CHAPTER 12
Trading Strategies Involving Options

Practice Questions

12.1
An investor can create a butterfly spread by buying call options with strike prices of \$15 and \$20 and selling two call options with strike prices of \$$17\frac{1}{2}$. The initial investment is $4 + 0.5 - 2 \times 2 = \$0.5$. The following table shows the variation of profit with the final stock price:

Stock Price, S_T	*Profit*
$S_T < 15$	$-\frac{1}{2}$
$15 < S_T < 17\frac{1}{2}$	$(S_T - 15) - \frac{1}{2}$
$17\frac{1}{2} < S_T < 20$	$(20 - S_T) - \frac{1}{2}$
$S_T > 20$	$-\frac{1}{2}$

12.2
A strangle is created by buying both options. The pattern of profits is as follows:

Stock Price, S_T	*Profit*
$S_T < 45$	$(45 - S_T) - 5$
$45 < S_T < 50$	-5
$S_T > 50$	$(S_T - 50) - 5$

12.3
A bull spread using calls provides a profit pattern with the same general shape as a bull spread using puts (see Figures 12.2 and 12.3 in the text). Define p_1 and c_1 as the prices of put and call with strike price K_1 and p_2 and c_2 as the prices of a put and call with strike price K_2. From put−call parity

$$p_1 + S = c_1 + K_1 e^{-rT}$$

$$p_2 + S = c_2 + K_2 e^{-rT}$$

Hence,

$$p_1 - p_2 = c_1 - c_2 - (K_2 - K_1)e^{-rT}$$

This shows that the initial investment when the spread is created from puts is less than the initial investment when it is created from calls by an amount $(K_2 - K_1)e^{-rT}$. In fact, as mentioned in the text, the initial investment when the bull spread is created from puts is

negative, while the initial investment when it is created from calls is positive.
The profit when calls are used to create the bull spread is higher than when puts are used by $(K_2 - K_1)(1-e^{-rT})$. This reflects the fact that the call strategy involves an additional risk-free investment of $(K_2 - K_1)e^{-rT}$ over the put strategy. This earns interest of $(K_2 - K_1)e^{-rT}(e^{rT} - 1) = (K_2 - K_1)(1-e^{-rT})$.

12.4
An aggressive bull spread using call options is discussed in the text. Both of the options used have relatively high strike prices. Similarly, an aggressive bear spread can be created using put options. Both of the options should be out of the money (i.e., they should have relatively low strike prices). The spread then costs very little to set up because both of the puts are worth close to zero. In most circumstances, the spread will provide zero payoff. However, there is a small chance that the stock price will fall fast so that on expiration both options will be in the money. The spread then provides a payoff equal to the difference between the two strike prices, $K_2 - K_1$.

12.5
A bull spread is created by buying the $30 put and selling the $35 put. This strategy gives rise to an initial cash inflow of $3. The outcome is as follows:

Stock Price	*Payoff*	*Profit*
$S_T \geq 35$	0	3
$30 \leq S_T < 35$	$S_T - 35$	$S_T - 32$
$S_T < 30$	-5	-2

A bear spread is created by selling the $30 put and buying the $35 put. This strategy costs $3 initially. The outcome is as follows:

Stock Price	*Payoff*	*Profit*
$S_T \geq 35$	0	-3
$30 \leq S_T < 35$	$35 - S_T$	$32 - S_T$
$S_T < 30$	5	2

12.6
Define c_1, c_2, and c_3 as the prices of calls with strike prices K_1, K_2 and K_3. Define p_1, p_2 and p_3 as the prices of puts with strike prices K_1, K_2 and K_3. With the usual notation

$$c_1 + K_1 e^{-rT} = p_1 + S$$

$$c_2 + K_2 e^{-rT} = p_2 + S$$

$$c_3 + K_3 e^{-rT} = p_3 + S$$

Hence,

$$c_1 + c_3 - 2c_2 + (K_1 + K_3 - 2K_2)e^{-rT} = p_1 + p_3 - 2p_2$$

Because $K_2 - K_1 = K_3 - K_2$, it follows that $K_1 + K_3 - 2K_2 = 0$ and

$$c_1 + c_3 - 2c_2 = p_1 + p_3 - 2p_2$$

The cost of a butterfly spread created using European calls is therefore exactly the same as the cost of a butterfly spread created using European puts.

12.7

A straddle is created by buying both the call and the put. This strategy costs \$10. The profit/loss is shown in the following table:

Stock Price	*Payoff*	*Profit*
$S_T > 60$	$S_T - 60$	$S_T - 70$
$S_T \le 60$	$60 - S_T$	$50 - S_T$

This shows that the straddle will lead to a loss if the final stock price is between \$50 and \$70.

