Chapter 10 – Logistic Regression

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Data Mining for Business Intelligence

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Logistic Regression 分类评定回归 （逻辑回归）

- Extends idea of linear regression to situation where outcome variable is categorical 将线性回归的思想用在类别型数据上。

- Widely used, particularly where a structured model is useful to explain (=*profiling*) or to predict 得到广泛应用，尤其使用在结构化的模型上，用于解释（分类分析）或者预测。

- We focus on binary classification 我们聚焦于二元分类（Y = 0 或者 Y = 1）

i.e. Y=0 or Y=1

The Logit

Goal: Find a function of the predictor variables that relates them to a 0/1 outcome 找到预测变量的函数并将他们与0/1结果联系起来。

- Instead of *Y* as outcome variable (like in linear regression), we use a function of Y called the *logit* 不同于线性回归中用Y作为结果变量，我们使用一个Y的函数（称为Logit函数）

- Logit can be modeled as a linear function of the predictors Logit函数可以是预测变量线性函数。

- The logit can be mapped back to a probability, which, in turn, can be mapped to a class。 Logit可以映射为概率并进一步映射为类别。

Step 1: Logistic Response Function
分类评定反应函数

p = probability of belonging to class 1 属于类别1的概率

Need to relate p to predictors with a function that guarantees $0 \leq p \leq 1$ 需要把概率和预测因子通过函数联系起来并保证概率值取值在0到1之间。

Standard linear function (as shown below) does not: 标准的线性函数做不到这一点。

$$p = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots \beta_q x_q$$

q = number of predictors

The Fix:
use *logistic response function* 使用分类评定反应函数

$$p = \frac{1}{1 + \ e^{-(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ \ldots \beta_q x_q)}}$$

Equation 10.2 in textbook

Step 2: The Odds 胜算几率

The odds of an event are defined as:

一个事件的胜算几率是:

eq. 10.4

$$Odds = \frac{p}{1-p}$$

p = probability of event

Or, given the odds of an event, the probability of the event can be computed by:

事件发生概率也可以从事件胜算几率算出来。

eq. 10.3

$$p = \frac{Odds}{1+Odds}$$

We can also relate the Odds to the predictors:

eq. 10.5 $$Odds = e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_q x_q}$$

To get this result, substitute 10.2 into 10.4

Step 3: Take log on both sides

This gives us the logit:

$$\log(Odds) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_q x_q$$

log(Odds) = *logit* (eq. 10.6)

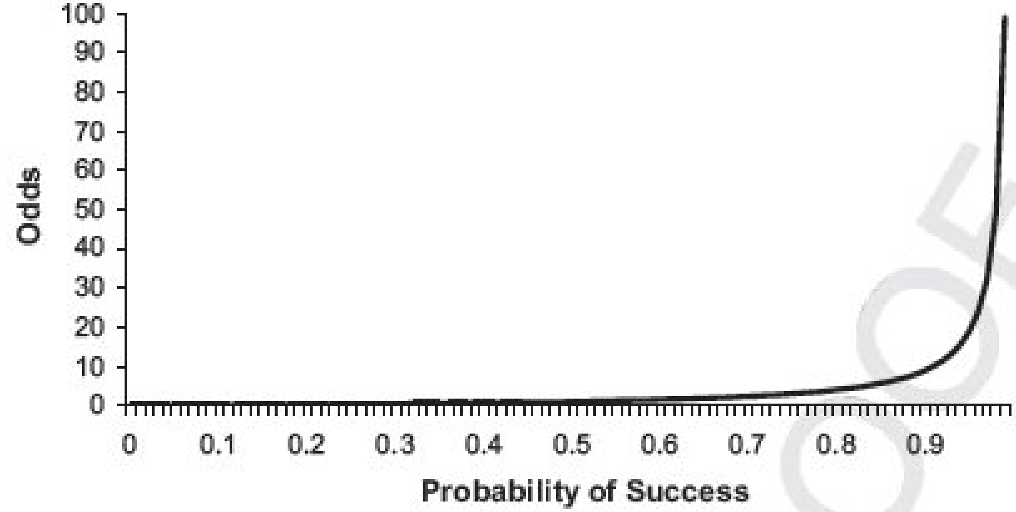
Logit, cont.

