Green Functions of Nonperturbtive QCD

(Part I)

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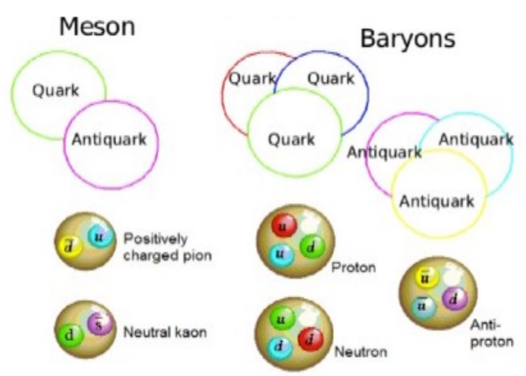
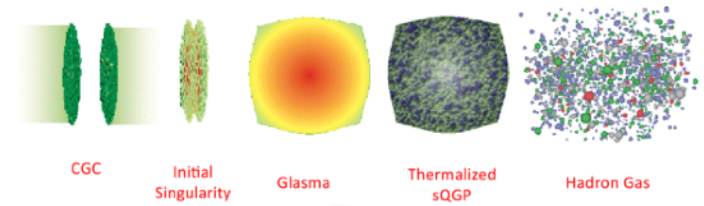
Frontier

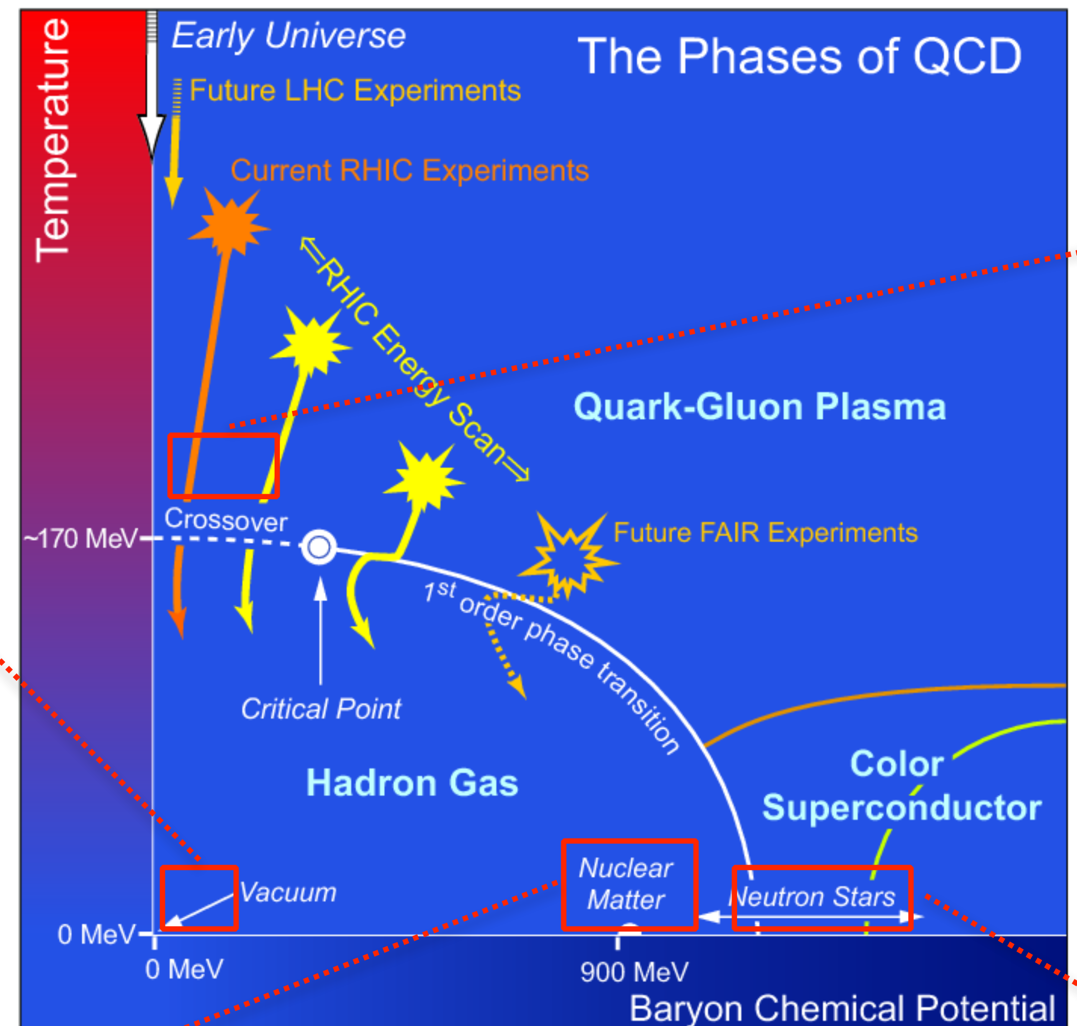
Frontier: QCD topics

Hadron

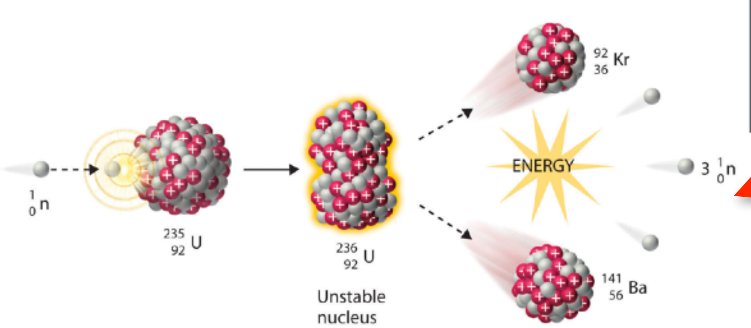


QGP

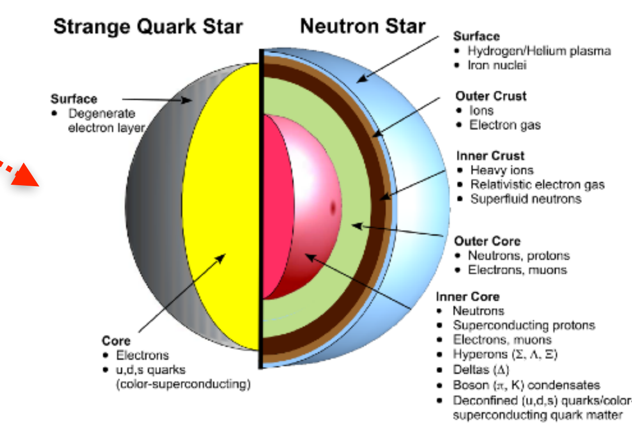




Nucleus



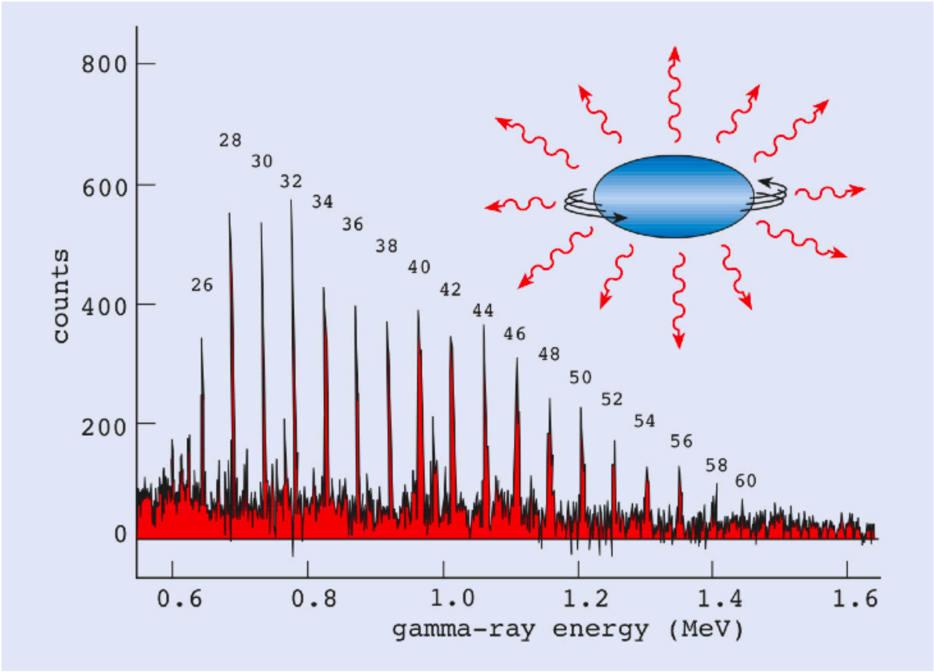
Compact Star



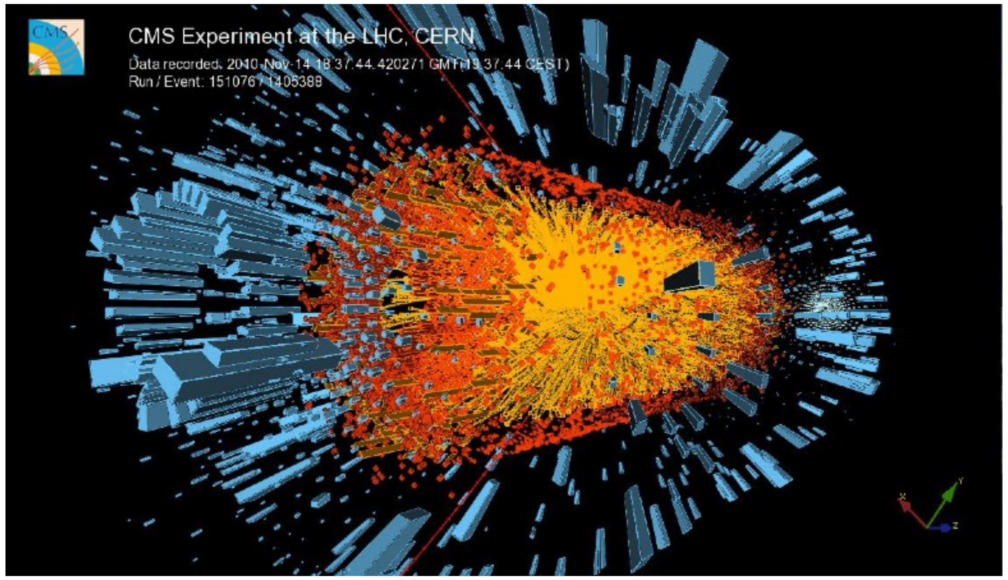
