CHAPTER 17
Options on Stock Indices and Currencies

Practice Questions

17.1
The lower bound is given by equation 17.1 as

$$300e^{-0.03\times0.5} - 290e^{-0.08\times0.5} = 16.90$$

17.2
In this case, $u = 1.0502$ and $p = 0.4538$. The tree is shown in Figure S17.1. The value of the option if it is European is \$0.0235; the value of the option if it is American is \$0.0250.

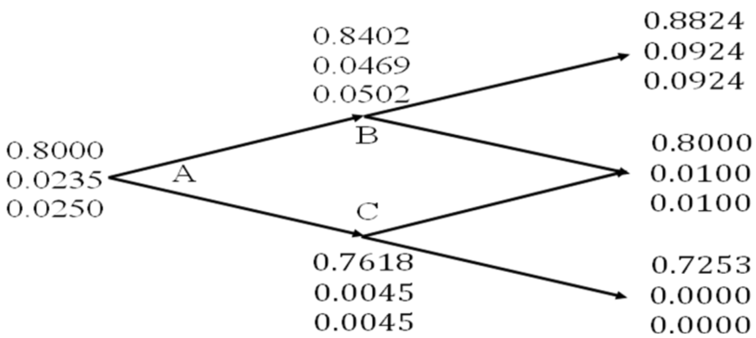

Figure S17.1: *Tree to evaluate European and American call options in Problem 17.2. At each node, upper number is the stock price; next number is the European call price; final number is the American call price*

17.3
A range forward contract allows a corporation to ensure that the exchange rate applicable to a transaction will not be worse than one exchange rate and will not be better than another exchange rate. In this case, a corporation would buy a put with the lower exchange rate and sell a call with the higher exchange rate.

17.4
In this case, $S_0 = 250$, $K = 250$, $r = 0.10$, $\sigma = 0.18$, $T = 0.25$, $q = 0.03$ and

$$d_1 = \frac{\ln(250/250) + (0.10 - 0.03 + 0.18^2/2)0.25}{0.18\sqrt{0.25}} = 0.2394$$

$$d_2 = d_1 - 0.18\sqrt{0.25} = 0.1494$$

and the call price is

$$250N(0.2394)e^{-0.03\times0.25} - 250N(0.1494)e^{-0.10\times0.25}$$

$$= 250\times0.5946e^{-0.03\times0.25} - 250\times0.5594e^{-0.10\times0.25}$$

or 11.15.

17.5

In this case, $S_0 = 0.52$, $K = 0.50$, $r = 0.04$, $r_f = 0.08$, $\sigma = 0.12$, $T = 0.6667$, and

$$d_1 = \frac{\ln(0.52/0.50) + (0.04 - 0.08 + 0.12^2/2)0.6667}{0.12\sqrt{0.6667}} = 0.1771$$

$$d_2 = d_1 - 0.12\sqrt{0.6667} = 0.0791$$

and the put price is

$$0.50N(-0.0791)e^{-0.04\times 0.6667} - 0.52N(-0.1771)e^{-0.08\times 0.6667}$$

$$= 0.50 \times 0.4685e^{-0.04\times 0.6667} - 0.52 \times 0.4297e^{-0.08\times 0.6667}$$

$$= 0.0162$$

17.6

A put option to sell one unit of currency A for K units of currency B is worth

$$Ke^{-r_B T}N(-d_2) - S_0 e^{-r_A T}N(-d_1)$$

where

$$d_1 = \frac{\ln(S_0/K) + (r_B - r_A + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0/K) + (r_B - r_A - \sigma^2/2)T}{\sigma\sqrt{T}}$$

and r_A and r_B are the risk-free rates in currencies A and B, respectively. The value of the option is measured in units of currency B. Defining $S_0^* = 1/S_0$ and $K^* = 1/K$

$$d_1 = \frac{-\ln(S_0^*/K^*) - (r_A - r_B - \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{-\ln(S_0^*/K^*) - (r_A - r_B + \sigma^2/2)T}{\sigma\sqrt{T}}$$

