Lab 2: Power System Optimization

1. Objective

- Get familiar with the basic concept of OPF model
- Get familiar with the use of YAIMIP to model, optimize and analyze OPF model

2. Demo Code (example.m)

YALMIP is a MATLAB toolbox for optimization modeling and is not shipped with any low-level solvers. Solvers should be installed as described in the solver manuals. Fortunately, YALMIP is compatible with almost all solvers on the market, such as GUROBI, COPT, GLPK, etc. This lab uses YAMLIP and open-source mixed integer programming solver GLPK to solve (See the appendix for the installation tutorial).

The following piece of code introduces essentially everything you need to start with the lab. A classical problem in scheduling and linear programming is the economic dispatch problem. In this problem, our task is to control the power output of generators to meet forecasted power demand with minimal costs. We have several different power plants with operating costs and various constraints. We will start with a very simple model, and then expand this model with more advanced features. To make the code easy to read, we will write it in a verbose non-vectorized format [1].

$$\begin{aligned} &\min \quad \sum_{t=1}^{T}\sum_{g=1}^{N} c_g x_{g,t} \\ &s.t \begin{cases} P_g^{\min} \le x_{g,t} \le P_g^{\max} & \forall g \in \{1,\cdots,N\}, t \in \{1,\cdots,T\} \\ \sum_{g=1}^{N} x_{g,t} = Pd_t & \forall t \in \{1,\cdots,T\} \end{cases} \end{aligned} \tag{1}$$

Data

Assume that there are N=3 generators. Each of these generators has a maximum power output limit, and a minimum power output limit. Our scheduling problem is solved over T=6 time intervals and the forecasted power demand is given. The cost of i-th generator for one time interval is given by a linear function $c_g x_{g,t}$ where $x_{g,t}$ is the output power from generator g at the time interval of t.

```
Nunits = 3;  %% Numbers of power generating plants
Horizon = 6; %% Time
Pmax = [100;50;25]; %% Maximum power capacity
Pmin = [20;10;0];   %% Minimum power capacity
C = [10 30 50];   %% Linear cost price
Pforecast = [80 130 160 150 120 80]; %% The forecasted power demand
```

Model

The model description starts with the definition of variables, where x is a continuous variable representing the power output of generator g at time interval t.

```
x      = sdpvar(Nunits,Horizon,'full');
```

Then, we use for-loops to add constraints one by one.

```
Constraints = [];
for k = 1:Horizon
  Constraints = [Constraints, Pmin <= x(:,k) <= Pmax];
end
```

The total demand in each time interval must be equal to the total generation.

```
for k = 1:Horizon
  Constraints = [Constraints, sum(x(:,k)) == Pforecast(k)];
end
```

The total operation cost over the forecasted horizon is set as the objective function.

```
Objective = 0;
for k = 1:Horizon
  Objective = Objective + C*x(:,k);
end
```

Optimize and Analyze

Once the model description is complete, we can solve the problem and display the results.

```
ops = sdpsettings('solver', 'glpk');
%% Set the solver to glpk
optimize(Constraints, Objective, ops)
bar(value(P)','stacked')
ylim([0,170])
ylabel('Power/MW')
xlabel('Time/h')
legend('Unit 1','Unit 2','Unit 3');
```

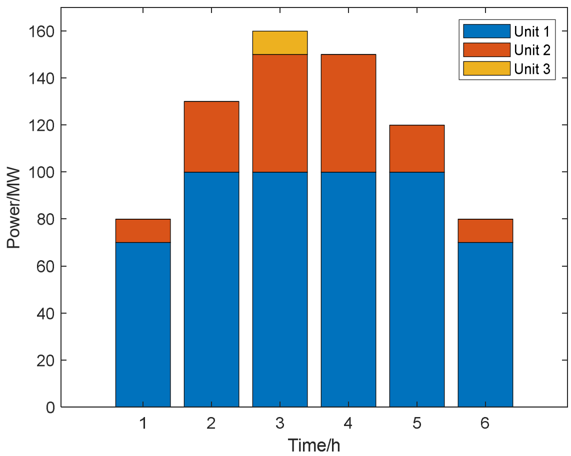


Figure 1. The Output of Each Unit

Now, it is up to you to develop your own code and complete the following three tasks.