Few-body

Many-body

Frontier: Experiments

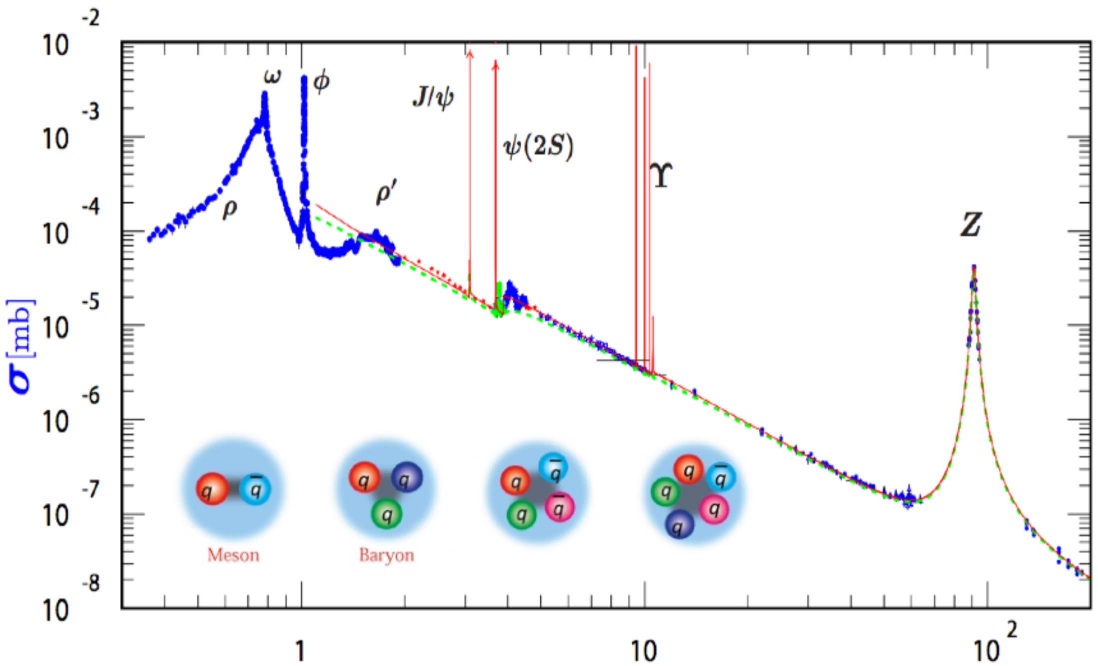


Novel states of nuclei

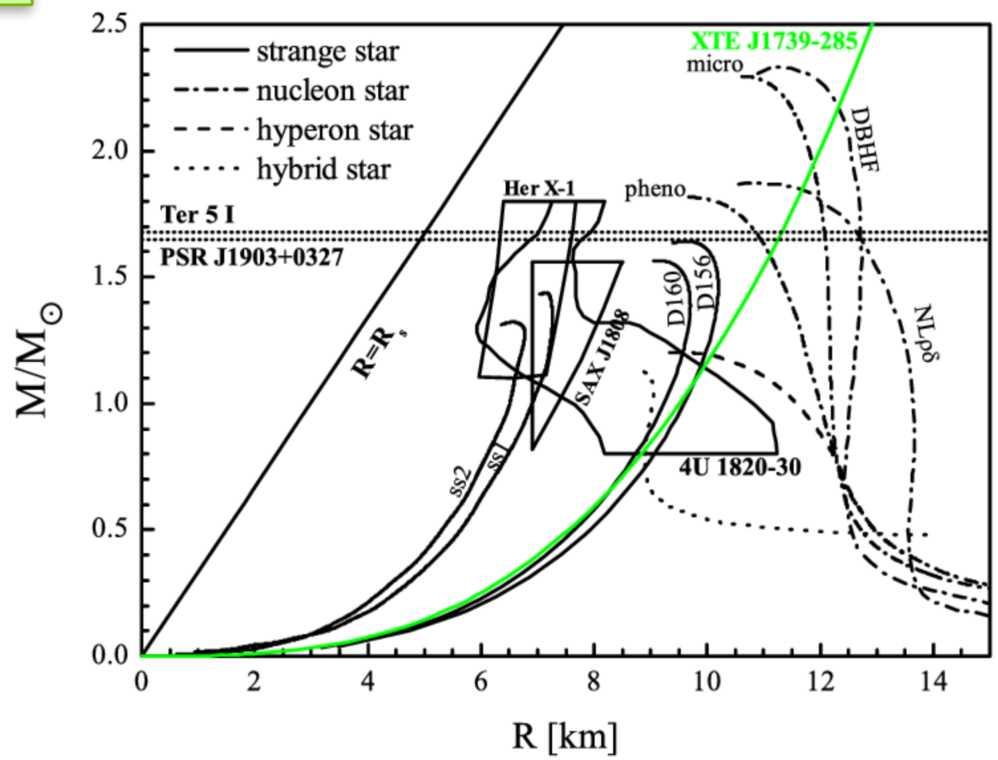


relativistic heavy-ion collision

Solve QCD

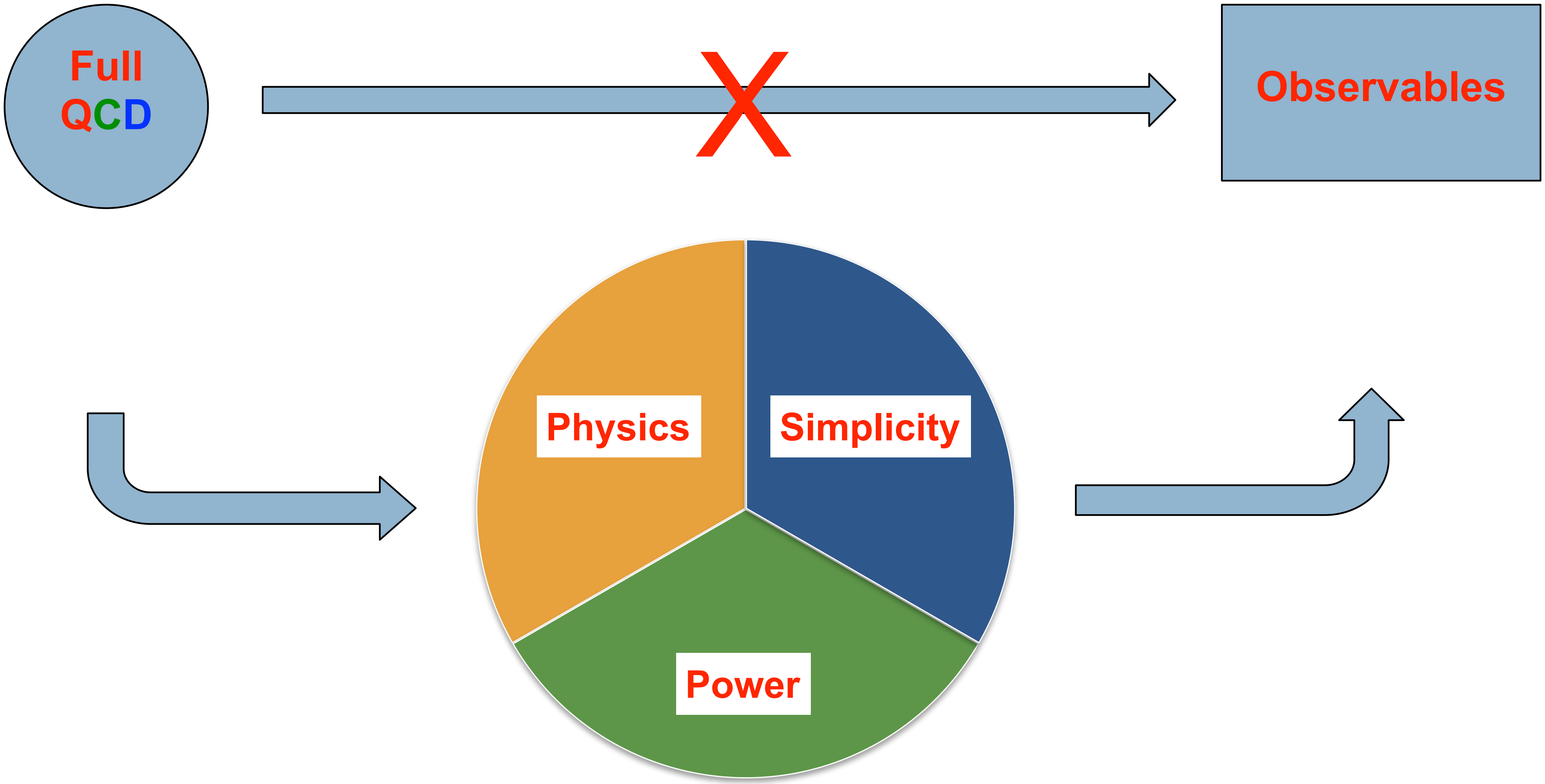


e⁺e⁻ hadronic annihilation



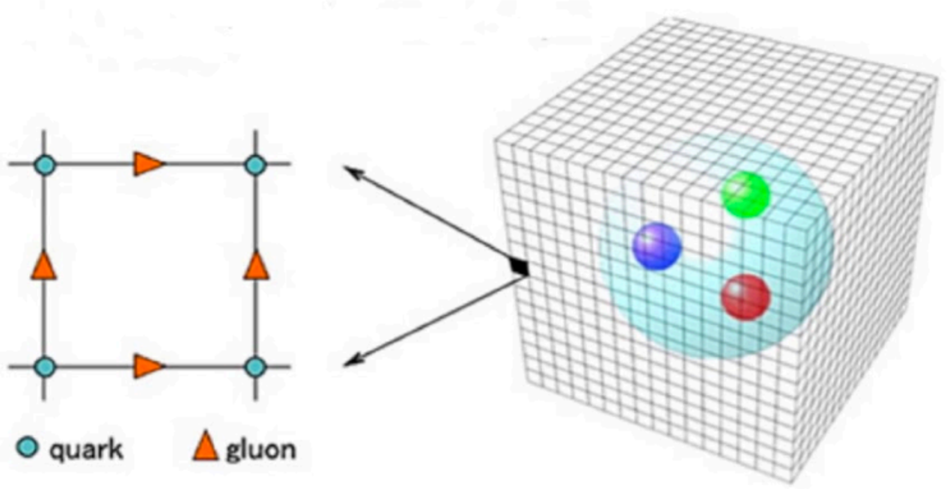
mass-radius relation of compact stars

Frontier: Nonperturbative QCD



Lattice QCD, NJL, Sum rules, AdS/QCD, LFQCD, Effective theories, …, Functional methods

