Supplemental Material for *Full Quantum Theory for Magnon Transport in Two-sublattice Magnetic Insulators and Magnon Junctions*

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I. DERIVATION OF EQ. (2)

The Hamiltonian of FIMI or AFMI, considering the nearest neighbor and next-nearest neighbor Heisenberg exchange interactions, can be expressed as follows:

$$\begin{aligned}\hat{H} = & -J_{AB}\sum_{<i,m>}\hat{\boldsymbol{S}}_i\cdot\hat{\boldsymbol{S}}_m - J_A\sum_{\ll i,j\gg}\hat{\boldsymbol{S}}_{\boldsymbol{i}}\cdot\hat{\boldsymbol{S}}_j \\ & -J_B\sum_{\ll m,n\gg}\hat{\boldsymbol{S}}_m\cdot\hat{\boldsymbol{S}}_{\boldsymbol{n}} - h_{\rm ext}\left(\sum_i\mu_A\hat{S}_i^z\right. \\ & \left.+\sum_m\mu_B\hat{S}_m^z\right)\end{aligned} \tag{S1}$$

Where $< >$ denotes summing over nearest sites, $\ll\gg$ denotes summing over next-nearest sites. J_{AB} and $J_{A(B)}$ represent the nearest and next-nearest Heisenberg exchange interactions strength, respectively. $\boldsymbol{S}_{\boldsymbol{i(m)}}$ is the spin in A(B) sublattice, $\mu_{A(B)}$ is the magnetic moment in A(B) sublattice. h_{ext} is applied magnetic field along the z direction. Using Holstein-Primakoff (HP) transformation, we can get

$$\begin{aligned}&\hat{S}_i^+=\sqrt{2S_A-\hat{a}_i^\dagger\hat{a}_i}\,\hat{a}_i,\ \ \hat{S}_i^-=\hat{a}_i^\dagger\sqrt{2S_A-\hat{a}_i^\dagger\hat{a}_i}\\ &\hat{S}_i^z=S_A-\hat{a}_i^\dagger\hat{a}_i,\ \ \hat{S}_m^+=\hat{b}_m^\dagger\sqrt{2S_B-\hat{b}_m^\dagger\hat{b}_m}\\ &\hat{S}_m^-=\sqrt{2S_B-\hat{b}_m^\dagger\hat{b}_m}\,\hat{b}_m,\hat{S}_m^z=\hat{b}_m^\dagger\hat{b}_m-S_B\end{aligned} \tag{S2}$$

where $\hat{a}_i\left(\hat{b}_m\right),\hat{a}_i^\dagger\left(\hat{b}_m^\dagger\right)$ are magnon annihilation and creation operators in A(B) sublattice, respectively. By substituting Eq. (S2) into Eq. (S1), we can reform the Hamiltonian of FIMI or AFMI using magnon annihilation and creation operators and get

$$\begin{aligned}\hat{H}= & -J_{AB}\sqrt{S_AS_B}\sum_{<i,m>}\left(\hat{a}_i\hat{b}_m+\hat{a}_i^\dagger\hat{b}_m^\dagger\right)\\ & -J_A\sum_{\ll i,j\gg}S_A\left(\hat{a}_i\hat{a}_j^\dagger+\hat{a}_i^\dagger\hat{a}_j\right)\\ & -J_B\sum_{\ll m,n\gg}S_B\left(\hat{b}_m^\dagger\hat{b}_n+\hat{b}_m b_n^\dagger\right)\\ & -\sum_i\left(J_{AB}S_BN_n-2J_AS_AN_{nn}-h_{ext}\mu_A\right)\hat{a}_i^\dagger\hat{a}_j\\ & -\sum_m\left(J_{AB}S_AN_n-2J_BS_BN_{nn}+h_{ext}\mu_B\right)\hat{b}_m^\dagger\hat{b}_n+const\end{aligned} \tag{S3}$$