3. Tasks

3.1 Single-period OPF

3.1.1 Formulation

Single-period OPF only considers one time interval and dispatches all the online generators to supply the demand while minimizing the total operating cost.

- **Objective function**

For the purpose of simplicity, we use linear cost function for each generator as shown in (2).

$$f = \sum_{i=1}^{NG} (c_i^o x_i + c_i^l) \tag{2}$$

where, x_i is the power output from generator *i*; c_i^o and c_i^l is cost coefficients of generator *i*. *NG* is the total number of generators.

- **Constraints**

a) Nodal power balance and branch flow constraints

We start with B-theta based DC power flow model and embed it into the formulation as a set of equality constraints[2].

$$\sum_{j=1}^{NB} B_{ij}^{bus} \theta_j = \sum_{j=1}^{NB} C_j^g x_j - Pd_i \quad \forall i \in \{1, \cdots, NB\} \tag{3}$$

where, θ_j is the voltage angle of bus j; Pd_i is the load of bus j; NL is the number of branches; C^g is the generator connection matrix, whose elements at (i, j) can be defined as one if generator j is located at bus i and set as zero otherwise. Please note everything should be in per unit (i.e., all power related quantities should be divided by baseMVA), so that theta here can be with physical meaning and in the unit of radian.

Then we use B^f matrix to obtain branch flow within a box constraints [$-F^{\max}$, $+F^{\max}$].

$$-F_i^{\max} \le \sum_{j=1}^{NB} B_{ij}^f \theta_j \le F_i^{\max} \quad \forall i \in \{1, \cdots, NL\} \tag{4}$$

where B^f is defined as:

$$B^f = B^{ff} (C^f - C^t) \tag{5}$$

$$B^{bus} = (C^f - C^t)^T B^f \tag{6}$$

B^{ff} is a sparse diagonal matrix, which can be defined such that its $(i, i)th$ element is the susceptance of branch l. The sparse connection matrices C^f and C^t used in building the system admittance matrices can be defined as follows. The $(i, j)th$ element of C^f and the $(i, k)th$ element of C^t are equal to 1 for each branch l, where branch l connects from bus *j* to bus *k*. All other elements of C^f and C^t are 0.

b) Generator capacity constraints

$$P_i^{\min} \le x_i \le P_i^{\max} \quad \forall i \in \{1, \cdots, NG\} \tag{7}$$

where, $P_i^{\max}$ and $P_i^{\min}$ are the maximum and minimum output of generator i.

3.1.2 Task for single-period OPF

A modified IEEE 39-bus system is selected as test system, which can be found in the attachment (case39ee.m).

The required tasks include:

a) Use YALMIP to establish the single-period OPF model above and optimize using GLPK solver;

b) Use "rundcopf" function in MATPOWER to solve the same problem again so as to verify the results from Step a).

c) Set the load ratio to 0.8, 1, and 1.1 and observe the change in branch loading rate and generator power output, analyze the influence of load ratio on OPF solutions.

Tips:

a) Branch loading rate can be calculated as follows:

$$r_l = \frac{F_l}{F_l^{\max}} = \frac{\sum_j^{NB} B_{lj}^f \theta_j}{F_l^{\max}} \quad \forall l \in \{1, \cdots, NL\} \tag{8}$$

b) Matrices related to *B* in (3) and (4) can be easily obtained using "makeBdc" function in MATPOWER.

3.2 Multi-period OPF

3.2.1 Formulation

In this task, we upgrade the single-period OPF model into multi-period, while consider wind power generation as well as generator ramping constraints.