Frontier: Lattice QCD

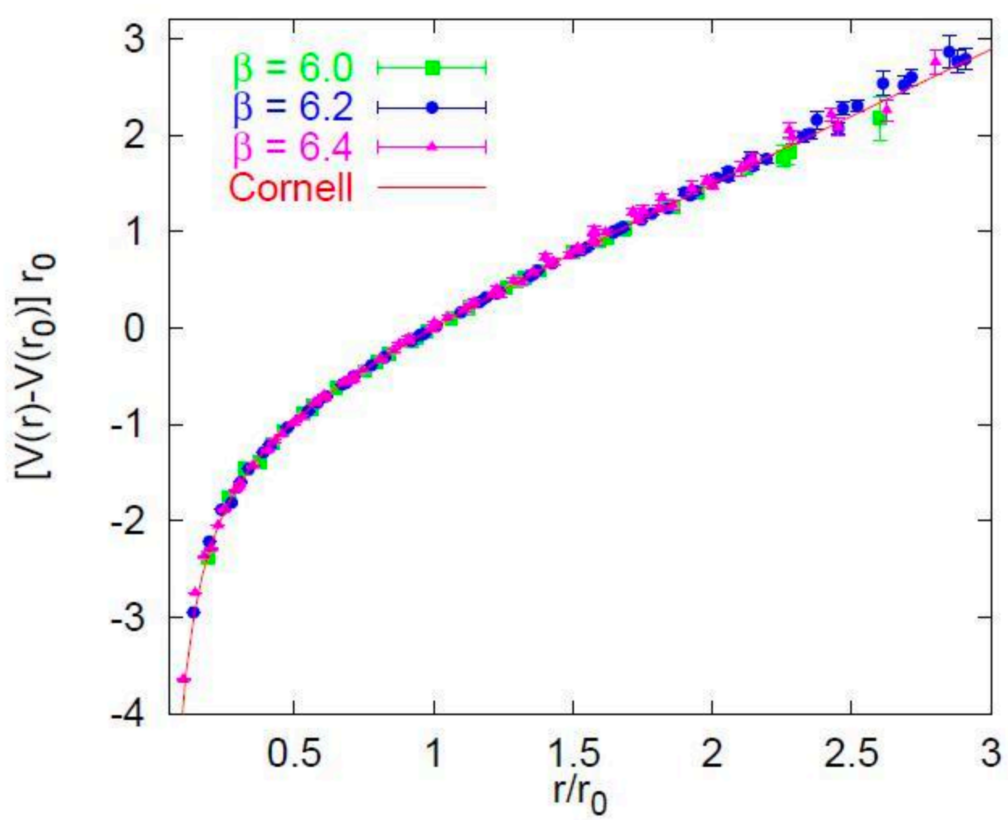


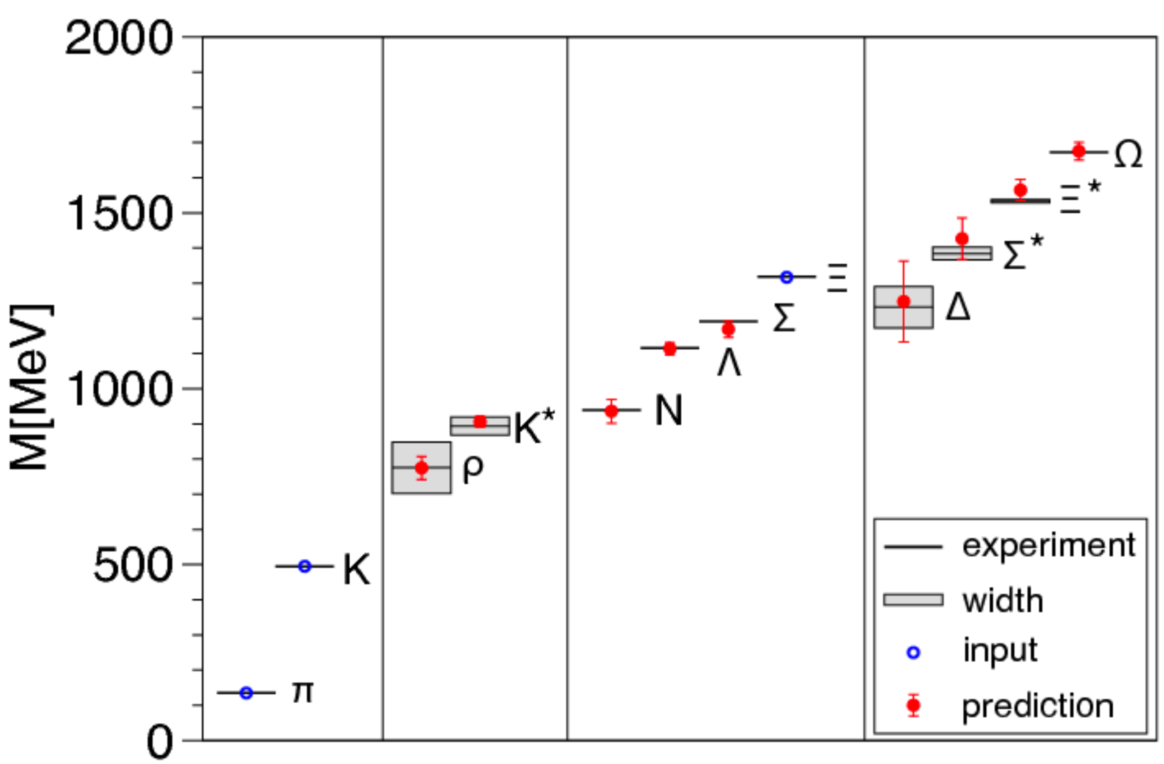
$$S = \int d^4x \left(\frac{1}{4} F^2 - \bar{q} M q \right)$$

$$Z = \int DA \det M \, e^{-S}$$

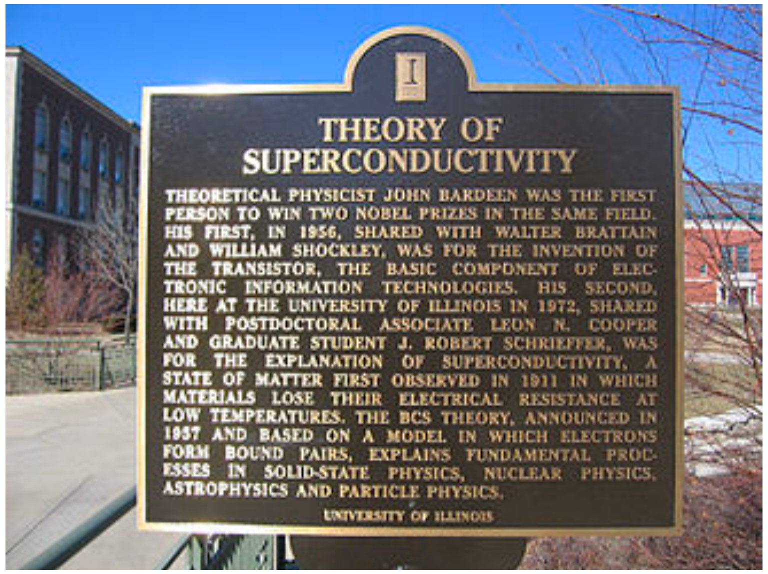
sign problem

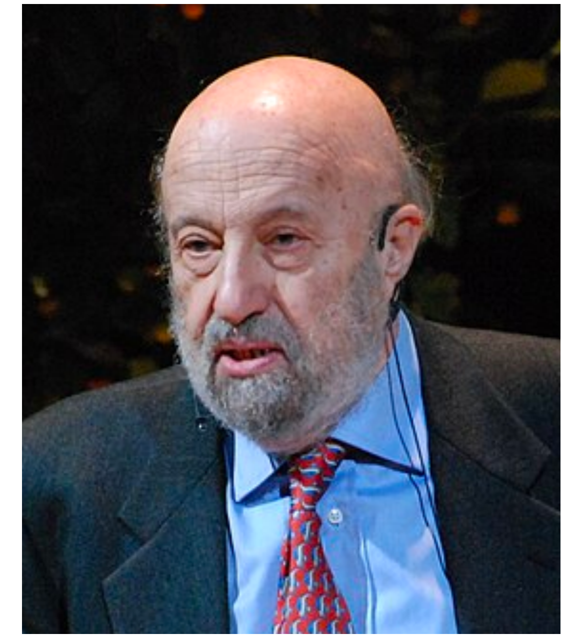
Kenneth Wilson



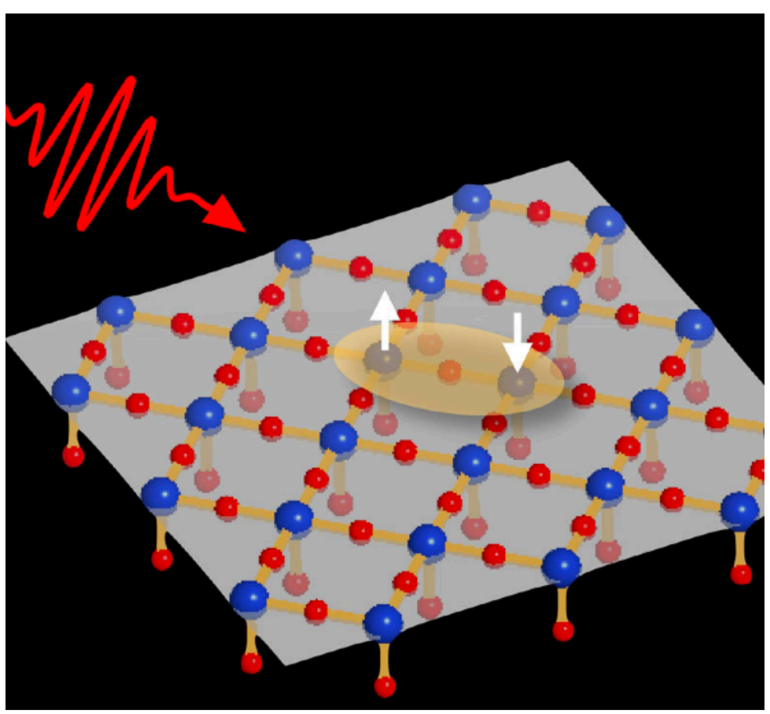


Frontier: Nambu—Jona-Lasinio model





Yoichiro Nambu **Giovanni Jona-Lasinio**



$$\mathscr{L} = i\bar{q}_L\gamma\cdot\partial q_L + i\bar{q}_R\gamma\cdot\partial q_R + \lambda(\bar{q}_L q_R)(\bar{q}_R q_L)$$