The put price is therefore

$$S_0 K[S_0^* e^{-r_B T}N(d_1^*) - K^* e^{-r_A T}N(d_2^*)$$

where

$$d_1^* = -d_2 = \frac{\ln(S_0^*/K^*) + (r_A - r_B + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2^* = -d_1 = \frac{\ln(S_0^*/K^*) + (r_A - r_B - \sigma^2/2)T}{\sigma\sqrt{T}}$$

This shows that put option is equivalent to KS_0 call options to buy 1 unit of currency A for $1/K$ units of currency B. In this case, the value of the option is measured in units of currency A. To obtain the call option value in units of currency B (the same units as the value of the put option was measured in) we must divide by S_0. This proves the result.

17.7

Lower bound for European option is

$$S_0 e^{-r_f T} - Ke^{-rT} = 1.5e^{-0.09\times 0.5} - 1.4e^{-0.05\times 0.5} = 0.069$$

Lower bound for American option is

$$S_0 - K = 0.10$$

17.8

In this case, $S_0 = 250$, $q = 0.04$, $r = 0.06$, $T = 0.25$, $K = 245$, and $c = 10$. Using put–call parity

$$c + Ke^{-rT} = p + S_0 e^{-qT}$$

or

$$p = c + Ke^{-rT} - S_0 e^{-qT}$$

Substituting,

$$p = 10 + 245e^{-0.25\times 0.06} - 250e^{-0.25\times 0.04} = 3.84$$

The put price is 3.84.

17.9

In this case, $S_0 = 696$, $K = 700$, $r = 0.07$, $\sigma = 0.3$, $T = 0.25$ and $q = 0.04$. The option can be valued using equation (17.5).

$$d_1 = \frac{\ln(696/700) + (0.07 - 0.04 + 0.09/2)\times 0.25}{0.3\sqrt{0.25}} = 0.0868$$

$$d_2 = d_1 - 0.3\sqrt{0.25} = -0.0632$$

and

$$N(-d_1) = 0.4654, \quad N(-d_2) = 0.5252$$

The value of the put, p, is given by:

$$p = 700e^{-0.07\times 0.25} \times 0.5252 - 696e^{-0.04\times 0.25} \times 0.4654 = 40.6$$

that is, it is \$40.6.

17.10

Following the hint, we first consider:

Portfolio A: A European call option plus an amount K invested at the risk-free rate.

Portfolio B: An American put option plus e^{-qT} of stock with dividends being reinvested in the stock.

Portfolio A is worth $c + K$ while portfolio B is worth $P + S_0 e^{-qT}$. If the put option is exercised at time $\tau (0 \le \tau < T)$, portfolio B becomes:

$$K - S_\tau + S_\tau e^{-q(T-\tau)} \le K$$

where S_τ is the stock price at time τ. Portfolio A is worth

$$c + Ke^{r\tau} \ge K$$

Hence, portfolio A is worth at least as much as portfolio B. If both portfolios are held to maturity (time T), portfolio A is worth

$$\max(S_T - K, 0) + Ke^{rT}$$

$$= \max(S_T, K) + K(e^{rT} - 1)$$

Portfolio B is worth $\max(S_T, K)$. Hence portfolio A is worth more than portfolio B.

Because portfolio A is worth at least as much as portfolio B in all circumstances

$$P + S_0 e^{-qT} \le c + K$$

Because $c \le C$,

$$P + S_0 e^{-qT} \le C + K$$

or

$$S_0 e^{-qT} - K \le C - P$$

This proves the first part of the inequality.

