Green Functions of Nonperturbtive QCD

(Part II)

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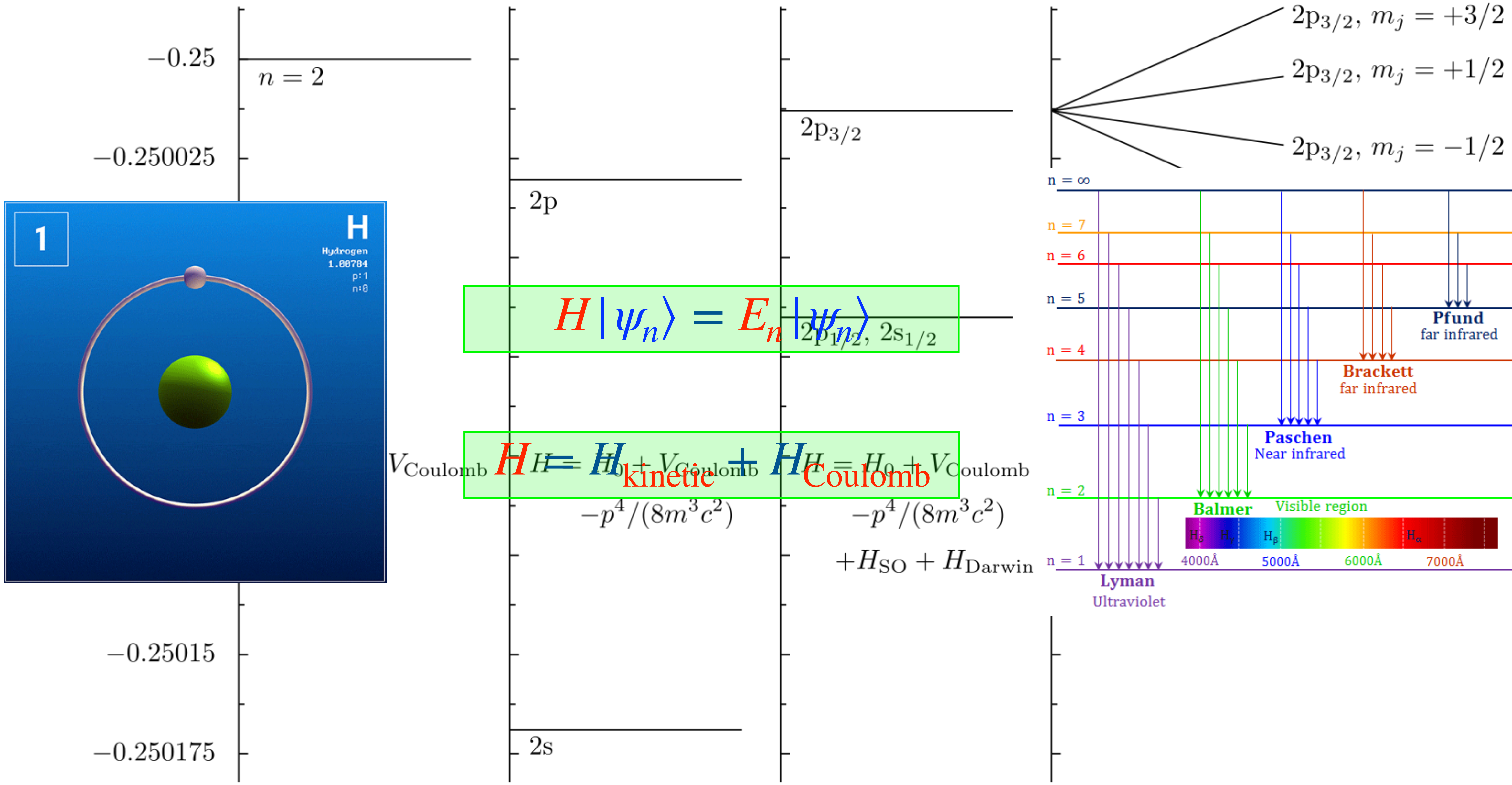
Introduction

Introduction: Few-body problem



$$H\,|\psi_n\rangle = E_n\,|\psi_n\rangle$$

$$H = H_{\text{kinetic}} + H_{\text{Coulomb}}$$

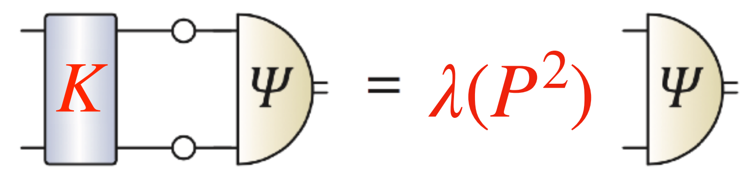
$$H = H_{\text{kinetic}} + H_{\text{Coulomb}} + H_{\text{spin-orbit}} + H_{\text{relativistic}} + H_{\text{QED}}$$

Introduction: Few-body problem

Quantum Mechanics → **Quantum Field Theory**

Schroedinger equation → Bound-state equation

$$H\,|\psi\rangle = E\,|\psi\rangle$$



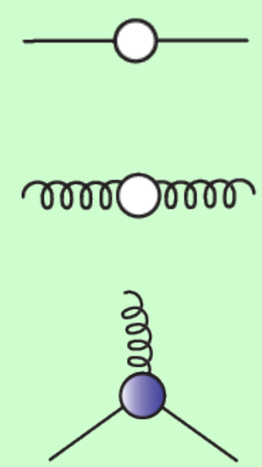
H_{kinetic}

H_{Coulomb}

$H_{\text{spin-orbit}}$

$H_{\text{relativistic}}$

H_{QED}



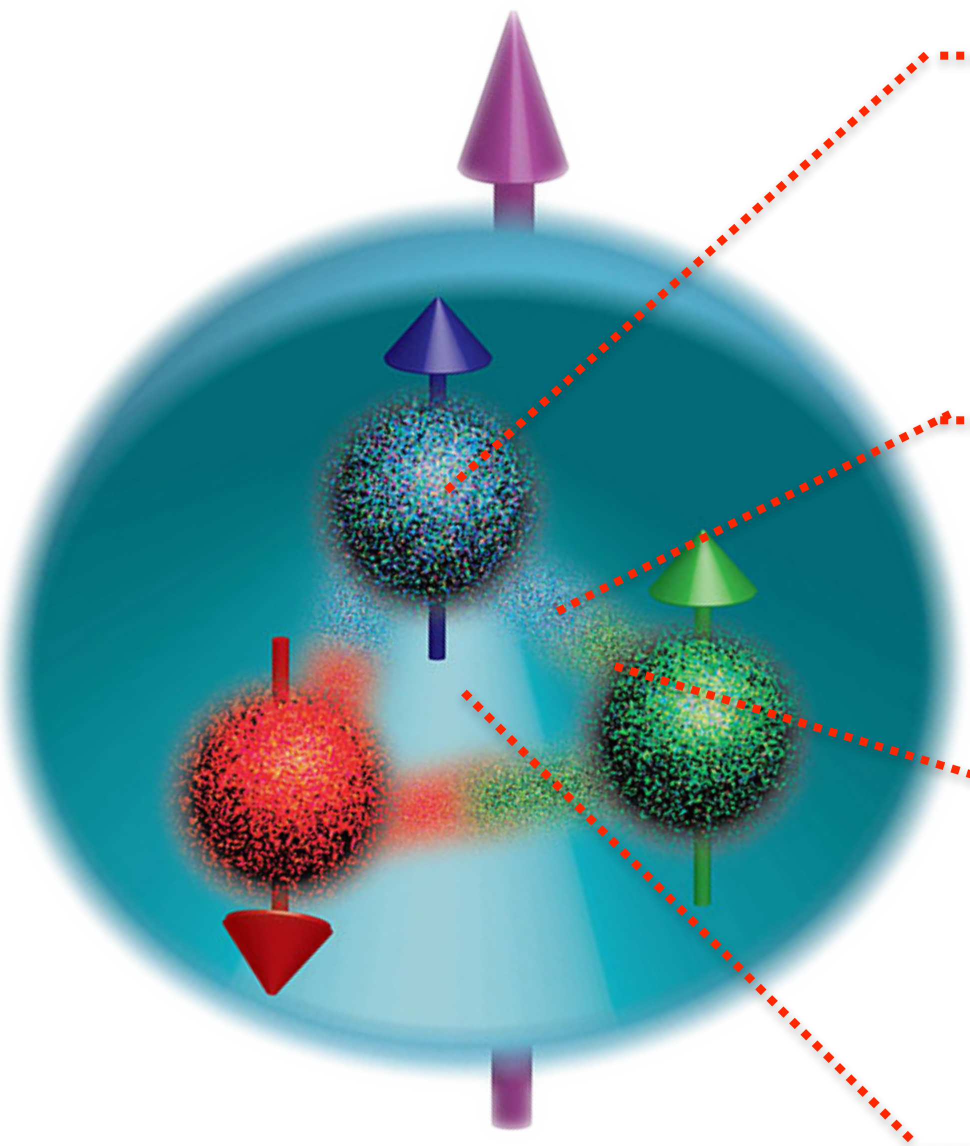
Quark

Gluon

Vertex

Architecture

Introduction: Few-body problem



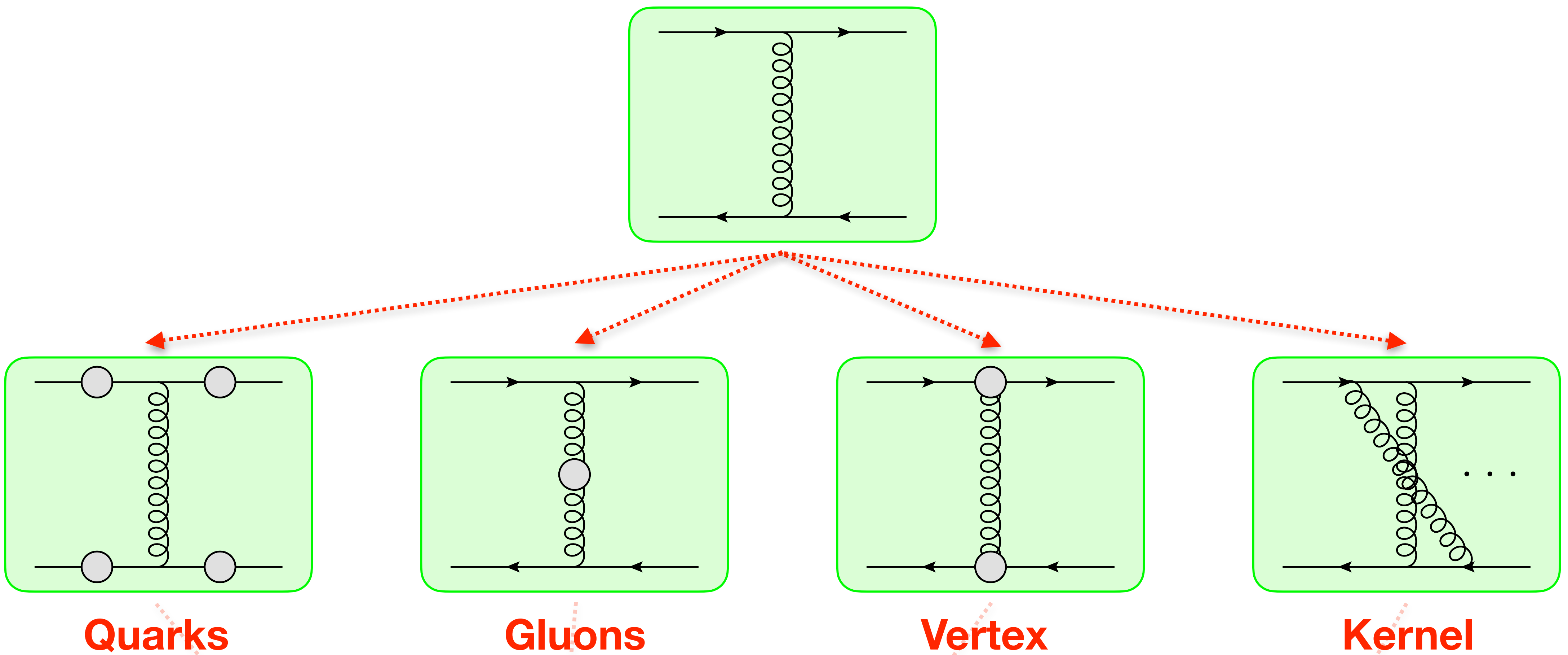
Quasi-particle quark

Quasi-particle gluon

Dressed coupling vertex

Ways of gluon-exchange

Introduction: Few-body problem



$$H = H_{\rm kinetic} + H_{\rm Coulomb} + H_{\rm spin-orbit} + H_{\rm relativistic} + H_{\rm QED}$$