where N_n, N_{nn} are the numbers of nearest and the next-nearest sites respectively. In the case of a one-dimensional atomic chain model, $N_n=N_{nn}=2$. Using Fourier transformation for magnon annihilation and creation operators

$$\begin{aligned}&\hat{a}_i=\tfrac{1}{\sqrt{N}}\sum_k e^{i\boldsymbol{k}\cdot\boldsymbol{R}_i}\hat{a}_k,\hat{a}_i^\dagger=\tfrac{1}{\sqrt{N}}\sum_k e^{-i\boldsymbol{k}\cdot\boldsymbol{R}_i}\hat{a}_k^\dagger,\\ &\hat{b}_m=\tfrac{1}{\sqrt{N}}\sum_k e^{-i\boldsymbol{k}\cdot\boldsymbol{R}_m}\hat{b}_k,\hat{b}_m^\dagger=\tfrac{1}{\sqrt{N}}\sum_k e^{i\boldsymbol{k}\cdot\boldsymbol{R}_m}\hat{b}_k^\dagger\end{aligned} \tag{S4}$$

we can get

$$\begin{aligned}\hat{H}\ &=\sum_k\left[-J_{AB}\sqrt{S_AS_B}\gamma_{k,n}\left(\hat{a}_k\hat{b}_k+\hat{a}_k^\dagger\hat{b}_k^\dagger\right)\right]\\ &\quad+\sum_k\left(-2J_AS_A\gamma_{k,nn}\hat{a}_k^\dagger\hat{a}_k-2J_BS_B\gamma_{k,nn}\hat{b}_k^\dagger\hat{b}_k\right)\\ &\quad+\sum_k\left(-J_{AB}S_BN_n+2J_AS_AN_{nn}+h_{ext}\mu_A\right)\hat{a}_k^\dagger\hat{a}_k\\ &\quad+\sum_k\left(-J_{AB}S_AN_n+2J_bS_bN_{nn}-h_{ext}\mu_b\right)\hat{b}_k^\dagger\hat{b}_k+const\\ &=\ \sum_k\left(-2J_AS_A\gamma_{k,nn}-J_{AB}S_BN_n+2J_AS_AN_{nn}+h_{ext}\mu_A\right)\hat{a}_k^\dagger\hat{a}_k\\ &\quad+\sum_k\left(-2J_BS_B\gamma_{k,nn}-J_{AB}S_AN_n+2J_BS_BN_{nn}-h_{ext}\mu_B\right)\hat{b}_k^\dagger\hat{b}_k\\ &\quad+\sum_k\left[-J_{AB}\sqrt{S_AS_B}\gamma_{k,n}\left(\hat{a}_k\hat{b}_k+\hat{a}_k^\dagger\hat{b}_k^\dagger\right)\right]+const\\ &\equiv\ \sum_k\left[A_k\hat{a}_k^\dagger\hat{a}_k+B_k\hat{b}_k^\dagger\hat{b}_k+C_k\left(\hat{a}_k\hat{b}_k+\hat{a}_k^\dagger\hat{b}_k^\dagger\right)\right]+const\end{aligned} \tag{S5}$$

Where $\gamma_{k,n}=\sum_{\delta_{\boldsymbol{n}}}e^{-i\boldsymbol{k}\cdot\boldsymbol{\delta_n}}$, $\gamma_{k,nn}=\sum_{\delta_{\boldsymbol{nn}}}e^{-i\boldsymbol{k}\cdot\boldsymbol{\delta_{nn}}}$. For one-dimensional atomic chain model, $\gamma_{k,n}=2cos\left(ka\right)$, $\gamma_{k,nn}=2cos\left(2ka\right)$, where a is the distance between nearest sites, $A_k\equiv-2J_AS_A\gamma_{k,nn}-J_{AB}S_BN_n+2J_AS_AN_{nn}+h_{ext}\mu_A$, $B_k\equiv-2J_BS_B\gamma_{k,nn}-J_{AB}S_AN_n+2J_BS_BN_{nn}-h_{ext}\mu_B$, $C_k\equiv-J_{AB}\sqrt{S_AS_B}\gamma_{k,n}$.