■ **Objective function**

Change the objective function (2) to a multi-period cost function (9). Also, another term f_2 is added to consider the cost of wind curtailment.

$$f_1 = \sum_{t=1}^{T} \sum_{i=1}^{NG} (c_i^o x_{i,t} + c_i^l) \tag{9}$$

$$f_2 = \lambda \sum_{t=1}^{T} (Pw_t - x_t^w) \tag{10}$$

$$f = f_1 + f_2 \tag{11}$$

where, λ is the penalty of wind curtailment, $\lambda = 10$; x_t^w is the wind generation output at time t; Pw_t is the predicted wind generation output at time t.

■ **Constraints**

a) Nodal power balance and branch flow constraints

Based on the formulation of (3), the output of wind generator is added into nodal power balance constraints as (12), with additional term added into the nodal power balance constraints.

$$\sum_{j=1}^{NB} B_{ij}^{bus}\theta_{j,t} = \sum_{j=1}^{NB} C_j^g x_{j,t} + C^w x_t^w - Pd_{i,t} \quad \forall i \in \{1,\cdots,NB\}, t \in \{1,\cdots,T\} \tag{12}$$

$$-F_i^{\max} \le \sum_{j=1}^{NB} B_{ij}^f \theta_{j,t} \le F_i^{\max} \quad \forall i \in \{1,\cdots,NL\}, t \in \{1,\cdots,T\} \tag{13}$$

where, C^w is the wind generator connection matrix, which definition is similar to C^s.

b) Generator capacity constraints

It is basically the same as (7) but with a new dimension of time added.

$$P_i^{\min} \le x_{i,t} \le P_i^{\max} \quad \forall i \in \{1,\cdots,NG\}, t \in \{1,\cdots,T\} \tag{14}$$

where, $P_i^{\max}$ and $P_i^{\min}$ are the maximum and minimum output power of generation i at time interval t.

c) Generator ramping constraints

$$-RD_i \le x_{i,t} - x_{i,t-1} \le RU_i \quad \forall i \in \{1,\cdots,NG\}, t \in \{2,\cdots,T\} \tag{15}$$

where, RU_i is upward ramp limit of generator i; RD_i is downward ramp limit of generator i;

$$RD_i = RU_i = \eta P_i^{\max} \tag{16}$$

where, $\eta = 0.7$.

d) Wind curtailment constraints

$$0 \le x_t^w \le Pw_t \quad \forall t \in \{1,\cdots,T\} \tag{17}$$

where, Pw_t is the predicted wind generation output at time t.

3.2.2 Tasks for multi-period OPF

A modified IEEE 39-bus system is selected as test system, wind generator is installed on bus 14. Load data and wind perdition data is stored in the "Data for multi-period OPF" sheet of the DATA.xlsx. Set the load ratio to 1. Please complete the following tasks:

a) Suppose the capacity of the wind generator is 10% of the sum of generator capacity (excluding wind generator itself), use YALMIP to establish the multi-period OPF model above and optimize, then observe the branch loading rate and the power output of generators for different time intervals.

b) Increase the capacity of the wind generator to 25% and 50% of the sum of generator capacity (excluding wind generator itself), use YALMIP to establish the multi-period OPF model above and optimize, then observe the branch loading rate and the power output of generators for different time intervals.

c) Summarize and find out the potential impact of increasing wind power penetration level on system operation.

3.3 Security-constrained unit commitment (SCUC)

3.3.1 Formulation

Security-constrained unit commitment (SCUC) problem is one of most important tools in modern power system control center and electricity market. Also, we consider an energy storage system in this model.

