Chapter 17 – Smoothing Methods
数据平滑方法

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Data Mining for Business Intelligence

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Smoothing is “data driven”
数据平滑是数据驱动的

- Regression methods assume underlying unchanging structure (linear, exponential, polynomial) 回归方法假设数据存在不变的结构（线性、指数、多项式）

- Smoothing derives forecasts based directly on the data alone (e.g. averaging), with no mathematical structural assumptions 数据平滑直接从数据中推出预测值（例如平均值），未对数据结构做假设。

- Suitable where the components (trend, seasonality) change over time 适合于时间序列构成要素（趋势、季节性）随时间发生变化的情况。

Simple moving average (MA) 简单的MA

Set window width “w”; take average of the w values.
设置时间窗口宽度w，对时间窗口内的值取平均。

For centered moving average, window is centered around forecast point 对于居中移动平均，窗口中央应该是被预测值。

For w=5, the forecast for t_3 averages the values $t_1 \cdots t_5$

Not useful for future forecasts 这种方法对预测未来值帮助不大。

For future forecasts, use “trailing average” = the value being forecast is at the end of the window 对未来值的预测使用“追踪平均”：被预测值在时间窗口之后。

Choosing window width 选择时间窗口宽度

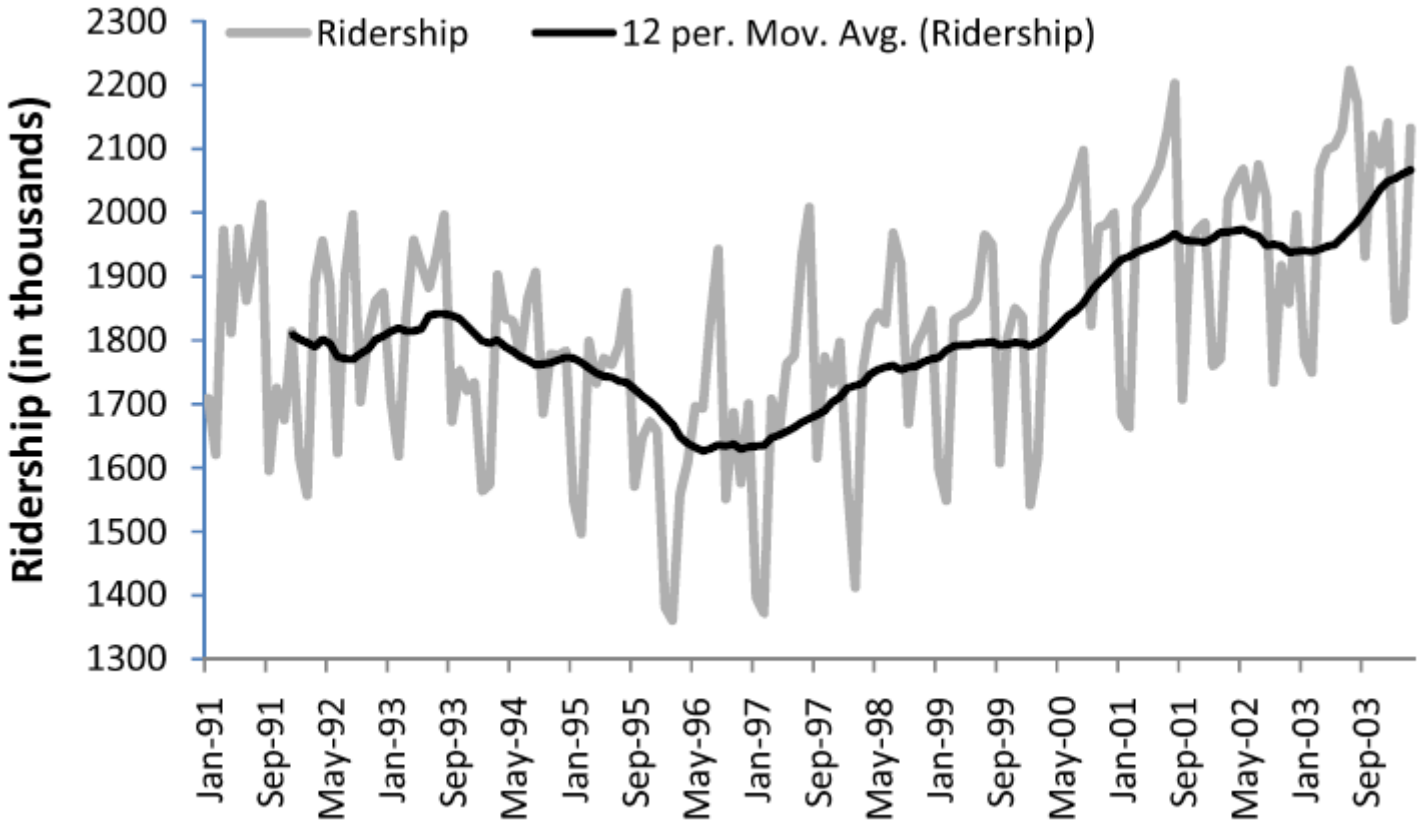
Goal is to suppress seasonality and noise 目标是压缩季节性和噪音