So, the logit is a linear function of predictors x_1, x_2, ··· 所以logit是预测因子的线性函数

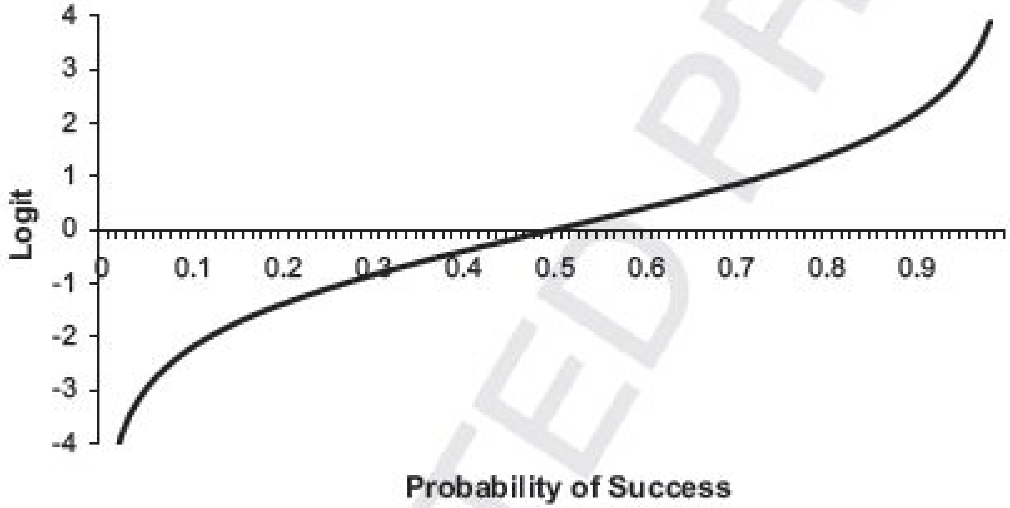
- Takes values from -infinity to +infinity 取值范围是负无穷大到正无穷大

Review the relationship between logit, odds and probability

Odds (a) and Logit (b) as function of P



(a)



(b)

Example

Personal Loan Offer 个人贷款审查

Outcome variable: accept bank loan (0/1) 结果变量：提供银行贷款（接受申请）或者不提供银行贷款（拒绝申请）。

Predictors: Demographic info, and info about their bank relationship 预测因子：人口统计信息，以及客户与银行的关系信息。

Data preprocessing 处理数据

- Partition 60% training, 40% validation 60%数据用于训练，40%用于验证。

- Create 0/1 dummy variables for categorical predictors 建立虚拟变量。

$$EducProf = \begin{cases} 1 \text{ if education is } \textit{Professional} \\ 0 \text{ otherwise} \end{cases}$$

$$EducGrad = \begin{cases} 1 \text{ if education is at } \textit{Graduate} \text{ level} \\ 0 \text{ otherwise} \end{cases}$$

$$Securities = \begin{cases} 1 \text{ if customer has securities account in bank} \\ 0 \text{ otherwise} \end{cases}$$

$$CD = \begin{cases} 1 \text{ if customer has CD account in bank} \\ 0 \text{ otherwise} \end{cases}$$

$$Online = \begin{cases} 1 \text{ if customer uses online banking} \\ 0 \text{ otherwise} \end{cases}$$

$$CreditCard = \begin{cases} 1 \text{ if customer holds Universal Bank credit card} \\ 0 \text{ otherwise} \end{cases}$$

