CHAPTER 19
The Greek Letters

Practice Questions

19.1
A short position in 1,000 options has a delta of -700 and can be made delta neutral with the purchase of 700 shares.

19.2
In this case, $S_0 = K$, $r = 0.1$, $\sigma = 0.25$, and $T = 0.5$. Also,

$$d_1 = \frac{\ln(S_0 / K) + (0.1 + 0.25^2 / 2)0.5}{0.25\sqrt{0.5}} = 0.3712$$

The delta of the option is $N(d_1)$ or 0.64.

19.3
To hedge an option position, it is necessary to create the opposite option position synthetically. For example, to hedge a long position in a put, it is necessary to create a short position in a put synthetically. It follows that the procedure for creating an option position synthetically is the reverse of the procedure for hedging the option position.

19.4
The strategy costs the trader 0.10 each time the stock is bought or sold. The total expected cost of the strategy, in present value terms, must be \$4. This means that the expected number of times the stock will be bought or sold is approximately 40. The expected number of times it will be bought is approximately 20 and the expected number of times it will be sold is also approximately 20. The buy and sell transactions can take place at any time during the life of the option. The above numbers are therefore only approximately correct because of the effects of discounting. Also, the estimate is of the number of times the stock is bought or sold in the risk-neutral world, not the real world.

19.5
The holding of the stock at any given time must be $N(d_1)$. Hence, the stock is bought just after the price has risen and sold just after the price has fallen. (This is the buy high sell low strategy referred to in the text.) In the first scenario, the stock is continually bought. In the second scenario, the stock is bought, sold, bought again, sold again, etc. The final holding is the same in both scenarios. The buy, sell, buy, sell... situation clearly leads to higher costs than the buy, buy, buy... situation. This problem emphasizes one disadvantage of creating options synthetically. Whereas the cost of an option that is purchased is known up front and depends on the forecasted volatility, the cost of an option that is created synthetically is not known up front and depends on the volatility actually encountered.

19.6
The delta of a European futures call option is usually defined as the rate of change of the option price with respect to the futures price (not the spot price). It is

$$e^{-rT} N(d_1)$$

In this case, $F_0 = 8$, $K = 8$, $r = 0.12$, $\sigma = 0.18$, $T = 0.6667$

$$d_1 = \frac{\ln(8/8) + (0.18^2/2) \times 0.6667}{0.18\sqrt{0.6667}} = 0.0735$$

$N(d_1) = 0.5293$ and the delta of the option is

$$e^{-0.12\times 0.6667} \times 0.5293 = 0.4886$$

The delta of a short position in 1,000 futures options is therefore -488.6.

19.7

In order to answer this problem, it is important to distinguish between the rate of change of the option with respect to the futures price and the rate of change of its price with respect to the spot price.

The former will be referred to as the futures delta; the latter will be referred to as the spot delta. The futures delta of a nine-month futures contract to buy one ounce of silver is by definition 1.0. Hence, from the answer to Problem 19.6, a long position in nine-month futures on 488.6 ounces is necessary to hedge the option position.

The spot delta of a nine-month futures contract is $e^{0.12\times 0.75} = 1.094$ assuming no storage costs. (This is because silver can be treated in the same way as a non-dividend-paying stock when there are no storage costs. $F_0 = S_0 e^{rT}$ so that the spot delta is the futures delta times e^{rT})

Hence, the spot delta of the option position is $-488.6 \times 1.094 = -534.6$. Thus, a long position in 534.6 ounces of silver is necessary to hedge the option position.

The spot delta of a one-year silver futures contract to buy one ounce of silver is $e^{0.12} = 1.1275$. Hence, a long position in $e^{-0.12} \times 534.6 = 474.1$ ounces of one-year silver futures is necessary to hedge the option position.

19.8

A long position in either a put or a call option has a positive gamma. From Figure 19.8, when gamma is positive, the hedger gains from a large change in the stock price and loses from a small change in the stock price. Hence the hedger will fare better in case (b).