Ground states → **Excited states** → M

Introduction: Many-body problem

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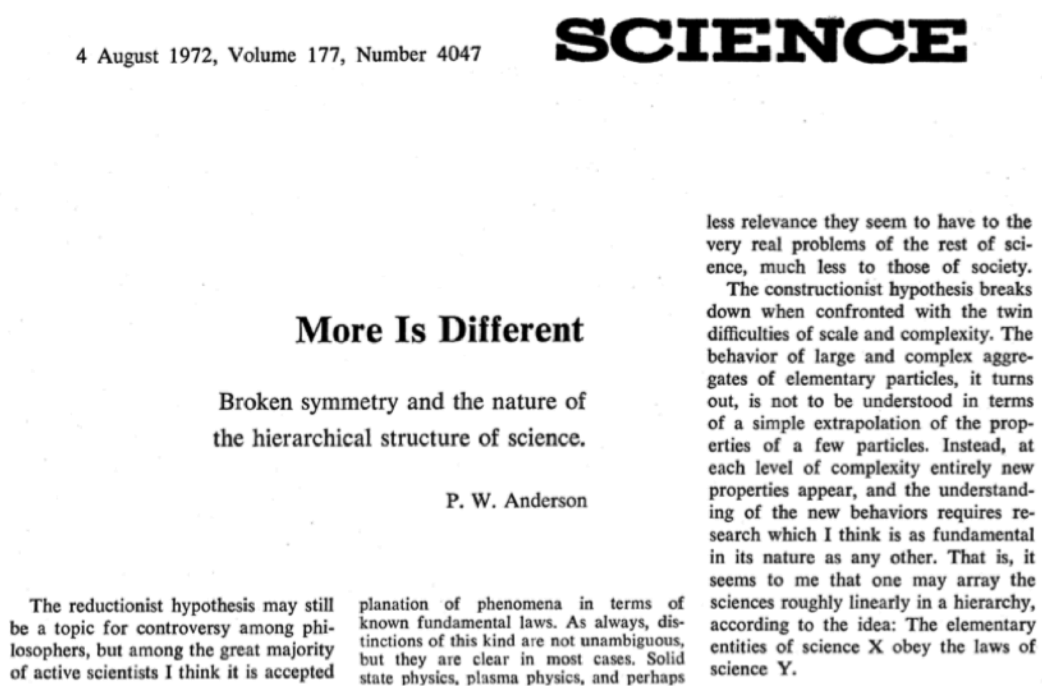

4 August 1972, Volume 177, Number 4047

SCIENCE

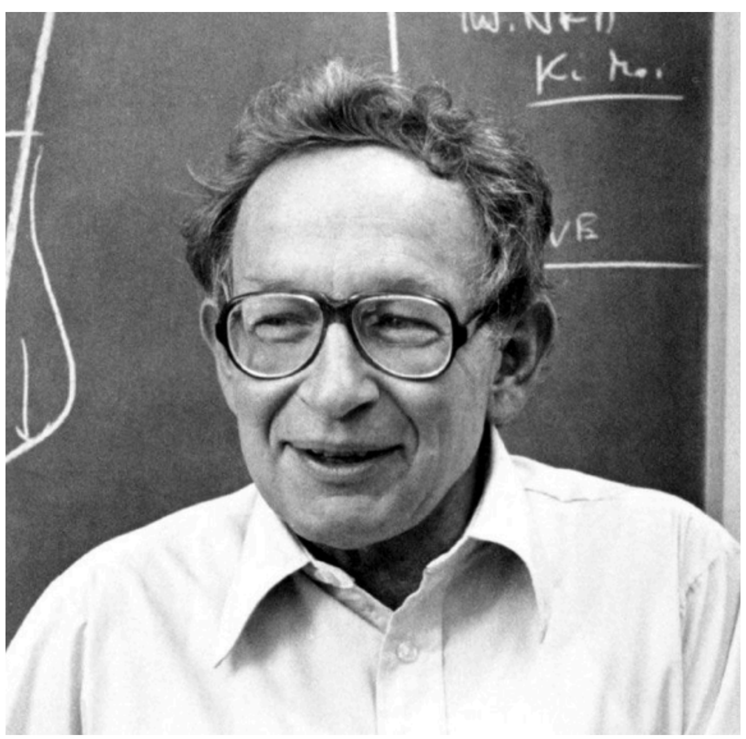
More Is Different

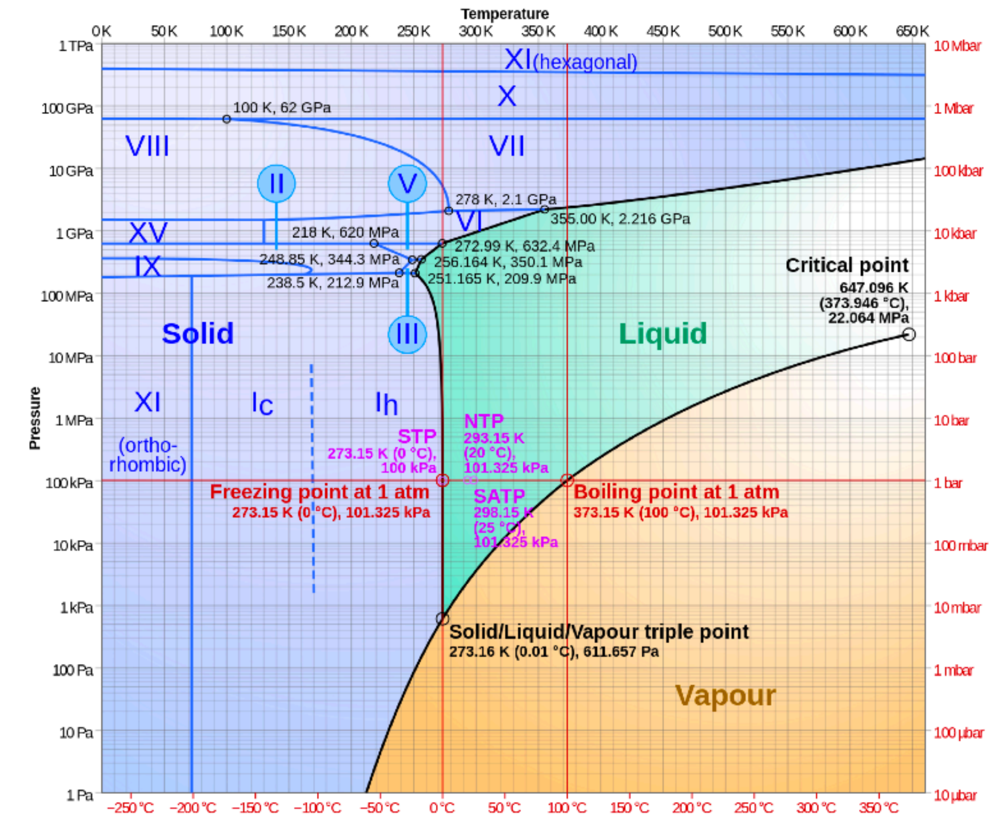
Broken symmetry and the nature of the hierarchical structure of science.

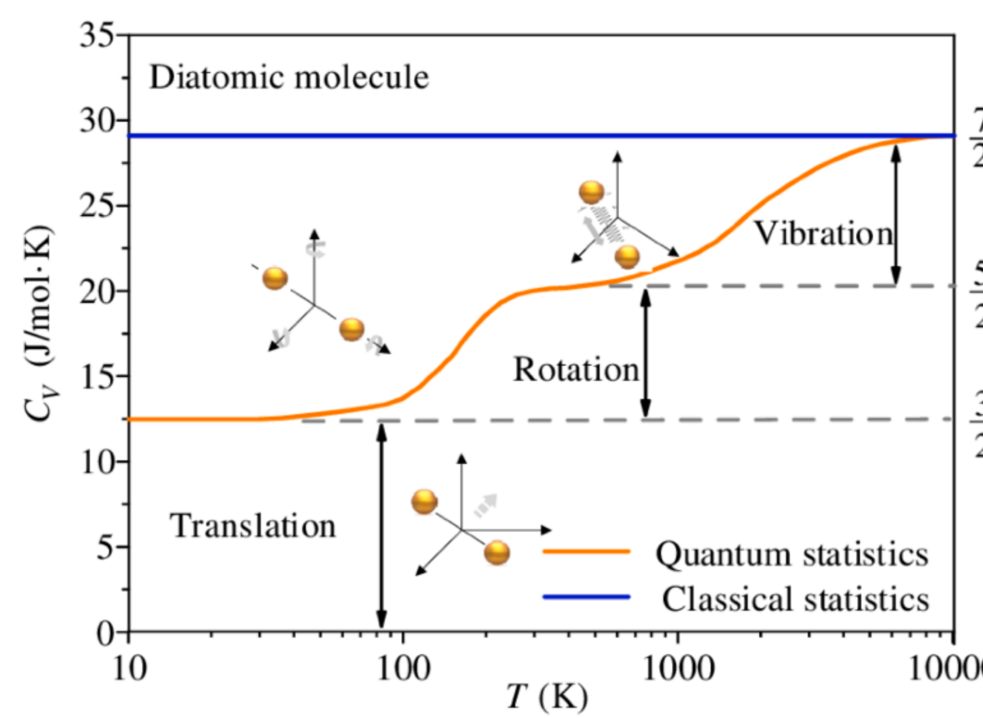
P. W. Anderson

The reductionist hypothesis may still be a topic for controversy among philosophers, but among the great majority of active scientists I think it is accepted planation of phenomena in terms of known fundamental laws. As always, distinctions of this kind are not unambiguous, but they are clear in most cases. Solid state physics, plasma physics, and perhaps less relevance they seem to have to the very real problems of the rest of science, much less to those of society.

The constructionist hypothesis breaks down when confronted with the twin difficulties of scale and complexity. The behavior of large and complex aggregates of elementary particles, it turns out, is not to be understood in terms of a simple extrapolation of the properties of a few particles. Instead, at each level of complexity entirely new properties appear, and the understanding of the new behaviors requires research which I think is as fundamental in its nature as any other. That is, it seems to me that one may array the sciences roughly linearly in a hierarchy, according to the idea: The elementary entities of science X obey the laws of science Y.







Philip W. Anderson

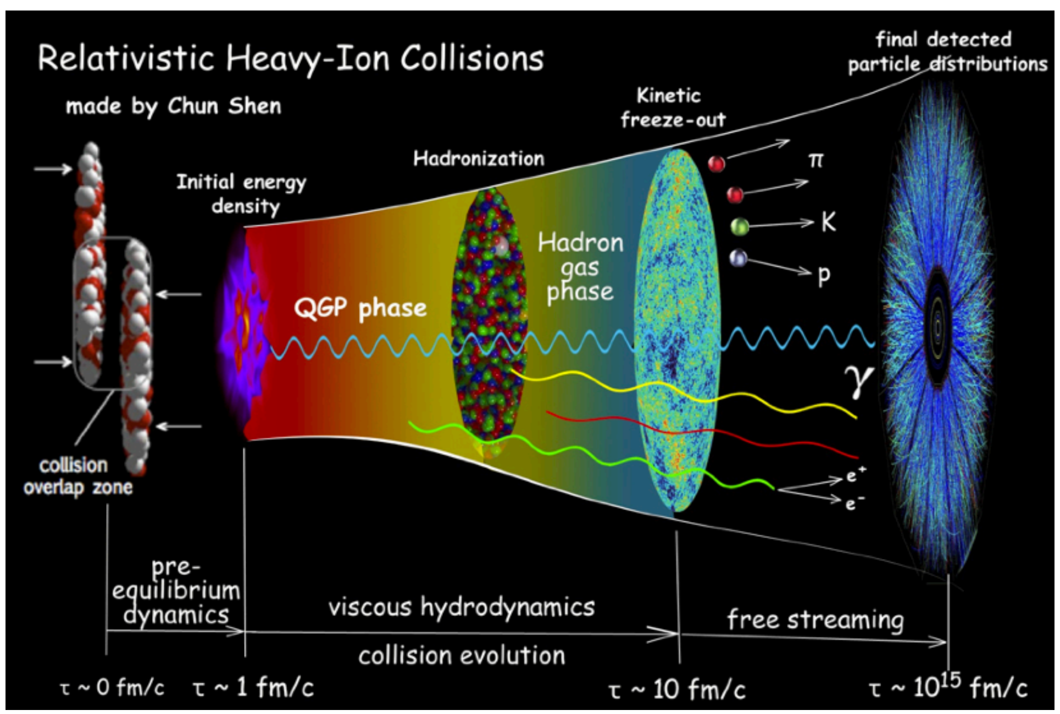
Introduction: Many-body problem

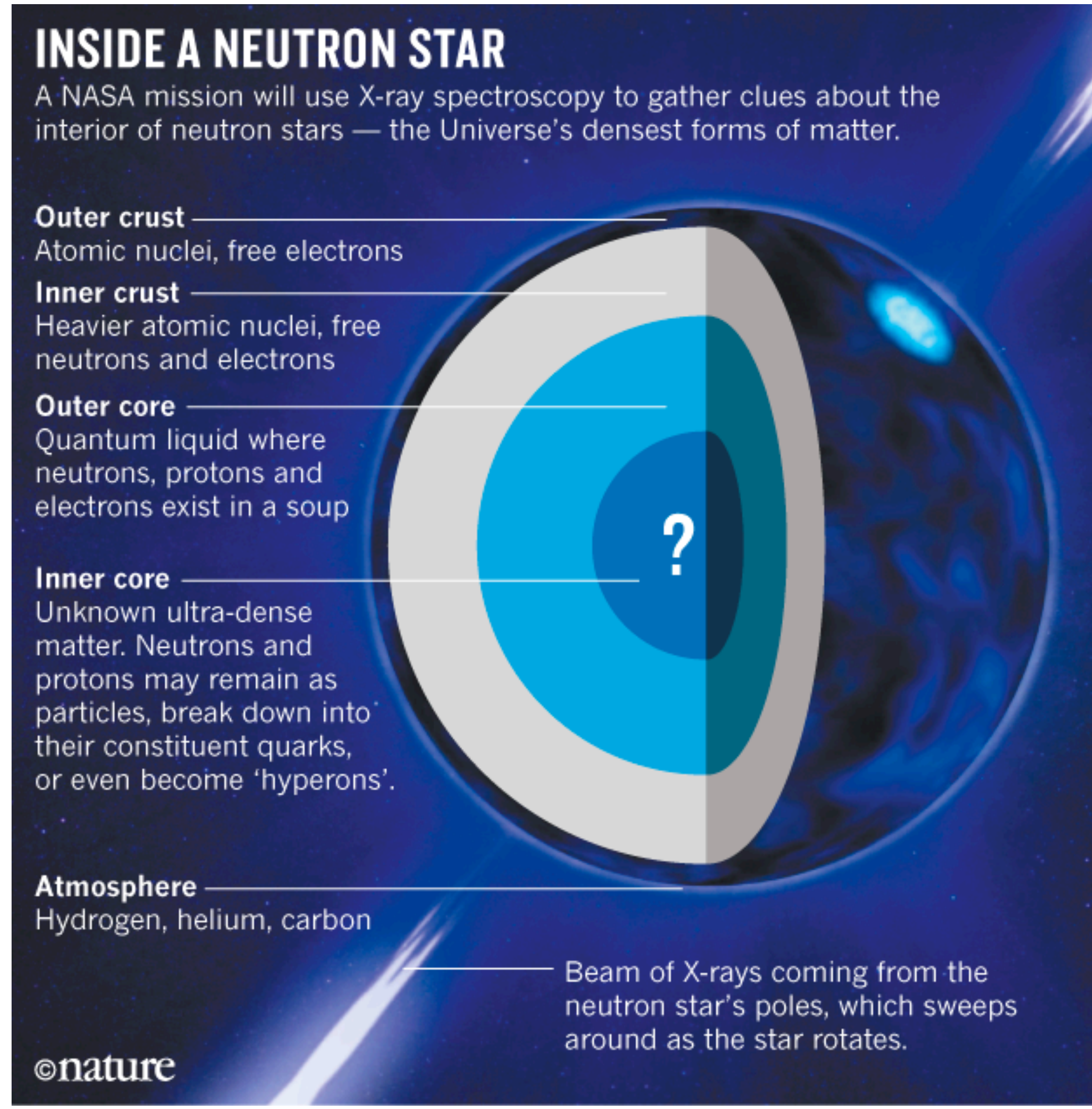
Handbook of QCD/Volume 3

THE CONDENSED MATTER PHYSICS OF QCD

KRISHNA RAJAGOPAL AND FRANK WILCZEK
Center for Theoretical Physics, Massachusetts Institute of Technology
Cambridge, MA USA 02139