12.8

The bull spread is created by buying a put with strike price K_1 and selling a put with strike price K_2. The payoff is calculated as follows:

Stock Price	*Payoff from Long Put*	*Payoff from Short Put*	*Total Payoff*
$S_T \ge K_2$	0	0	0
$K_1 < S_T < K_2$	0	$S_T - K_2$	$-(K_2 - S_T)$
$S_T \le K_1$	$K_1 - S_T$	$S_T - K_2$	$-(K_2 - K_1)$

12.9

Possible strategies are:

Strangle
Straddle
Strip
Strap
Reverse calendar spread
Reverse butterfly spread

The strategies all provide positive profits when there are large stock price moves. A strangle is less expensive than a straddle, but requires a bigger move in the stock price in order to provide a positive profit. Strips and straps are more expensive than straddles but provide bigger profits in certain circumstances. A strip will provide a bigger profit when there is a large downward stock price move. A strap will provide a bigger profit when there is a large upward stock price move. In the case of strangles, straddles, strips and straps, the profit increases as the size of the stock price movement increases. By contrast, in a reverse calendar spread and a reverse butterfly spread, there is a maximum potential profit regardless of the size of the stock price movement.

12.10
Suppose that the delivery price is K and the delivery date is T. The forward contract is created by buying a European call and selling a European put when both options have strike price K and exercise date T. This portfolio provides a payoff of $S_T - K$ under all circumstances where S_T is the stock price at time T. Suppose that F_0 is the forward price. If $K = F_0$, the forward contract that is created has zero value. This shows that the price of a call equals the price of a put when the strike price is F_0.

12.11
A box spread is a bull spread created using calls together with a bear spread created using puts. With the notation in the text, it consists of a) a long call with strike K_1, b) a short call with strike K_2, c) a long put with strike K_2, and d) a short put with strike K_1. a) and d) give a long forward contract with delivery price K_1; b) and c) give a short forward contract with delivery price K_2. The two forward contracts taken together give the payoff of $K_2 - K_1$.

12.12
The result is shown in Figure S12.1. The profit pattern from a long position in a call and a put when the put has a higher strike price than a call is much the same as when the call has a higher strike price than the put. Both the initial investment and the final payoff are much higher in the first case

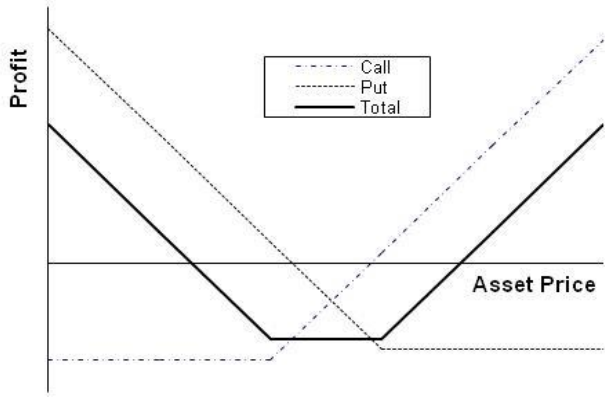

Figure S12.1: *Profit Pattern in Problem 12.12*

12.13
To use DerivaGem, select the first worksheet and choose Currency as the Underlying Type. Select Black−Scholes European as the Option Type. Input exchange rate as 0.64, volatility as 15%, risk-free rate as 5%, foreign risk-free interest rate as 4%, time to exercise as 1 year, and exercise price as 0.60. Select the button corresponding to call. Do not select the implied volatility button. Hit the Enter key and click on calculate. DerivaGem will show the price of the option as 0.0618. Change the exercise price to 0.65, hit Enter, and click on calculate again. DerivaGem will show the value of the option as 0.0352. Change the exercise price to 0.70, hit Enter, and click on calculate. DerivaGem will show the value of the option as 0.0181.
Now select the button corresponding to put and repeat the procedure. DerivaGem shows the

values of puts with strike prices 0.60, 0.65, and 0.70 to be 0.0176, 0.0386, and 0.0690, respectively.

The cost of setting up the butterfly spread when calls are used is therefore

$$0.0618 + 0.0181 - 2 \times 0.0352 = 0.0095$$

The cost of setting up the butterfly spread when puts are used is

$$0.0176 + 0.0690 - 2 \times 0.0386 = 0.0094$$

Allowing for rounding errors, these two are the same.

