①

1维单粒子时序~~Green~~格林函数

$$iG(x,t;x',t') = \langle T[\psi(x,t)\psi^\dagger(x',t')]\rangle = \mathrm{Tr}\,\rho T[\psi(x,t)\psi(x',t')].$$

平衡多体理论

零温下. $\rho = |\Psi_0\rangle\langle\Psi_0|$

$$iG(x,t;x',t') = \langle\Psi_0|T[\psi(x,t)\psi^\dagger(x',t')]|\Psi_0\rangle \quad \text{(海森堡绘景)}.$$

相互作用绘景中 $\hat\psi(t) = U(t)\hat\psi(0)$.

$$\underline{U(t) = T\cdot\exp[-i\int_0^t \hat V(t_1)dt_1]}.$$

$$\hat\psi(t) = S(t,t')\hat\psi(t').$$

$$S(t,t') = U(t)U^\dagger(t')$$

$$\frac{\partial S(t,t')}{\partial t} = -i\hat V(t)S(t,t') \Rightarrow S(t,t') = T\exp[-i\int_{t'}^t dt_1\hat V(t_1)]$$

$U(t) = e^{iH_0t}e^{-iHt}$ 证明.

$e^{-iH(t)}\psi(t) = e^{-iH_0t}\hat\psi(t)$.

$\psi(0) = \psi(t) = e^{iHt}e^{-iH_0t}U(t)\hat\psi(0)$

$\Rightarrow U^\dagger = e^{iHt}e^{-iH_0t}$.

系统哈密顿量 $H = H_0 + V$, H_0基态 ϕ_0.

$$\psi(0) = S(0,-\infty)\phi_0.$$

假设 $\psi(\infty) = S(\infty,0)\psi(0) = e^{iL}\phi_0$.

$$\begin{aligned} G_\lambda(t) &= e^{iHt}G_\lambda e^{-iHt} \\ &= e^{iHt}e^{-iH_0t}\hat G_\lambda(t)e^{iH_0t}e^{-iHt} \\ &= U^\dagger(t)\hat G_\lambda(t)U(t). \\ &= \underline{S(0,t)\hat G_\lambda(t)S(t,0)} \end{aligned}$$

$$\begin{aligned} iG(x,t;x',t') &= \theta(t-t')\langle\Psi_0|\psi(x,t)\psi^\dagger(x',t')|\Psi_0\rangle \\ &\quad - \theta(t'-t)\langle\Psi_0|\psi^\dagger(x',t')\psi(x,t)|\Psi_0\rangle \end{aligned}$$

②

$$\theta(t-t')\ \langle\phi_0|S(-\infty,0)\ S(0,t)\ \hat{\psi}(x,t)\ S(t,0)\ S(0,t')\ \hat{\psi}^{+}(x',t')\ S(t',0)\ S(0,-\infty)|\phi_0\rangle.$$

$S(-\infty,\infty)S(\infty,0)$

$$=\theta(t-t')\ \frac{\langle\phi_0|S(-\infty,\infty)\hat{\psi}(x,t)\ \hat{\psi}^{+}(x',t')|\phi_0\rangle}{\langle\phi_0|S(\infty,-\infty)|\phi_0\rangle}$$

$S(\infty,-\infty)|\phi_0\rangle=e^{iL}|\phi_0\rangle$

$S(\infty,0)=e^{iL}\ S(-\infty,0).$

$S(0,\infty)=e^{-iL}\ S(0,-\infty)$

$\therefore S(0,-\infty)=e^{iL}\ S(0,\infty)$

$$iG(x,t;x',t')=\frac{\langle\Phi_0|T[S(\infty,-\infty)\ \psi(x,t)\psi^{+}(x',t')]|\Phi_0\rangle}{\langle\Phi_0|S(\infty,-\infty)|\Phi_0\rangle.}$$ (相互作用表象).

$\rho=\sum_{\Phi} P_{\Phi}|\Phi\rangle\langle\Phi|.$

$iG_{\Phi}(x,t;x',t')=\langle\Phi|T[\psi(x,t)\psi^{+}(x',t')]|\Phi\rangle$

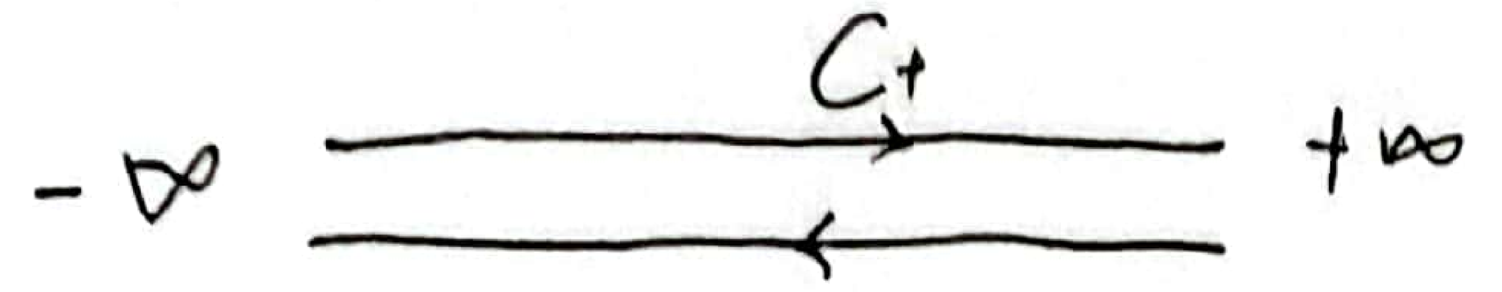
总格林函数 $G=\sum_{\Phi} P_{\Phi}G_{\Phi}.$

$\hat{\psi}(x,t)=e^{iHt}\psi(x)e^{-iHt}$

零温时 $H_{\varepsilon}=H+e^{-\varepsilon|t|}H'$

③. ~~Swr~~

Schwinger-keldysh Contour



$$iG_\Phi(x,t;x',t') = \langle \Phi(-\infty)|_I S(-\infty,\infty)\, T[S(\infty,-\infty)\hat\psi(x,t)\hat\psi^\dagger(x',t')]|\Phi(-\infty)\rangle_I$$

$$C = C_+ \cup C_-$$

使用回路 C 中的变量 τ 和时序

$$iG_\Phi(x,\tau;x',\tau') = \langle \Phi(-\infty)|_I\, T_c[S_c(-\infty,-\infty)\hat\psi(x,\tau)\hat\psi^\dagger(x',\tau')]|\Phi(-\infty)\rangle_I$$

$$S_c(-\infty,-\infty) \equiv T_c \exp\left(-i\oint_c d\tau_1 \hat H'(\tau_1)\right).$$

$$iG(x,\tau;x',\tau') = \mathrm{Tr}\,\rho(-\infty)\, T_c[S_c(-\infty,-\infty)\hat\psi(x,\tau)\hat\psi^\dagger(x',\tau')].$$