■ **Objective function**

$$f_1 = \sum_{t=1}^{T}\sum_{i=1}^{NG}(c_i^o x_{i,t} + c_i^l u_{i,t}) + \sum_{t=2}^{T}\sum_{i=1}^{NG} c_i^s y_{i,t} \tag{18}$$

$$f_2 = \lambda \sum_{t=1}^{T}(Pw_t - x_t^w) \tag{19}$$

$$f = f_1 + f_2 \tag{20}$$

where, $x_{i,t}$ is the power output from generator i at time interval t. $u_{i,t}$ is a binary variable (either 0 or 1) representing the on-off status of generator i at time interval t. $y_{i,t}$ is a binary variable representing the generator i start up at time interval t. Therefore, the startup cost of generators is added here, c_i^s is the startup cost of generator i.

■ **Constraints**

a) Nodal power balance and branch flow constraints

Based on (12), the output of energy storage is added as a term into the equality constraints.

$$\sum_{j=1}^{NB} B_{ij}^{bus}\theta_{j,t} = \sum_{j=1}^{NB} C_j^g x_{j,t} + C^w x_t^w + C^e(x_t^s - x_t^c) - Pd_{i,t} \quad \forall i \in \{1,\cdots,NB\}, t \in \{1,\cdots,T\} \tag{21}$$

$$-F_i^{\max} \le \sum_{j=1}^{NB} B_{ij}^f \theta_{j,t} \le F_i^{\max} \quad \forall i \in \{1,\cdots,NL\}, t \in \{1,\cdots,T\} \tag{22}$$

where, $\theta_{j,t}$ is the angle of bus j at time t; $Pd_{i,t}$ is the load of bus j at time t; x_t^c and x_t^s represents charge and discharge power of ESS at time t, respectively; C^e is the energy storage connection matrix, which definition is similar to C^s.

b) Generator capacity constraints

The generator on/off binary variable $u_{i,t}$ is added into the box constraints (14) so that $x_{i,t}$ is forced to 0 when $u_{i,t}$ is 0 (i.e., the generator i is off in time interval t).

$$P_i^{\min} u_{i,t} \le x_{i,t} \le P_i^{\max} u_{i,t} \quad \forall i \in \{1,\cdots,NG\}, t \in \{1,\cdots,T\} \tag{23}$$

where, $P_i^{\max}$ and $P_i^{\min}$ are the maximum and minimum output of generator i at time interval t.

c) Generator ramping constraints

This is the same as (15).

$$-RD_i \le x_{i,t} - x_{i,t-1} \le RD_i \quad \forall i \in \{1,\cdots,NG\}, t \in \{2,\cdots,T\} \tag{24}$$

where, RD_i is ramping limits of generator i. The value of RD_i is defined as (16).

d) Wind generation constraints

This is the same as (17).

$$0 \le x_t^w \le Pw_t \quad \forall t \in \{1, \cdots, T\} \tag{25}$$

where, Pw_t is the predicted wind generation output at time interval t.

e) Generator minimum up/down time constraints

Generators are required to stay in the on/off status for an extended period before they can be switched off/on. Constraints (26) guarantees $y_{i,t}$ take the appropriate values when the unit starts up. Constraints (27) is the minimum up time constraint. Constraints (28) is the minimum down time constraint.

$$y_{i,t} \ge u_{i,t} - u_{i,t-1} \quad \forall i \in \{1, \cdots, NG\}, t \in \{2, \cdots, T\} \tag{26}$$

$$\sum_{k=t}^{t+TO_i-1} u_{i,k} \ge UT_i y_{i,t} \quad \forall i \in \{1, \cdots, NG\}, t \in \{2, \cdots, T - UT_i + 1\} \tag{27}$$

$$\sum_{k=t}^{t+TS_i-1} (1 - u_{i,k}) \ge UD_i (u_{i,t-1} - u_{i,t}) \quad \forall i \in \{1, \cdots, NG\}, t \in \{2, \cdots, T - UD_i + 1\} \tag{28}$$

where, UT_i is minimum up time of generator i; UD_i is minimum down time of generator i.

f) Energy storage constraints

Constraints (29)-(33) represent the operating behavior of an energy storage system (ESS).