Important progress in understanding the behavior of hadronic matter at high density has been achieved recently, by adapting the techniques of condensed matter theory. At asymptotic densities, the combination of asymptotic freedom and BCS theory make a rigorous analysis possible. New phases of matter with remarkable properties are predicted. They provide a theoretical laboratory within which chiral symmetry breaking and confinement can be studied at weak coupling. They may also play a role in the description of neutron star interiors. We discuss the phase diagram of QCD as a function of temperature and density, and close with a look at possible astrophysical signatures.



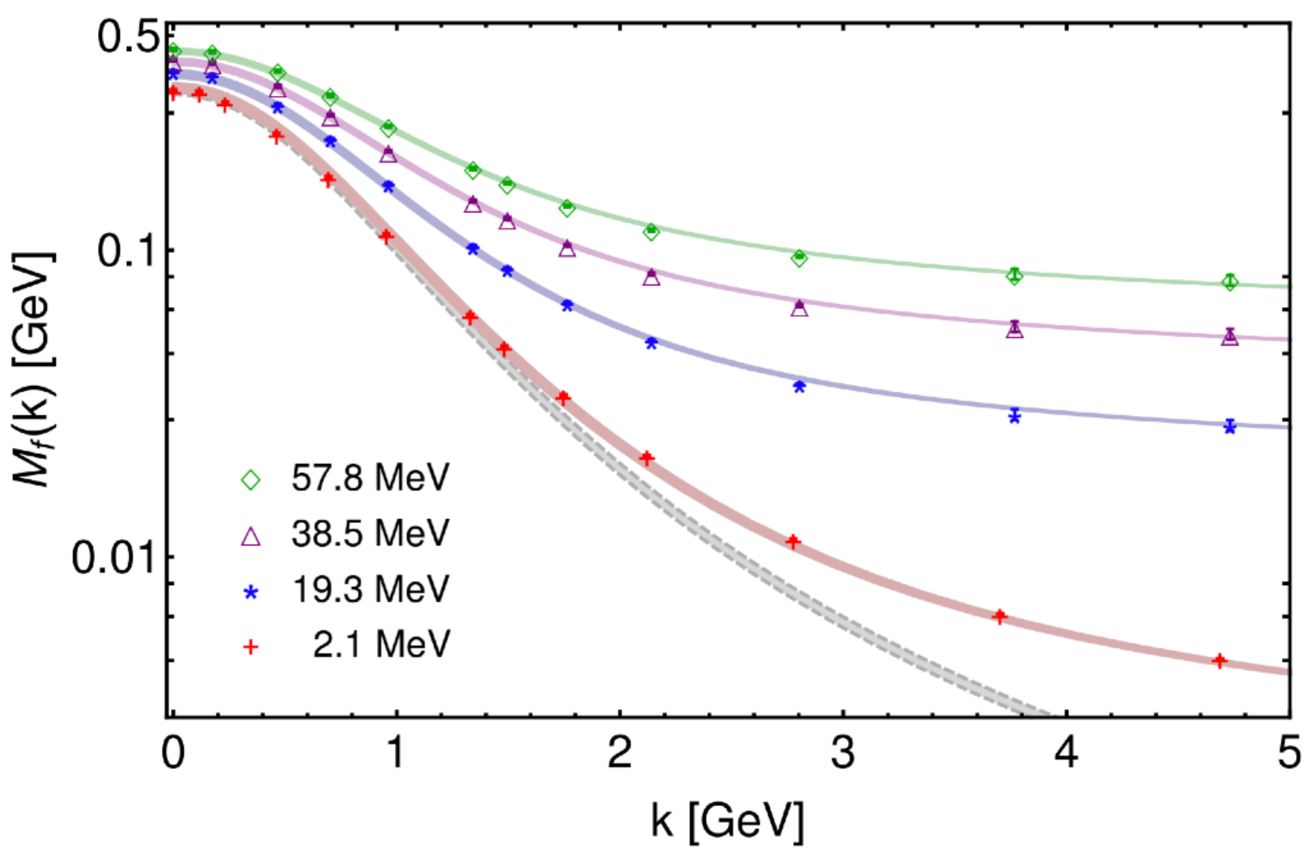


Development

Development: Gluon

$$S(p) = \frac{1}{i\gamma \cdot pA(p^2) + B(p^2)} = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

Chang, Yang, et. al., PRD 104, 094509 (2021)



Vacuum — invisible dispersive medium

✦ **Now:**

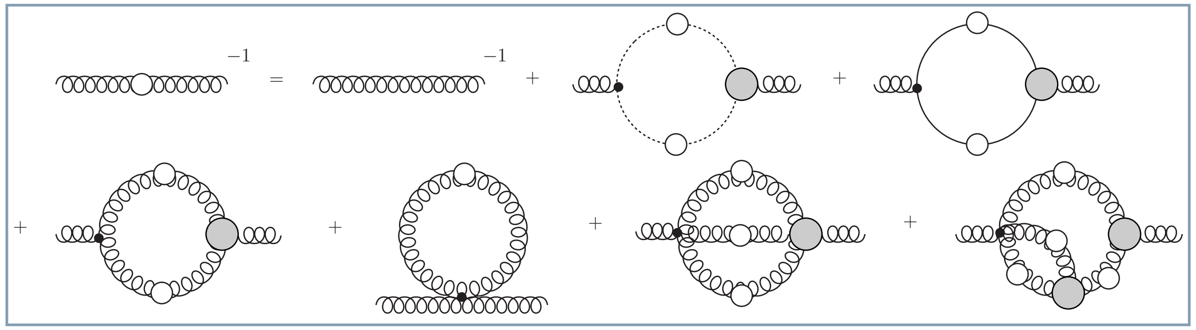
1. The quark's **effective mass** runs with its momentum.
2. The most **constituent mass** of a light quark comes from a cloud of gluons.

✦ **Next:**

1. What is the **infrared scale** of quark mass function?
2. How does the **transition** connect the non-perturbative and perturbative regions?

Development: Gluon

Gluon gap equation: Aguilar, Binosi, Papavassiliou and Rodriguez-Quintero

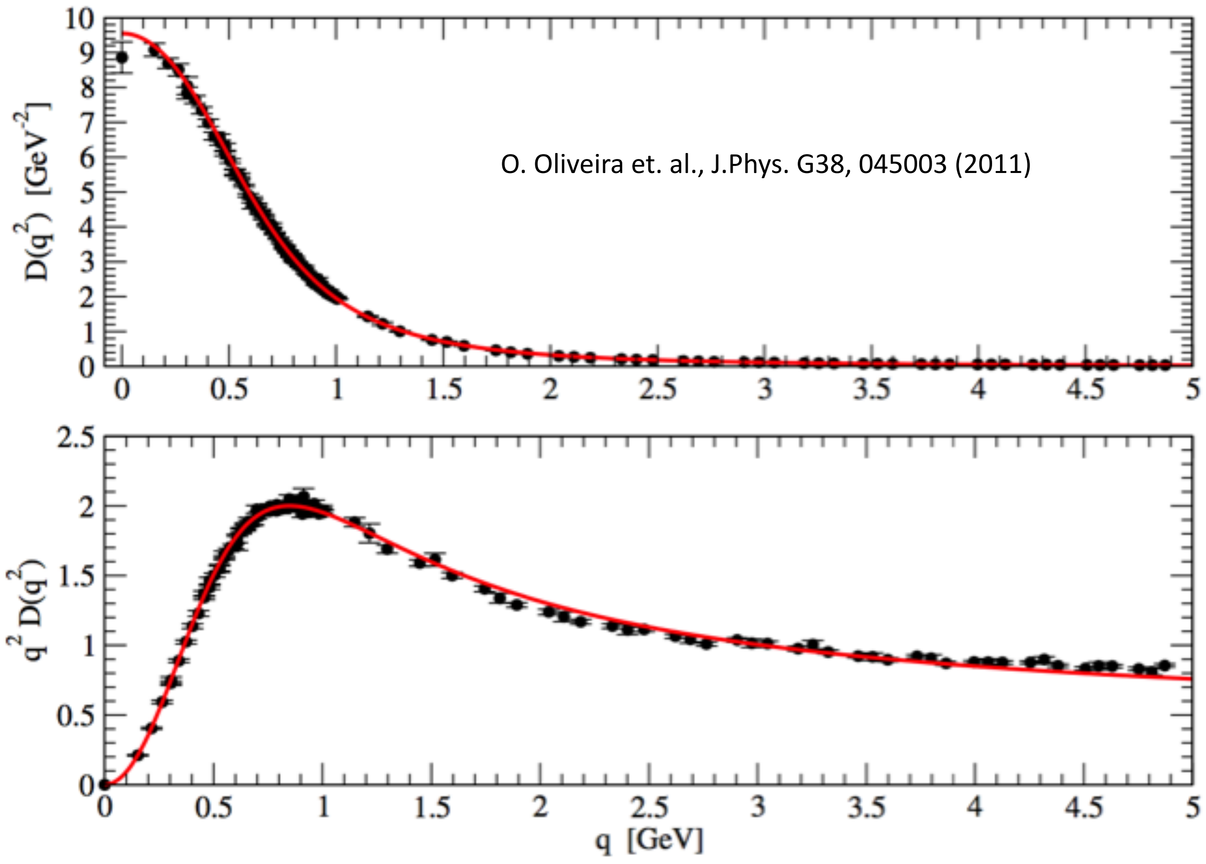


- The interaction can be decomposed: ***gluon running mass*** + ***effective running coupling***

$$g^2 D_{\mu\nu}(k) = \mathcal{G}(k^2)\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$

$$\mathcal{G}(k^2) \approx \frac{4\pi\alpha_{RL}(k^2)}{k^2 + m_g^2(k^2)}$$

Lattice QCD simulations:



- In QCD: Gluons are ***cannibals*** – a particle species whose members become **massive** by eating each other!