Single Predictor Model 单因素模型

Modeling loan acceptance on income (*x*) 使用收入作为预测因子对贷款申请批准进行建模。

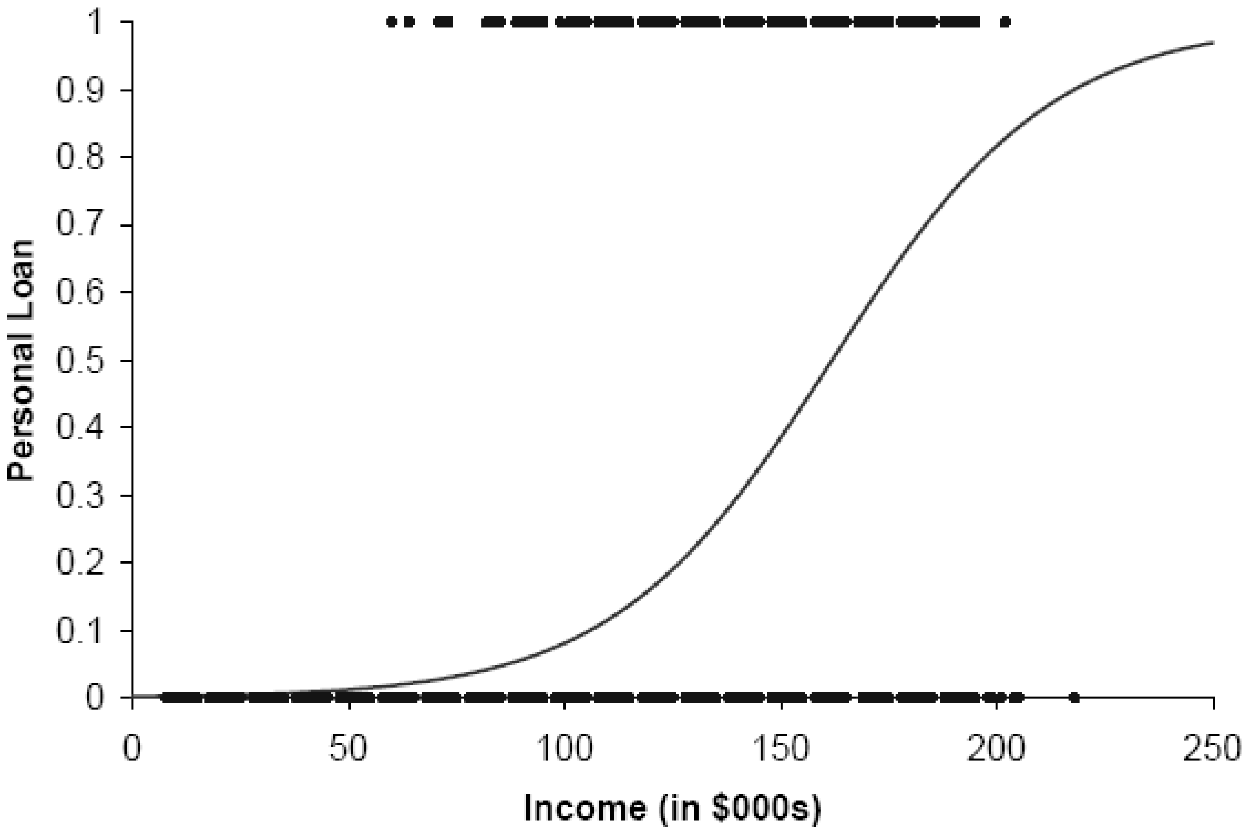
$$\text{Prob}(Personal\ Loan = Yes \mid Income = x) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}}$$

Fitted coefficients (more later): b_0 = -6.3525, b_1 = -0.0392

$$P(Personal\ Loan = Yes \mid Income = x) = \frac{1}{1 + e^{6.3525 - 0.0392x}}$$

Seeing the Relationship

$$P(Personal\ Loan = Yes \mid Income = x) = \frac{1}{1 + e^{6.3525 - 0.0392x}}$$



Last step - classify 最后一步: 分类

Model produces an estimated probability of being a "1"

模型计算记录属于类别1的概率。

- Convert to a classification by establishing cutoff level

建立截值以便根据概率值将记录转化为类别。

- If estimated prob. > cutoff, classify as "1"

  如果估计概率值大于截值则将记录归为类别1。

Ways to Determine Cutoff 决定截值的方法

- 0.50 is popular initial choice

通常选择0.50作为截值。

- Additional considerations (see Chapter 5) 附加考虑的因素
  - Maximize classification accuracy

    使分类准确率最大
  - Maximize sensitivity (subject to min. level of specificity)

    给定最小特异度情况下使敏感度达到最大
  - Minimize false positives (subject to max. false negative rate)

    给定最大假阴性率情况下使假阳性率达到最小
  - Minimize expected cost of misclassification (need to specify costs)使误分类的期望成本达到最小（需要设定成本）

Example, cont.