Constraints (29) is the charging/discharging state constraint of ESS, which ensures that the ESS can only work at either charging or discharging condition. Constraints (30) and (31) confine the maximum power during charging or discharging. Constraints (32) and (33) represents state-of-charge (SOC) operation of ESS.

$$u_t^c + u_t^s \le 1 \quad \forall t \in \{1, \cdots, T\} \tag{29}$$

$$0 \le x_t^c \le P_c^{\max} u_t^c \quad \forall t \in \{1, \cdots, T\} \tag{30}$$

$$0 \le x_t^s \le P_s^{\max} u_t^s \quad \forall t \in \{1, \cdots, T\} \tag{31}$$

$$E_t = E_{t-1} + \eta_c x_t^c - \eta_s x_t^s \quad \forall t \in \{1, \cdots, T\} \tag{32}$$

$$\varepsilon E^{cap} \le E_t \le E^{cap} \quad \forall t \in \{1, \cdots, T\} \tag{33}$$

where, E_t represents the stored energy of ESS at time t; x_t^c and x_t^s represents charge and discharge power of ESS at time t, respectively; u_t^c and u_t^s are binary variables and they define the charging/discharging status of ESS, e.g., if $u_t^c = 1$, the ESS is charging from the grid; E^{cap} is the maximum capacity of ESS; η_c and η_s are the efficiency coefficient of ESS during charging and discharging, which are both set to 0.9; εE^{cap} is the minimum stored energy of ESS, which is set as $\varepsilon E^{cap} = 0.1 E^{cap}$.

3.3.2 Tasks for SCUC

Based on the IEEE 39-bus system, wind generator is installed on bus 14 and ESS is installed on bus 13 and initial stored energy $E_0 = 0.2 E^{cap}$. Load data and wind perdition data is stored in the " Data for SCUC" sheet of the DATA.xlsx. Set the load ratio to 1. SOC is defined as follows:

$$SOC = {E_t}/{E^{cap}} \tag{34}$$

Please complete the following tasks:

a) Suppose the capacity of the wind generator is 10% of the sum of generator capacity (excluding wind generator itself), use YALMIP to establish the model above and optimize, then observe the branch loading rate, the generator output, the wind output as well as SOC changes in different time intervals.

b) Increase the capacity of the wind generator to 25% and 50% of the sum of generator capacity (excluding wind generator itself), use YALMIP to establish the model above and optimize, then observe the branch loading rate, the generator output, the wind output as well as SOC changes in different time intervals.

c) Summarize and find out the potential impact of increasing wind power penetration level on system operation.

d) Identify the role of ESS by comparing the UC solution with and without the ESS installation in the test case.

References

[1] "Unit commitment," *YALMIP*, Sep. 16, 2016. https://yalmip.github.io/example/unitcommitment/ (accessed Sep. 24, 2022).

[2] Zimmerman Ray Daniel, Carlos Edmundo Murillo-Sánchez, and Robert John Thomas, "MATPOWER: Steady-State Operations, Planning, and Analysis Tools for Power Systems Research and Education," *IEEE T Power Syst*, vol. 26, no. 1, pp. 12–19, 2011, doi: 10/b94mfs.

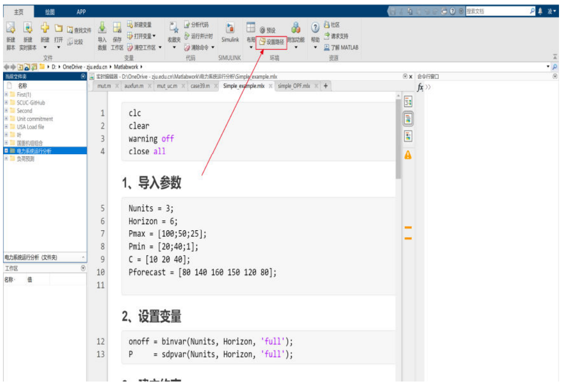
[3] "COPT," https://shanshu.ai/solver (accessed Sep. 27, 2022).