Typically choose window width = season length 通常选择的时间窗口宽度是 季节长度。

In Excel:

Add Trendline > Moving Average

Amtrak data – moving average



CENTERED MOVING AVERAGE WITH WINDOW $W = 12$, OVERLAID ON AMTRAK RIDERSHIP SERIES. THIS HELPS VISUALIZE TRENDS

Moving Average for Forecasting

Shortcomings

Suppresses seasonality, but does not forecast seasonal component

Lags behind trends

Thus, simple Moving Average useful only for series that lack trend and seasonality

Coping with these shortcomings

- Use regression model to de-trend and de-seasonalize

- Use Moving Average to forecast the de-trended and de-seasonalized series

- Add trend and seasonality back to the forecast

Simple exponential smoothing

Like MA, except use weighted average of all past values, instead of simple average in a window

Forecast at time $t+1$:

$$F_{t+1} = aY_t + a(1-a)Y_{t-1} + a(1-a)^2Y_{t-2} + \cdots$$

Equivalent to

$$F_{t+1} = F_t + aE_t$$

E is forecast error at time t

Smoothing parameter *α*

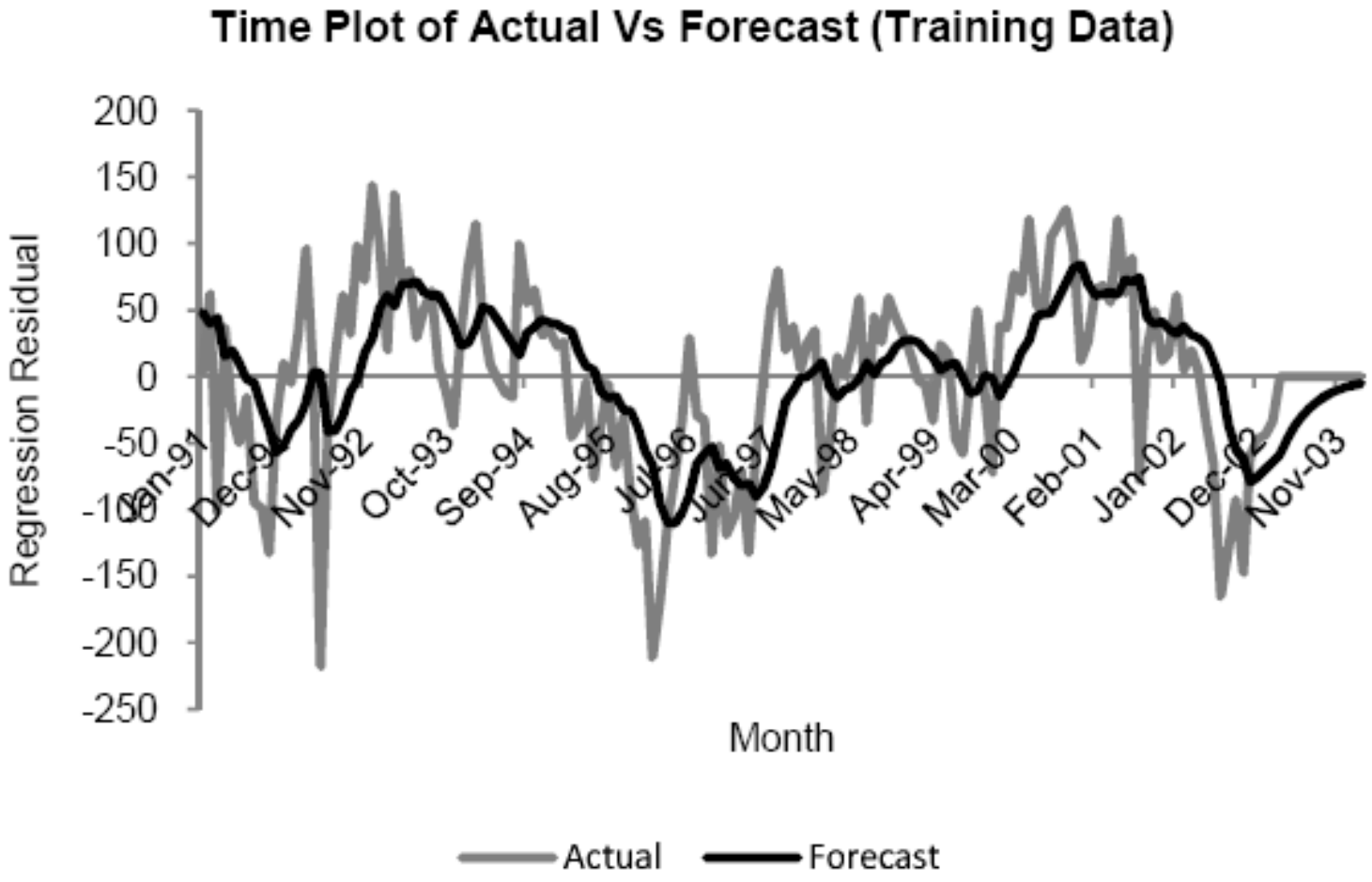
Simple exponential smoother corrects based on error

- If last period forecast was too high, next period is adjusted down
- If last period forecast was too low, next period is adjusted up

Amount of correction depends on value of α

- Value close to 1 = fast learning, close to 0 = low learning
- Default in XLMiner = 0.2

Amtrak – Simple Exponential Smoothing (forecasted residual vs. actual residual)



Using the smoothed forecast

- We have a regression model that has already forecast a value for April, incorporating trend and seasonality (Amtrak ridership)