(1.7) $\rho = \sum_\Phi P_\Phi |\Phi\rangle\langle\Phi|$

(1.15) $|\Phi\rangle = S(0,\pm\infty)|\Phi(\pm\infty)\rangle_I$

$$\rho = S(0,-\infty)\,\rho(-\infty)\,S(-\infty,0).$$

$H\rho = E\rho.$

$\underline{\underline{\rho = |\Phi\rangle\langle\Phi|}}$.

$tr(\rho A)$ $\qquad\qquad \sum_i \langle\phi_i| \frac{e^{-\beta H}}{Z} A|\phi_i\rangle$

$= \sum_n \langle n|\Phi\rangle\langle\Phi|A|n\rangle.$

子剧论

$$\langle\hat O\rangle = \sum_j \langle e_j|\hat\rho\hat O|e_j\rangle = \sum_{ij} P_i \langle e_j|\phi_i\rangle\langle\phi_i|\hat O|e_j\rangle = \sum_i P_i\langle\phi_i|\hat O|\phi_i\rangle.$$

ⓟ.

$$iG(x,\tau;x',\tau') = \mathrm{Tr}\left(\rho\, T_c[S_c(0,0)\hat\psi(x,\tau)\hat\psi^\dagger(x',\tau')]\right).$$

$$iG(x,\tau;x',\tau') = \mathrm{Tr}\, \rho(t_0)\, T_c[S_c(t_0,t_0)\hat\psi(x,\tau)\hat\psi^\dagger(x',\tau')].$$

$$U(t) = T\exp\left(-i\int_0^t dt_1\, \mathcal{H}(t_1)\right).$$

$$S(t,t') = T\exp\left(-i\int_{t'}^{t} dt_1\, \hat{H}'(t_1)\right)$$

$A(t) = e^{iH(t-t_0)} A e^{-iH(t-t_0)}$ Heisenberg picture.

$\hat A(t) = e^{iH_0(t-t_0)} A e^{-iH_0(t-t_0)}$ Interaction picture.

$$S(t_0-i\beta,t_0) = e^{iH_0(-i\beta)}\, U(t_0-i\beta,t_0)$$
$$= e^{\beta H_0}\, U(t_0-i\beta,t_0).$$

$$U(t_0-i\beta,t_0) = e^{-\beta H}$$

$$\Rightarrow S(t_0-i\beta,t_0) = e^{\beta H_0}\cdot e^{-\beta H}.$$

ρ 等 $\dfrac{e^{-\beta H}}{Z} = \dfrac{e^{-\beta H_0} S(t_0-i\beta,t_0)}{Z}.$

$$iG(1,1') = \frac{1}{Z}\mathrm{Tr}\, e^{-\beta H_0} S(t_0-i\beta,t_0)\, T_c[S_c(t_0,t_0)\hat\psi_H(1)\hat\psi_H^\dagger(1')].$$

$\hat\psi(x,t) = e^{iHt}\psi(x)e^{-iHt}$, $\cancel{1\equiv(x,\tau)}$ $1\equiv(x,\tau)$, $1'\equiv(x',\tau')$

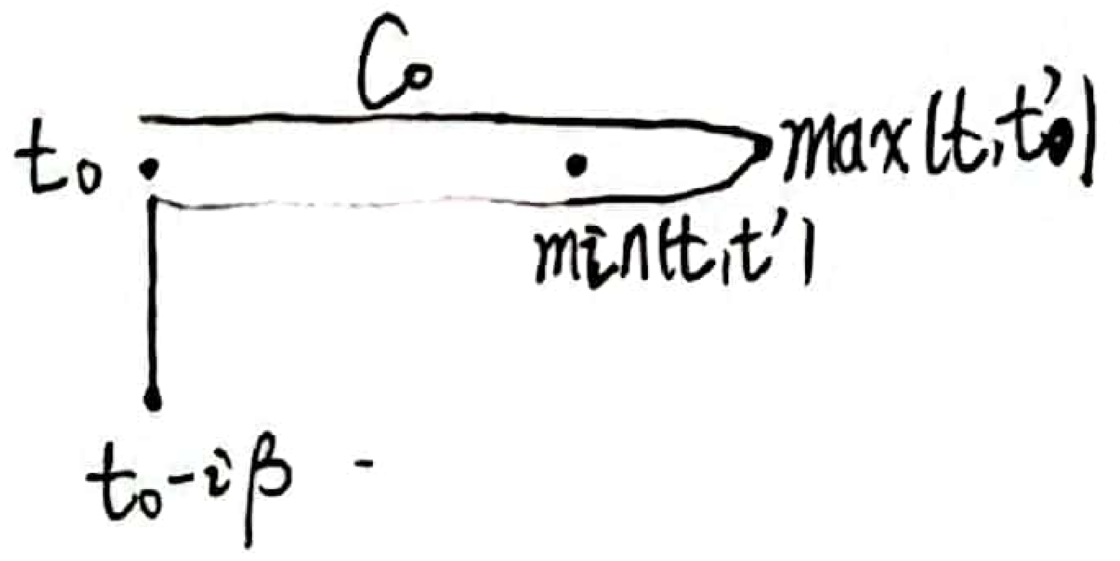
$$\hat\psi_H(x,t) = S(t_0,t)\hat\psi(x,t)S(t,t_0)$$

$$\mathcal{H}(t) = \underbrace{H}_{\text{平衡项}} + \underbrace{H'(t)}_{\text{扰动项}}$$

$$H = \underbrace{H_0}_{\text{无相互作用项}} + \underbrace{V(t)}_{\text{相互作用项}}$$

$\cancel{iG(1,1')}$

$$iG(1,1') = \frac{1}{Z}\mathrm{Tr}\, e^{-\beta H} S(t_0-i\beta,t_0)\, T_c[S_c(t_0,t_0)\, S(t_0,t)\hat\psi(1)S(t,t_0)\, S(t_0,t')\hat\psi^\dagger(1')S(t',t_0)].$$



Kadanoff-Baym contour.

