# Game Theory Homework 3

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# Qn 2

## 1)

### a) Monopoly

Maximize $(1-p_1-p_2)p_1+(1-p_1-p_2-0.1)p_2=(1-p_1-p_2)(p_1+p_2)-0.1p_2$ where $p_1, p_2$ are the quantities of the product of the old firm and the new firm respectively.

The payoff is maximized when $p_2=0$ and $p_1=0.5$, which is $u_1=0.25$ and $u_2=0$

### b) *Cournot* duopoly

Payoff of the old firm(monopolist) $u_1=(1-p_1-p_2)p_1$, the payoff is maximized when $p_1=(1-p_2)/2$,

Payoff of the new firm $u_2=(1-p_1-p_2-0.1)p_2$, the payoff is maximized when $p_2=(0.9-p_1)/2$,

so $p_1=\frac{11}{30}, p_2=\frac{4}{15}, u_1=\frac{121}{900}, u_2=\frac{16}{225}$

### c) Aggressive behavior

The old firm(monopolist) wants the new entrant to make losses, so $u_2=(1-p_1-p_2-0.1)p_2\le 0$ for every $0\le p_2\le 1$

That is $u_2=(0.9-p_1-\frac{0.9-p_1}{2})\frac{0.9-p_1}{2}\le 0$ , solve $p_1=0.9$.

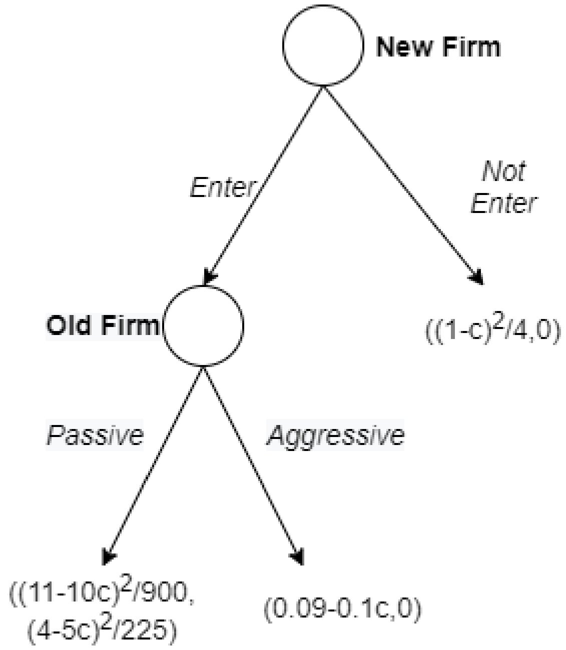
$u_1=(1-0.9)0.9=0.09, u_2=0$

## 2)

**Monopoly:** $u_1=(1-p_1-c)p_1$, the payoff is maximized when $p_1=\frac{1-c}{2}$, the maximum payoff is $u_1=\frac{(1-c)^2}{4}$

**Cournot duopoly**: $p_1=(1-c-p_2)/2$, $p_2=(0.9-c-p_1)/2$, solve $p_1=\frac{11-10c}{30}$,$p_2=\frac{4-5c}{15}$, $u_1=(1-c-p_1-p_2)p_1=\frac{(11-10c)^2}{900}, u_2=\frac{(4-5c)^2}{225}$

**Aggressive behavior**: $u_2=(0.9-c-p_1-p_2)p_2\le 0$ for every $0\le p_2\le 1$. (**The old firm wants the new entrant to earn nothing**) That is $p_1=0.9-c$, $u_1=0.09-0.1c$




## 3)

### a)

The old firm would be **passive** if $(11-10c)^2/900 > 0.09-0.1c$

$f(c) = (11-10c)^2 - (81-90c) = 100c^2 - 130c + 40$. $f(0.5) = f(0.8) = 0$, solve $(11-10c)^2/900 > 0.09 - 0.1c$ when $c > 0.8$ or $c < 0.5$

In this case, the new entrant would **enter** the market because $\frac{(4-5c)^2}{225} > 0$. The SPE of the game is (**Passive, Enter**). The threat of aggressive behavior is not credible because anyway the new entrant would enter the market

### b)

Otherwise ($0.5 < c < 0.8$), the old firm would be **aggressive**. The new entrant is indifferent in choosing **Enter** or **Not Enter** because anyway its payoff is 0. The SPE of the game is (**Aggressive, Enter**) and (**Aggressive, Not Enter**). The threat of aggressive behavior is still not credible because it cannot prevent the new entrant from entering the market.

### c)

If $c = 0.5$, the old firm's payoff is always$(11-10c)^2/900 = 0.09-0.1c = 0.04$ no matter being **Passive or Aggressive**.

The new entrant will choose **Enter** because $\frac{(4-5c)^2}{225} > 0$, the SPE of the game is (**Passive, Enter**) and (**Aggressive, Enter**)

### d)

If $c = 0.8$, the old firm's payoff is always $(11-10c)^2/900 = 0.09-0.1c = 0.01$ no matter being **Passive or Aggressive**.

The new entrant is in different in choosing **Enter** and **Not Enter** because anyway its payoff is 0. In this case, any action pair is SPE.

In all cases, the threat of aggressive behavior is not credible.

## 4)

| | Enter | Not Enter |
|---|---|---|
| **Passive** | $(\frac{(11-10c)^2}{900}, \frac{(4-5c)^2}{225})$ | $(\frac{(1-c)^2}{4}, 0)$ |
| **Aggressive** | $(0.09 - 0.1c, 0)$ | $(\frac{(1-c)^2}{4}, 0)$ |

If $0 \leq c < 0.1426$, because

$0.09 + 0.8c - c^2 < \frac{(1-c)^2}{4}$ when $0 \leq c < 0.1426$.

### a)

If $c > 0.8$ or $c < 0.5$, we can prove that $0.09 - 0.1c < \frac{(1-c)^2}{4}$.

**Prove**: $f(c) = 100 \times (\frac{(1-c)^2}{4} - (0.09 - 0.1c)) = 25c^2 - 40c + 16 = (5c-4)^2 > 0$

(**Aggressive, Not Enter**) is a pure strategy Nash equilibrium that is not subgame perfect equilibrium

For other value of $c$, there is no pure strategy Nash equilibrium that is not subgame perfect equilibrium.

## 5)

The SPNE will be the same as the answer to 3) because each market's payoff is independent to other market's payoff.