- Estimates of *b*' s are derived through an iterative process called *maximum likelihood estimation*

通过一个反复最大似然估计的过程来估计beta变量取值。

- Let' s include all 12 predictors in the model now

让我们把所有12个预测因子放入模型。

- XLMiner' s output gives coefficients for the logit, as well as odds for the individual terms

XLMiner可以给出logit回归的系数和每一项的胜算几率。

The Regression Model

Input variables	Coefficient	Std. Error	p-value	Odds
Constant term	-13.20165825	2.46772742	0.00000009	*
Age	-0.04453737	0.09096102	0.62439483	0.95643985
Experience	0.05657264	0.09005365	0.5298661	1.05820346
Income	0.0657607	0.00422134	0	1.06797111
Family	0.57155931	0.10119002	0.00000002	1.77102649
CCAvg	0.18724874	0.06153848	0.00234395	1.20592725
Mortgage	0.00175308	0.00080375	0.02917421	1.00175464
Securities Account	-0.85484785	0.41863668	0.04115349	0.42534789
CD Account	3.46900773	0.44893095	0	32.10486984
Online	-0.84355801	0.22832377	0.00022026	0.43017724
CreditCard	-0.96406376	0.28254223	0.00064463	0.38134006
EducGrad	4.58909273	0.38708162	0	98.40509796
EducProf	4.52272701	0.38425466	0	92.08635712

Figure 10.3: Logistic regression coefficient table for personal loan acceptance as a function of 12 predictors.

Estimated Equation for Logit (Equation 10.9)

$$
\begin{aligned}
\text{logit} = & -13.201 - 0.045 Age + 0.057 Experience + 0.066 Income + 0.572 Family \\
& + 0.18724874 CCAvg + 0.002 Mortgage - 0.855 Securities + 3.469 CD \\
& - 0.844 Online - 0.964 Credit\ Card + 4.589 EducGrad + 4.523 EducProf
\end{aligned}
$$

Equation for Odds (Equation 10.10)

$$\begin{aligned}\text{odds}(Personal\ Loan = \text{Yes}) \;=\; & e^{-13.201}(0.956)^{Age}\ (1.058)^{Experience}\ (1.068)^{Income} \\ & \cdot\ (1.771)^{Family}\ (1.206)^{CCAvg}\ (1.002)^{Mortgage} \\ & \cdot\ (0.425)^{Securities}\ (32.105)^{CD}\ (0.430)^{Online} \\ & \cdot\ (0.381)^{CreditCard}(98.405)^{EducGrad}\ (92.086)^{EducProf}\end{aligned}$$

Converting to Probability

$$p = \frac{Odds}{1 + Odds}$$

Interpreting Odds, Probability

For predictive classification, we typically use probability with a cutoff value 为了做预测性分类，我们通常使用概率估计值并确定一个截值。

For explanatory purposes, odds have a useful interpretation: 胜算几率可用于解释模型。

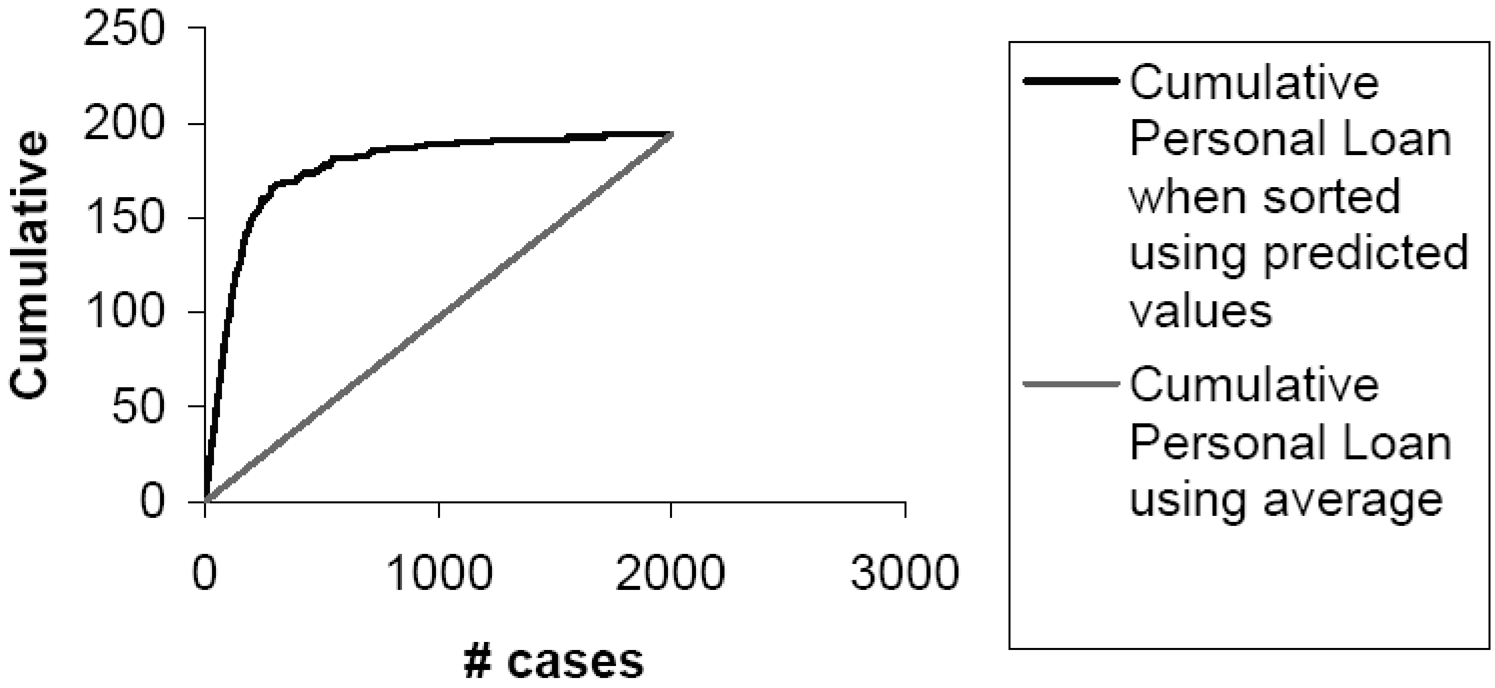
- If we increase x_1 by one unit, holding x_2, $x_3 \cdots x_q$ constant, then 如果我们增加x_1一个单位，并保留其他变量x_2, $x_3 \cdots x_q$为常数。
- b_1 is the factor by which the odds of belonging to class 1 increase 那么记录属于1的胜算几率增加b_1倍。

