_1 - 2(S_T - K_2) = K_2 - K_1 - (S_T - K_2) \ge 0$

$S_T > K_3$: Portfolio Value $=S_T - K_1 - 2(S_T - K_2) + S_T - K_3 = K_2 - K_1 - (K_3 - K_2) = 0$

The value is always either positive or zero at the expiration of the option. In the absence of arbitrage possibilities, it must be positive or zero today. This means that

$$c_1 + c_3 - 2c_2 \ge 0$$

or

$$c_2 \le 0.5(c_1 + c_3)$$

Note that students often think they have proved this by writing down

$$c_1 \le S_0 - K_1 e^{-rT}$$

$$2c_2 \le 2(S_0 - K_2 e^{-rT})$$

$$c_3 \le S_0 - K_3 e^{-rT}$$

and subtracting the middle inequality from the sum of the other two. But they are deceiving themselves. Inequality relationships cannot be subtracted. For example, $9>8$ and $5>2$, but it is not true that $9-5>8-2$!

11.26

The corresponding result is

$$p_2 \le 0.5(p_1 + p_3)$$

where p_1, p_2 and p_3 are the prices of European put option with the same maturities and strike prices K_1, K_2 and K_3, respectively. This can be proved from the result in Problem 11.25 using put–call parity. Alternatively, we can consider a portfolio consisting of a long position in a put option with strike price K_1, a long position in a put option with strike price K_3, and a short position in two put options with strike price K_2. The value of this portfolio in different situations is given as follows:

$S_T \leq K_1$: Portfolio Value $= K_1 - S_T - 2(K_2 - S_T) + K_3 - S_T = K_3 - K_2 - (K_2 - K_1) = 0$

$K_1 < S_T \leq K_2$: Portfolio Value $= K_3 - S_T - 2(K_2 - S_T) = K_3 - K_2 - (K_2 - S_T) \geq 0$

$K_2 < S_T \leq K_3$: Portfolio Value $= K_3 - S_T$

$S_T > K_3$: Portfolio Value $= 0$

Because the portfolio value is always zero or positive at some future time the same must be true today. Hence,

$$p_1 + p_3 - 2p_2 \geq 0$$

or

$$p_2 \leq 0.5(p_1 + p_3)$$