①

$$|\infty\rangle = S(\infty,-\infty)|-\infty\rangle.$$

$$\langle t_0|\rho(t_0)|t_0\rangle = 1 = \frac{1}{Z}\langle t_0|e^{-\beta H_0} S(t_0-i\beta, t_0)|t_0\rangle.$$

$$= \frac{1}{Z}\mathrm{Tr}\, e^{-\beta H_0} T_c[S_{c^*}(t_0-i\beta,t_0) S_c(t_0,t_0)].$$

$$S_{c^*}(t_0-i\beta,t_0) = T_{c^*}\exp(-i\int_{c^*} d\tau_1 \hat{V}(\tau_1)).$$

Neglet of Initial Correlations and Schwinger-Keldysh Limit.

$$iG(1,1') = \langle\Phi_0|T_c[S_c^H(-\infty,\infty)\, S_c(-\infty,-\infty)\, \underbrace{\hat{\psi}(1)\,\hat{\psi}^+(1')}]|\Phi_0\rangle.$$

关于H_0的相互作用表象.

$$S_c^H(-\infty,-\infty) \equiv T_c\exp(-i\oint_c d\tau_1 \hat{V}(\tau_1)).$$

$$S_c(-\infty,-\infty) \equiv T_c\exp(-i\oint_c d\tau_1 \hat{H}'(\tau_1)).$$

Initial correlations with arbitrary initial density matrix.

$$iG(x,\tau;x',\tau') = \mathrm{Tr}\,\rho(t_0) T_c[S_c(t_0,t_0)\hat{\psi}(x,\tau)\hat{\psi}^+(x',\tau')].$$

$\rho(t_0) = \frac{e^{-\lambda B}}{\mathrm{Tr}e^{-\lambda B}}$, λ is not β, B is not Hermitian

Relation to Real-Time Green's Functions.

real-time Green function

$$iG(x,t;x',t') = \langle T[\psi(x,t)\psi^+(x',t')]\rangle \doteq \mathrm{Tr}\,\rho\, T[\psi(x,t)\psi^+(x',t)].$$

⑥.

$$G(x,\tau;x',\tau') = \begin{cases} G^T(x,t;x',t') \equiv -i\langle T[\psi(x,t)\psi^\dagger(x',t')]\rangle, & \text{if } \tau,\tau'\in C_+ \\ G^<(x,t;x',t') \equiv i\langle \psi^\dagger(x',t')\psi(x,t)\rangle, & \text{if } \tau\in C_+,\ \tau'\in C_- \\ G^>(x,t;x',t') \equiv -i\langle \psi(x,t)\psi^\dagger(x',t')\rangle, & \text{if } \tau\in C_-,\ \tau'\in C_+ \\ G^{\tilde T}(x,t;x',t') \equiv -i\langle \tilde T[\psi(x,t)\psi^\dagger(x',t')]\rangle, & \text{if } \tau,\tau'\in C_- \end{cases}$$

2x2 Green function matrix.

$$G = \begin{pmatrix} G^T & G^< \\ G^> & G^{\tilde T} \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}.$$

$$\underline{G^R = G^T - G^<} = G^> - G^{\tilde T} = -i\theta(t-t')\,[\psi(x,t)\psi^\dagger(x',t') + \psi^\dagger(x',t')\psi(x,t)].$$

$$\underline{G^A = G^T - G^>} = G^< - G^{\tilde T} = i\theta(t'-t)\psi(x,t)\psi^\dagger(x',t') + i\theta(t'-t)\psi^\dagger(x',t')\psi(x,t)$$

$$= i\theta(t'-t)\{\psi(x,t),\psi^\dagger(x',t')\}.$$

$$G^K = G^> + G^< = G^T + G^{\tilde T} = -i\langle[\psi(x,t),\psi^\dagger(x',t')]\rangle$$

$$\hat G = L\tau^3 G L^\dagger = \begin{pmatrix} G^R & G^K \\ 0 & G^A \end{pmatrix}.$$

$$G^T - G^< = -i\theta(t-t')\psi(x,t)\psi^\dagger(x',t') + i\theta(t'-t)\psi^\dagger(x',t')\psi(x,t).$$
$$-i\psi^\dagger(x',t)\psi(x,t).$$

$$(AB)^> = A^R B^> + A^> B^A.$$

$$C = AB.$$

$$C^<(t,t') = \int_C d\tau_1 A(t,\tau_1)B(\tau_1,t').$$

$$= \int_{-\infty}^{\infty} A^T(t,t_1)B^<(t_1,t')\,dt_1 + \int_{\infty}^{-\infty} A^<(t,t_1)B^{\tilde T}(t_1,t)\,dt_1$$

$$= \int_{-\infty}^{\infty} dt_1\,[(A^R(t,t_1) + A^<(t,t_1))B^<(t_1,t') - A^<(t,t_1)(B^<(t_1,t') - B^A(t_1,t))].$$

$$= \int_{-\infty}^{\infty} dt_1\,[A^R(t,t_1)B^<(t_1,t') - A^<(t,t_1)B^A(t_1,t)]$$

⑦. Quantum Kinetic Equation.