Development: Gluon

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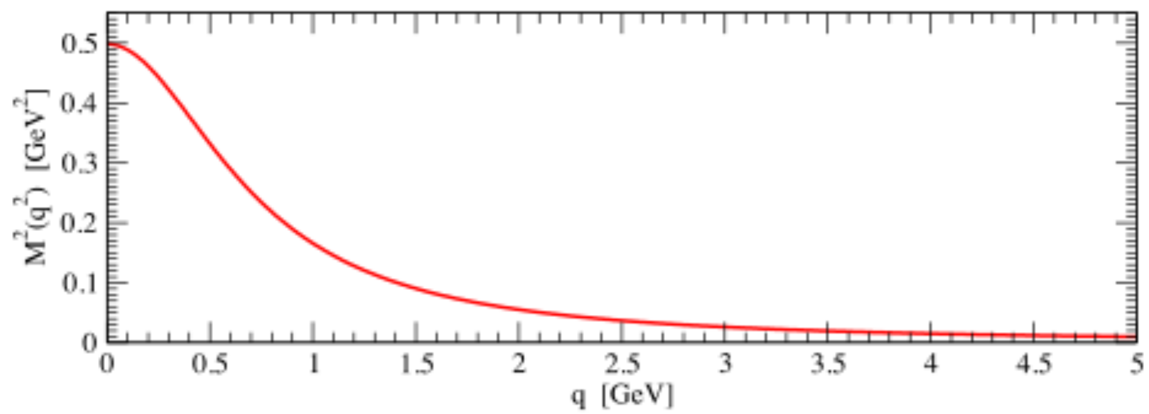
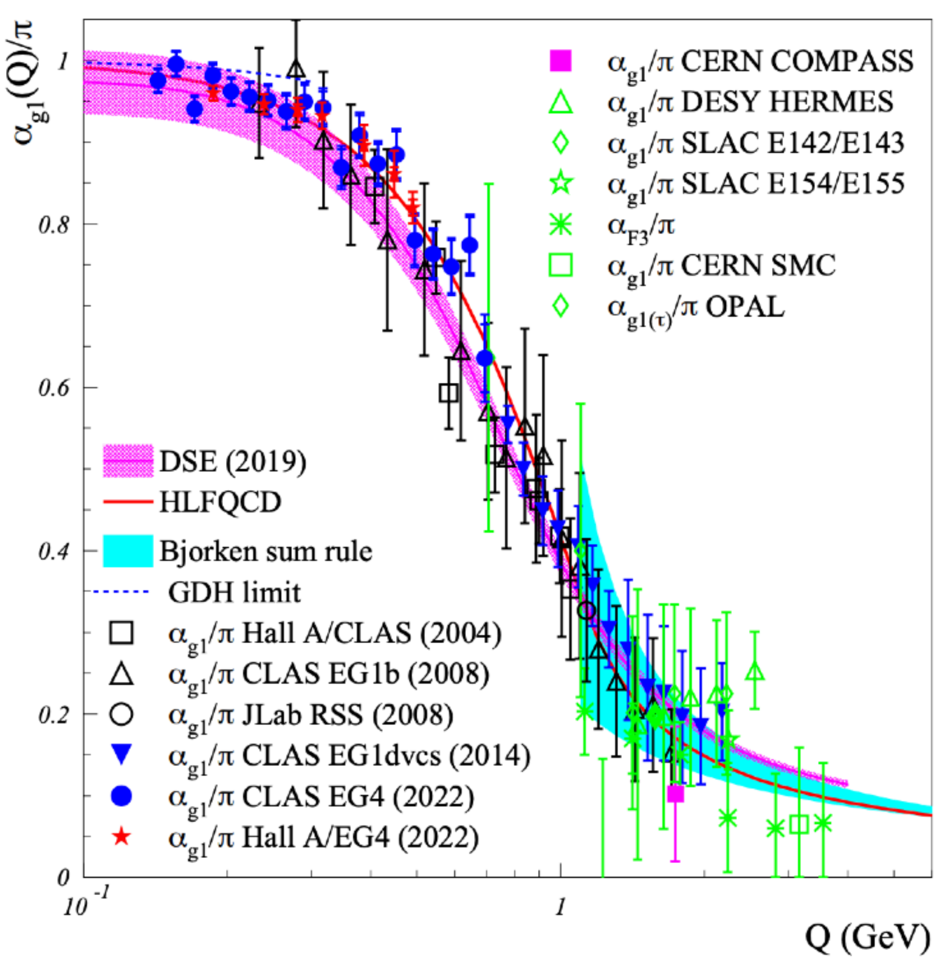
Gluon mass function: O. Oliveira et. al., J.Phys. G38, 045003 (2011)



Running coupling: Deur, Brodsky, Roberts, PPNP, 104081 (2024)



✦ **Now:**

1. The dressed gluon can be well parameterized by a **mass scale**

$$m_g^2(k^2) = \frac{M_g^4}{M_g^2 + k^2}$$

2. The effective running coupling **saturates** in the infrared limit.

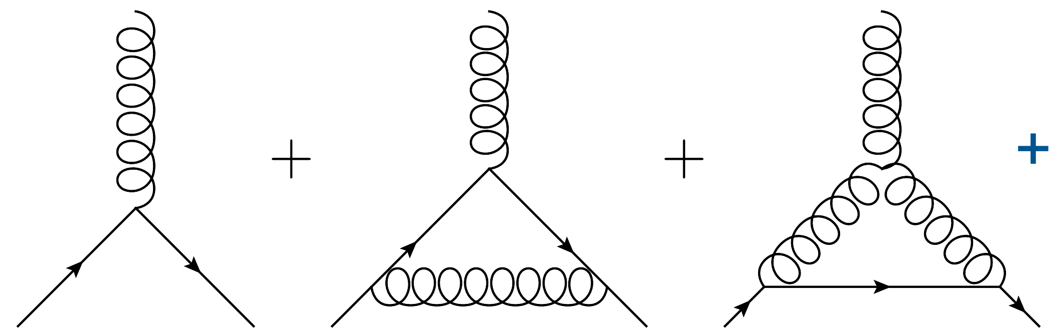
Lattice input + DSE correction

✦ **Next:**

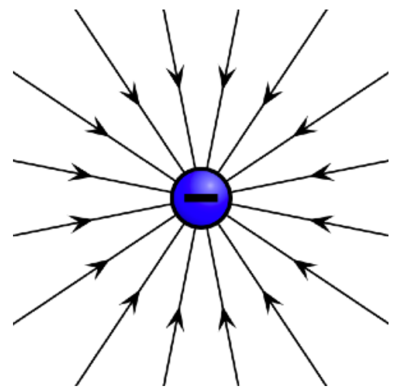
1. What is the **mass scale** of gluon?

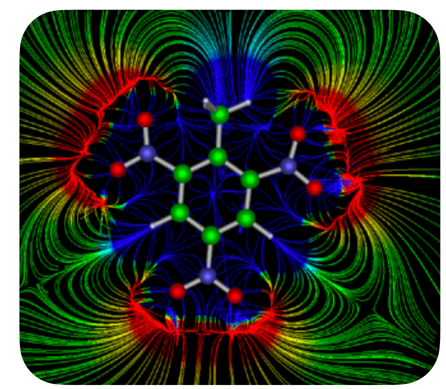
2. What is the **infrared magnitude** of running coupling?

Development: Vertex

Quark-gluon vertex:  + … = ?

point charge





distributed charge

- The **Dirac** and **Pauli** terms: for an on-shell fermion, the vertex can be decomposed by two form factors:

$$\Gamma^\mu(P',P) = \gamma^\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}}{2M_f} Q^\nu F_2(Q^2)$$

- The form factors express (color-)**charge** and (color-)**magnetization** densities. And the so-called **anomalous magnetic moment** is proportional to the Pauli term.

See, e.g., QIN et al, PLB722, 384 (2013)

Development: Vertex

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Spacetime

- Poincaré symmetry

Fields

- Gauge symmetry
- Chiral symmetry

“Symmetry dictates interaction.” — CN Yang

- **Gauge symmetry:** Longitudinal WGTI

$$iq_\mu \Gamma_\mu(k,q) = S^{-1}(k) - S^{-1}(p)$$

- **Lorentz symmetry + :** Transverse WGTIs

$$\begin{aligned} q_\mu \Gamma_\nu(k,p) - q_\nu \Gamma_\mu(k,p) = {} & S^{-1}(p)\sigma_{\mu\nu} + \sigma_{\mu\nu} S^{-1}(k) \\ & + 2im\Gamma_{\mu\nu}(k,p) + t_\lambda \epsilon_{\lambda\mu\nu\rho}\Gamma^A_\rho(k,p) \\ & + A^V_{\mu\nu}(k,p)\,, \end{aligned}$$

$$\begin{aligned} q_\mu \Gamma^A_\nu(k,p) - q_\nu \Gamma^A_\mu(k,p) = {} & S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu} S^{-1}(k) \\ & + t_\lambda \epsilon_{\lambda\mu\nu\rho}\Gamma_\rho(k,p) \\ & + V^A_{\mu\nu}(k,p)\,, \qquad \sigma^5_{\mu\nu} = \sigma_{\mu\nu}\gamma_5 \end{aligned}$$

The WGTIs of the vertices can be decoupled and (partially) solved.

See, e.g., PLB722, 384 (2013)

Development: Vertex

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Equation of Motion (DSEs)

$$S = \frac{1}{i\gamma \cdot p A(p^2) \; + \; B(p^2)}$$

$$\gamma^{\mu}\Sigma_A \; + \; i\sigma_{\mu\nu}Q^{\nu}\Delta_B \; + \; \text{etc.} \sim \Gamma^{\mu}$$

Ward identities (Symmetries)

✦ **Now:**

1. There is a dynamic chiral symmetry breaking (**DCSB**) **feedback**.

2. The **appearance** of the vertex is dramatically modified by the **dynamics**.

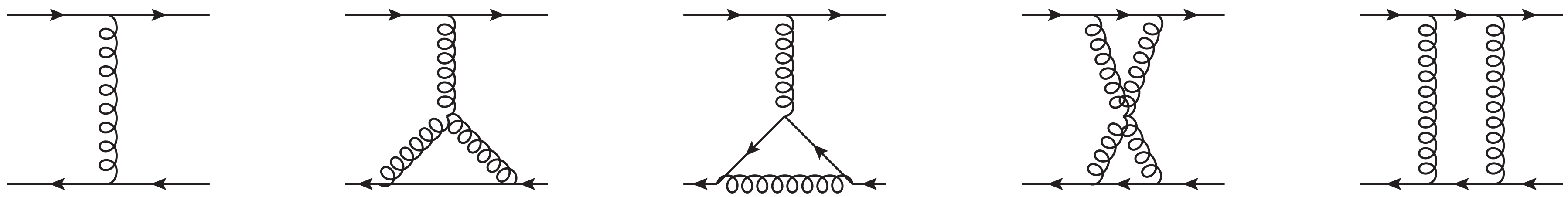
✦ **Next:**

1. What are the exact **strengths** of the terms in the vertex?

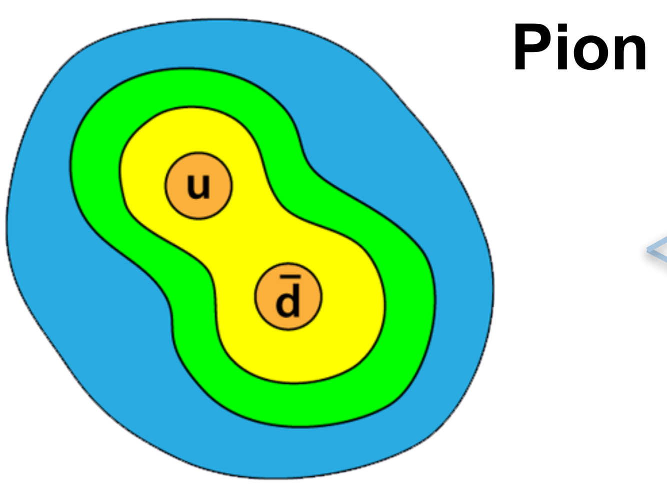
2. What the exact **behaviors** of the form factors in the vertex?

See, e.g., PLB722, 384 (2013)

Development: Kernel



+ … = ?



✦ **Bound state** of quark and anti-quark, but abnormally light:

$$M_\pi \ll M_u + M_{\bar{d}}$$

✦ **Goldstone's theorem:** If a generic continuous symmetry is spontaneously broken, then new massless scalar particles appear in the spectrum of possible excitations.

✦ The **discrete** and **continuous** **symmetries** strongly constrain the kernel:

Poincaré symmetry C-, P-, T-symmetry	Gauge symmetry Chiral symmetry

Development: Kernel

✦ In the chiral limit, the color-singlet axial-vector WGTI (**chiral symmetry**) is written as

$$\boxed{\partial_\mu J^\mu = 0} \qquad P_\mu \Gamma_{5\mu}(k,P) = S^{-1}\left(k+\frac{P}{2}\right) i\gamma_5 + i\gamma_5 S^{-1}\left(k-\frac{P}{2}\right)$$

✦ Assuming **DCSB**, i.e., the mass function is nonzero, we have the following equation

$$\lim_{P\to 0} P_\mu \Gamma_{5\mu}(k,P) = 2i\gamma_5 B(k^2) \neq 0$$

✦ The axial-vector vertex must involve a pseudo scalar pole (Goldstone's theorem)

$$\Gamma_{5\mu}(k,P) \sim \frac{2i\gamma_5 f_\pi E_\pi(k^2) P_\mu}{P^2} \propto \frac{P_\mu}{P^2}$$

$$\boxed{f_\pi E_\pi(k^2) = B(k^2)}$$

Pion exists if, and only if, **mass** is dynamically generated.

Two-body problem solved, almost completely, once solution of **one-body** problem is known.