- Smoothing forecast for April ’ 04 residual = -4.001

- Implies we should adjust regression forecast downward by 4001 riders

Double exponential smoothing 二次指数平滑

Incorporates trend

K-step ahead forecast is derived from the level (L) and trend (T) estimates at time *t*

$$F_{t+k} = L_t + kT_t$$

where

$$L_t = \alpha Y_t + (1-\alpha)(L_{t-1} + T_{t-1})$$

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1}$$

Holt Winter’s exponential smoothing

- Extension of double exponential smoothing

- Incorporate both trend and seasonality

Holt Winter’s forecast for time t+k

Adds seasonality to double exponential

For M seasons (e.g., $M=7$ for weekly), forecast is

$$F_{t+k} = (L_t + kT_t)S_{t+k-M}$$

L = level, T = trend, S = season

Updating L, T and S

$$L_t = \alpha Y_t / S_{t-M} + (1-\alpha)(L_{t-1} + T_{t-1})$$

$$T_t = \beta\,(L_t - L_{t-1}) + (1-\beta) T_{t-1}$$

$$S_t = \gamma Y_t / L_t + (1-\gamma) S_{t-M}.$$

More on the updating equations

- First equation like double exponential, except it uses the seasonally-adjusted value at time *t*

- Second equation identical to double exponential

- Third equation means seasonal index is updated with weighted avg. of previous cycle index, and current trend adjusted value

Note: seasonality is modeled as multiplicative

Summary

- Smoothing methods rely on local data, not mathematical structure

- Simple smoothing does not account for trend and seasonality, but can be combined with model-based forecasts to improve the forecast

- Holt-Winter’s smoothing incorporates seasonality and trend