19.9

A short position in either a put or a call option has a negative gamma. From Figure 19.8, when gamma is negative, the hedger gains from a small change in the stock price and loses from a large change in the stock price. Hence, the hedger will fare better in case (a).

19.10

In this case, $S_0 = 0.80$, $K = 0.81$, $r = 0.08$, $r_f = 0.05$, $\sigma = 0.15$, $T = 0.5833$

$$d_1 = \frac{\ln(0.80/0.81) + \left(0.08 - 0.05 + 0.15^2/2\right) \times 0.5833}{0.15\sqrt{0.5833}} = 0.1016$$

$$d_2 = d_1 - 0.15\sqrt{0.5833} = -0.0130$$

$$N(d_1)=0.5405;\ N(d_2)=0.4948$$

The delta of one call option is $e^{-r_f T} N(d_1) = e^{-0.05\times 0.5833} \times 0.5405 = 0.5250$.

$$N'(d_1) = \frac{1}{\sqrt{2\pi}} e^{-d_1^2/2} = \frac{1}{\sqrt{2\pi}} e^{-0.00516} = 0.3969$$

so that the gamma of one call option is

$$\frac{N'(d_1)e^{-r_fT}}{S_0\sigma\sqrt{T}} = \frac{0.3969\times 0.9713}{0.80\times 0.15\times\sqrt{0.5833}} = 4.206$$

The vega of one call option is

$$S_0\sqrt{T}N'(d_1)e^{-r_fT} = 0.80\sqrt{0.5833}\times 0.3969\times 0.9713 = 0.2355$$

The theta of one call option is

$$-\frac{S_0N'(d_1)\sigma e^{-r_fT}}{2\sqrt{T}} + r_fS_0N(d_1)e^{-r_fT} - rKe^{-rT}N(d_2)$$

$$= -\frac{0.8\times 0.3969\times 0.15\times 0.9713}{2\sqrt{0.5833}}$$

$$+0.05\times 0.8\times 0.5405\times 0.9713 - 0.08\times 0.81\times 0.9544\times 0.4948$$

$$= -0.0399$$

The rho of one call option is

$$KTe^{-rT}N(d_2)$$

$$= 0.81\times 0.5833\times 0.9544\times 0.4948$$

$$= 0.2231$$

Delta can be interpreted as meaning that, when the spot price increases by a small amount (measured in cents), the value of an option to buy one yen increases by 0.525 times that amount. Gamma can be interpreted as meaning that, when the spot price increases by a small amount (measured in cents), the delta increases by 4.206 times that amount. Vega can be interpreted as meaning that, when the volatility (measured in decimal form) increases by a small amount, the option's value increases by 0.2355 times that amount. When volatility increases by 1% (= 0.01), the option price increases by 0.002355. Theta can be interpreted as meaning that, when a small amount of time (measured in years) passes, the option's value decreases by 0.0399 times that amount. In particular, when one calendar day passes, it decreases by $0.0399/365 = 0.000109$. Finally, rho can be interpreted as meaning that, when the interest rate (measured in decimal form) increases by a small amount, the option's value increases by 0.2231 times that amount. When the interest rate increases by 1% (= 0.01), the options value increases by 0.002231.

19.11

Assume that S_0, K, r, σ, T, q are the parameters for the option held and S_0, K^*, r, σ, T^*, q are the parameters for another option. Suppose that d_1 has its usual meaning and is calculated on the basis of the first set of parameters while d_1^* is the value of d_1 calculated on the basis of the second set of parameters. Suppose further that w of the second option are held for each of the first option held. The gamma of the portfolio is:

$$\alpha\left[\frac{N'(d_1)e^{-qT}}{S_0\sigma\sqrt{T}} + w\frac{N'(d_1^*)e^{-qT^*}}{S_0\sigma\sqrt{T^*}}\right]$$

where α is the number of the first option held.