12.14

Assume that the investment in the index is initially \$100. (This is a scaling factor that makes no difference to the result.) DerivaGem can be used to value an option on the index with the index level equal to 100, the volatility equal to 20%, the risk-free rate equal to 4%, the dividend yield equal to 1%, and the exercise price equal to 100. For different times to maturity, T, we value a call option (using Black−Scholes European) and the amount available to buy the call option, which is $100 - 100e^{-0.04 \times T}$. Results are as follows:

Time to maturity, T	*Funds Available*	*Value of Option*
1	3.92	9.32
2	7.69	13.79
5	18.13	23.14
10	32.97	33.34
11	35.60	34.91

This table shows that the answer is between 10 and 11 years. Continuing the calculations, we find that if the life of the principal-protected note is 10.35 year or more, it is profitable for the bank. (Excel's Solver can be used in conjunction with the DerivaGem functions to facilitate calculations.)

12.15

If volatility is zero, the option to purchase the stock portfolio for \$1,000 when there are no dividends is worth 1,000 minus the present value of 1,000 (= \$164.73 in the example). When volatility is greater than zero (as it is in practice), the option is worth more than this. The \$164.73 that the provider of the principal protected note has available to purchase an option is therefore not enough.

12.16

The initial investment is \$2.60. The total payoff is (a) \$4, (b) \$1, and (c) 0.

12.17

The trader makes a profit if the total payoff is less than \$7. This happens when the price of the asset is between \$33 and \$57.

12.18

A butterfly spread is created by buying the \$55 put, buying the \$65 put and selling two of the \$60 puts. This costs $3 + 8 - 2 \times 5 = \$1$ initially. The following table shows the profit/loss from the strategy.

Stock Price	*Payoff*	*Profit*
$S_T \geq 65$	0	-1
$60 \leq S_T < 65$	$65 - S_T$	$64 - S_T$
$55 \leq S_T < 60$	$S_T - 55$	$S_T - 56$
$S_T < 55$	0	-1

The butterfly spread leads to a loss when the final stock price is greater than \$64 or less than \$56.

12.19
There are two alternative profit patterns for part (a). These are shown in Figures S12.2 and S12.3. In Figure S12.2, the long maturity (high strike price) option is worth more than the short maturity (low strike price) option. In Figure S12.3, the reverse is true. There is no ambiguity about the profit pattern for part (b). This is shown in Figure S12.4.

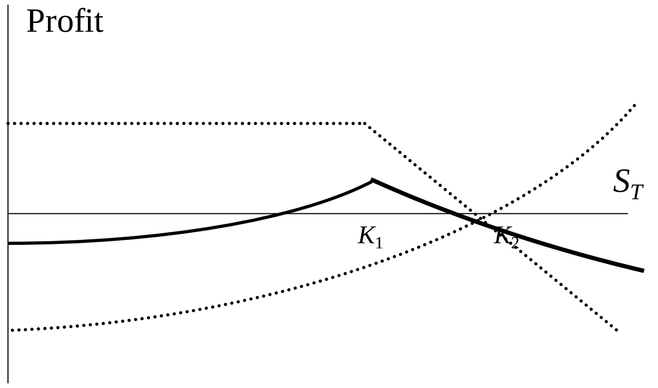

Figure S12.2: *Investor's Profit/Loss in Problem 12.19a when long maturity call is worth more than short maturity call*

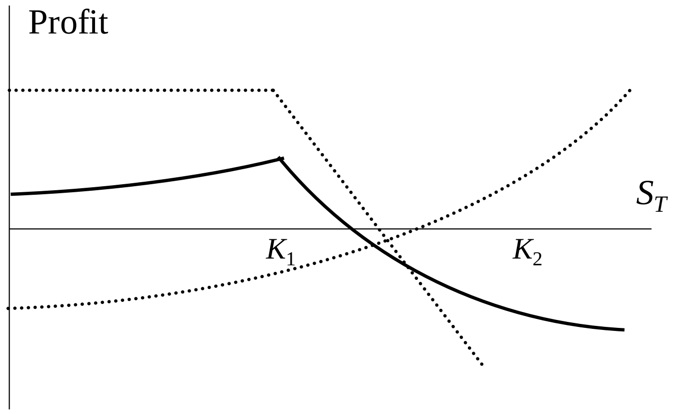

Figure S12.3 *Investor's Profit/Loss in Problem 12.19a when short maturity call is worth more than long maturity call*

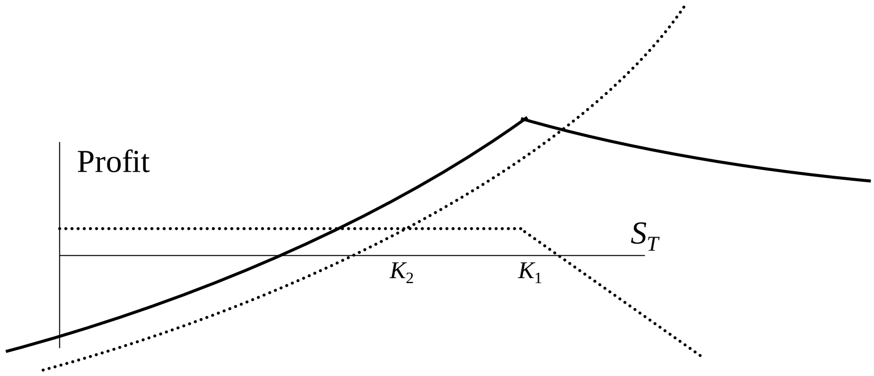

Figure S12.4 *Investor's Profit/Loss in Problem 12.19b*

12.20

The variation of an investor's profit/loss with the terminal stock price for each of the four strategies is shown in Figure S12.5. In each case, the dotted line shows the profits from the components of the investor's position and the solid line shows the total net profit.