Model independent

Gauge independent

Scheme independent

See, e.g., PLB733, 202 (2014)

Development: Kernel

✦ General decomposition:

$$
\begin{aligned}
K^{(2)} = &\left[K_{L0}^{(+)} \otimes K_{R0}^{(-)}\right] + \left[K_{L0}^{(-)} \otimes K_{R0}^{(+)}\right] + \left[K_{L1}^{(-)} \otimes_+ K_{R1}^{(-)}\right] \\
&+ \left[K_{L1}^{(+)} \otimes_+ K_{R1}^{(+)}\right] + \left[K_{L2}^{(-)} \otimes_- K_{R2}^{(-)}\right] + \left[K_{L2}^{(+)} \otimes_- K_{R2}^{(+)}\right]
\end{aligned}
$$

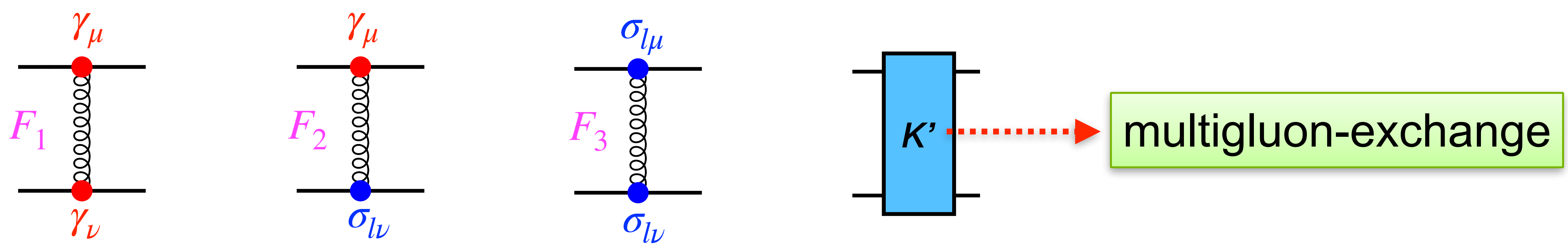
$$
\text{with} \quad \gamma_5 K^{(\pm)} \gamma_5 = \pm K^{(\pm)}, \quad \otimes_\pm := \tfrac{1}{2}(\otimes \pm \gamma_5 \otimes \gamma_5)
$$

✦ Decomposed WTI:

$$
\begin{aligned}
\Sigma_B(k_+) &= \int_{dq} \left\{K_{L0}^{(+)}[\Delta_{\sigma_A}^{\pm}]K_{R0}^{(-)} - K_{L1}^{(-)}[\sigma_B(q_+)]K_{R1}^{(-)} + K_{L1}^{(+)}[\sigma_B(q_-)]K_{R1}^{(+)}\right\} \\
0 &= \int_{dq} \left\{K_{L0}^{(+)}[\sigma_B(q_-)]K_{R0}^{(-)} - K_{L0}^{(-)}[\sigma_B(q_+)]K_{R0}^{(+)} + K_{L2}^{(+)}[\Delta_{\sigma_A}^{\pm}]K_{R2}^{(+)}\right\} \\
[\Sigma_A(k_+) - \Sigma_A(k_-)] &= \int_{dq} \left\{K_{L0}^{(+)}[-\sigma_B(q_+)]K_{R0}^{(-)} + K_{L0}^{(-)}[\sigma_B(q_-)]K_{R0}^{(+)} + K_{L2}^{(-)}[\Delta_{\sigma_A}^{\pm}]K_{R2}^{(-)}\right\} \\
-\Sigma_B(k_-) &= \int_{dq} \left\{K_{L0}^{(-)}[\Delta_{\sigma_A}^{\pm}]K_{R0}^{(+)} + K_{L1}^{(-)}[\sigma_B(q_-)]K_{R1}^{(-)} + K_{L1}^{(+)}[-\sigma_B(q_+)]K_{R1}^{(+)}\right\}
\end{aligned}
$$

✦ Now:

1. A realistic kernel must involves the Dirac and Pauli structures:



✦ Next:

1. How to further **pin down structures** of the kernel?

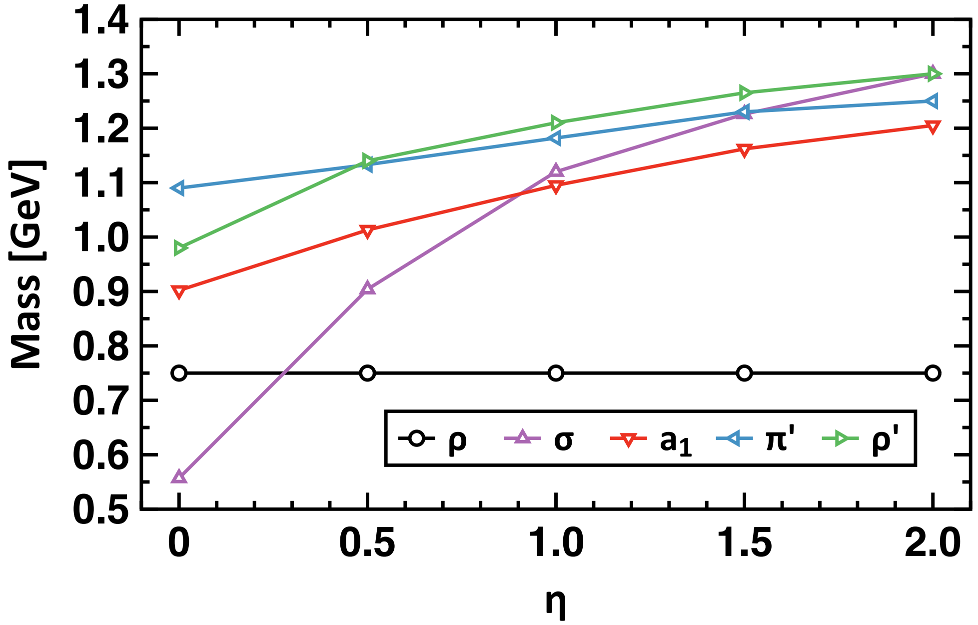
2. How to **simplify the kernel** for more practical applications?

See, e.g., QIN et al, CPL 38 (2021) 7, 071201

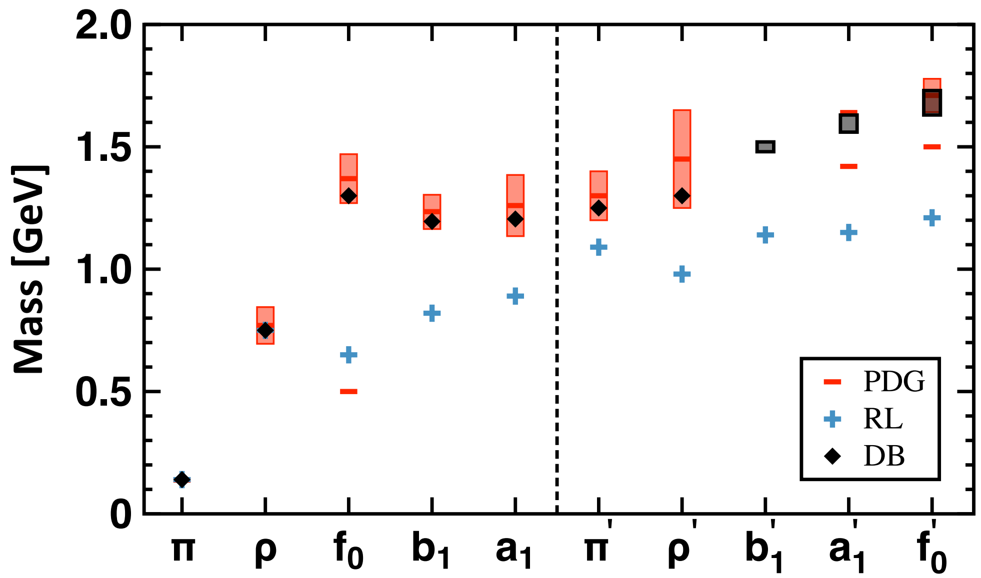
Few-body

Few-body: Meson spectra

- Impact of the Pauli term (AM):



- Light-flavor meson spectrum:



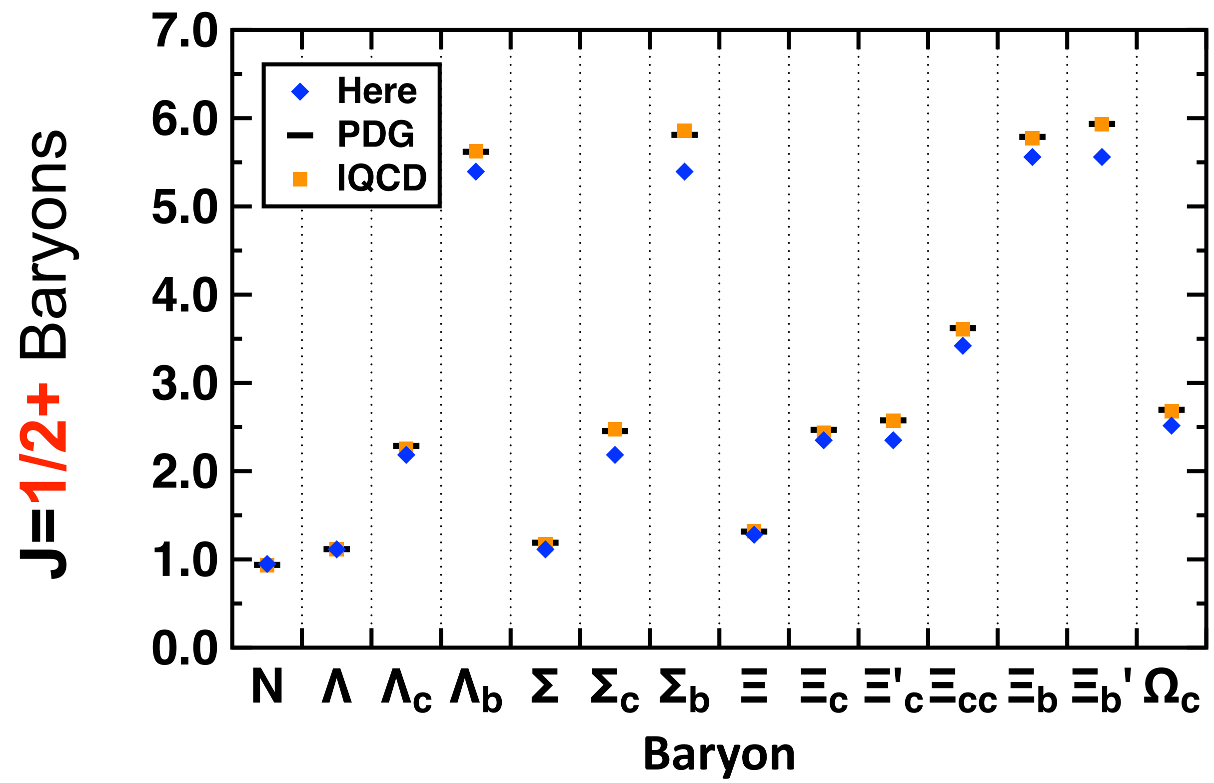
- With increasing the AM strength, the $a_1-\rho$ mass-splitting rises very rapidly. From a quark model perspective, the DCSB-enhanced kernel increases spin-orbit repulsion.

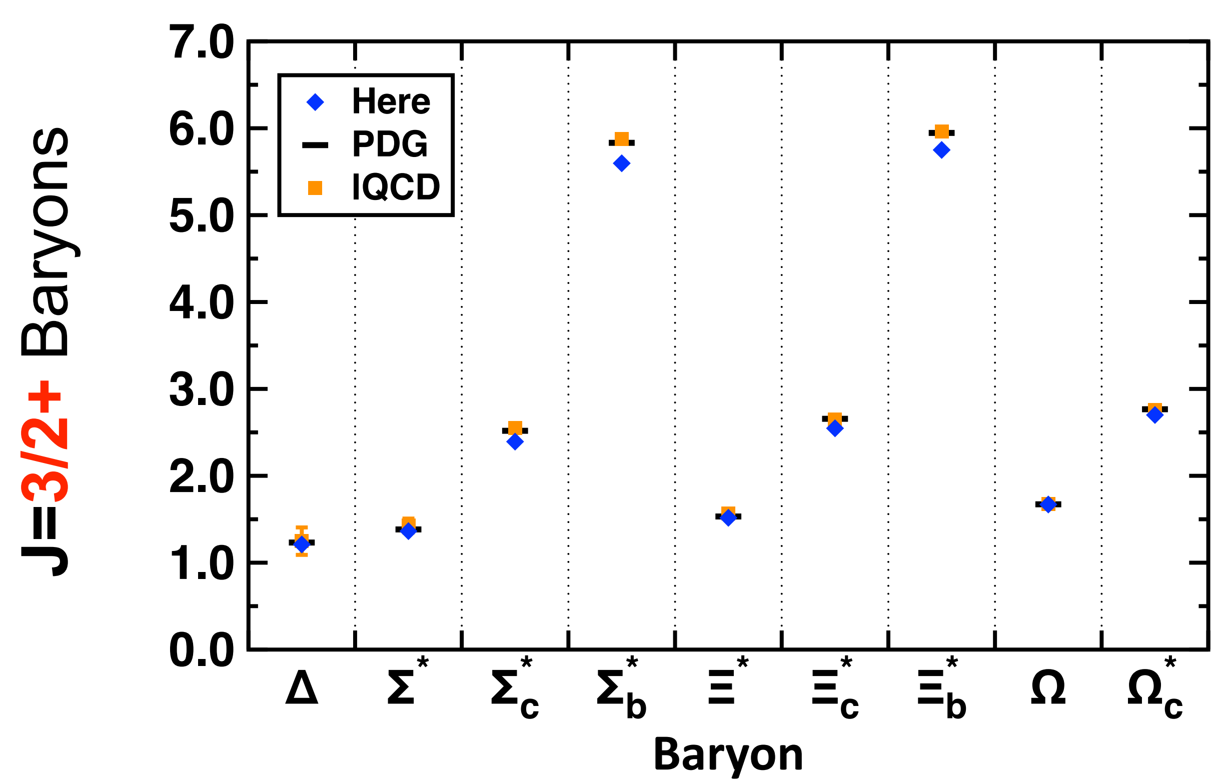
- The spin-orbit boosted quark-core mass of the f_0 is greater than the empirical value, and matches the estimated result obtained using chiral perturbation theory.

- The magnitude and ordering of radial excitation states can be fixed with the DCSB-enhanced kernel.