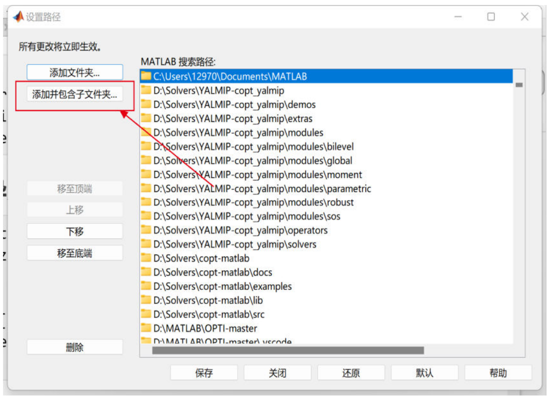
APPENDIX – INSTALLATION OF DEPENDENT SOFTWARE

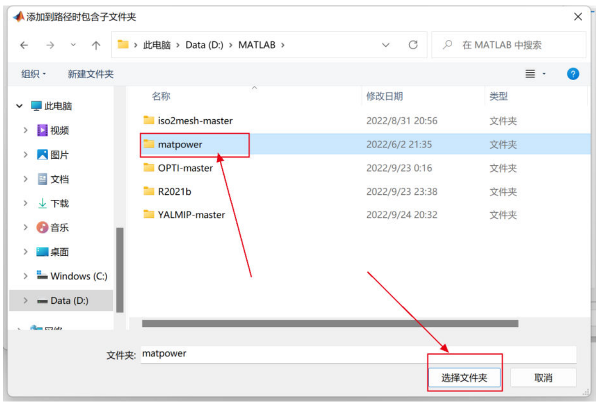
1. MATPOWER

You can download from https://matpower.org/ or use the version in the attachment.

Unzip the downloaded compressed package and save it to your own folder. Set the folder in the MATLAB environment path.



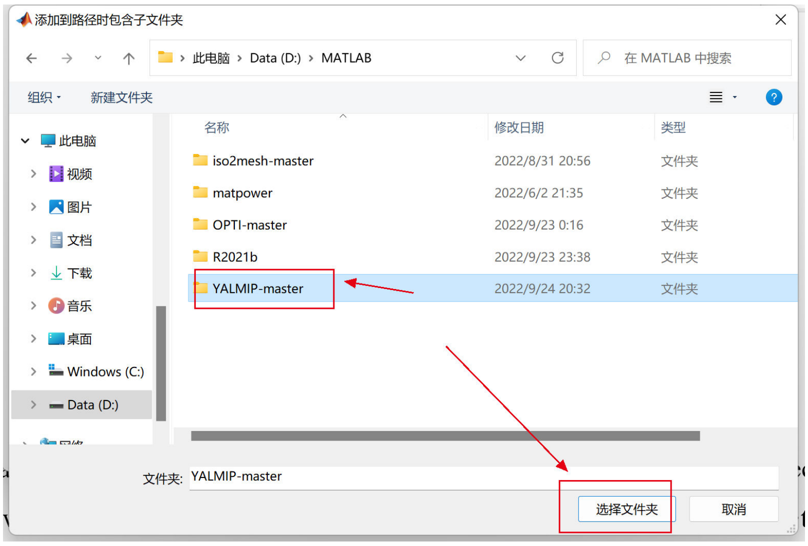




2. YAMLIP

You can download from https://yalmip.github.io/ or use the version in the attachment.

Unzip the downloaded compressed package and save it to your own folder. Set the folder in the MATLAB environment path.



3. GLPK

You can download from https://github.com/blegat/glpkmex or use the version in the attachment.

Unzip the downloaded compressed package and save it to your own folder. Set the folder in the MATLAB environment path.