Using Bogoliubov transformation

$$\begin{aligned}&\hat{a}_k=u_k\hat{\alpha}_k+v_k\hat{\beta}_k^\dagger,\hat{a}_k^\dagger=u_k\hat{\alpha}_k^\dagger+v_k\hat{\beta}_k,\\ &\hat{b}_k=u_k\hat{\beta}_k+v_k\hat{\alpha}_k^\dagger,\hat{b}_k^\dagger=u_k\hat{\beta}_k^\dagger+v_k\hat{\alpha}_k,\\ &\hat{\alpha}_k=u_k\hat{a}_k-v_k\hat{b}_k^\dagger,\hat{\alpha}_k^\dagger=u_k\hat{a}_k^\dagger-v_k\hat{b}_k,\\ &\hat{\beta}_k=u_k\hat{b}_k-v_k\hat{a}_k^\dagger,\hat{\beta}_k^\dagger=u_k\hat{b}_k^\dagger-v_k\hat{a}_k\end{aligned} \tag{S6}$$

With commutation relationship $\left[\hat{\alpha}_k,\hat{\alpha}_{k'}^\dagger\right]=\left[\hat{\beta}_k,\hat{\beta}_{k'}^\dagger\right]=\delta_{k,k'}$, $[\hat{\alpha}_k,\hat{\beta}_k']=[\hat{\alpha}_k,\hat{\beta}_{k'}^\dagger]=[\hat{\alpha}_k^\dagger,\hat{\beta}_k']=[\hat{\alpha}_k^\dagger,\hat{\beta}_{k'}^\dagger]=0$ and relationship $u_k^2-v_k^2=1$, we can get

$$\begin{aligned}\hat{H}= & \sum_k[(A_ku_k^2+B_kv_k^2+2C_ku_kv_k)\hat{\alpha}_k^\dagger\hat{\alpha}_k\\ & +\left(A_ku_kv_k+B_ku_kv_k+C_k\left(u_k^2+v_k^2\right)\right)\hat{\alpha}_k^\dagger\hat{\beta}_k^\dagger\\ & +\left(A_ku_kv_k+B_ku_kv_k+C_k\left(u_k^2+v_k^2\right)\right)\hat{\alpha}_k\hat{\beta}_k\\ & +(A_kv_k^2+B_ku_k^2+2C_ku_kv_k)\hat{\beta}_k^\dagger\hat{\beta}_k]+const\end{aligned} \tag{S7}$$

Take

$$\begin{aligned}&u_k=-\sqrt{\tfrac{1}{2}+\tfrac{A_k+B_k}{2\sqrt{(A_k+B_k)^2-4C_k^2}}},\\ &v_k=\sqrt{-\tfrac{1}{2}+\tfrac{A_k+B_k}{2\sqrt{(A_k+B_k)^2-4C_k^2}}}\end{aligned} \tag{S8}$$

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$$\begin{aligned}\hat{H} &= \sum_k \left[\frac{A_k - B_k}{2} + \frac{\sqrt{(A_k^2+B_k^2)-4C_k^2}}{2}\right] \hat{\alpha}_k^\dagger \hat{\alpha}_k \\ &\equiv + \left[\frac{-A_k + B_k}{2} + \frac{\sqrt{(A_k^2+B_k^2)-4C_k^2}}{2}\right] \hat{\beta}_k^\dagger \hat{\beta}_k + const \\ &\equiv \sum_k w_k^\alpha \hat{\alpha}_k^\dagger \hat{\alpha}_k + w_k^\beta \hat{\beta}_k^\dagger + const\end{aligned} \tag{S9}$$

II. CALCULATION OF MAGNON JUNCTION EFFECT

The Hamiltonian of the magnon junction is composed of five items

$$\begin{aligned}\hat{H} = &\ \hat{H}_{FMI1} + \hat{H}_{AFMI} + \hat{H}_{FMI2} \\ &+ \hat{H}_{FMI1,AFMI} + \hat{H}_{FMI2,AFMI}\end{aligned} \tag{S10}$$