经典 Boltzmann 方程.

$$\frac{df}{dt} = \left(\frac{\partial f}{\partial T}\right)_{coll} \equiv I[f]$$

$$I[f] = \left(\frac{\partial}{\partial T} + \vec{v}\cdot\nabla_R + \vec{F}\cdot\nabla_p\right) f$$

量子力学情况下分布函数还有一个能量组元: $f = f(p, \omega, \vec{R}, T)$.

$$\left(\frac{\partial}{\partial T} + \vec{v}\cdot\nabla_R + \vec{F}\cdot\nabla_p + \frac{d\omega}{dT}\frac{\partial}{\partial\omega}\right) f = I[f].$$

Keldysh Equation.

$$G^R = G^T - G^<.$$

$$G^T - G^< = G_0^T - G_0^< + (G_0^T - G_0^<) U (G^T - G^<) + (G^T - G^<) \Sigma^R (G_0^T - G_0^<)$$

$$\cancel{= G_0^T - G_0^<}$$

$$G^> = G_0^> + (G_0 U G)^> + (G_0 \Sigma G)^>.$$

$$U(t) = U(t,t')\,\delta(t-t').$$

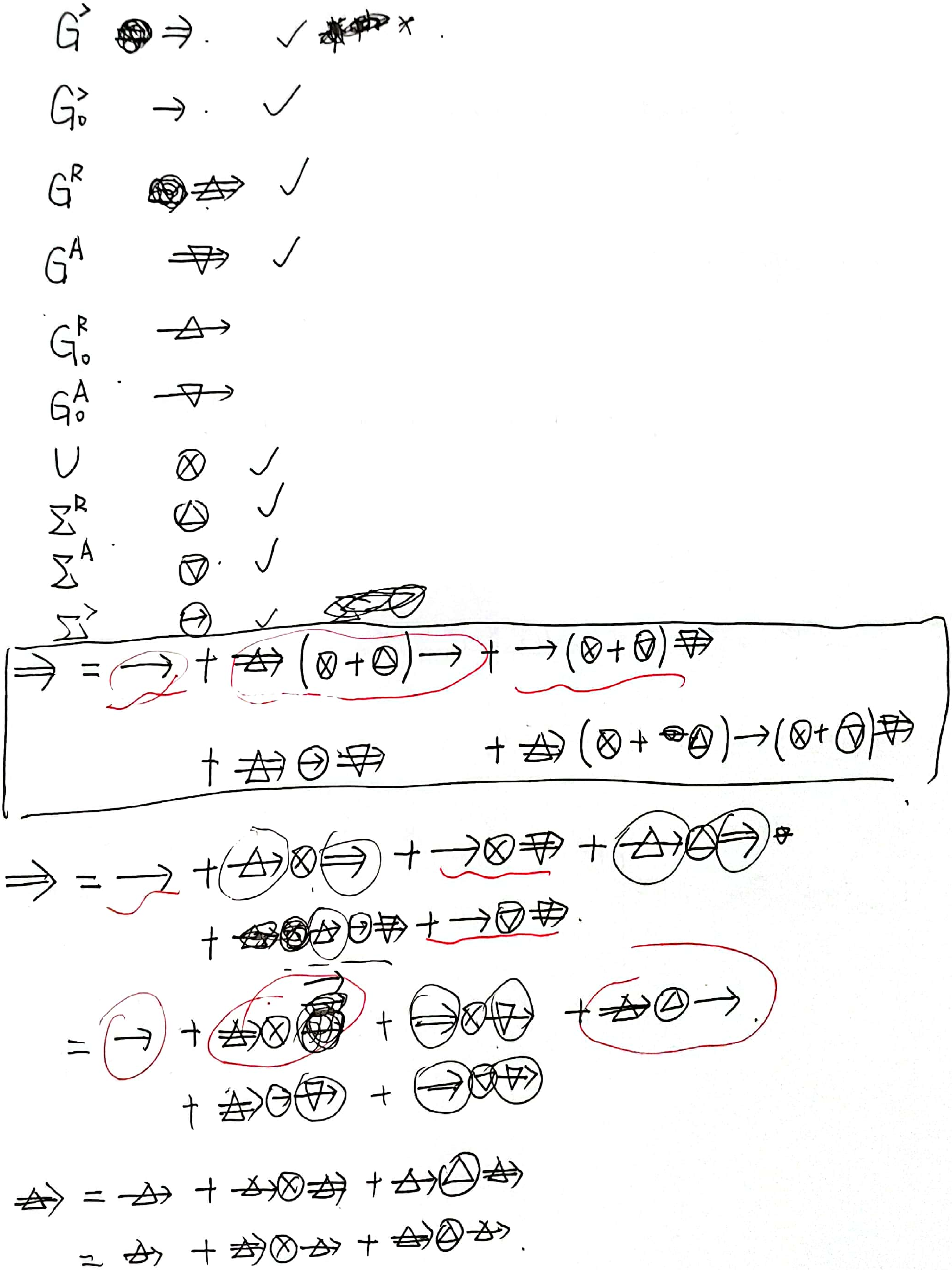
~~长達~~

Kledysh Equations.