Since we require gamma to be zero,

$$w = -\frac{N'(d_1)e^{-q(T-T^*)}}{N'(d_1^*)}\sqrt{\frac{T^*}{T}}$$

The vega of the portfolio is

$$\alpha\left[S_0\sqrt{T}N'(d_1)e^{-q(T)}+wS_0\sqrt{T^*}N'(d_1^*)e^{-q(T^*)}\right]$$

Since we require vega to be zero,

$$w=-\sqrt{\frac{T}{T^*}}\frac{N'(d_1)e^{-q(T-T^*)}}{N'(d_1^*)}$$

Equating the two expressions for w

$$T^*=T$$

Hence, the maturity of the option held must equal the maturity of the option used for hedging.

19.12

The fund is worth \$300,000 times the value of the index. When the value of the portfolio falls by 5% (to \$342 million), the value of the index also falls by 5% to 1140. The fund manager therefore requires European put options on 300,000 times the index with exercise price 1140.

a) $S_0=1200$, $K=1140$, $r=0.06$, $\sigma=0.30$, $T=0.50$ and $q=0.03$. Hence,

$$d_1=\frac{\ln(1200/1140)+\left(0.06-0.03+0.3^2/2\right)\times 0.5}{0.3\sqrt{0.5}}=0.4186$$

$$d_2=d_1-0.3\sqrt{0.5}=0.2064$$

$$N(d_1)=0.6622;\quad N(d_2)=0.5818$$

$$N(-d_1)=0.3378;\quad N(-d_2)=0.4182$$

The value of one put option is

$$\begin{aligned}&1140e^{-rT}N(-d_2)-1200e^{-qT}N(-d_1)\\&=1140e^{-0.06\times0.5}\times0.4182-1200e^{-0.03\times0.5}\times0.3378\\&=63.40\end{aligned}$$

The total cost of the insurance is therefore,

$$300,000\times 63.40=\$19,020,000$$

b) From put–call parity

$$S_0e^{-qT}+p=c+Ke^{-rT}$$

or,

$$p=c-S_0e^{-qT}+Ke^{-rT}$$

This shows that a put option can be created by selling (or shorting) e^{-qT} of the index, buying a call option and investing the remainder at the risk-free rate of interest. Applying this to the situation under consideration, the fund manager should:

1. Sell $360e^{-0.03\times0.5}=\$354.64$ million of stock.
2. Buy call options on 300,000 times the index with exercise price 1140 and maturity in six months.
3. Invest the remaining cash at the risk-free interest rate of 6% per annum.

This strategy gives the same result as buying put options directly.

c) The delta of one put option is

$$e^{-qT}[N(d_1)-1]$$
$$=e^{-0.03\times0.5}(0.6622-1)$$
$$-0.3327$$

This indicates that 33.27% of the portfolio (i.e., \$119.77 million) should be initially sold and invested in risk-free securities.

d) The delta of a nine-month index futures contract is

$$e^{(r-q)T}=e^{0.03\times0.75}=1.023$$

The spot short position required is

$$\frac{119,770,000}{1200}=99,808$$

times the index. Hence, a short position in

$$\frac{99,808}{1.023\times250}=390$$

futures contracts is required.

19.13

When the value of the portfolio goes down 5% in six months, the total return from the portfolio, including dividends, in the six months is

$$-5+2=-3\%$$

that is, -6% per annum. This is 12% per annum less than the risk-free interest rate. Since the portfolio has a beta of 1.5, we would expect the market to provide a return of 8% per annum less than the risk-free interest rate; that is, we would expect the market to provide a return of -2% per annum. Since dividends on the market index are 3% per annum, we would expect the market index to have dropped at the rate of 5% per annum or 2.5% per six months; that is, we would expect the market to have dropped to 1170. A total of $450,000=(1.5\times300,000)$ put options on the index with exercise price 1170 and exercise date in six months are therefore required.