For the second part consider:
Portfolio C: An American call option plus an amount Ke^{-rT} invested at the risk-free rate.
Portfolio D: A European put option plus one stock with dividends being reinvested in the stock.
Portfolio C is worth $C + Ke^{-rT}$ while portfolio D is worth $p + S_0$. If the call option is exercised at time $\tau\,(0 \le \tau < T)$, portfolio C becomes:

$$S_\tau - K + Ke^{-r(T-\tau)} < S_\tau$$

while portfolio D is worth

$$p + S_\tau e^{q(\tau - t)} \ge S_\tau$$

Hence, portfolio D is worth more than portfolio C. If both portfolios are held to maturity (time T), portfolio C is worth $\max(S_T, K)$ while portfolio D is worth

$$\max(K - S_T, 0) + S_T e^{qT}$$

$$= \max(S_T, K) + S_T(e^{qT} - 1)$$

Hence, portfolio D is worth at least as much as portfolio C.
Since portfolio D is worth at least as much as portfolio C in all circumstances:

$$C + Ke^{-rT} \le p + S_0$$

Since $p \le P$,

$$C + Ke^{-rT} \le P + S_0$$

or

$$C - P \le S_0 - Ke^{-rT}$$

This proves the second part of the inequality. Hence,

$$S_0 e^{-qT} - K \le C - P \le S_0 - Ke^{-rT}$$

17.11
This follows from put–call parity and the relationship between the forward price, F_0, and the spot price, S_0.

$$c + Ke^{-rT} = p + S_0 e^{-r_f T}$$

and

$$F_0 = S_0 e^{(r - r_f)T}$$

so that

$$c + Ke^{-rT} = p + F_0 e^{-rT}$$

If $K = F_0$ this reduces to $c = p$. The result that $c = p$ when $K = F_0$ is true for options on all underlying assets, not just options on currencies. An at-the-money option is frequently defined as one where $K = F_0$ (or $c = p$) rather than one where $K = S_0$.

17.12
The volatility of a stock index can be expected to be less than the volatility of a typical stock. This is because some risk (i.e., return uncertainty) is diversified away when a portfolio of stocks is created. In capital asset pricing model terminology, there exists systematic and unsystematic risk in the returns from an individual stock. However, in a stock index, unsystematic risk has been diversified away and only the systematic risk contributes to volatility.

17.13
The cost of portfolio insurance increases as the beta of the portfolio increases. This is because portfolio insurance involves the purchase of a put option on the portfolio. As beta increases, the volatility of the portfolio increases causing the cost of the put option to increase. When index options are used to provide portfolio insurance, both the number of options required and the strike price increase as beta increases.

17.14
If the value of the portfolio mirrors the value of the index, the index can be expected to have dropped by 10% when the value of the portfolio drops by 10%. Hence, when the value of the portfolio drops to \$54 million the value of the index can be expected to be 1,080. This indicates that put options with an exercise price of 1080 should be purchased. The options should be on:

$$\frac{60,000,000}{1200} = \$50,000$$

times the index. Each option contract is for \$100 times the index. Hence, 500 contracts should be purchased.

17.15
When the value of the portfolio falls to \$54 million the holder of the portfolio makes a capital loss of 10%. After dividends are taken into account, the loss is 7% during the year. This is 12% below the risk-free interest rate. According to the capital asset pricing model, the expected excess return of the portfolio above the risk-free rate equals beta times the expected excess return of the market above the risk-free rate.
Therefore, when the portfolio provides a return 12% below the risk-free interest rate, the market's expected return is 6% below the risk-free interest rate. As the index can be assumed to have a beta of 1.0, this is also the excess expected return (including dividends) from the index. The expected return from the index is therefore -1% per annum. Since the index provides a 3% per annum dividend yield, the expected movement in the index is -4%. Thus, when the portfolio's value is \$54 million, the expected value of the index is $0.96 \times 1200 = 1152$. Hence, European put options should be purchased with an exercise price of 1152. Their maturity date should be in one year.
The number of options required is twice the number required in Problem 17.14. This is because we wish to protect a portfolio which is twice as sensitive to changes in market conditions as the portfolio in Problem 17.14. Hence, options on \$100,000 (or 1,000 contracts) should be purchased. To check that the answer is correct, consider what happens when the value of the portfolio declines by 20% to \$48 million. The return including dividends is -17%. This is 22% less than the risk-free interest rate. The index can be expected to provide a return (including dividends) which is 11% less than the risk-free interest rate, that is, a return of -6%. The index can therefore be expected to drop by 9% to 1092. The payoff from

the put options is $(1152-1092)\times 100,000=\$6$ million. This is exactly what is required to restore the value of the portfolio to $54 million.