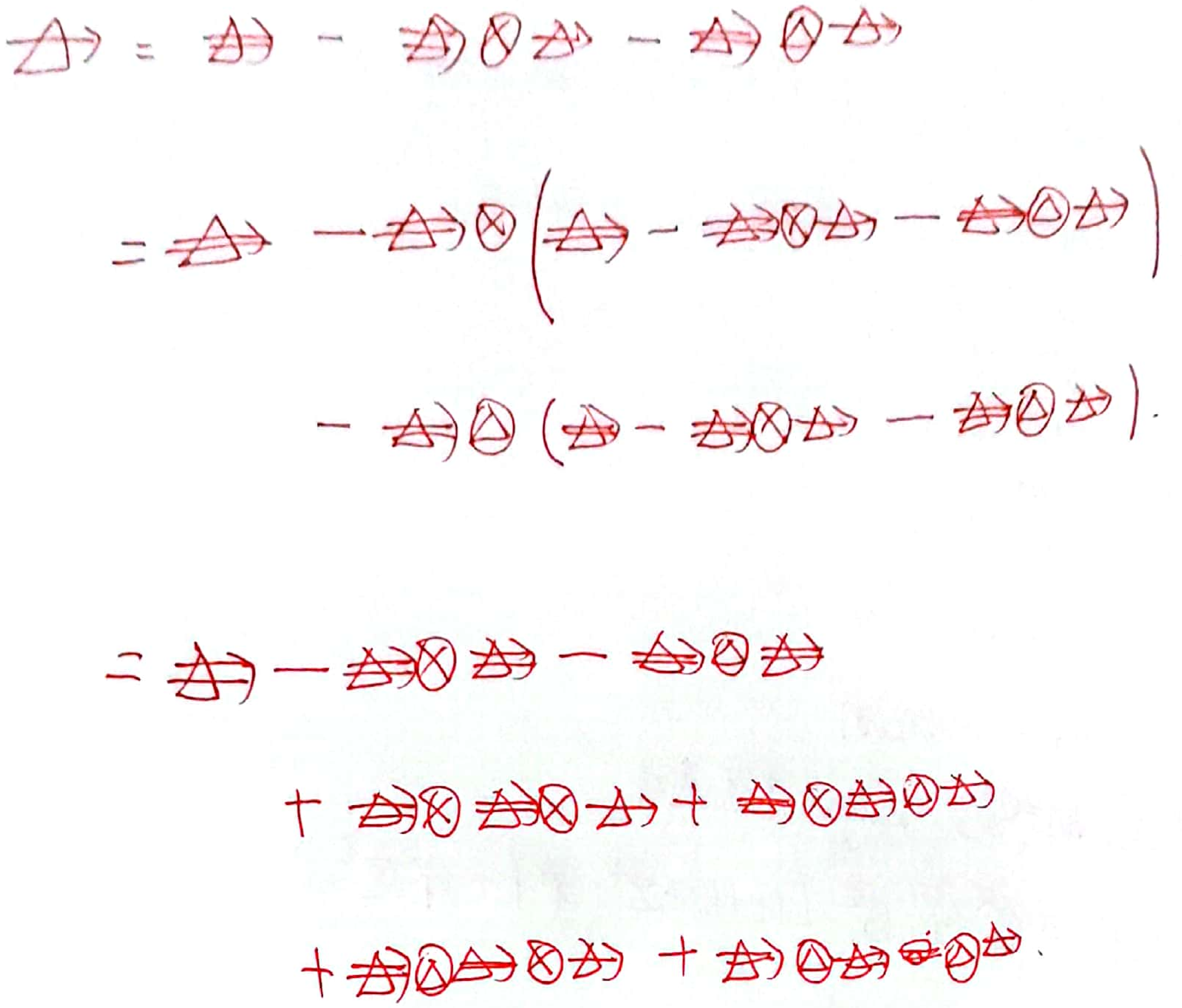
$$\begin{cases} G^{\gtrless} = G_0^{\gtrless} + G_0^R U G^{\gtrless} + G_0^{\gtrless} U G^A + G_0^R \Sigma^R G^{\gtrless} \\ \qquad + G_0^R \Sigma^{\gtrless} G^A + G_0^{\gtrless} \Sigma^A G^A. \\ G^{\gtrless} = G_0^{\gtrless} + G^R U G_0^{\gtrless} + G^{\gtrless} U G_0^A + G^R \Sigma^R G_0^{\gtrless} \\ \qquad + G^R \Sigma^{\gtrless} G_0^A + G^{\gtrless} \Sigma^A G_0^A. \end{cases}$$

⑧ $G^> = G_0^> + G^R(U+\Sigma^R)G_0^> + G_0^>(U+\Sigma^A)G^A + G^R\Sigma^>G^A$

$\quad + G^R(U+\Sigma^R)G_0^>(U+\Sigma^A)G^A \cancel{+G^R\Sigma^>G^A}$



⑨.



一般形式的 Keldysh equation.

$$G^{\lessgtr} = [1+G^R(U+\Sigma^R)]G_0^{\lessgtr}[1+(U+\Sigma^A)G^A] + G^R\Sigma^{\lessgtr}G^A.$$

$$\langle\psi|\hat{p}|\phi\rangle = \int \psi^*(x)\frac{\hbar}{i}\frac{\partial}{\partial x}\phi(x)\,dx$$

$$= \int \psi^*(x)\frac{\hbar}{i}\,\partial\phi(x)$$

$$= \psi^*(x)\phi(x)\Big|_{-\infty}^{\infty} - \frac{\hbar}{i}\int\phi(x)\frac{\partial\psi^*}{\partial x}\cdot dx$$

$$= -\frac{\hbar}{i}\int\phi(x)\frac{\partial\psi^*(x)}{\partial x}\,dx.$$

$$\langle\psi|\hat{p}|\phi\rangle^* = \frac{\hbar}{i}\int\phi^*(x)\frac{\partial\psi(x)}{\partial x}\,dx.$$

$$= \langle\phi|\hat{p}^\dagger|\psi\rangle.$$

$$\hat{p}^\dagger = \frac{\hbar}{i}\frac{\partial}{\partial x} = \hat{p}.$$

10.
$$(i\partial_\tau - H_0(x,-i\nabla))G_0(x,\tau;x',\tau') = \delta(x-x')\delta(\tau-\tau').$$

$$G_0(x,\tau;x',\tau')(-i\partial_{\tau'} - H_0(x',i\nabla')) = \delta(x-x')\delta(\tau-\tau').$$

we can write in simplified notation.

$$\hat{G}_0^{-1}G_0^{RA} = 1.$$
$$\hat{G}_0^{-1}G_0^{\gtrless} = 0$$
$$G_0^{RA}\hat{G}_0^{-1} = 1$$
$$G_0^{\gtrless}\hat{G}_0^{-1} = 0.$$

式中 $G_0 = i\partial_t - H_0(x,-i\nabla)$.

对一个有限大小的系统，keldysh 方程变成

$$G^{\gtrless} = G^R\hat{G}_0^{-1}G_0^{\gtrless}[1+(U+\Sigma^A)G^A] + G^R\Sigma^{\gtrless}G^A.$$
$$= G^R\Sigma^{\gtrless}G^A.$$

$$\hat{G}_0^{-1}G^{\gtrless} = UG^{\gtrless} + \Sigma^R G^{\gtrless} + \Sigma^{\gtrless}G^A$$
$$G^{\gtrless}(\hat{G}_0^{-1}-U) = G^R\Sigma^{\gtrless} + G^{\gtrless}\Sigma^A.$$
$$\Rightarrow [\hat{G}_0^{-1}-U, G^{\gtrless}] = \Sigma^R G^{\gtrless} + \Sigma^{\gtrless}G^A - G^R\Sigma^{\gtrless} - G^{\gtrless}\Sigma^A.$$

非平衡谱函数 $A \equiv i(G^R - G^A)$

散射速率 $\Gamma \equiv i(\Sigma^R - \Sigma^A)$.

$$\{\Sigma^> - \cancel{\Sigma^<}, G^<\} - \{G^> - \cancel{G^<}, \Sigma^<\}.$$

$$[\hat{G}_0^{-1} - U - \mathrm{Re}\Sigma, G^<] - [\Sigma^<, \mathrm{Re}G] = \frac{i}{2}(\{\Sigma^>, G^<\} - \{G^>, \Sigma^<\}).$$

Wigner 表示和梯度展开.