a) $S_0=1200$, $K=1170$, $r=0.06$, $\sigma=0.3$, $T=0.5$ and $q=0.03$. Hence,

$$d_1=\frac{\ln(1200/1170)+\left(0.06-0.03+0.09/2\right)\times0.5}{0.3\sqrt{0.5}}=0.2961$$
$$d_2=d_1-0.3\sqrt{0.5}=0.0840$$

$$N(d_1)=0.6164;\quad N(d_2)=0.5335$$

$$N(-d_1)=0.3836;\quad N(-d_2)=0.4665$$

The value of one put option is

$$Ke^{-rT}N(-d_2)-S_0e^{-qT}N(-d_1)$$
$$=1170e^{-0.06\times0.5}\times0.4665-1200e^{-0.03\times0.5}\times0.3836$$
$$=76.28$$

The total cost of the insurance is therefore,

$$450,000 \times 76.28 = \$34,326,000$$

Note that this is significantly greater than the cost of the insurance in Problem 19.12.

b) As in Problem 19.12, the fund manager can 1) sell \$354.64 million of stock, 2) buy call options on 450,000 times the index with exercise price 1170 and exercise date in six months, and 3) invest the remaining cash at the risk-free interest rate.

c) The portfolio is 50% more volatile than the index. When the insurance is considered as an option on the portfolio, the parameters are as follows: $S_0 = 360$, $K = 342$, $r = 0.06$, $\sigma = 0.45$, $T = 0.5$ and $q = 0.04$

$$d_1 = \frac{\ln(360/342) + \left(0.06 - 0.04 + 0.45^2/2\right) \times 0.5}{0.45\sqrt{0.5}} = 0.3517$$

$$N(d_1) = 0.6374$$

The delta of the option is

$$\begin{aligned} &e^{-qT}[N(d_1) - 1] \\ &= e^{-0.04\times 0.5}(0.6374 - 1) \\ &= -0.355 \end{aligned}$$

This indicates that 35.5% of the portfolio (i.e., \$127.8 million) should be sold and invested in riskless securities.

d) We now return to the situation considered in (a) where put options on the index are required. The delta of each put option is

$$\begin{aligned} &e^{-qT}(N(d_1) - 1) \\ &= e^{-0.03\times 0.5}(0.6164 - 1) \\ &= -0.3779 \end{aligned}$$

The delta of the total position required in put options is $-450,000 \times 0.3779 = -170,000$. The delta of a nine month index futures is (see Problem 19.12) 1.023. Hence, a short position in

$$\frac{170,000}{1.023 \times 250} = 665$$

index futures contracts.

19.14

a) For a call option on a non-dividend-paying stock,

$$\Delta = N(d_1)$$

$$\Gamma = \frac{N'(d_1)}{S_0 \sigma \sqrt{T}}$$

$$\Theta = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2)$$

Hence, the left-hand side of equation (19.4) is:

$$
\begin{aligned}
&= -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2) + rS_0N(d_1) + \frac{1}{2}\sigma S_0 \frac{N'(d_1)}{\sqrt{T}} \\
&= r[S_0N(d_1) - Ke^{-rT}N(d_2)] \\
&= r\Pi
\end{aligned}
$$

b) For a put option on a non-dividend-paying stock,

$$\Delta = N(d_1) - 1 = -N(-d_1)$$

$$\Gamma = \frac{N'(d_1)}{S_0\sigma\sqrt{T}}$$

$$\Theta = -\frac{S_0N'(d_1)\sigma}{2\sqrt{T}} + rKe^{-rT}N(-d_2)$$

Hence, the left-hand side of equation (19.4) is:

$$
\begin{aligned}
&-\frac{S_0N'(d_1)\sigma}{2\sqrt{T}} + rKe^{-rT}N(-d_2) - rS_0N(-d_1) + \frac{1}{2}\sigma S_0\frac{N'(d_1)}{\sqrt{T}} \\
&= r[Ke^{-rT}N(-d_2) - S_0N(-d_1)] \\
&= r\Pi
\end{aligned}
$$

c) For a portfolio of options, Π, Δ, Θ and Γ are the sums of their values for the individual options in the portfolio. It follows that equation (19.4) is true for any portfolio of European put and call options.