17.16
The implied dividend yield is the value of q that satisfies the put–call parity equation. It is the value of q that solves

$$154+1400e^{-0.05\times 0.5}=34.25+1500e^{-0.5q}$$

This is 1.99%.

17.17
A total return index behaves like a stock paying no dividends. In a risk-neutral world, it can be expected to grow on average at the risk-free rate. Forward contracts and options on total return indices should be valued in the same way as forward contracts and options on non-dividend-paying stocks.

17.18
The put–call parity relationship for European currency options is

$$c+Ke^{-rT}=p+Se^{-r_fT}$$

To prove this result, the two portfolios to consider are:
Portfolio A: One call option plus one discount bond which will be worth K at time *T*.
Portfolio B: One put option plus e^{-r_fT} of foreign currency invested at the foreign risk-free interest rate.
Both portfolios are worth $\max(S_T, K)$ at time T. They must therefore be worth the same today. The result follows.

17.19
In portfolio A, the cash, if it is invested at the risk-free interest rate, will grow to K at time T. If $S_T>K$, the call option is exercised at time T and portfolio A is worth S_T. If $S_T<K$, the call option expires worthless and the portfolio is worth K. Hence, at time *T*, portfolio A is worth

$$\max(S_T, K)$$

Because of the reinvestment of dividends, portfolio B becomes one share at time T. It is, therefore, worth S_T at this time. It follows that portfolio A is always worth as much as, and is sometimes worth more than, portfolio B at time T. In the absence of arbitrage opportunities, this must also be true today. Hence,

$$c+Ke^{-rT}\geq S_0e^{-qT}$$

or

$$c\geq S_0e^{-qT}-Ke^{-rT}$$

This proves equation (17.1).
In portfolio C, the reinvestment of dividends means that the portfolio is one put option plus one share at time *T*. If $S_T<K$, the put option is exercised at time T and portfolio C is worth K. If $S_T>K$, the put option expires worthless and the portfolio is worth S_T. Hence, at time T, portfolio C is worth

$$\max(S_T, K)$$

Portfolio D is worth K at time T. It follows that portfolio C is always worth as much as, and is sometimes worth more than, portfolio D at time T. In the absence of arbitrage opportunities, this must also be true today. Hence,

$$p + S_0 e^{-qT} \geq Ke^{-rT}$$

or

$$p \geq Ke^{-rT} - S_0 e^{-qT}$$

This proves equation (17.2).
Portfolios A and C are both worth $\max(S_T, K)$ at time T. They must, therefore, be worth the same today, and the put–call parity result in equation (17.3) follows.

17.20
There is no way of doing this. A natural idea is to create an option to exchange K euros for one yen from an option to exchange Y dollars for 1 yen and an option to exchange K euros for Y dollars. The problem with this is that it assumes that either both options are exercised or that neither option is exercised. There are always some circumstances where the first option is in-the-money at expiration while the second is not and vice versa.

17.21
We assume the time to maturity is 0.1667 years. We set the asset price and strike price equal to 270, the risk-free rate equal to 0.25% , the dividend yield equal to 2% and the call option price equal to 5.35. DerivaGem gives the implied volatility as 13.07%.
From put call parity (equation 17.3) the price of the put, p, is given by

$$5.35 + 270e^{-0.0025\times0.1667} = p + 270^{-0.02\times0.1667}$$

so that p=6.14. DerivaGem shows that the implied volatility is 13.07% (as for the call).
A European call has the same implied volatility as a European put when both have the same strike price and time to maturity. This is formally proved in Chapter 20.

17.22
(a) The price is 14.39 as indicated by the tree in Figure S17.2.
(b) The price is 14.97 as indicated by the tree in Figure S17.3.