Wigner表示.
$$\begin{cases} \vec{r} \equiv \vec{x}_1 - \vec{x}_2, & \vec{R} \equiv \dfrac{\vec{x}_1 + \vec{x}_2}{2} \\ t \equiv t_1 - t_2, & T \equiv \frac{1}{2}(t_1 + t_2). \end{cases}$$

⑪.

快速变化的变量做傅里叶变换.

缓慢 ——— 梯度展开.

$$C(k,\Omega,R,T) \equiv \int dt \int d\vec{r}\, e^{i(\Omega t - \vec{k}\cdot\vec{r})}\, C(r,t,R,T).$$

$$[\hat{G}_0^{-1}-U, G^<]_{r,t,R,T} = i\frac{\partial}{\partial T} G^<(r,t,R,T) - H_0(R+\tfrac{1}{2}r, -i(\tfrac{1}{2}\nabla_R + \nabla_r))G^<$$
$$+ G^< H_0(R-\tfrac{1}{2}\vec{r}, i(\tfrac{1}{2}\nabla_R - \nabla_r)) + eE\cdot r G^<.$$

导数是平移的生成器.

$$\therefore f(\vec{R}+\vec{a}, T+S) = e^{\vec{a}\cdot\nabla_R} e^{S\partial_T} \cdot f(\vec{R},T).$$

Quantum Boltzmann Equation.

Kadanoff-Byam equation.

$$[\hat{G}_0^{-1} - U - \mathrm{Re}\Sigma, G^<] = [\Sigma^<, \mathrm{Re}G] + \tfrac{1}{2}(\{\Sigma^>, G^<\} - \{G^>, \Sigma^<\}).$$

QBE with Electric and Magnetic Field.

$$\vec{A}(x) = -\tfrac{1}{2}\vec{x}\times\vec{B}.$$

$$H_0(-i\nabla) \to H_0(-i\nabla - e\vec{A}) = H_0(-i\nabla + \tfrac{1}{2}e\vec{x}\times\vec{B}).$$

$$H_0(x_1, -i\nabla_1 - e\vec{A}_1) \to H_0(R+\tfrac{1}{2}r, -i(\tfrac{1}{2}\nabla_R + \nabla_r) + \tfrac{1}{2}e(R+\tfrac{1}{2}r)\times\vec{B}).$$

After Fourier transformation

$$H_0(R \pm \tfrac{i}{2}\nabla_k, k + \tfrac{1}{2}eR\times\vec{B} \mp \tfrac{i}{2}(\nabla_R + \tfrac{1}{2}eB\times\nabla_k))$$

Mahan-Hänsch transformation.

$$H_0(R \pm \tfrac{i}{2}\nabla_k, k + \tfrac{1}{2}eR\times\vec{B} \mp \tfrac{i}{2}(\nabla_R + eE\tfrac{\partial}{\partial w} + \tfrac{1}{2}eB\times\nabla_k)).$$

Kinematical momentum.

$$p = k + \tfrac{1}{2}eR\times B = k - eA$$

Final expression

$$H_0(R + \tfrac{i}{2}\nabla_p, p \mp \tfrac{i}{2}(\nabla_R + eE\tfrac{\partial}{\partial w} + eB\times\nabla_p)).$$

(12). the most general form of QBE

$$[\hat{G}_0^{-1}-U, G^<]_{p,\omega,R,T} = [\mathrm{Re}\Sigma, G^<]_{p,\omega,R,T} + [\Sigma^<, \mathrm{Re}G]_{p,\omega,R,T}$$
$$+\frac{1}{2}\left(\{\Sigma^>, G^<\}_{p,\omega,R,T} - \{G^>,\Sigma^<\}_{p,\omega,R,T}\right).$$

One-Band Spinless Electrons.

$$H_0(-i\nabla) = \frac{(-i\nabla)^2}{2m}$$

$$[\hat{G}_0^{-1}-U, G^<]_{p,\omega,R,T} = i\frac{\partial G^<}{\partial T} - \frac{\left(p-\frac{i}{2}(\nabla_R+eE\frac{\partial}{\partial\omega}+eB\times\nabla_p)\right)^2}{2m}G^<$$
$$+G^<\frac{\left(p+\frac{i}{2}(\nabla_R+eE\frac{\partial}{\partial\omega}+eB\times\nabla_p)\right)^2}{2m} \qquad +iEe\nabla_p G^<.$$

$$= i\frac{\partial G^<}{\partial T} + i\,v(\nabla_R+eE\frac{\partial}{\partial\omega})G^<$$
$$+\underline{i\,v\cdot eB\times\nabla_p G^<} \; +iEe\nabla_p G^<.$$

Applications of nonequilibrium formalism

Nonequilibrium transport through a quantum dot.