Loan Example: Evaluating Classification Performance

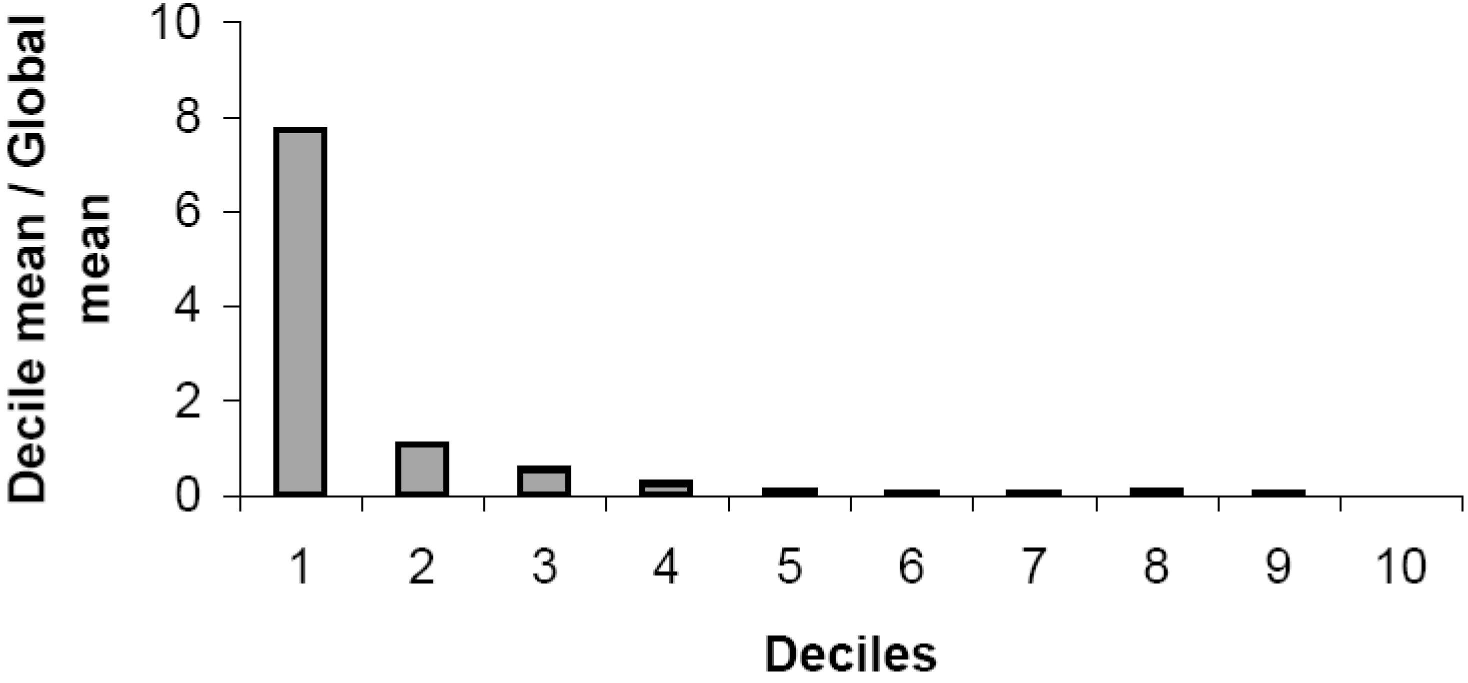
Performance measures: Confusion matrix and % of misclassifications　预测表现的评价指标：混淆矩阵和错误分类的百分比。