19.15

A currency is analogous to a stock paying a continuous dividend yield at rate r_f. The differential equation for a portfolio of derivatives dependent on a currency is (see equation 17.6)

$$\frac{\partial \Pi}{\partial t} + (r - r_f)S\frac{\partial \Pi}{\partial S} + \frac{1}{2}\sigma^2S^2\frac{\partial^2\Pi}{\partial S^2} = r\Pi$$

Hence,

$$\Theta + (r - r_f)S\Delta + \frac{1}{2}\sigma^2S^2\Gamma = r\Pi$$

Similarly, for a portfolio of derivatives dependent on a futures price, F (see equation 18.6)

$$\Theta + \frac{1}{2}\sigma^2F^2\Gamma = r\Pi$$

19.16

We can regard the position of all portfolio insurers taken together as a single put option. The three known parameters of the option, before the 23% decline, are $S_0 = 70$, $K = 66.5$, $T = 1$. Other parameters can be estimated as $r = 0.06$, $\sigma = 0.25$ and $q = 0.03$. Then:

$$d_1 = \frac{\ln(70/66.5) + (0.06 - 0.03 + 0.25^2/2)}{0.25} = 0.4502$$

$$N(d_1) = 0.6737$$

The delta of the option is

$$e^{-qT}[N(d_1)-1]$$
$$=e^{-0.03}(0.6737-1)$$
$$=-0.3167$$

This shows that 31.67% or \$22.17 billion of assets should have been sold before the decline. These numbers can also be produced from DerivaGem by selecting Underlying Type and Index and Option Type as Black–Scholes European.

After the decline, $S_0 = 53.9$, $K = 66.5$, $T = 1$, $r = 0.06$, $\sigma = 0.25$ and $q = 0.03$.

$$d_1 = \frac{\ln(53.9/66.5)+(0.06-0.03+0.25^2/2)}{0.25} = -0.5953$$

$$N(d_1) = 0.2758$$

The delta of the option has dropped to

$$e^{-0.03\times 0.5}(0.2758-1)$$
$$=-0.7028$$

This shows that cumulatively 70.28% of the assets originally held should be sold. An additional 38.61% of the original portfolio should be sold. The sales measured at pre-crash prices are about \$27.0 billion. At post-crash prices, they are about \$20.8 billion.

19.17

With our usual notation, the value of a forward contract on the asset is $S_0e^{-qT} - Ke^{-rT}$. When there is a small change, ΔS, in S_0 the value of the forward contract changes by $e^{-qT}\Delta S$. The delta of the forward contract is therefore e^{-qT}. The futures price is $S_0e^{(r-q)T}$. When there is a small change, ΔS, in S_0 the futures price changes by $\Delta Se^{(r-q)T}$. Given the daily settlement procedures in futures contracts, this is also the immediate change in the wealth of the holder of the futures contract. The delta of the futures contract is therefore $e^{(r-q)T}$. We conclude that the deltas of a futures and forward contract are not the same. The delta of the futures is greater than the delta of the corresponding forward by a factor of e^{rT}. (Business Snapshot 5.2 is related to this question.)

19.18

The delta indicates that when the value of the exchange rate increases by \$0.01, the value of the bank's position increases by $0.01\times 30{,}000 = \$300$. The gamma indicates that when the exchange rate increases by \$0.01, the delta of the portfolio decreases by $0.01\times 80{,}000 = 800$. For delta neutrality, 30,000 CAD should be shorted. When the exchange rate moves up to 0.93, we expect the delta of the portfolio to decrease by $(0.93-0.90)\times 80{,}000 = 2{,}400$ so that it becomes 27,600. To maintain delta neutrality, it is therefore necessary for the bank to unwind its short position 2,400 CAD so that a net 27,600 have been shorted. As shown in the text (see Figure 19.8), when a portfolio is delta neutral and has a negative gamma, a loss is experienced when there is a large movement in the underlying asset price. We can conclude that the bank is likely to have lost money.