NM | QD | NM

Hamiltonian:

$$\hat{H} = \sum_{k,\alpha\in L,R}\varepsilon_{k\alpha}\hat{C}^+_{k\alpha}\hat{C}_{k\alpha} + \hat{H}_{QD} + \sum_{k,\alpha,n}\left(t_{k\alpha n}\hat{C}^+_{k\alpha}\hat{d}_n + t^*_{k\alpha n}\hat{d}^+_n\hat{C}_{k\alpha}\right).$$

$$\hat{H}_{QD} = \begin{cases}\sum_{m,n}\hat{d}^+_m\hat{d}_n h_{mn}, & \text{noninteracting dot.}\\ \sum_\sigma \varepsilon_\sigma \hat{d}^+_\sigma\hat{d}_\sigma + U n_\downarrow n_\uparrow, & \text{Anderson model.}\end{cases}$$

(3). Expression of the current.

$$J_\alpha(t) = -e\langle \dot{N}_\alpha \rangle.$$

$$[A, BC] = ABC - BCA$$
$$= B[A,C] + [A,B]C$$
$$= \cancel{BAC} - BCA + ABC - \cancel{BAC}$$

$$N_\alpha = \sum_k \hat{C}^\dagger_{k\alpha} \hat{C}_{k\alpha}$$

Heisenberg equation.

$$i\hbar \dot{N}_\alpha = [N_\alpha, H].$$

$$\frac{i\hbar J_\alpha(t)}{-e} = [\sum_k \hat{C}^\dagger_{k\alpha}\hat{C}_{k\alpha}, \cancel{\sum_{k,\alpha} \hat{C}^\dagger_{k\alpha}\hat{C}_{k\alpha} \cdot \varepsilon_{k\alpha}} + \hat{H}_{QD} + \sum_{k,\alpha,n}(t_{k\alpha n}\hat{C}^\dagger_{k\alpha}\hat{d}_n + t^*_{k\alpha n}\hat{d}^\dagger_n \hat{C}_{k\alpha})]$$

$$\sum_{k_1}\sum_{k_2,\alpha_2,n} [\hat{C}^\dagger_{k_1\alpha}\hat{C}_{k_1\alpha}, t_{k_2\alpha_2 n}\hat{C}^\dagger_{k_2\alpha_2}\hat{d}_n] = \sum t_{k_2\alpha n}\hat{C}^\dagger_{k_1\alpha}[\hat{C}_{k_1\alpha}, \hat{C}^\dagger_{k_2\alpha_2}\hat{d}_n] + t_{k_2\alpha n}[\hat{C}^\dagger_{k_1\alpha}, \hat{C}^\dagger_{k_2\alpha_2}\hat{d}_n]\hat{C}_{k_1\alpha}$$

$$= \sum_{k,n} t_{k\alpha n}\hat{C}^\dagger_{k\alpha}\hat{d}_n$$

$$\sum_{k_1}\sum_{k_2,\alpha_2,n} [\hat{C}^\dagger_{k_1\alpha}\hat{C}_{k_1\alpha}, t^*_{k_2\alpha_2 n}\hat{d}^\dagger_n\hat{C}_{k_2\alpha_2}] = \sum_{k,n} t^*_{k\alpha n}\hat{d}^\dagger_n\hat{C}_{k\alpha}$$

$$\therefore \frac{i\hbar J_\alpha(t)}{-e} = \sum_{k,n}(t_{k\alpha n}\hat{C}^\dagger_{k\alpha}\hat{d}_n + t^*_{k\alpha n}\hat{d}^\dagger_n\hat{C}_{k\alpha}).$$

$$\Rightarrow J_\alpha(t) = \frac{-e}{i\hbar}\sum_{k,n}(t_{k\alpha n}\hat{C}^\dagger_{k\alpha}\hat{d}_n + t^*_{k\alpha n}\hat{d}^\dagger_n\hat{C}_{k\alpha}).$$

$$= \frac{e}{\hbar}\sum_{k,n} i(t_{k\alpha n}\hat{C}^\dagger_{k\alpha}\hat{d}_n + t^*_{k\alpha n}\hat{d}^\dagger_n\hat{C}_{k\alpha}) = \frac{e}{\hbar}\sum_{k,n} 2\,\mathrm{Re}(i\, t_{k\alpha n}\hat{C}^\dagger_{k\alpha}\hat{d}_n)$$

$$= \frac{2e}{\hbar}\mathrm{Re}\sum_{k,n} t_{k\alpha n}\underbrace{G^<_{n,k\alpha}(t,t)}_{\equiv\, i\langle\hat{C}^\dagger_{k\alpha}\hat{d}_n\rangle}.$$

④ $\hat{H} = \hat{H} + \hat{H}'$ $\rightarrow$ perturbation.

$\hookrightarrow$ unperturbed Hamiltonian.

$$\hat{H} = \sum_{k,\alpha} \varepsilon_{k\alpha} \hat{C}^{+}_{k\alpha} \hat{C}_{k\alpha} + \hat{H}_{QD}$$

$$\hat{H}' = \sum_{k,\alpha,n} \left(t_{k\alpha n} \hat{C}^{+}_{k\alpha} \hat{d}_n + t^{*}_{k\alpha n} \hat{d}^{+}_n \hat{C}_{k\alpha} \right)$$

$$\boxed{G_{nk\alpha}(\tau,\tau') = -i\, d_n(\tau)\, e^{-i\int \hat{H} dt} \cdot \hat{C}^{+}_{k\alpha}(\tau')}$$

$$= d_n(\tau) \cdot \sum_{L} \frac{(-i)^{L}}{L!} \oint_{C_1} \cdots \oint_{C_L} H(\tau_L)\, d\tau_L \cdot \hat{C}^{+}_{k\alpha}(\tau').$$

$$\langle T_c \{ \hat{d}_n(t)\, H'(\tau_1) \cdots H'(\tau_L)\, \hat{C}^{+}_{k\alpha}(t') \} \rangle$$

$$= \prod_{i=1}^{L} \sum_{k_i \alpha_i n_i} \langle T_c \{ \hat{d}_n(t)\, \hat{d}^{+}_{n_i}(\tau_i)\, \hat{C}_{k_i\alpha_i}(\tau_i)\, \hat{C}^{+}_{k\alpha}(t') \} \rangle \cdot t^{*}_{k_i \alpha_i n_i}$$

Interacting Fields — Wick's theorem

For a real scalar field $\phi(x) = \int \frac{d^3 \vec{p}}{(2\pi)^3} \cdot \frac{1}{\sqrt{2\omega_p}} \left(a_p e^{-ipx} + a_p^{+} e^{ipx} \right)$.

$$T[\phi(x)\phi(y)] = \begin{cases} \phi(x)\phi(y), & x > y \\ \phi(y)\phi(x), & x < y. \end{cases}$$