More useful in this example: lift　本例中更有用的评价指标是提升图。

Lift chart (validation dataset)



Decile-wise lift chart (validation dataset)



Multicollinearity 多重共线性

Problem: As in linear regression, if one predictor is a linear combination of other predictor(s), model estimation will fail 如同在线性回归中出现的问题一样，如果一个预测因子是其他预测因子的线性组合，模型估计就会失败。

- Note that in such a case, we have at least one redundant predictor 注意在这种情况下，我们有至少一个冗余的预测因子。

Solution: Remove extreme redundancies (by dropping predictors via variable selection – see next, or by data reduction methods such as PCA) 解决方法：移除这个极端的冗余预测因子（通过变量选择或者通过主成分分析法）

Variable Selection 变量选择

This is the same issue as in linear regression 这与线性回归出现的问题是一样的。

- The number of correlated predictors can grow when we create derived variables such as interaction terms (e.g. *Income x Family)*, to capture more complex relationships 如果我们新建变量（比如交互作用变量 Income X Family）来捕捉更复杂的关系，那么相关的预测因子数目可能会增加。
- Problem: Overly complex models have the danger of overfitting 问题：过度复杂的模型有过度拟合的危险。
- Solution: Reduce variables via automated selection of variable subsets (as with linear regression) 解决方法：通过自动选择变量子集方法减少变量个数（如同在线性回归中做的那样）。

P-values for Predictors 预测因子的p值

- Test null hypothesis that coefficient = 0 检测零假设：变量系数等于0。

- Useful for review to determine whether to include variable in model 这对检视并决定是否将该变量放入模型中很有用。

Complete Example:
Predicting Delayed Flights DC to NY

Variables

Outcome: delayed or not-delayed

Predictors:

- Day of week
- Departure time
- Origin (DCA, IAD, BWI)
- Destination (LGA, JFK, EWR)
- Carrier
- Weather (1 = bad weather)

Data Preprocessing

Create binary dummies for the categorical variables

Partition 60%-40% into training/validation

The Fitted Model (not all 28 variables shown)

Input variables	Coefficient	Std. Error	p-value	Odds
Constant term	-2.76648855	0.60903645	0.00000556	*
Weather	16.94781685	472.3040772	0.97137541	22926812
ORIGIN_BWI	0.31663841	0.407509	0.43715307	1.37250626
ORIGIN_DCA	-0.52621925	0.37920129	0.1652271	0.59083456
DEP_TIME_BLK_0700-0759	0.17635399	0.52038968	0.73469388	1.19286025
DEP_TIME_BLK_0800-0859	0.37122276	0.4879483	0.44678667	1.44950593
DEP_TIME_BLK_0900-0959	-0.2891154	0.61024719	0.6356656	0.74892575
DEP_TIME_BLK_1000-1059	-0.84254718	0.65849793	0.20072155	0.4306123
DEP_TIME_BLK_1100-1159	0.26919952	0.62188113	0.66510242	1.30891633
DEP_TIME_BLK_1200-1259	0.39577994	0.47712085	0.40681183	1.48554242
DEP_TIME_BLK_1300-1359	0.23689635	0.49711299	0.63368666	1.26730978
DEP_TIME_BLK_1400-1459	0.94953001	0.4257178	0.02571949	2.58449459
DEP_TIME_BLK_1500-1559	0.81428736	0.47320139	0.08528619	2.25756645
DEP_TIME_BLK_1600-1659	0.73656398	0.46096623	0.11007198	2.08874631
DEP_TIME_BLK_1700-1759	0.80683631	0.42013136	0.05480258	2.24080753
DEP_TIME_BLK_1800-1859	0.65816337	0.56922781	0.2475834	1.93124211
DEP_TIME_BLK_1900-1959	1.40413988	0.47974923	0.00342446	4.07202291
DEP_TIME_BLK_2000-2059	0.94785261	0.63308424	0.1343417	2.580163
DEP_TIME_BLK_2100-2159	0.76115495	0.45146817	0.09180449	2.14074731
DEST_EWR	-0.33785093	0.31752595	0.28732395	0.7133016
DEST_JFK	-0.66931868	0.2657896	0.01179471	0.5120573
CARRIER_CO	1.81500936	0.53502011	0.0006928	6.14113379
CARRIER_DH	1.25616693	0.52265555	0.016242	3.51193428
CARRIER_DL	0.41380161	0.33544913	0.21736139	1.51255703
CARRIER_MQ	1.73093832	0.32989427	0.00000015	5.64594936