Where $\hat{H}_{FMI1}$, $\hat{H}_{AFMI}$, $\hat{H}_{FMI2}$ are Hamiltonian of FMI1, AFMI and FMI2, respectively, and $\hat{H}_{FMI1,AFMI}$, $\hat{H}_{FMI2,AFMI}$ are coupling between FMI1 and AFMI, FMI2 and AFMI, respectively. Only on-site and next-nearest transition energy are considered. For parallel state:

$$\hat{H}_{FMI1(2)} = \sum_{i,j} [(A_0^{FMI1(2)}\ \delta_{i,j} + A_2^{FMI1(2)}\ \delta_{i,j\pm 2})] \hat{\alpha}_i^\dagger \hat{\alpha}_j \tag{S11}$$

$$\begin{aligned}\hat{H}_{AFMI} = &\ \sum_{i,j} [(A_0^{AFMI}\ \delta_{i,j} + A_2^{AFMI}\ \delta_{i,j\pm 2}) \hat{\alpha}_i^\dagger \hat{\alpha}_j \\ &+ (B_0^{AFMI} \delta_{i,j} + B_2^{AFMI} \delta_{i,j\pm 2}) \hat{\beta}_i^\dagger \hat{\beta}_j]\end{aligned} \tag{S12}$$

$$\begin{aligned}\hat{H}_{FMI1(2),AFMI} = &\ J_{FMI1(2),AFMI} (\hat{\alpha}_{end(1),FMI1(2)}^\dagger \\ &\hat{\alpha}_{1(end),AFMI} + \hat{\alpha}_{end(1),FMI1(2)}^\dagger \\ &\hat{\beta}_{1(end),AFMI}) + H.c.\end{aligned} \tag{S13}$$

, and for antiparallel state, all the $\hat{\alpha}$ ($\hat{\alpha}^\dagger$) in Hamiltonian of $\hat{H}_{FMI2}$ and $\hat{\alpha}_{FMI2}$ ($\hat{\alpha}_{FMI2}^\dagger$) in Hamiltonian of $\hat{H}_{FMI2,AFMI}$ are replaced by $\hat{\beta}$ ($\hat{\beta}^\dagger$) and $\hat{\beta}_{FMI2}$ ($\hat{\beta}_{FMI2}^\dagger$).

We can use Eqs. (11∼12, S10∼S13) to calculate magnon currents in three parts of magnon junction. The boundary conditions are set to be that magnon currents are continuous at interface between FMI1 and AFMI1, between AFMI and FMI2. And the magnon current injected from NM1 to FMI1 is set to be zero, which excluded the influence of spin current injected from NM1 on output magnon current. The simulation parameters are set to be on-site energy $A_0^{FMI1} = A_0^{FMI2} = A_0^{AFMI} = B_0^{AFMI} = 0.5$ eV, nearest transition energy $A_1^{FMI1} = A_1^{FMI2} = -0.5$ eV, $A_1^{AFMI} = B_1^{AFMI} = -0.25$ eV, coupling energy of two types of magnons $J_{FMI1,AFMI} = J_{FMI2,AFMI} = 1$ eV, spin chemical potential of two NMs layer $\mu_{NM1} = \mu_{NM2} = 0$, temperature $k_B T_{NM1} = 0.026$ eV, $T_{FMI1} = 0.9\ T_{NM1}$, $T_{AFMI} = 0.8\ T_{NM1}$, $T_{FMI2} = 0.7\ T_{NM1}$, $T_{NM2} = 0.6\ T_{NM1}$, total site number $N_{FMI1} = N_{AFMI} = N_{FMI2} =$ 20, coupling with two NMs layers $\eta^{L(R)} = 8$ and Gilbert damping constant $\alpha_{FMI1} = \alpha_{FMI2} = 0.01$, $\alpha_{AFMI} = 0.001$.