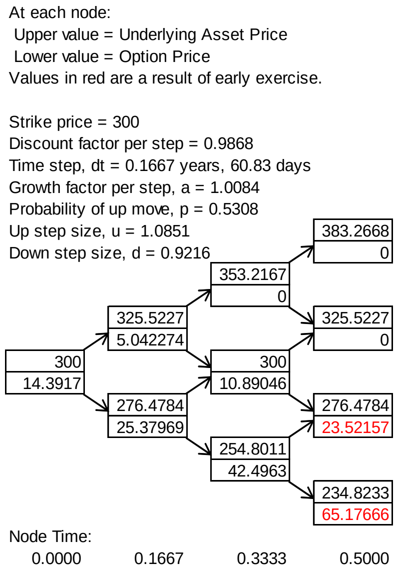

Figure S17.2: *Tree for valuing the European option in Problem 17.22*

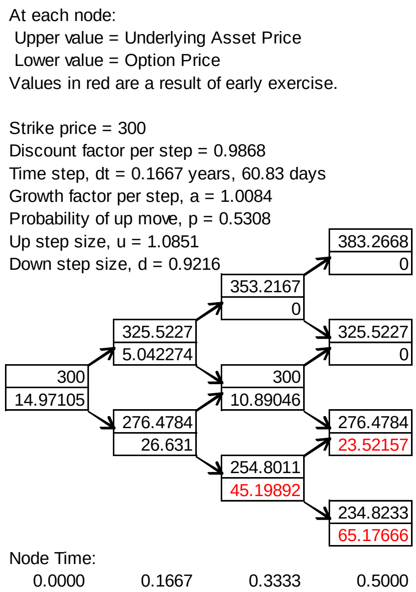

Figure S17.3: *Tree for valuing the American option in Problem 17.22*

17.23

In this case, S_0 =0.75, K=0.75, $r = 0.05$, $r_f = 0.04$, $\sigma = 0.08$ and $T = 0.75$. The option can be valued using equation (17.8)

$$d_1 = 0.1429, \qquad d_2 = 0.0736$$

and

$$N(d_1) = 0.5568, \quad N(d_2) = 0.5293$$

The value of the call, c, is given by

$$c = 0.75e^{-0.04\times0.75} \times 0.5568 - 0.75e^{-0.05\times0.75} \times 0.5293 = 0.0229$$

that is, it is 2.29 cents. From put–call parity

$$p + S_0 e^{-r_f T} = c + Ke^{-rT}$$

so that

$$p = 0.0229 + 0.75\mathrm{e}^{-0.05\times0.75} - 0.75\mathrm{e}^{-0.04\times0.75} = 0.0174$$

The option to buy US\$0.75 with C\$1.00 is the same as the same as an option to sell one Canadian dollar for US\$0.75. This means that it is a put option on the Canadian dollar and its price is US\$0.0174 or 1.74 cents.

17.24

(a) From the formula at the end of Section 17.4

$$q = -\frac{1}{0.25}\ln\frac{78 - 26 + 950e^{-0.04\times0.25}}{1000} = 0.0299$$

The dividend yield is 2.99%.

(b) We can calculate the implied volatility using either the call or the put. The answer (given by DerivaGem) is 24.68% in both cases.

17.25

The price of currency B expressed in terms of currency A is $1/S$. From Ito's lemma, the process followed by $X = 1/S$ is

$$dX = [(r_B - r_A)S \times (-1/S^2) + 0.5\sigma^2 S^2 \times (2/S^3)]dt + \sigma S \times (-1/S^2)dz$$

or

$$dX = [r_A - r_B + \sigma^2]Xdt - \sigma Xdz$$

Symmetry arguments would suggest that it should be

$$dX = [r_A - r_B]Xdt - \sigma Xdz$$

This is Siegel's paradox and is discussed further in Business Snapshot 30.1.

17.26

In this case, the guarantee is valued as a put option with S_0 = 1,000, K = 1,000, r = 5%, q = 1%, σ = 15%, and T=10. The value of the guarantee is given by equation (17.5) as 38.46 or 3.8% of the value of the portfolio.