NJL: χ-symmetry

$$\Phi(\vec{x}) = \frac{1}{N_c}\mathrm{Tr}\left\{T\exp\left[i\int_0^\beta d\tau A_4\left(\vec{x},\tau\right)\right]\right\}$$

Gauge invariant,
Not center invariant

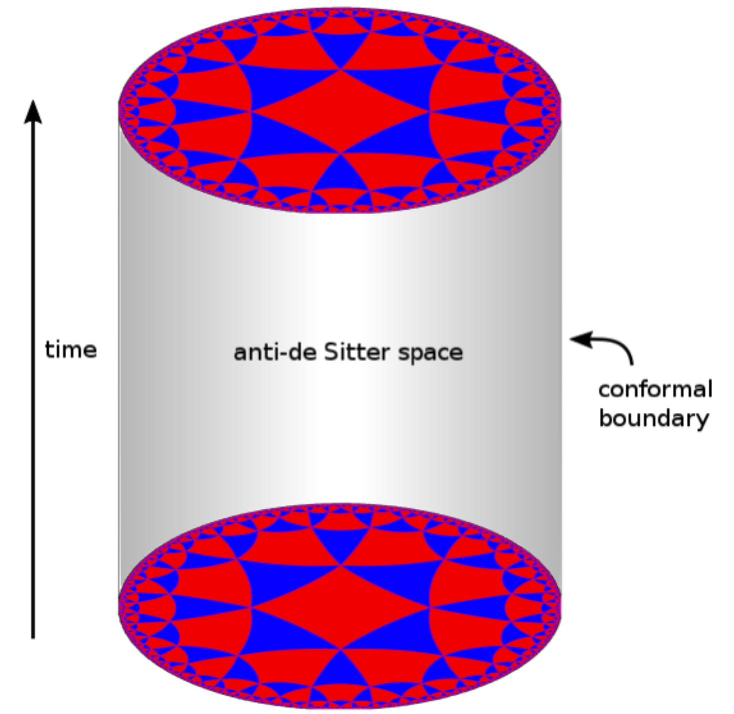
$$|\Phi(\vec{x})| = \exp(-\beta F)$$

PNJL: χ-symmetry & confinement

Frontier: Ads/QCD

Mapping: String theory vs QFT

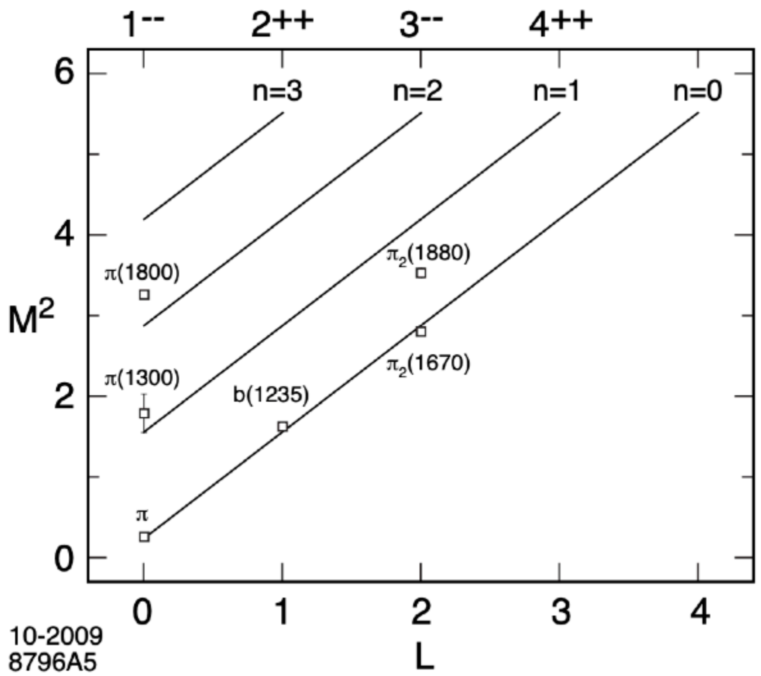
$$\textit{String theory on } AdS_5 \times S^5 \quad \simeq \quad \mathcal{N}=4,\ SU(N) \textit{ gauge theory in 4D}$$



AdS/CFT: strong/weak coupling duality

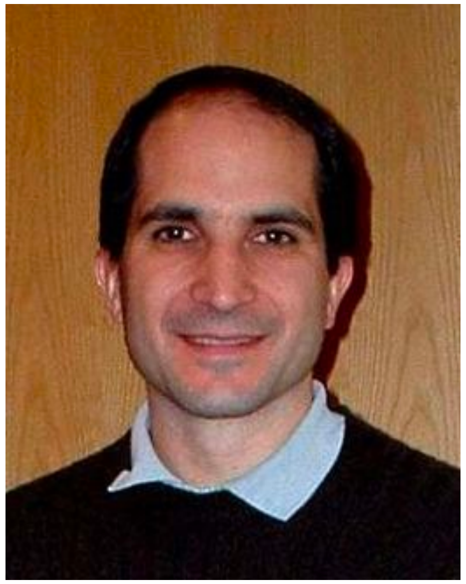
$$\lambda = g_{YM}^2 N \qquad \lambda \sim \left(\frac{\ell_{AdS}}{\ell_{string}}\right)^4 \qquad \frac{\ell_{AdS}^{d-1}}{G_N} \sim \left(\frac{\ell_{AdS}}{\ell_P}\right)^{d-1} \sim N^2$$

AdS/QCD: holographic theory



$$\frac{\eta}{s} = \frac{1}{4\pi}$$

No proof



Juan Maldacena

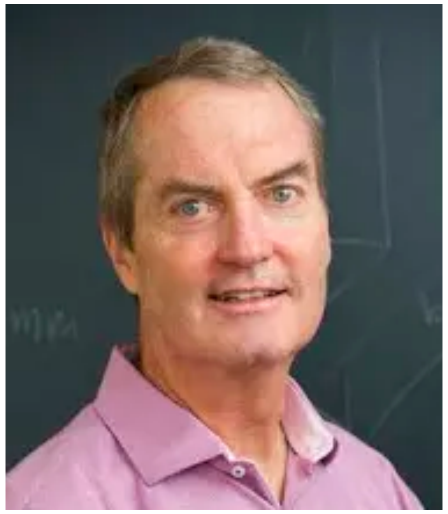
Frontier: Functional methods

Startpoint: QFT partition function **solvable** and **traceable** approximation to QCD

$$Z_{QFT}[\phi, j] = \int D\phi e^{-S[\phi, j]}$$

Differential version:

Functional renormalization equation



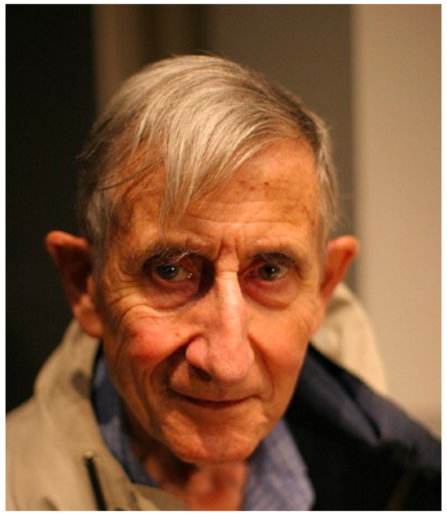
Joseph Polchinski

Christof Wetterich

Integral version:

Dyson-Schwinger equation

NOT limited in HEP



Freeman Dyson

Julian Schwinger

Framework

Framework: Equation of motion

Classical Mechanics	Quantum Field Theory
Generalized coord.	Fields on spacetime
$\frac{\delta S[q]}{\delta q} = 0$	$\left\langle \frac{\delta S[\phi(x)]}{\delta \phi(x)} \right\rangle = 0$
Euler-Lagrange Equation	Dyson-Schwinger Equations

Degrees of freedom

Principle of Least Action

Equations of Motion (EoM)

Framework: Green function

Properties of Green function:

See, e.g., arXiv:hep-ph/0208074, arXiv:0909.0703

- express particle scattering
- encode all observables
- use Feynman diagrams

$$G^{(n)}(x_1, ..., x_n) = \langle\Omega|T[\phi(x_1)...\phi(x_n)]|\Omega\rangle = \frac{\int[\mathcal{D}\phi]e^{-S[\phi]}\phi(x_1)...\phi(x_n)}{\int[\mathcal{D}\phi]e^{-S[\phi]}}$$

Generating function:

- Partition function of QFT

$$Z[J] = \int \mathcal{D}\phi\, e^{-S[\phi]+\int d^4x J(x)\phi(x)}$$

$$G^{(n)}(x_1, ..., x_n) = \frac{1}{Z[0]}\frac{\delta^n Z[J]}{\delta J(x_1)...\delta J(x_n)}\bigg|_{J=0}$$