Boundary condition is a nonlinear system of first order equations, and we can get a rough solution of μ_{FMI1}, μ_{AFMI}, μ_{FMI2}. Here we use one-dimensional atomic chain model and assume that the spin chemical potential affect the transport of magnon in AFMI or FIMI through change the on-site energy of magnons. Different part's retarded self-energy has the following formalism: For FMI1, the left, right and center retarded self-energys are

$$\Sigma_{L,FMI1\ i,j}^R(\varepsilon) = -\frac{i\eta_L(\varepsilon - \mu_{NM1})}{\hbar} \delta_{i,1}\delta_{j,1} \tag{S14}$$

$$\begin{aligned}\Sigma_{R,FMI1\ i,j}^R(\varepsilon) = &\ 2\frac{J_{FMI1,AFMI}^2}{(A_2^{AFMI})^2} [\frac{(\varepsilon - \mu_{AFMI})}{2} \\ &\pm \sqrt{\frac{(\varepsilon - \mu_{AFMI})^2}{4} - (A_2^{AFMI})^2}]/\hbar \\ &\delta_{i,N_{FMI1}}\delta_{j,N_{FMI1}}\end{aligned} \tag{S15}$$

The choice of solution is determined by the requirement that Im $\Sigma_{R,FMI1\ i,j}^R(\varepsilon) < 0$.

$$\Sigma_{C,FMI1\ i,j}^R(\varepsilon) = -\frac{i\alpha_{FMI1}(\varepsilon - \mu_{FMI1})}{\hbar} \delta_{i,j} \tag{S16}$$

For AFMI, the left, right and center retarded self-energys are

$$\begin{aligned}\Sigma_{L,AFMI\ i,j}^R(\varepsilon) = &\ \frac{J_{FMI1,AFMI}^2}{(A_2^{FMI1})^2} [\frac{(\varepsilon - \mu_{FMI1})}{2} \\ &\pm \sqrt{\frac{(\varepsilon - \mu_{FMI1})^2}{4} - (A_2^{FMI1})^2}]/\hbar \\ &\delta_{i,1}\delta_{j,1}\end{aligned} \tag{S17}$$

$$\begin{aligned}\Sigma_{R,AFMI\ i,j}^R(\varepsilon) = &\ \frac{J_{FMI2,AFMI}^2}{(A_2^{FMI2})^2} [\frac{(\varepsilon - \mu_{FMI2})}{2} \\ &\pm \sqrt{\frac{(\varepsilon - \mu_{FMI2})^2}{4} - (A_2^{FMI2})^2}]/\hbar \\ &\delta_{i,N_{AFMI}}\delta_{j,N_{AFMI}}\end{aligned} \tag{S18}$$

The choice of solution is determined by the requirement that Im $\Sigma_{L,AFMI\ i,j}^R(\varepsilon)$ and Im $\Sigma_{R,AFMI\ i,j}^R(\varepsilon) < 0$.

$$\Sigma_{C,AFMI\ i,j}^R(\varepsilon) = -\frac{i\alpha_{AFMI}(\varepsilon - \mu_{AFMI})}{\hbar} \delta_{i,j} \tag{S19}$$

For FMI2, the left, right and center retarded self-energys are

$$\begin{aligned}\Sigma_{L,FMI2\ i,j}^R(\varepsilon) = &\ 2\frac{J_{FMI2,AFMI}^2}{(A_2^{AFMI})^2} [\frac{(\varepsilon - \mu_{AFMI})}{2} \\ &\pm \sqrt{\frac{(\varepsilon - \mu_{AFMI})^2}{4} - (A_2^{AFMI})^2}]/\hbar \\ &\delta_{i,1}\delta_{j,1}\end{aligned} \tag{S20}$$

The choice of solution is determined by the requirement that Im $\Sigma_{L,FMI2\ i,j}^R(\varepsilon) < 0$.