$A = A(a_p, a_p^{+})$, $B = B(a_p, a_p^{+})$

$:AB:$ 的定义：将 $A \cdot B$ 展开为 a^{+} 与 a 的乘积，然后将所有的 a 移到 a^{+} 的右边

we can write $\phi(x) = \phi^{+}(x) + \phi^{-}(x)$

where $\phi^{+}(x) \equiv \int \frac{d^3 \vec{p}}{(2\pi)^3} \cdot \frac{1}{\sqrt{2\omega_p}} a_p e^{-ipx}$

$\phi^{-}(x) \equiv \int \frac{d^3 p}{(2\pi)^3} \cdot \frac{1}{\sqrt{2\omega_p}} a_p^{+} e^{ipx}$.

$$\Rightarrow \ :\phi(x)\phi(y): = \phi^{+}(x)\phi^{+}(y) + \phi^{-}(y)\phi^{+}(x) + \phi^{-}(x)\phi^{+}(y) + \phi^{-}(x)\phi^{-}(y).$$

⑮. $T\phi(x)\phi(y)$ 和 $:\phi(x)\phi(y):$ 的联系.

if $x>y$

$$T\phi(x)\phi(y) = (\phi^+(x)+\phi^-(x))\cdot(\phi^+(y)+\phi^-(y))$$
$$= :\phi(x)\phi(y): + [\phi^+(x),\phi^-(y)]$$
$$= :\phi(x)\phi(y): + D(x-y).$$

if $x<y$

$$T\phi(x)\phi(y) = :\phi(y)\phi(x): + D(y-x).$$
$$= :\phi(x)\phi(y): + D(y-x).$$

$$\Rightarrow T\phi(x)\phi(y) = :\phi(x)\phi(y): + \Delta_F(x-y).$$

Similarly, for complex scalar fields $\psi(x)$.

$$T\psi(x)\psi(y) = \psi(x)\psi(y) = :\psi(x)\psi(y):$$
$$T\psi^+(x)\psi^+(y) = :\psi^+(x)\psi^+(y):$$
$$T\psi(x)\psi^+(y) = :\psi(x)\psi^+(y): + \underbrace{\Delta_F(x-y)}_{\equiv \begin{cases} D(x-y), & x>y \\ D(y-x), & x<y. \end{cases}}$$

$\overbrace{AB} \equiv T(AB) - :AB:$ (contractor of two operators).

for real scalar field.

$$\overbrace{\phi(x)\phi(y)} = \Delta_F(x-y).$$

for complex scalar field.

$$\overbrace{\psi(x)\psi(y)} = 0$$
$$\overbrace{\psi^+(x)\psi^+(y)} = 0$$
$$\overbrace{\psi(x)\psi^+(y)} = \Delta_F(x-y).$$

⑪. Wick's theorem

For a string of fields $\phi(x_i) \equiv \phi_i$

$$T\phi_1\phi_2\cdots\phi_n = :\phi_1\phi_2\cdots\phi_n: + :\text{all possible contractions}:$$

e.g. $T\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) = :\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4):$

$$+ \phi_1\phi_2 :\phi_3\phi_4: + \phi_1\phi_3 :\phi_2\phi_4: + \phi_1\phi_4 :\phi_2\phi_3:$$

$$+ \phi_2\phi_3 :\phi_1\phi_4: + \phi_2\phi_4 :\phi_1\phi_3: + \phi_3\phi_4 :\phi_1\phi_2:$$

$$+ \phi_1\phi_2\ \phi_3\phi_4 + \phi_1\phi_3\ \phi_2\phi_4 + \phi_1\phi_4\ \phi_2\phi_3$$

$$G_{nk\alpha}(\tau,\tau') = \sum_{k_1\alpha_1 n_1} \oint_C d\tau_1 (-i) \sum_{l=1}^{\infty} \frac{(-i)^{l-1}}{(l-1)!} \oint_C d\tau_2 \cdots \oint_C d\tau_l$$

$$\times \langle T_c\{\hat{d}_n(\tau)\hat{H}'(\tau_2)\cdots\hat{H}'(\tau_l)\hat{C}^{+}_{k_1\alpha_1}(\tau_1)\}\rangle t^{*}_{k_1\alpha_1 n_1}$$

$$(-i)\langle T_c\{\hat{C}_{k_1\alpha_1}(\tau_1)\hat{C}_{k\alpha}(\tau')\}\rangle.$$

$$= \sum_m \oint_C d\tau_1 G_{nm}(\tau,\tau_1) t^{*}_{k\alpha m} g_{k\alpha}(\tau_1,\tau')$$

$$\cancel{\int_{-\infty}^{\infty} \frac{e^{i\omega t}}{\omega - i\delta - \varepsilon_{k\alpha}} d\omega = 2\pi i\, e^{it(i\delta + \varepsilon_{k\alpha})}}$$

$$\cancel{= 2\pi i\, e^{-\delta t + it\varepsilon_{k\alpha}}}.$$

$$\cancel{= ie^{i\varepsilon t}\sqrt{\frac{\pi}{2}}\left(\theta(t) + 2\right)}$$

$$\cancel{2\theta(t) - 1}$$

$$G_0^A(x,t;x',t') \equiv i\theta(t'-t)\langle\{\hat{\psi}(x,t),\underbrace{\hat{\psi}^{+}(x',t')}_{\text{(相互作用表象)}}\}\rangle$$

$$= i\theta(t'-t)\, e^{iH(t-t')}$$

$$G_0^A(\omega) = \int_{-\infty}^{\infty} i\theta(t) e^{-it(H+i\delta)} e^{i\omega t} dt \quad \frac{1}{\omega - H - i\delta}$$

17.

$$J = \frac{ie}{\hbar}\int \frac{d\omega}{2\pi}\,[f_L(\omega) - f_R(\omega)]\cdot \mathrm{Tr}\left(\frac{\Gamma_L(\omega)\Gamma_R(\omega)}{\Gamma_L(\omega)+\Gamma_R(\omega)}\right)[G^R(\omega) - G^A(\omega)].$$

$$\langle \hat{\psi}(t)|\hat{A}(t)|\hat{\psi}(t)\rangle$$

$$= \langle \psi(t)|\, e^{-iH_0 t}\; e^{iH_0 t} A\, e^{-iH_0 t}\; e^{iH_0 t}\, |\hat{\psi}(t_0)\rangle.$$