See, e.g., QIN et al, CPL 38 (2021) 7, 071201

Few-body: Baryon spectra



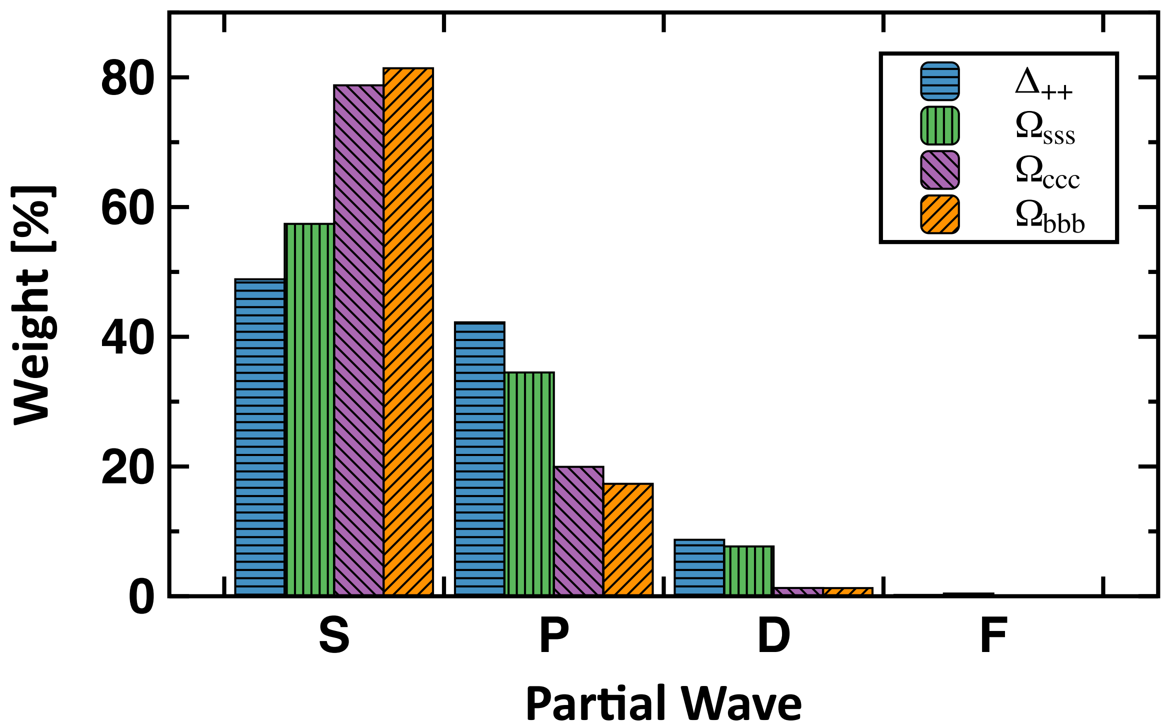


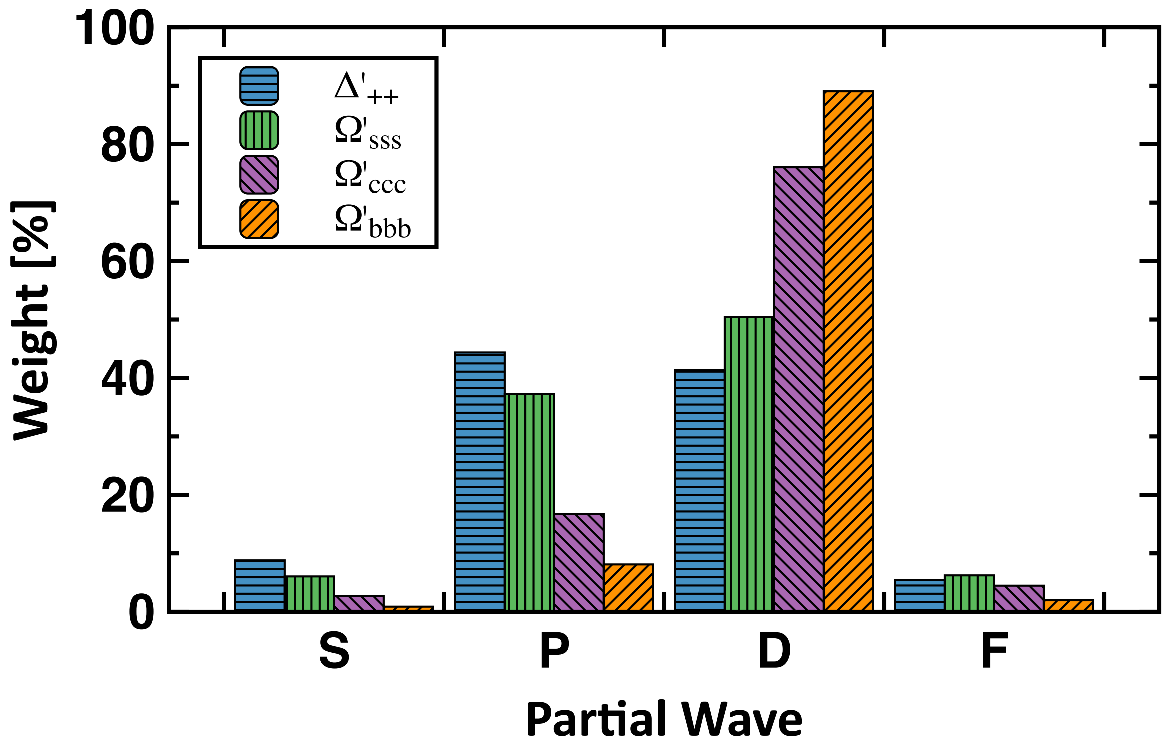
✦ The ground states of Nucleon and Delta families can be described by a simple kernel.

✦ The excited states and the parity partners require a DCSB-enhanced kernel.

See, e.g., Few-Body Syst 60, 26 (2019)

Few-body: Hadron structure





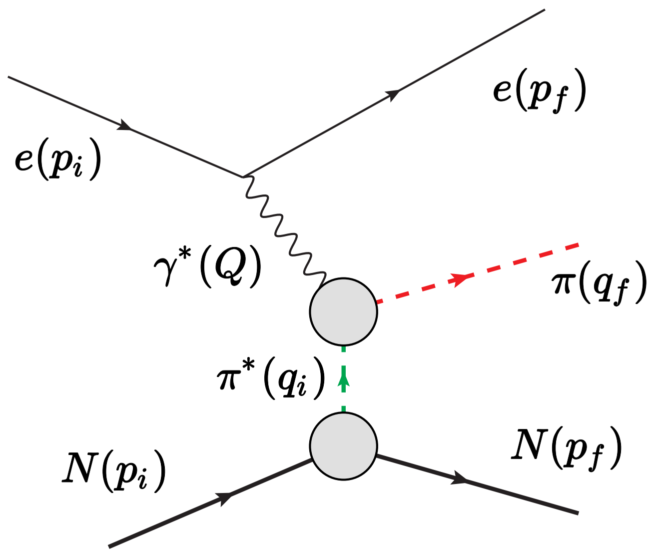
✓ S-waves dominate for ground states, but p-waves grow for light baryons.

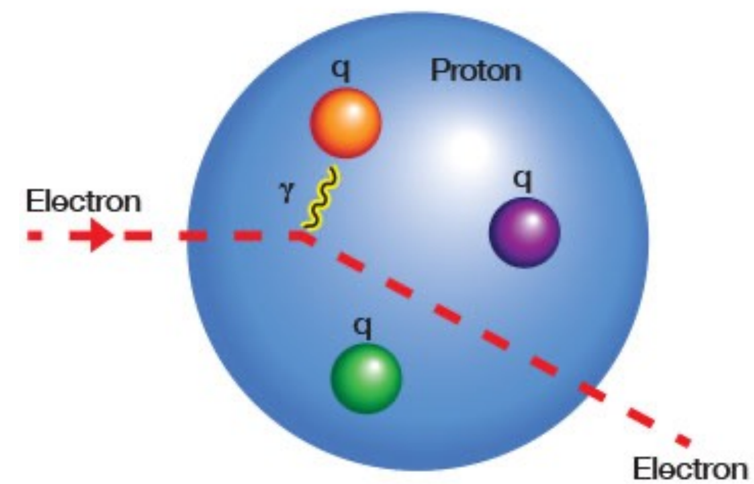
How NR quark model works

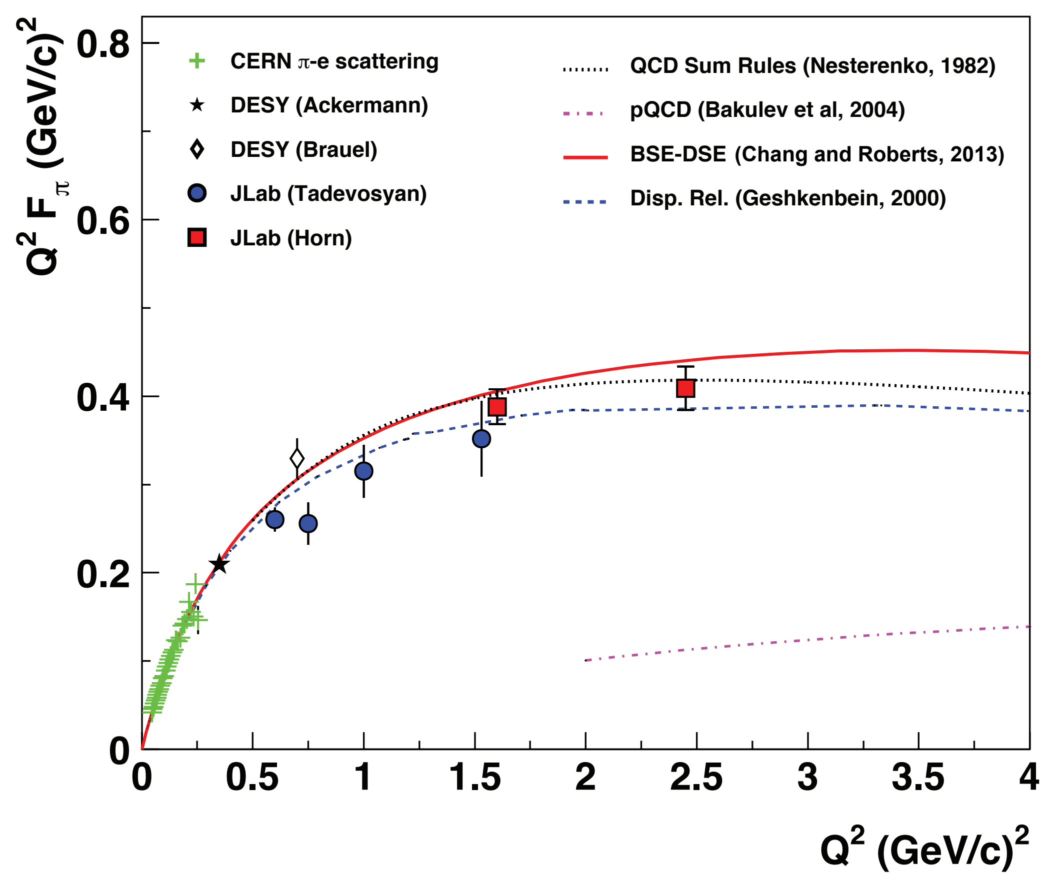
✓ D-waves dominate for excited states, but p-waves grow for light baryons.

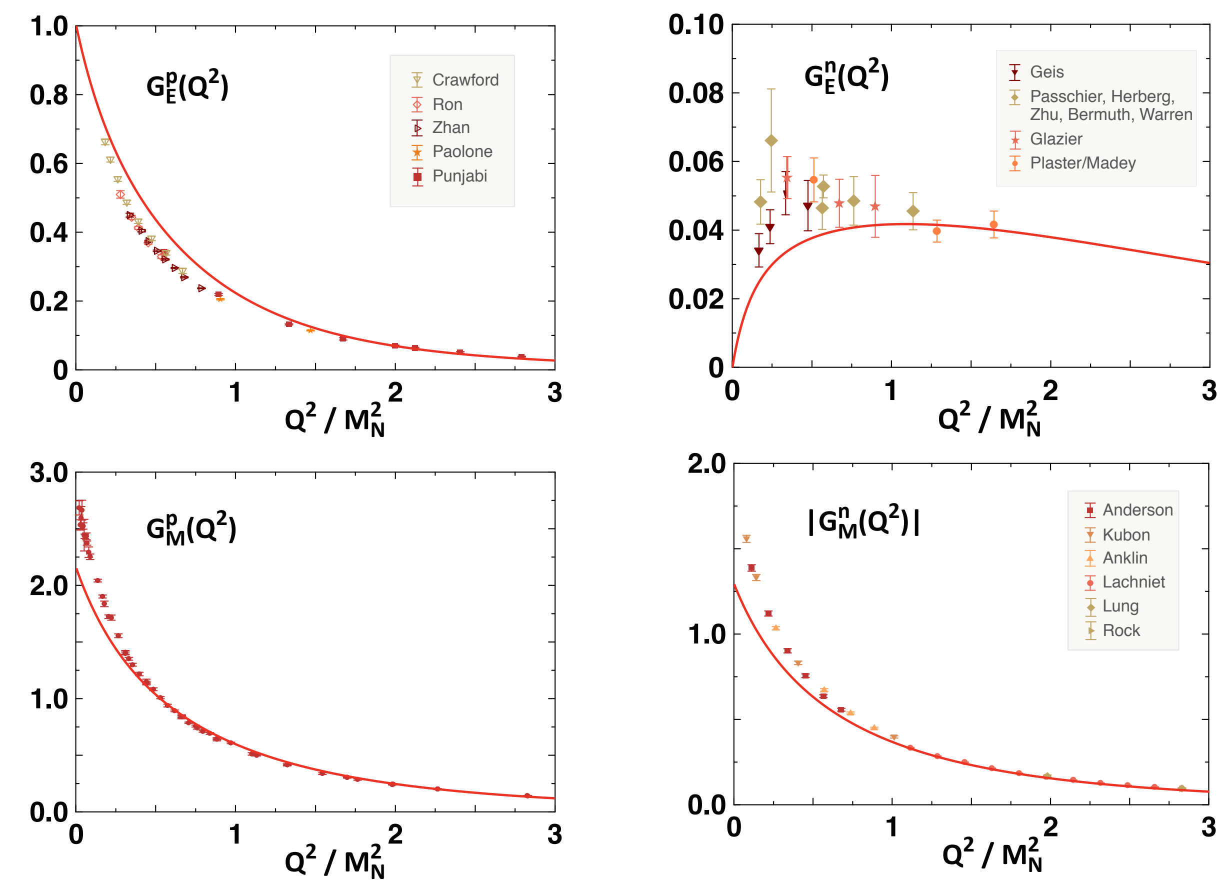
See, e.g., PRD 97, 114017 (2018)