- Identity of generating function:

$$0 = \int[\mathcal{D}\phi]\frac{\delta}{\delta\phi(x)}\exp\left\{-S[\phi] + \int d^4x J(x)\phi(x)\right\}$$

$$0 = \int[\mathcal{D}\phi]\left\{-\frac{\delta S[\phi]}{\delta\phi(x)} + J(x)\right\}\exp\left\{-S[\phi] + \int d^4x J(x)\phi(x)\right\}$$

- Generating equation of DSE:

$$\left\langle\frac{\delta S[\phi]}{\delta\phi(x)}\right\rangle_J = \frac{1}{Z[J]}\frac{\delta S[\delta/\delta J]}{\delta\phi(x)}Z[J] = J(x)$$

Framework: Symmetry breaking

- **Classical theory:**

$$S[\phi] \xrightarrow{\text{EoM}} O$$

- **Quantum theory:**

$$Z[\phi, j] = \int [D\phi] e^{-S[\phi]+j\phi} \xrightarrow{\text{EoM}} \{O, O', \cdots\}$$

Symmetric	$S[\phi]$	$[D\phi]$	$\{O\}$
Anomalous	$S[\phi]$	$[\cancel{D\phi}]$	$\{O\}$
Spontaneous	$S[\phi]$	$[D\phi]$	$\{O, O', \cdots\}$

Framework: Symmetry breaking

Symmetry	Anomaly	Fate
$SU(3)_C$	no	unbroken
$SU(N)_V \times U(1)_V$	no	unbroken
$U(1)_A$	yes	not a symmetry
$SU(N)_A$	no	spontaneously broken
Scale Invariance	yes	not a symmetry
CP	yes?	not a symmetry?

Framework: DSE of QCD

QCD Lagrangian:

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_i \big(i(\gamma^\mu D_\mu)_{ij} - m\delta_{ij}\big)\psi_j - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a$$

Quark gap equation:

- Functional differential of action:

$$\frac{\delta S}{\delta \bar{\psi}} = (i\not{\partial} - m_0)\psi + gA^a_\mu \gamma^\mu t^a \psi$$

- Generating equation of DSE:

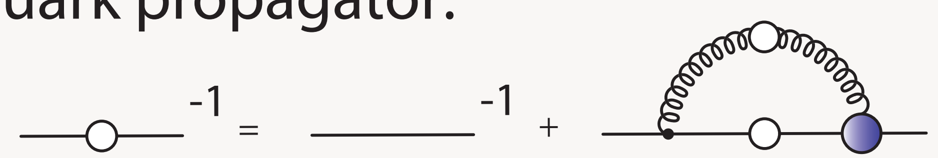
$$\frac{1}{Z}\left\{(i\not{\partial} - m_0)\frac{\delta}{\delta\bar{\eta}} + g\frac{\delta}{\delta J^\mu_a}\gamma^\mu t^a \frac{\delta}{\delta\bar{\eta}}\right\}\cdot Z = \eta(x)$$

- Extraction of two-point function:

$$(i\not{\partial} - m)\frac{\delta^2 W}{\delta\bar{\eta}_x \delta\eta_y}\bigg|_{j=0} - g\gamma^\mu t^a \frac{\delta^3 W}{\delta J^\mu_{ax}\delta\bar{\eta}_x\delta\eta_y}\bigg|_{j=0} = \delta(x-y)$$

$$Z[J] = \sum_{N=0}^{\infty}\frac{(-1)^N}{N!}W^N[J] = e^{-W[J]}$$

Quark propagator:

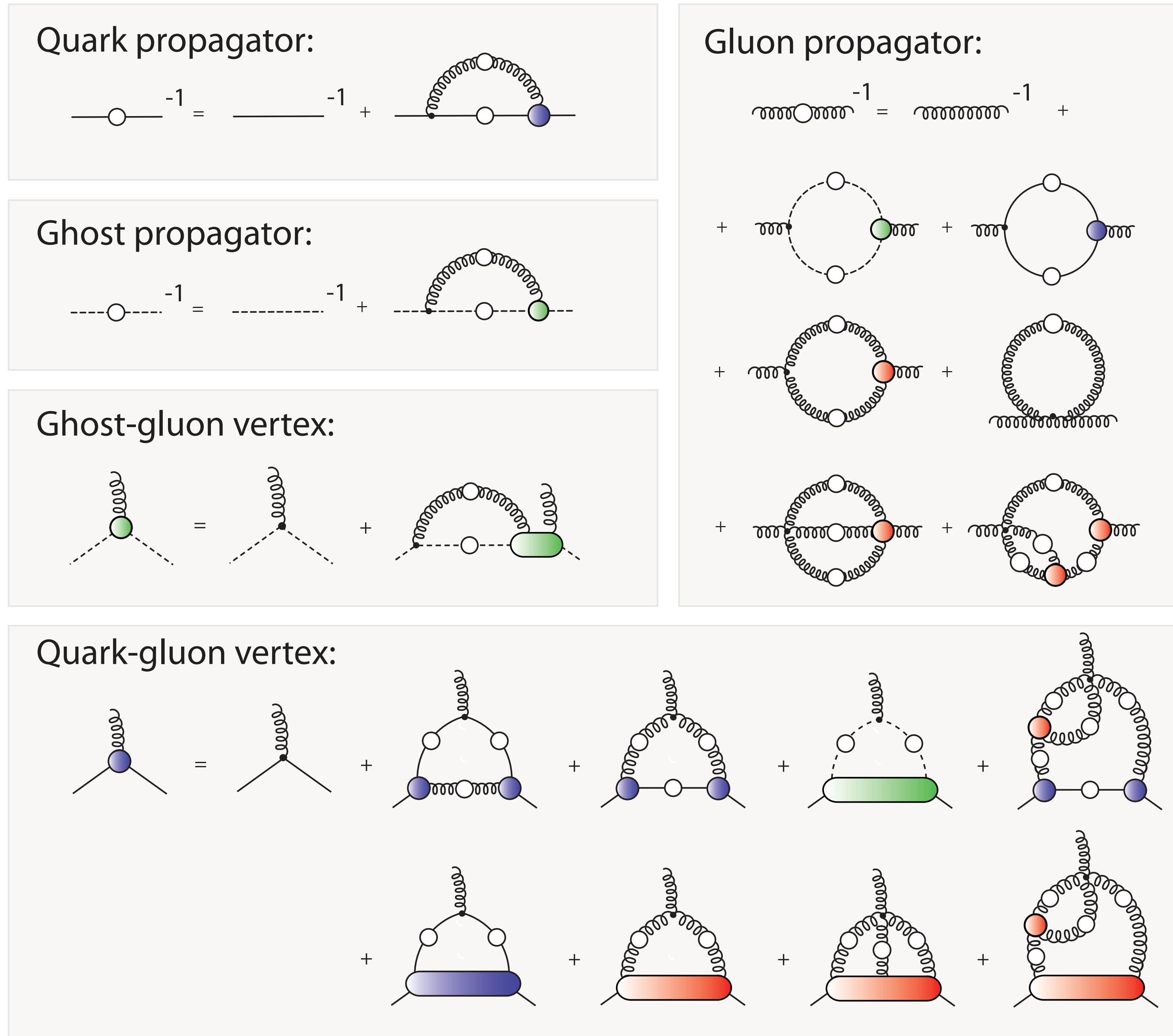


Framework: Disadvantage vs Advantage

Disadvantage:

✦ Most equations are very complicated.

✦ Multi-order Green functions couple together.



Advantage:

✦ Solid connection to QCD

✦ Controllable complexities

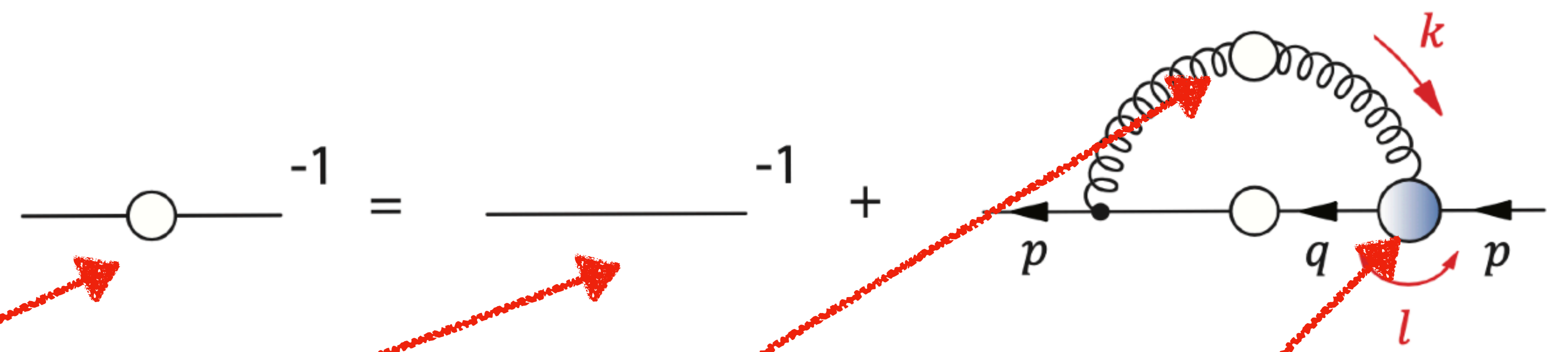
❑ ***Modeling*** ❑ **Truncation**

Formalism

Formalism: One-body equation

Quark gap equation:

- Quark self-energy



$$S^{-1}(p,\zeta) = Z_2(i\not{p} + Z_m m_0) + Z_1 \int_q^{\Lambda} g_0^2 D_{\mu\nu}^{ab}(p-q,\zeta)\frac{\lambda^a}{2}\gamma_\mu S(q,\zeta)\Gamma_\nu^b(p,q,\zeta)$$

Solution structure:

- Decomposition of Lorentz structure

$$S(p,\zeta) = \frac{1}{i\not{p}A(p^2,\zeta^2)+B(p^2,\zeta^2)} = \frac{Z(p^2,\zeta^2)}{i\not{p}+M(p^2)}$$