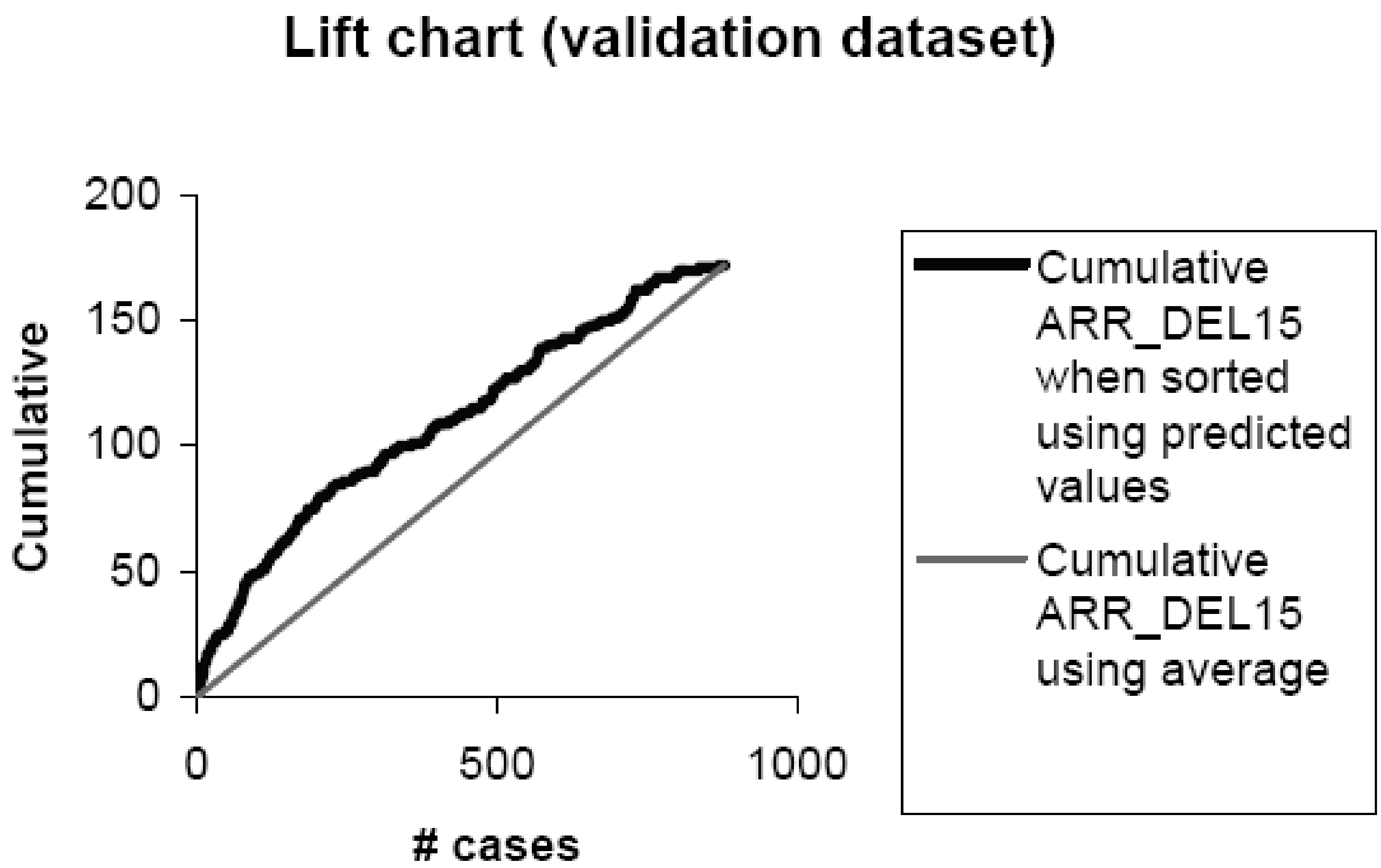
Model Output (Validation Data)