19.19

(a) For a non-dividend paying stock, put–call parity gives at a general time t:

$$p + S = c + Ke^{-r(T-t)}$$

Differentiating with respect to S:

$$\frac{\partial p}{\partial S}+1=\frac{\partial c}{\partial S}$$

or

$$\frac{\partial p}{\partial S}=\frac{\partial c}{\partial S}-1$$

This shows that the delta of a European put equals the delta of the corresponding European call less 1.0.

(b) Differentiating with respect to S again

$$\frac{\partial^2 p}{\partial S^2}=\frac{\partial^2 c}{\partial S^2}$$

Hence, the gamma of a European put equals the gamma of a European call.

(c) Differentiating the put–call parity relationship with respect to σ

$$\frac{\partial p}{\partial \sigma}=\frac{\partial c}{\partial \sigma}$$

showing that the vega of a European put equals the vega of a European call.

(d) Differentiating the put–call parity relationship with respect to t

$$\frac{\partial p}{\partial t}=rKe^{-r(T-t)}+\frac{\partial c}{\partial t}$$

This is in agreement with the thetas of European calls and puts given in Section 19.5 since $N(d_2)=1-N(-d_2)$.

19.20

The delta of the portfolio is

$$-1,000\times 0.50-500\times 0.80-2,000\times(-0.40)-500\times 0.70=-450$$

The gamma of the portfolio is

$$-1,000\times 2.2-500\times 0.6-2,000\times 1.3-500\times 1.8=-6,000$$

The vega of the portfolio is

$$-1,000\times 1.8-500\times 0.2-2,000\times 0.7-500\times 1.4=-4,000$$

(a) A long position in 4,000 traded options will give a gamma-neutral portfolio since the long position has a gamma of $4,000\times 1.5=+6,000$. The delta of the whole portfolio (including traded options) is then:

$4,000\times 0.6-450=1,950$

Hence, in addition to the 4,000 traded options, a short position of 1,950 in sterling is necessary so that the portfolio is both gamma and delta neutral.

(b) A long position in 5,000 traded options will give a vega-neutral portfolio since the long position has a vega of $5,000\times 0.8=+4,000$. The delta of the whole portfolio (including traded options) is then

$5,000\times 0.6-450=2,550$

Hence, in addition to the 5,000 traded options, a short position of 2,550 in sterling is necessary so that the portfolio is both vega and delta neutral.

19.21

Let w_1 be the position in the first traded option and w_2 be the position in the second traded option. We require:

$$6,000 = 1.5w_1 + 0.5w_2$$

$$4,000 = 0.8w_1 + 0.6w_2$$

The solution to these equations can easily be seen to be $w_1 = 3,200$, $w_2 = 2,400$. The whole portfolio then has a delta of

$$-450 + 3,200 \times 0.6 + 2,400 \times 0.1 = 1,710$$

Therefore, the portfolio can be made delta, gamma and vega neutral by taking a long position in 3,200 of the first traded option, a long position in 2,400 of the second traded option, and a short position of 1,710 in sterling.

19.22

The product provides a six-month return equal to

$$\max(0, 0.4R)$$

where R is the return on the index. Suppose that S_0 is the current value of the index and S_T is the value in six months.