Few-body: Hadron structure









See, e.g., PRL 111, 141802 (2013)

See, e.g., PRD 84, 014014 (2011)

Many-body

Many-body: Phase diagram

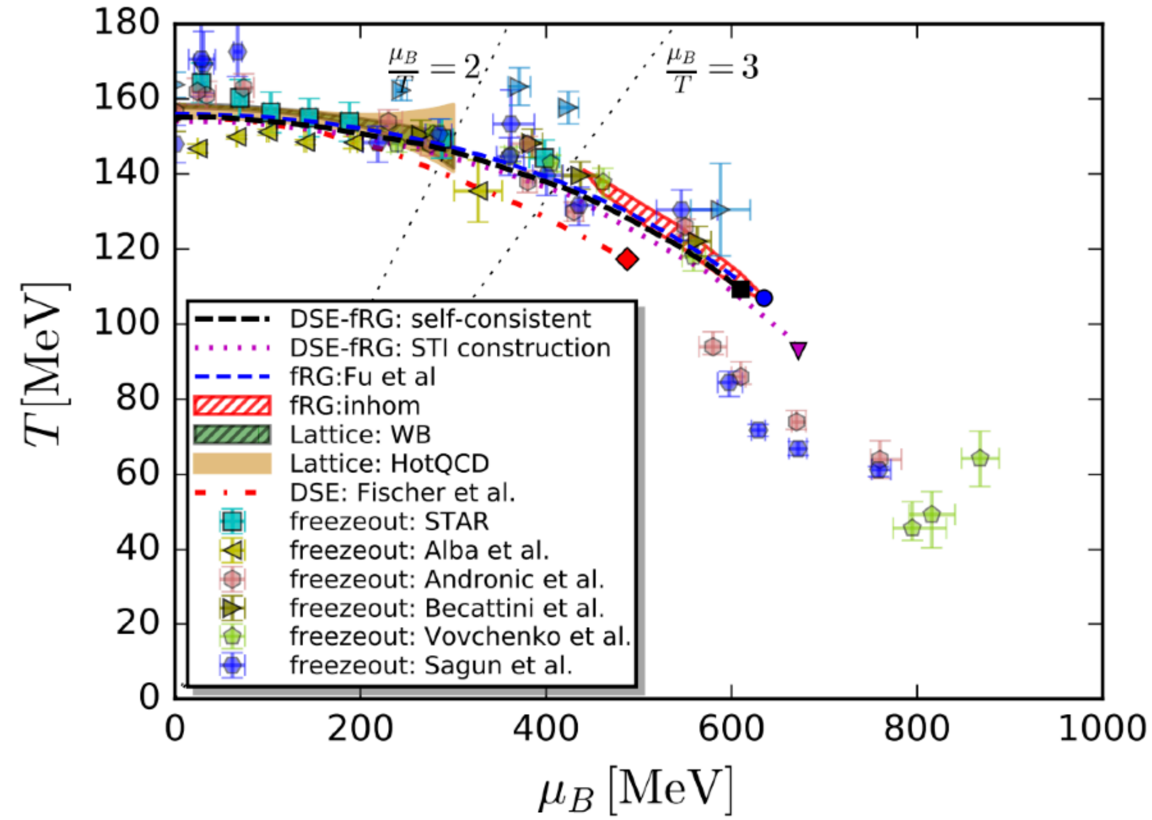
See, e.g., PRD 102, 034027 (2020)

✦ Converged critical points:

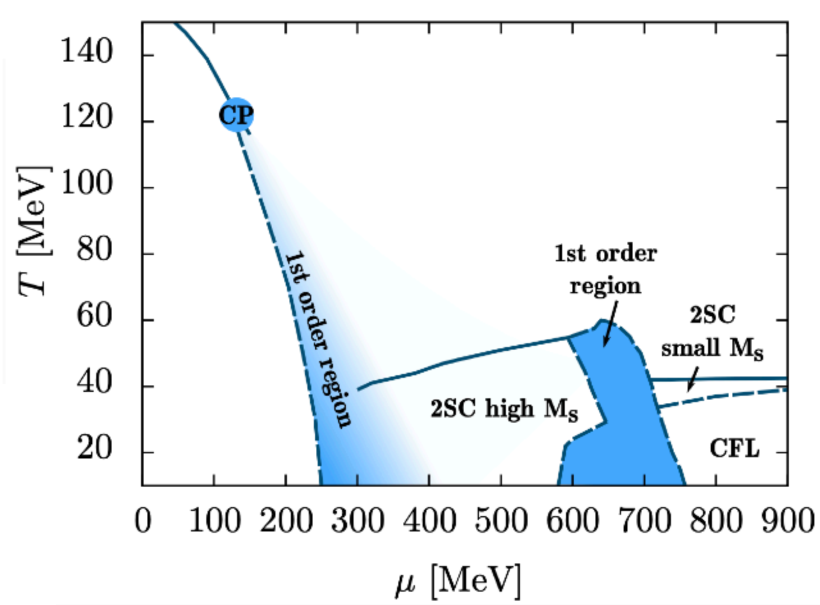
$$T_c \sim 150\mathrm{MeV}$$

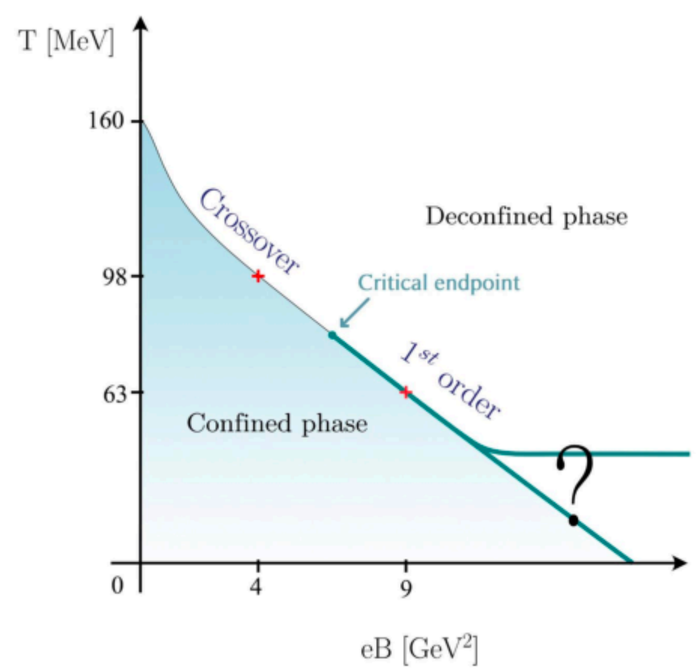
$$(T, \mu_B)_{\mathrm{CEP}} \sim (110, 600)\mathrm{MeV}$$

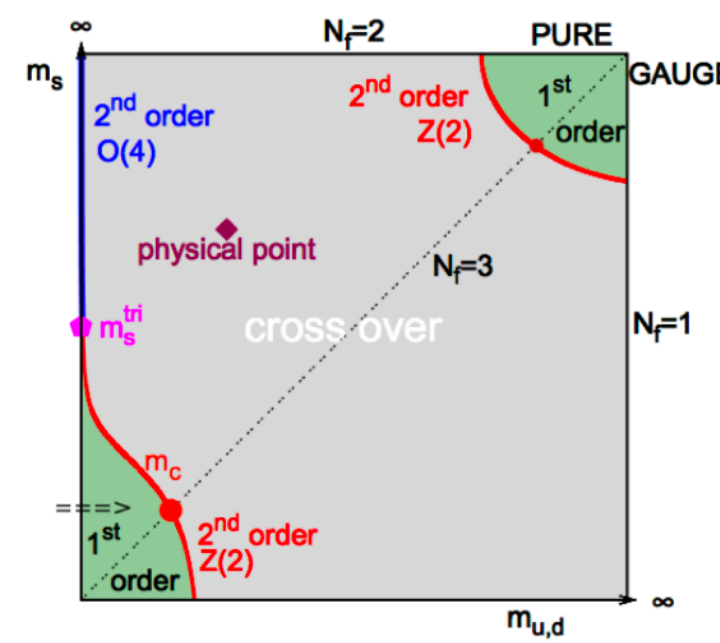
$$\left(\frac{\mu_B}{T}\right)_{\mathrm{CEP}} \sim 5.5$$



✦ More on phase diagram:



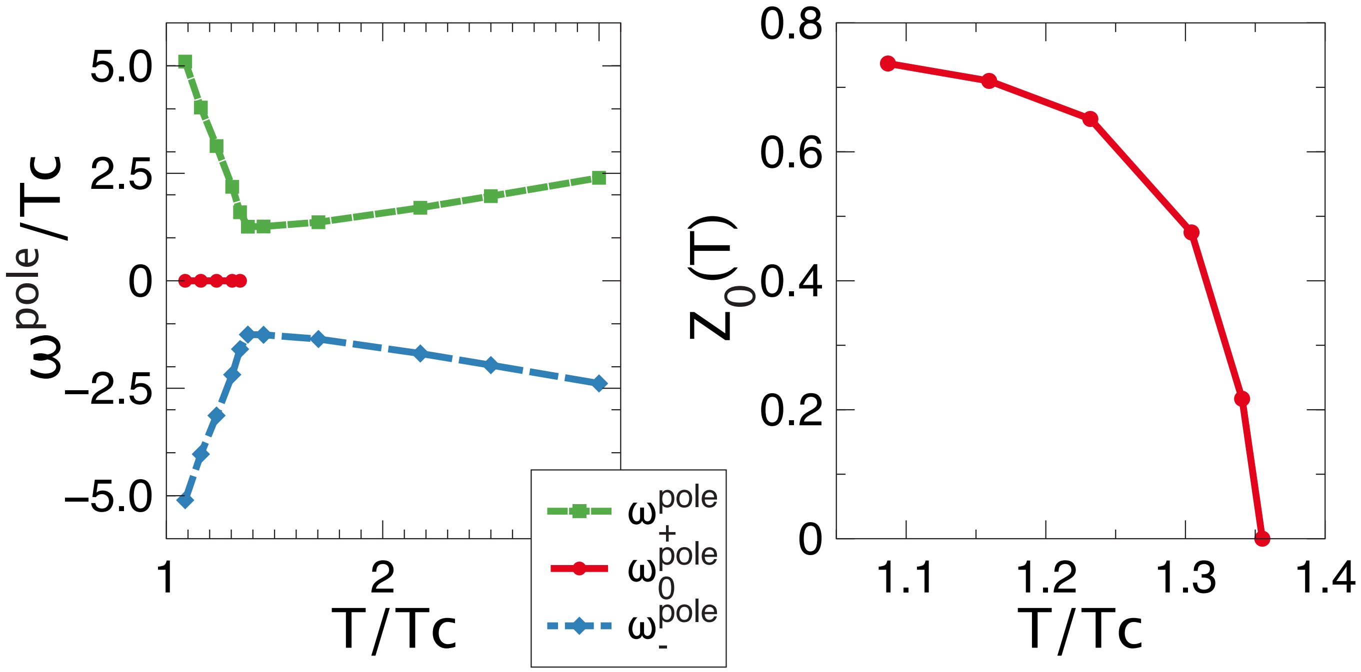




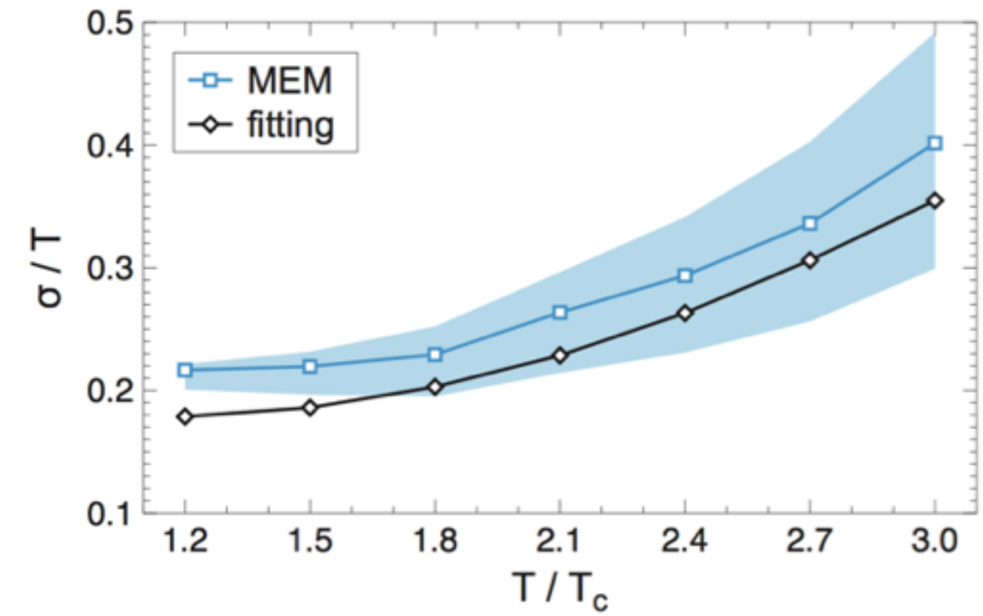
See, e.g., PPNP, 105, 1 (2019)

Many-body: Novel state

✦ Zero mode appears in the vicinity of T_c:



✦ Transport properties of sQGP:

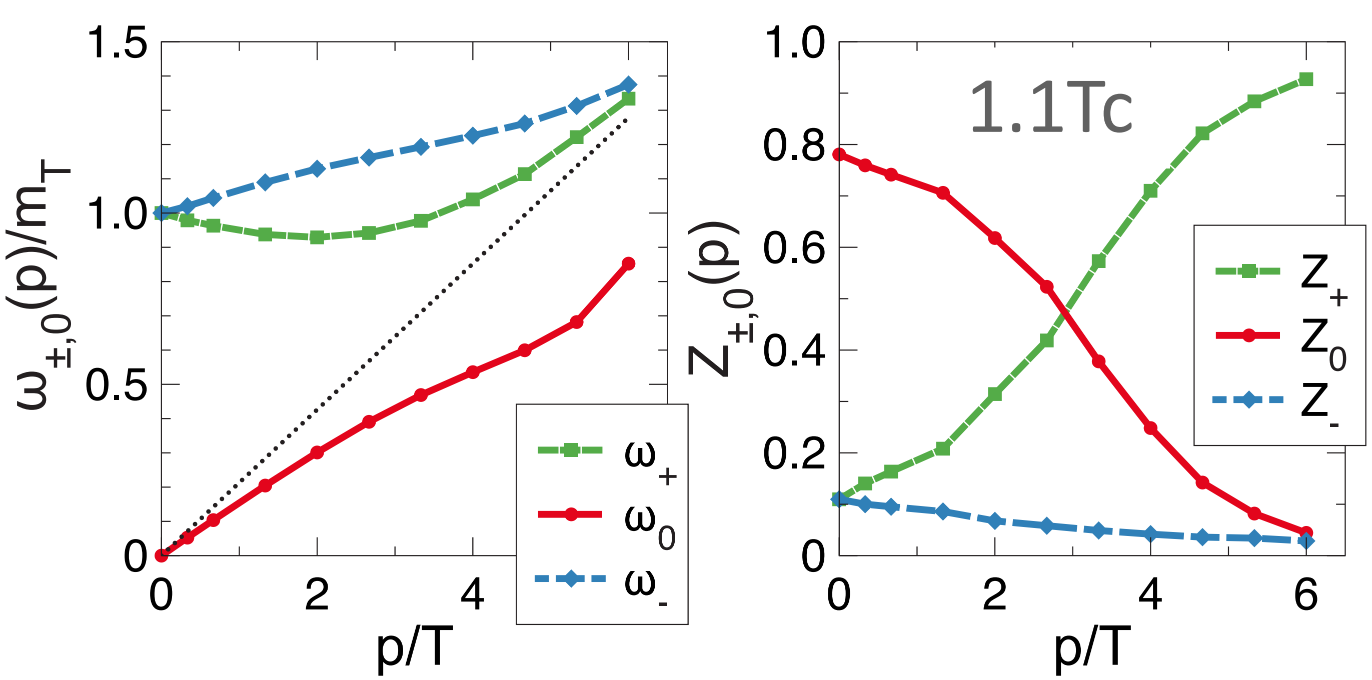


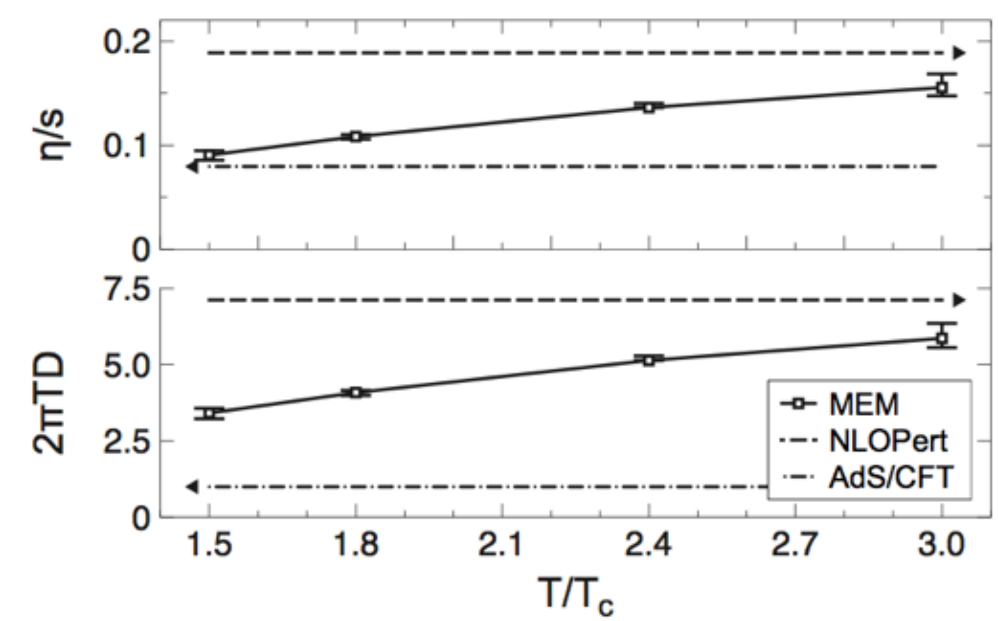
$\sigma/T \sim 0.4$ Aarts et. al., PRL 99 (2007)

$0.2 < \sigma/T < 0.3$ ($1\sim2T_c$) Amato et. al., PRL 111 (2013)

$\sigma/T \sim 9/(16\pi) \sim 0.18$ A. Atmaja, JHEP, 1008 (2010)

✦ Atypical dispersion relations of excitations:





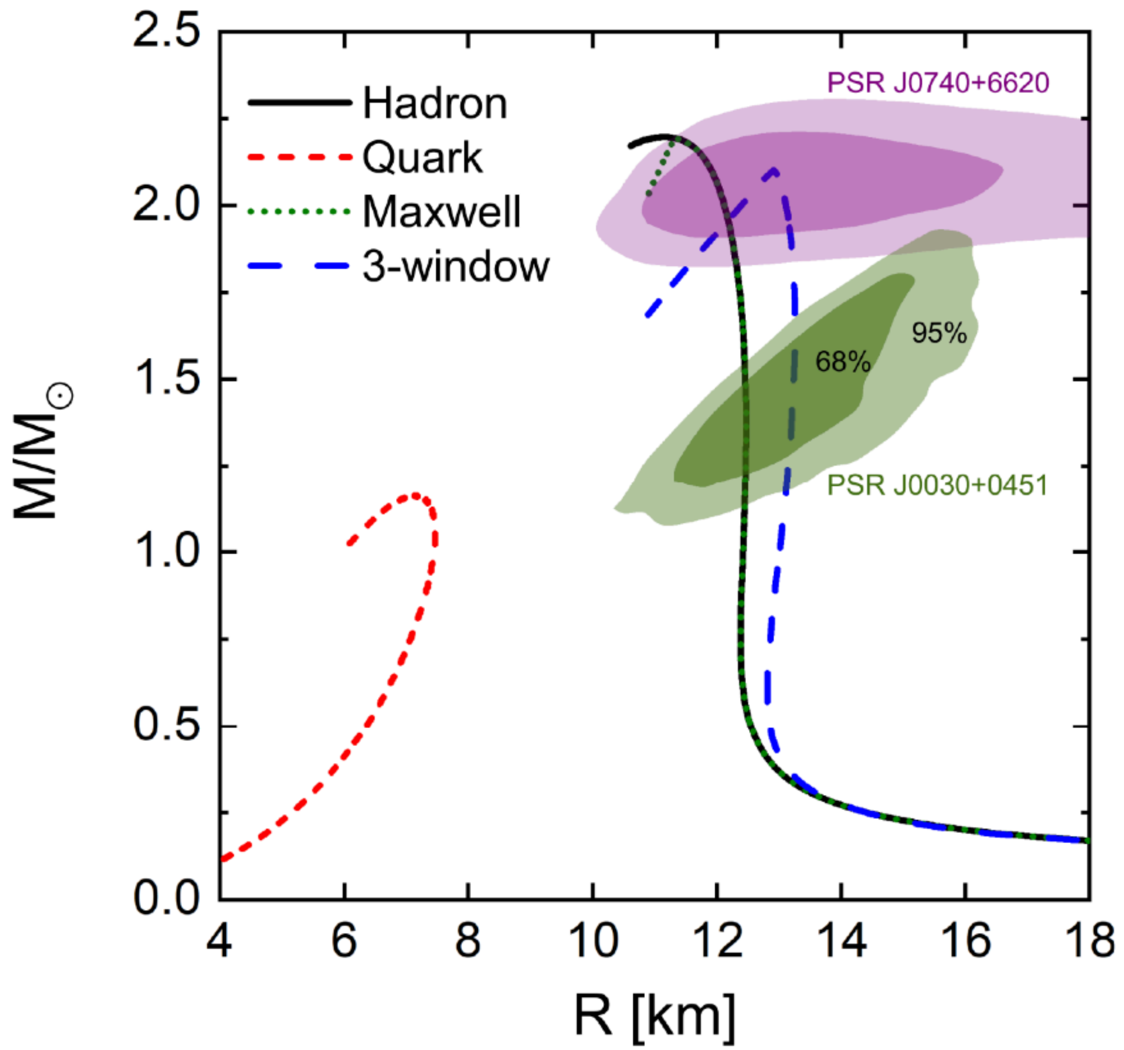
LQCD: $2\pi TD \sim 2$, Ding et. al, PRD, 86, (2012)

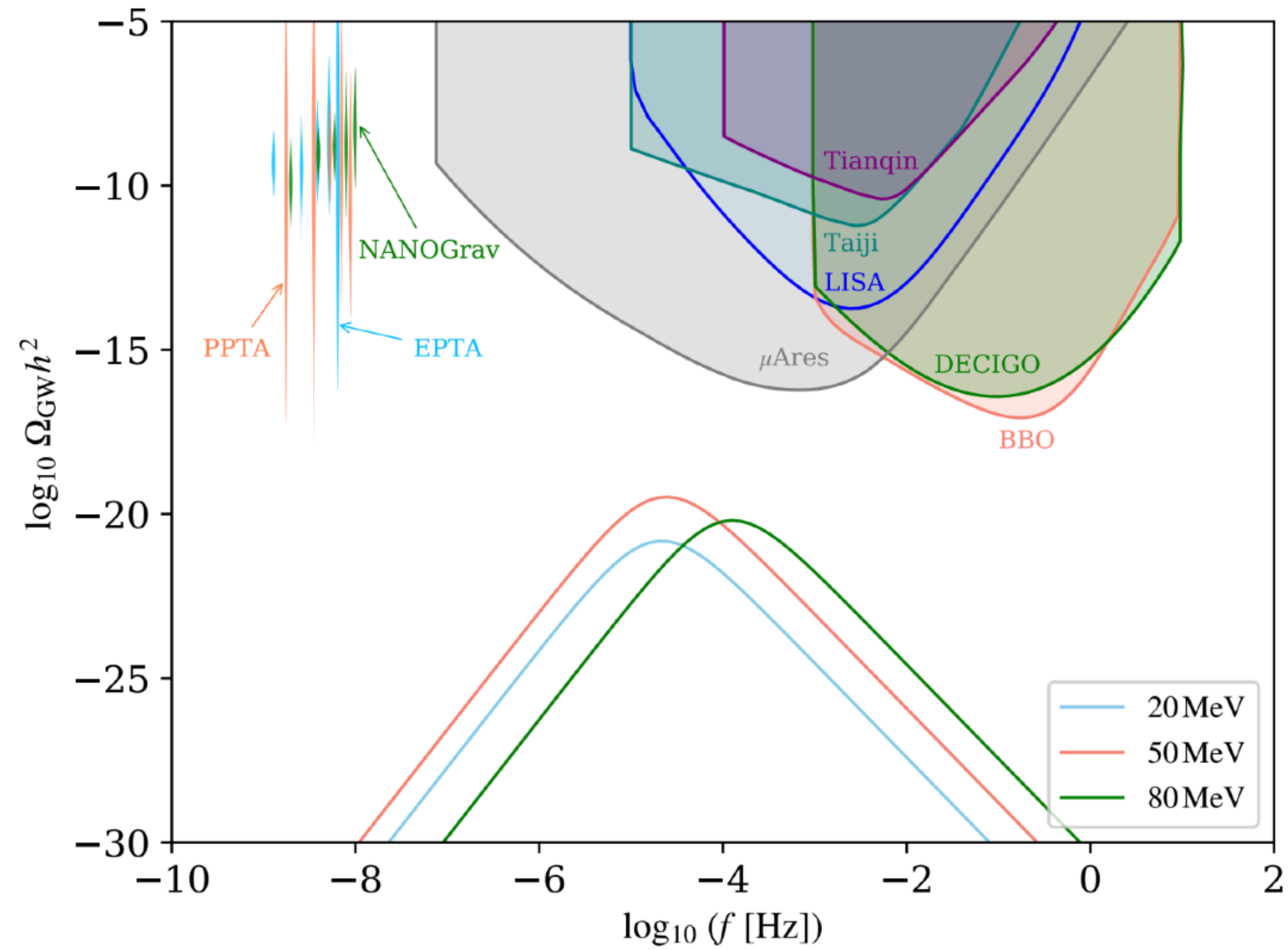
LQCD: $4 < 2\pi TD < 8$, Banerjee et. al., PRD, 85, (2012)

v_2: $4 < 2\pi TD < 6$, PHENIX, PRC, 84, (2011)

See, e.g., PRD84, 014017 (2011); PLB742, 358 (2015); PLB734, 157 (2014)

Many-body: Novel state





✦ Unified description of nuclear and quark matter

✦ Gravitational waves spectra of the first-order QCD phase transition

See, e.g., PRD 107, 103009 (2023), arXiv:2407.03795 (2024)

Summary

重庆大学 CHONGQING UNIVERSITY

✦ **Introduction**: why few-body and many-body problems are important for QCD

✦ **Development**: state-of-the-art understanding for quark, gluon, vertex, kernel

✦ **Few-body**: meson and baryon spectra, wave functions, hadron structures

✦ **Many-body**: phase diagram in the multi-dimensional space, novel states, GW