Cut off Prob.Val. for Success (Updatable)	0.5

Classification Confusion Matrix		
	Predicted Class	
Actual Class	delayed	non-delayed
delayed	18	154
non-delayed	3	705

Error Report			
Class	# Cases	# Errors	% Error
delayed	172	154	89.53
non-delayed	708	3	0.42
Overall	880	157	17.84

Lift Chart



Lift chart (validation dataset)
200
150
100
50
0
Cumulative
0
500
1000
cases
Cumulative ARR_DEL15 when sorted using predicted values
Cumulative ARR_DEL15 using average

After Variable Selection (Model with 7 Predictors)

Validation Data scoring - Summary Report

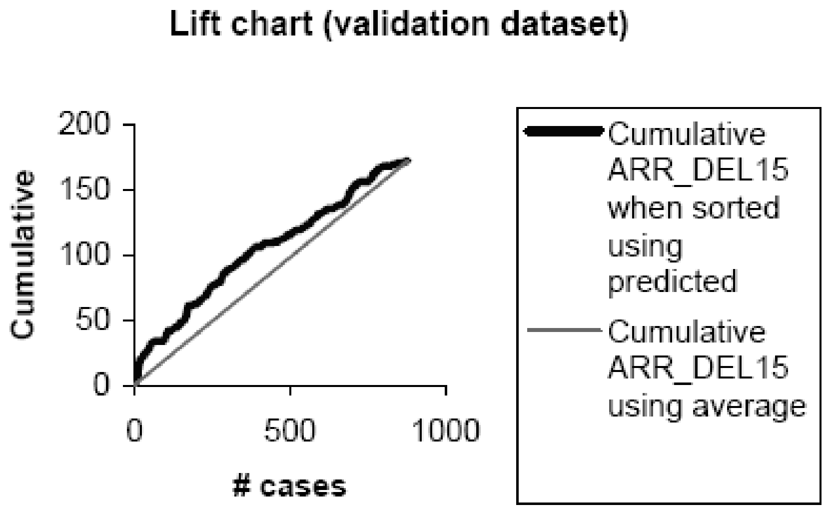
Cut off Prob.Val. for Success (Updatable)	0.5

Classification Confusion Matrix

	Predicted Class	
Actual Class	1	0
1	13	159
0	0	708

Error Report

Class	# Cases	# Errors	% Error
1	172	159	92.44
0	708	0	0.00
Overall	880	159	18.07



7-Predictor Model

Input variables	Coefficient	Std. Error	p-value	Odds
Constant term	-1.76942575	0.11373349	0	*
Weather	16.77862358	479.4146118	0.97208124	19358154
DEP_TIME_BLK_0600-0659	-0.62896502	0.36761174	0.08709048	0.53314334
DEP_TIME_BLK_0900-0959	-1.26741421	0.47863296	0.00809724	0.28155872
DEP_TIME_BLK_1000-1059	-1.37123489	0.52464402	0.00895813	0.25379336
DEP_TIME_BLK_1300-1359	-0.6303032	0.3188065	0.04803356	0.53243035
Sun-Mon	0.52237105	0.15871418	0.00099736	1.68602061
Carrier_CO_OH_MQ_RU	0.68775123	0.15049717	0.00000488	1.98923719

Note that Weather is unknown at time of prediction (requires weather forecast or dropping that predictor)

Summary

- Logistic regression is similar to linear regression, except that it is used with a categorical response 分类评定回归（逻辑回归）与线性回归类似，区别是分类评定回归使用类别型变量作为被解释变量。

- It can be used for explanatory tasks (=profiling) or predictive tasks (=classification) 可用于解释性任务或者预测性任务。

- The predictors are related to the response Y via a nonlinear function called the *logit* 预测因子与目标变量Y之间的关系是成为Logit的非线性关系。

- As in linear regression, reducing predictors can be done via variable selection 与线性回归一样，减少预测因子可以在变量选择中完成。

- Logistic regression can be generalized to more than two classes (not in XLMiner) 可以推广到多个类别的情形。