When an amount A is invested, the return received at the end of six months is:

$$A \max\left(0, 0.4\frac{S_T - S_0}{S_0}\right)$$

$$= \frac{0.4A}{S_0} \max(0, S_T - S_0)$$

This is $0.4A/S_0$ of at-the-money European call options on the index. With the usual notation, they have value:

$$\frac{0.4A}{S_0}[S_0 e^{-qT} N(d_1) - S_0 e^{-rT} N(d_2)]$$

$$= 0.4A[e^{-qT} N(d_1) - e^{-rT} N(d_2)]$$

In this case, $r = 0.08$, $\sigma = 0.25$, $T = 0.50$ and $q = 0.03$

$$d_1 = \frac{\left(0.08 - 0.03 + 0.25^2/2\right)0.50}{0.25\sqrt{0.50}} = 0.2298$$

$$d_2 = d_1 - 0.25\sqrt{0.50} = 0.0530$$

$$N(d_1) = 0.5909; \quad N(d_2) = 0.5212$$

The value of the European call options being offered is

$$0.4A(e^{-0.03\times0.5} \times 0.5909 - e^{-0.08\times0.5} \times 0.5212)$$

$$= 0.0325A$$

This is the present value of the payoff from the product. If an investor buys the product, the investor avoids having to pay $0.0325A$ at time zero for the underlying option. The cash flows to the investor are therefore,

Time 0: $-A + 0.0325A = -0.9675A$

After six months: $+A$

The return with continuous compounding is $2\ln(1/0.9675) = 0.066$ or 6.6% per annum. The product is therefore slightly less attractive than a risk-free investment.

19.23

(a)

$$FN'(d_1) = \frac{F}{\sqrt{2\pi}} e^{-d_1^2/2}$$

$$KN'(d_2) = KN'(d_1 - \sigma\sqrt{T}) = \frac{K}{\sqrt{2\pi}} e^{-(d_1^2/2) + d_1\sigma\sqrt{T} - \sigma^2 T/2}$$

Because $d_1\sigma\sqrt{T} = \ln(F/K) + \sigma^2 T/2$, the second equation reduces to

$$KN'(d_2) = \frac{K}{\sqrt{2\pi}} e^{-(d_1^2/2) + ln(F/K)} = \frac{F}{\sqrt{2\pi}} e^{-d_1^2/2}$$

The result follows.

(b)

$$\frac{\partial c}{\partial F} = e^{-rT} N(d_1) + e^{-rT} FN'(d_1) \frac{\partial d_1}{\partial F} - e^{-rT} KN'(d_2) \frac{\partial d_2}{\partial F}$$

Because

$$\frac{\partial d_1}{\partial F} = \frac{\partial d_2}{\partial F}$$

it follows from the result in (a) that

$$\frac{\partial c}{\partial F} = e^{-rT} N(d_1)$$

(c)

$$\frac{\partial c}{\partial \sigma} = e^{-rT} FN'(d_1) \frac{\partial d_1}{\partial \sigma} - e^{-rT} KN'(d_2) \frac{\partial d_2}{\partial \sigma}$$

Because $d_1 = d_2 + \sigma\sqrt{T}$

$$\frac{\partial d_1}{\partial \sigma} = \frac{\partial d_2}{\partial \sigma} + \sqrt{T}$$

From the result in (a), it follows that

$$\frac{\partial c}{\partial \sigma} = e^{-rT} FN'(d_1)\sqrt{T}$$

(d)

Rho is given by

$$\frac{\partial c}{\partial r} = -Te^{-rT}[FN(d_1) - KN(d_2)]]$$

or $-cT$.

Because $q = r$ in the case of a futures option there are two components to rho. One arises from differentiation with respect to r, the other from differentiation with respect to q.

19.24

For the option considered in Section 19.1, $S_0 = 49$, $K = 50$, $r = 0.05$, $\sigma = 0.20$, and $T = 20/52$. DerivaGem shows that $\Theta = -0.011795 \times 365 = -4.305$, $\Delta = 0.5216$, $\Gamma = 0.065544$, $\Pi = 2.4005$. The left hand side of equation (19.4)

$$-4.305 + 0.05 \times 49 \times 0.5216 + \frac{1}{2} \times 0.2^2 \times 49^2 \times 0.065544 = 0.120$$

The right hand side is

$$0.05 \times 2.4005 = 0.120$$

This shows that the result in equation (19.4) is satisfied.