$$\Sigma^R_{R,FMI2\ i,j}(\varepsilon) = -\frac{i\eta_R(\varepsilon-\mu_{NM2})}{\hbar}\delta_{i,N_{FMI2}}\delta_{j,N_{FMI2}} \tag{S21}$$

$$\Sigma^R_{C,FMI2\ i,j}(\varepsilon) = -\frac{i\alpha_{FMI2}(\varepsilon-\mu_{FMI2})}{\hbar}\delta_{i,j} \tag{S22}$$

III. DERIVATION OF $A_{2i+1} = B_{2i+1} = 0$

Let's take α mode magnon as an example, we will prove $A_{2i+1} = 0$. Similarly, we can also prove $B_{2i+1} = 0$ by using the same method. The expression for w_k^α is

$$w_k^\alpha = \frac{A_k - B_k}{2} + \frac{\sqrt{(A_k+B_k)^2 - 4C_k^2}}{2} \tag{S23}$$

where

$$\begin{aligned} A_k &\equiv -2J_AS_A\gamma_{k,nn} - J_{AB}S_BN_n + 2J_AS_AN_{nn} + h_{ext}\mu_A \\ B_k &\equiv -2J_BS_B\gamma_{k,nn} - J_{AB}S_AN_n + 2J_BS_BN_{nn} - h_{ext}\mu_B \\ C_k &\equiv -J_{AB}\sqrt{S_AS_B}\gamma_{k,n} \end{aligned} \tag{S24}$$

$J_{AB} < 0$ and $J_{A(B)} > 0$ represent the nearest and next-nearest exchange interactions in the A(B) sublattices, $S_{A(B)}$ is the spin in A(B) sublattice, $\mu_{A(B)}$ is the magnetic moment in A(B) sublattice. h_{ext} is applied magnetic field, N_n, N_{nn} are the numbers of nearest and the next-nearest sites. And in the case of one-dimensional atomic chain model

$$\gamma_{k,n} = 2\cos(ka), \gamma_{k,nn} = 2\cos(2ka) \tag{S25}$$

Substitute Eq. (S24) and (S25) in (S23), we can get

$$\begin{aligned} w_k^\alpha = &\frac{-4(J_AS_A-J_BS_B)\cos(2ka) - J_{AB}(S_B-S_A)N_n + 2(J_AS_A-J_BS_B)N_{nn} + h_{ext}(\mu_A-\mu_B)}{2} \\ &+ \frac{\sqrt{[-4(J_AS_A+J_BS_B)\cos(2ka) - J_{AB}(S_B+S_A)N_n + 2(J_AS_A+J_BS_B)N_{nn} + h_{ext}(\mu_A+\mu_B)]^2 - 8J_AS_AS_B(\cos(2ka)+1)}}{2} \end{aligned} \tag{S26}$$

And we can see from Eq. (S26) that $w^\alpha(\pi - x) = w^\alpha(x)$, since $cos(i(\pi - x)) = (-1)^i cos(ix)$, we can get that

$$\begin{aligned} A_i &= \tfrac{1}{2\pi}\int_{-\pi}^{\pi} w^\alpha(\pi - t)\cos(i(\pi-t))dt \\ &= (-1)^i\tfrac{1}{2\pi}\int_{-\pi}^{\pi} w^\alpha(t)\cos(it)dt \\ &= (-1)^i\tfrac{1}{2\pi}\left[\int_{-\pi}^{0} w^\alpha(t)\cos(it)dt + \int_0^{\pi} w^\alpha(t)\cos(it)dt\right] \end{aligned} \tag{S27}$$

And $\int_{-\pi}^{0} w^\alpha(t)cos(it)dt = \int_{\pi}^{2\pi} w^\alpha(t)cos(it)dt$, so we can get

$$A_i = (-1)^n\frac{1}{2\pi}\int_0^{2\pi} w^\alpha(t)\cos(it)dt = (-1)^i A_i \tag{S28}$$

So we can get $A_{2i+1} = 0$. Using the same method, we can prove $B_{2i+1} = 0$.