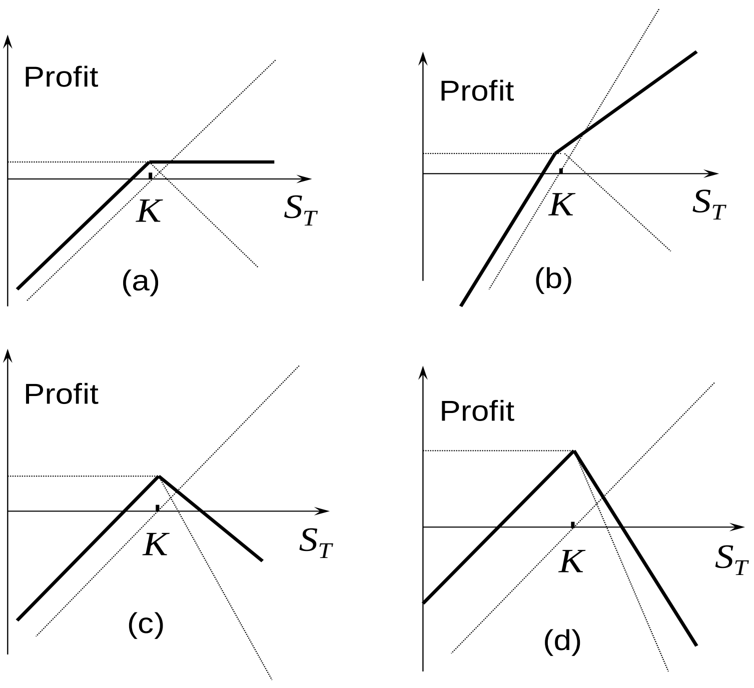

Figure S12.5 *Answer to Problem 12.20*

12.21

(a) A call option with a strike price of 25 costs 7.90 and a call option with a strike price of 30 costs 4.18. The cost of the bull spread is therefore $7.90-4.18=3.72$. The profits ignoring the impact of discounting are as follows:

Stock Price Range	*Profit*
$S_T \leq 25$	–3.72
$25 < S_T < 30$	$S_T - 28.72$
$S_T \geq 30$	1.28

(b) A put option with a strike price of 25 costs 0.28 and a put option with a strike price of 30 costs 1.44. The cost of the bear spread is therefore $1.44-0.28=1.16$. The profits ignoring the impact of discounting are as follows:

Stock Price Range	*Profit*
$S_T \leq 25$	+3.84
$25 < S_T < 30$	$28.84 - S_T$
$S_T \geq 30$	–1.16

(c) Call options with maturities of one year and strike prices of 25, 30, and 35 cost 8.92, 5.60, and 3.28, respectively. The cost of the butterfly spread is therefore $8.92+3.28-2\times 5.60=1.00$. The profits ignoring the impact of discounting are as follows:

Stock Price Range	*Profit*
$S_T \leq 25$	–1.00
$25 < S_T < 30$	$S_T - 26.00$
$30 \leq S_T < 35$	$34.00 - S_T$

(d) Put options with maturities of one year and strike prices of 25, 30, and 35 cost 0.70, 2.14, 4.57, respectively. The cost of the butterfly spread is therefore $0.70+4.57-2\times 2.14=0.99$. Allowing for rounding errors, this is the same as in (c). The profits are the same as in (c).

(e) A call option with a strike price of 30 costs 4.18. A put option with a strike price of 30 costs 1.44. The cost of the straddle is therefore $4.18+1.44=5.62$. The profits ignoring the impact of discounting are as follows:

Stock Price Range	*Profit*
$S_T \leq 30$	$24.38 - S_T$
$S_T > 30$	$S_T - 35.62$

(f) A six-month call option with a strike price of 35 costs 1.85. A six-month put option with a strike price of 25 costs 0.28. The cost of the strangle is therefore $1.85+0.28=2.13$. The profits ignoring the impact of discounting are as follows:

Stock Price Range	*Profit*
$S_T \leq 25$	$22.87 - S_T$
$25 < S_T < 35$	-2.13
$S_T \geq 35$	$S_T - 37.13$