- Equations of dressing functions

$$A(p^2) = 1 + \frac{1}{4p^2} tr[-i\not{p}\Sigma(p^2)], \quad B(p^2) = \frac{1}{4} tr[\Sigma(p^2)]$$

$$A_x = 1 + \frac{1}{6\pi^3}\int_0^\infty ydy \frac{\Theta_A(x,y;A_x,B_x,A_y,B_y)}{yA_y^2+B_y^2}$$

$$B_x = m + \frac{1}{6\pi^3}\int_0^\infty ydy \frac{\Theta_B(x,y;A_x,B_x,A_y,B_y)}{yA_y^2+B_y^2}$$

Formalism: One-body equation

Non-linear equation:

- Discrete momenta

$$\{p_i^2\} = \{10^{t_i}\}, \quad t_i \in [-\Lambda, \Lambda]$$

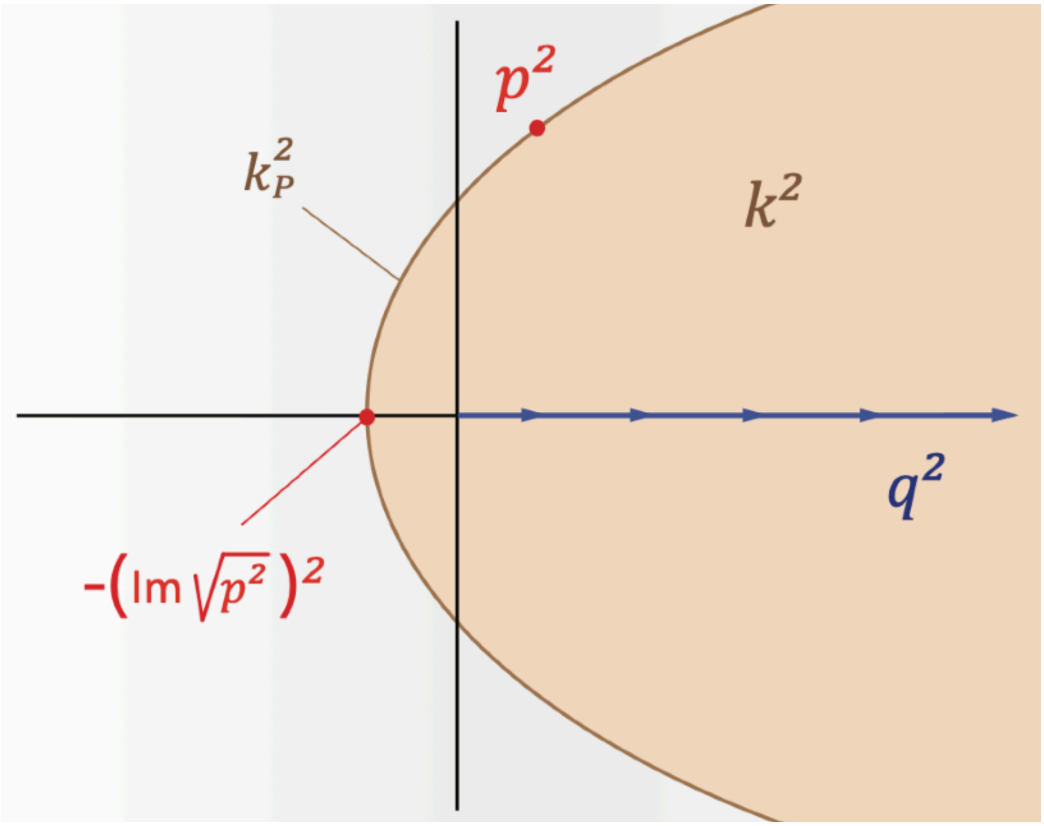
$$\vec{G}(\vec{R}) = \vec{R} - \vec{F}(\vec{R}), \quad H_{ij} = \delta_{ij} - \frac{\partial F_i(\vec{R})}{\partial R_j}$$

Trivial iteration, Newton's iteration, Broyden's iteration, …

- Complex momenta

$$P^2 = -M^2, \quad P_\mu = (\vec{P}, P_4), \quad P_4 = i\sqrt{\vec{P}^{\,2} + M^2}$$

$$q = (\vec{q}, q_4 + iM), \quad \Re(q^2) + M^2 \geqslant \frac{\Im^2(q^2)}{M^2}$$



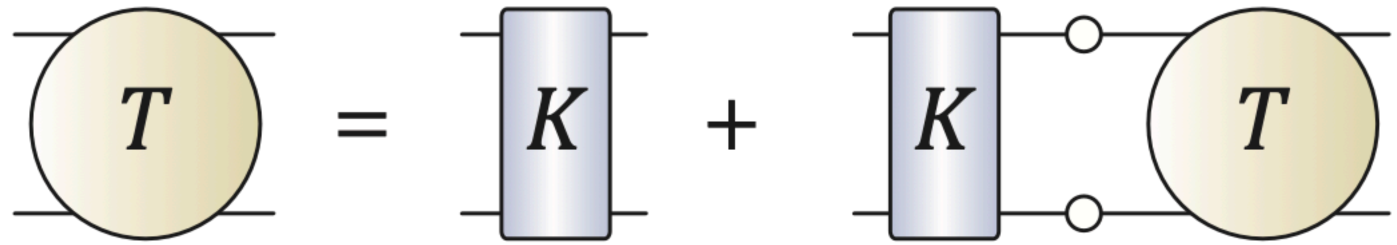
- Complex integral

$$F(q'^2) = \frac{\oint_C \frac{F(q^2)}{q^2 - q'^2} dq^2}{\oint_C \frac{1}{q^2 - q'^2} dq^2}$$

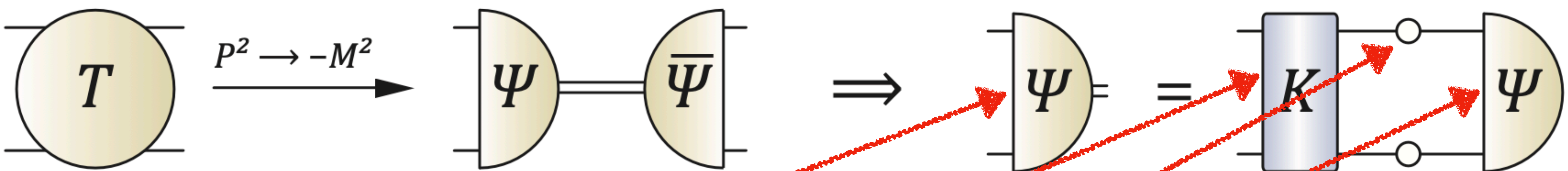
Formalism: Two-body equation

Bethe-Salpeter equation:

- DSE of T-matrix



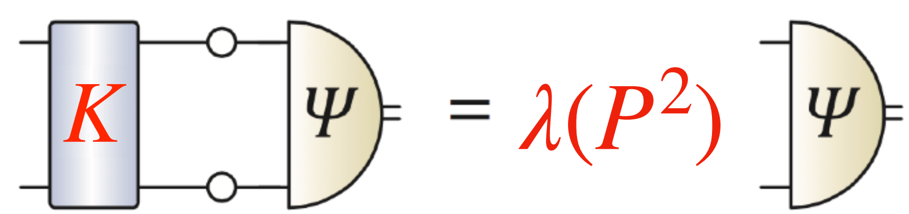
- Bound-state pole



$$\Gamma(k,P)_{\alpha\beta} = \int_q^{\Lambda} K^{\rho\sigma}_{\alpha\beta}(q,k,P)[S(q_+)\Gamma(q,P)S(q_-)]_{\rho\sigma}$$

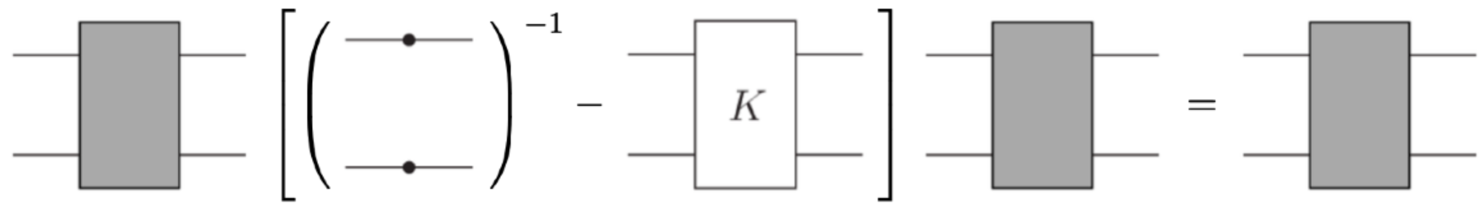
- Schrödinger equation

$$H|\psi\rangle = E|\psi\rangle$$



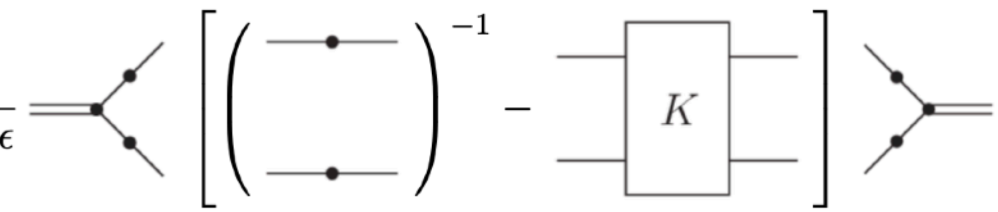
Formalism: Two-body equation

Linear equation:



- BSA normalization

$$\lim_{\text{on-shell}} \frac{i}{p^2 - m^2 + i\epsilon}$$



$$= 1$$

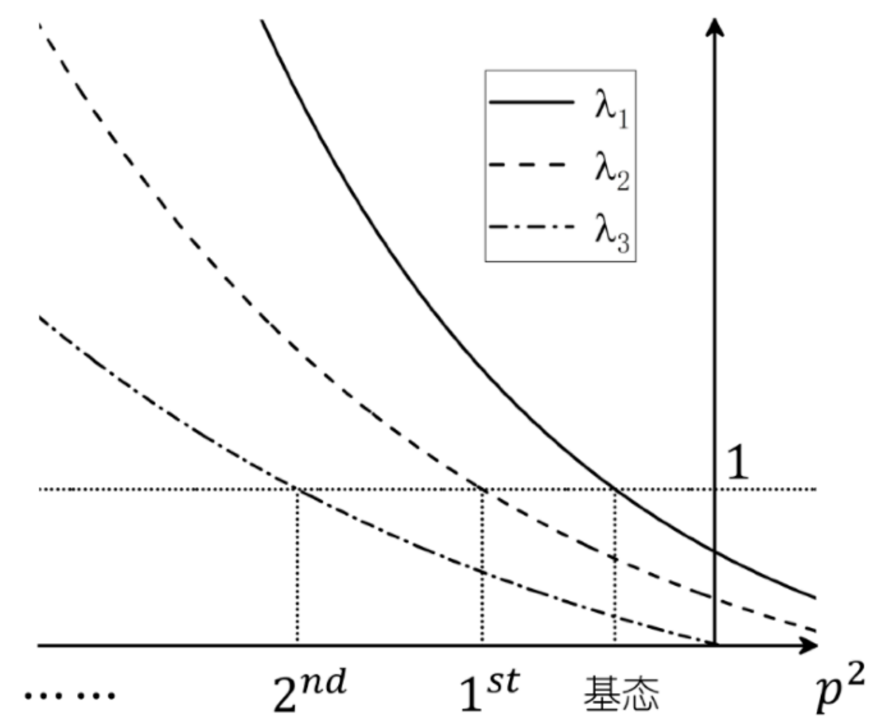
- Lorentz structure

$$\Gamma(P,q) \to \{\mathbb{1}, \slashed{P}, \slashed{q}, \slashed{P}\slashed{q}\} \times f_i(P^2, q^2, P\cdot q)$$

$$\Gamma_\mu(P,q) \to \{\mathbb{1}, \slashed{P}, \slashed{q}, \slashed{P}\slashed{q}\} \times \{\gamma_\mu, P_\mu, q_\mu\} \times f_i(P^2, q^2, P\cdot q)$$

$J=0$	$J=1$	
$\tau_1 = \mathbb{1}$	$\tau_1^\mu = \gamma^\mu$	$\tau_5^\mu = z\,(\gamma^\mu \hat{\slashed{q}}_T - \hat{q}^\mu)$
$\tau_2 = \hat{\slashed{P}}$	$\tau_2^\mu = \gamma^\mu \hat{\slashed{P}}$	$\tau_6^\mu = i\,\hat{q}^\mu \hat{\slashed{P}} - \frac{i}{2}\,\gamma^\mu [\hat{\slashed{q}}, \hat{\slashed{P}}]$
$\tau_3 = z\,\hat{\slashed{q}}_T$	$\tau_3^\mu = i\,\hat{q}^\mu$	$\tau_7^\mu = \hat{q}^\mu \hat{\slashed{q}}_T - \frac{1}{3}\,\hat{q}_T^2 \gamma^\mu$
$\tau_4 = i\,[\hat{\slashed{q}}, \hat{\slashed{P}}]$	$\tau_4^\mu = z\,\hat{q}^\mu \hat{\slashed{P}}$	$\tau_8^\mu = \frac{1}{3}\,\hat{q}_T^2 \gamma^\mu \hat{\slashed{P}} - \frac{1}{2}\,\hat{q}^\mu [\hat{\slashed{q}}, \hat{\slashed{P}}]$

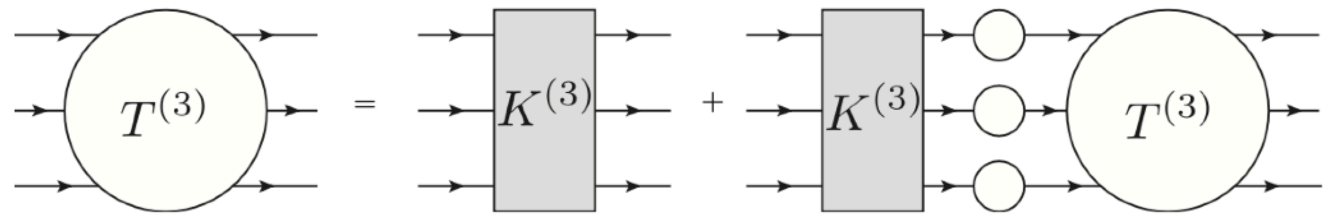
$$\lambda(P^2)\vec{F}_X(P^2) = \boldsymbol{K}_X(P^2)\vec{F}_X(P^2)$$



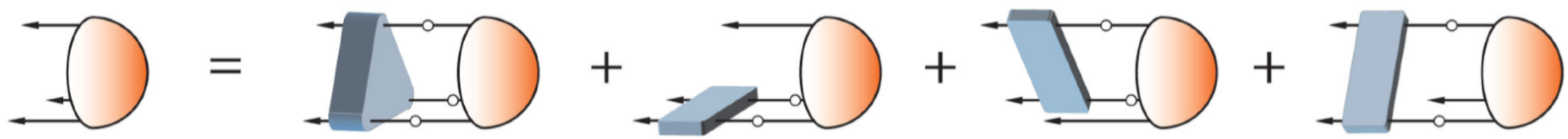
Formalism: Three-body equation

Faddeev equation:

- DSE of T-matrix



- Scatter kernel



Amplitude structure:

$$\Psi = \Psi_{\text{Color}} \otimes \Psi_{\text{Flavor}} \otimes \Psi_{\text{Dirac}}$$

Dirac (J): 4 x 4 x 4 x 4   Flavor (I): 2 x 2 x 2 x 2   Color (0): 3 x 3 x 3

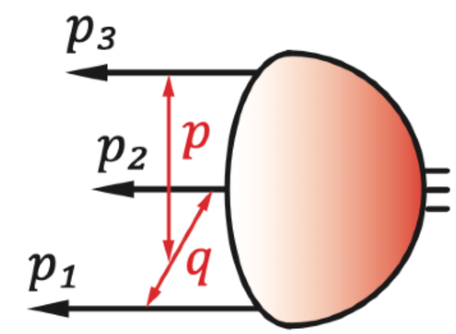
- Flavor structure

$2 \otimes 2 \otimes 2$	$T \backslash T_z$	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$
4	$\frac{3}{2}$	uuu	$\frac{1}{\sqrt{3}}(uud+udu+duu)$	$\frac{1}{\sqrt{3}}(udd+dud+ddu)$	ddd
2	$\frac{1}{2}$ (F_S)		$-\frac{1}{\sqrt{6}}(udu+duu-2uud)$	$\frac{1}{\sqrt{6}}(udd+dud-2ddu)$	
2	$\frac{1}{2}$ (F_A)		$\frac{1}{\sqrt{2}}(udu-duu)$	$\frac{1}{\sqrt{2}}(udd-dud)$	

Formalism: Three-body equation

- Dirac structure

Pauli-Lubanski operator



$$W_\mu = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}P^\nu J^{\alpha\beta}$$

$$J = S + L, \quad L = L_p + L_q$$

s	l	#
$3/2$	2	4
$3/2$	2	4
$3/2$	2	4
$3/2$	2	4

s	l	#
$3/2$	2	4
$3/2$	1	4
$3/2$	1	4
$3/2$	1	4

s	l	#
$1/2$	0	8
$1/2$	1	8
$1/2$	1	8
$1/2$	1	8

J=1/2: 64

J=3/2: 128

- Momentum structure

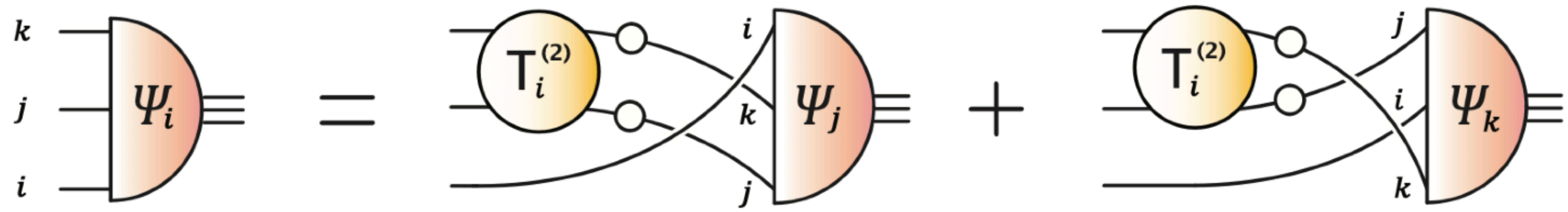
$$(P, q, p) \to (q^2, p^2, P\cdot q, P\cdot p, p\cdot q)$$

rest frame: 5-dim, 100+

Formalism: Three-body equation

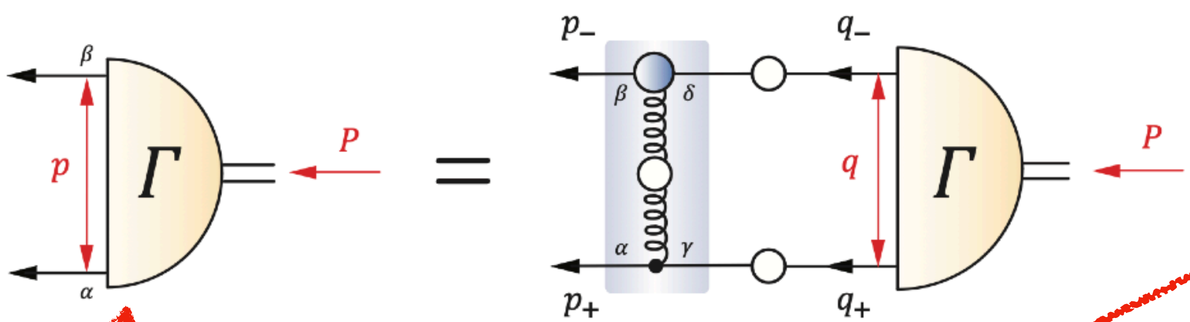
Diquark picture:

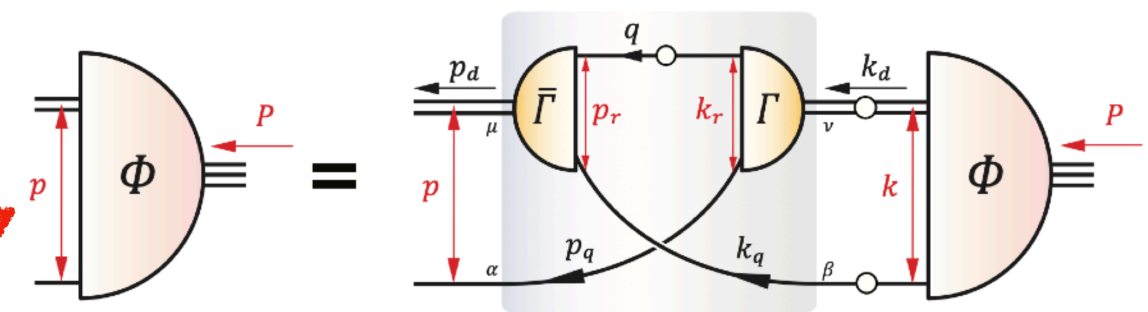
- Two-body matrix



- Diquark correlation

2 x two-body equation





$$\Gamma(k,P)_{\alpha\beta} = \int_q^{\Lambda} K^{\rho\sigma}_{\alpha\beta}(q,k;P)[S(q_+)\Gamma(q,P)S^T(q_-)]_{\rho\sigma}$$

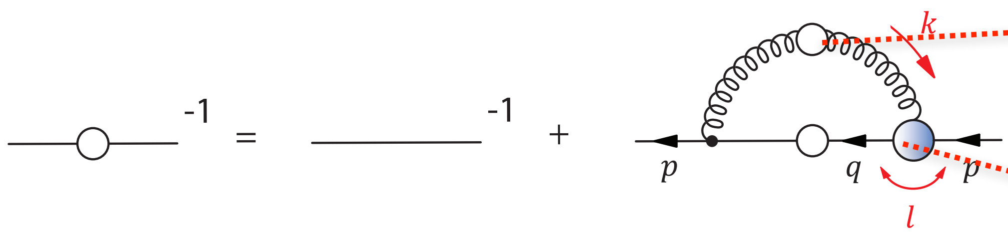
SC - PS **AV - VC**

$$\Phi^a_{\alpha\beta}(p,P) = \int_k^{\Lambda} \{\Gamma^b(k_r,k_d)S^T(q)\bar{\Gamma}^a(p_r,-p_d)S(k_q)\Phi^c(k,P)\}_{\alpha\beta} D^{bc}(k_d)$$

$$\sim \frac{1}{M^2}$$

Formalism: Summary of equations

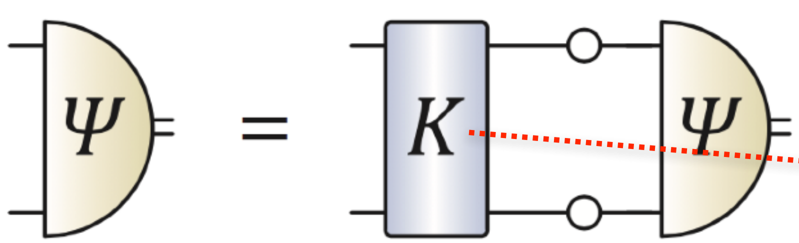
- **One-body gap equation**



Gluon propagator

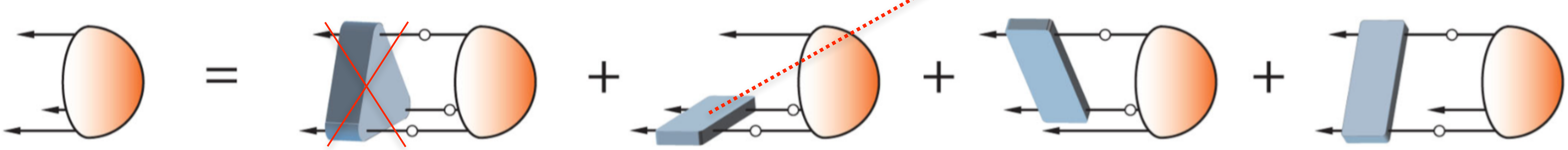
Quark-gluon vertex

- **Two-body Bethe-Salpeter equation**



Scattering Kernel

- **Three-body Faddeev equation**



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Trial

Trial: Gluon model

- Tensor structure

$$D_{\mu\nu}^{ab}(k) = \frac{\delta^{ab}}{k^2}\left[\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\mathcal{D}(k^2) + \xi\frac{k_\mu k_\nu}{k^2}\right]$$

$$g^2 D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\mathcal{G}(k^2)$$

- Simple model

$$\mathcal{G}(k^2) = \frac{1}{M_G^2}\theta(\Lambda^2 - k^2)$$

contact

$$\mathcal{G}(k^2) = 4\pi^2\eta^2\delta^4(k)$$

constant

- Realistic model

$$\mathcal{G}(k^2) = \mathcal{G}_{\rm IR}(k^2) + \frac{4\pi}{k^2}\alpha_{\rm pQCD}(k^2)$$

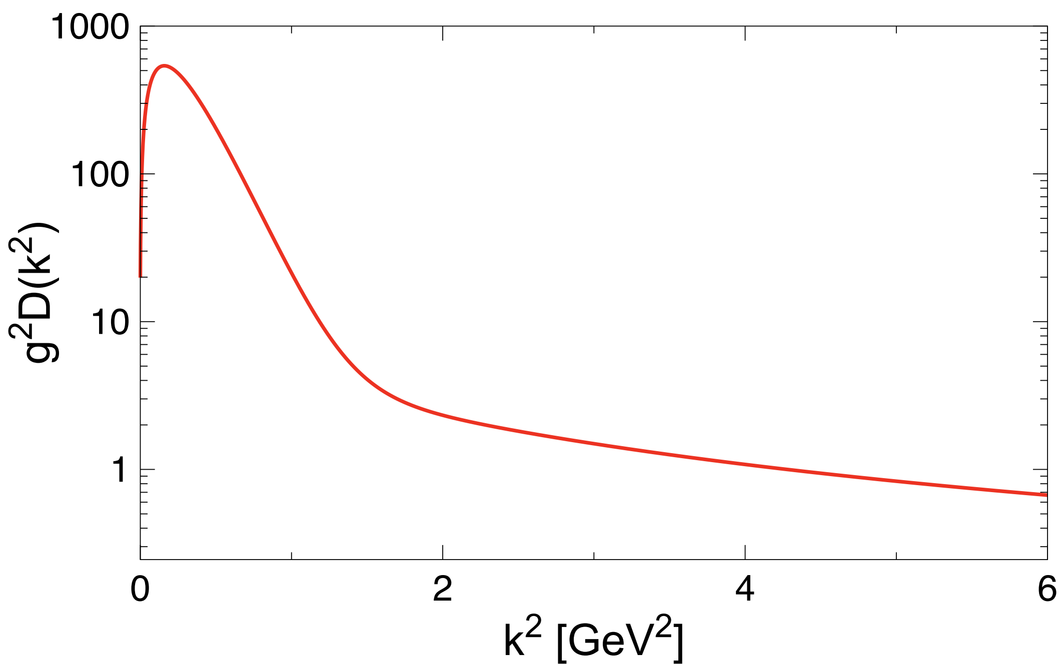
$$\alpha_{\rm pQCD}(k^2) = \frac{2\pi\gamma_m(1 - e^{-k^2/4m_t^2})}{\ln[\tau + (1 + k^2/\Lambda_{\rm QCD}^2)^2]}$$

Linear

$$\mathcal{G}_{\rm IR}(k^2) = 4\pi^2\frac{\chi^2}{k^4 + \Delta}$$

Exponential

$$\mathcal{G}_{\rm IR}(k^2) = \frac{4\pi^2}{\omega^6}Dk^2 e^{-k^2/\omega^2}$$



Trial: Vertex model

- Dirac structure

$$\Gamma_\mu(k,Q) = \{\gamma_\mu, k_\mu, Q_\mu\} \times \{\mathbf{1}, \not{k}, \not{Q}, \not{k}\not{Q}\}$$

- Simple model

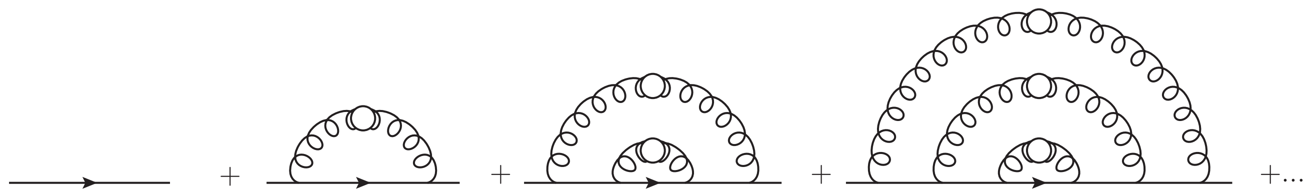
$$\Gamma_\mu^{\text{Bare}}(k,Q) = \gamma_\mu$$

- Fredholm equation

$$A(p^2,\zeta^2) = Z_2 + \frac{4}{3}\int_q^\Lambda \mathcal{G}(k^2)\frac{A(q^2,\zeta^2)}{q^2A^2(q^2,\zeta^2)+B^2(q^2,\zeta^2)}\left[\frac{p\cdot q}{p^2} + \frac{2(k\cdot p)(k\cdot q)}{p^2k^2}\right]$$
$$B(p^2,\zeta^2) = Z_4 m_0 + \frac{4}{3}\int_q^\Lambda \mathcal{G}(k^2)\frac{3B(q^2,\zeta^2)}{q^2A^2(q^2,\zeta^2)+B^2(q^2,\zeta^2)}$$

- Rainbow approximation

$$g^2 D_{\mu\nu}(k)\Gamma_\nu(k,Q) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\gamma_\nu \mathcal{G}(k^2)$$



Trial: Quark solution

- Using MOM renormalization scheme:

$$S^{-1}(\zeta) = i\not{\zeta} + m_f$$

$$\Sigma(p) = i\not{p}[A'(p^2) - 1] + B'(p^2)$$

- Subtraction scheme (I):

$$S^{-1}(p) = Z_2(i\not{p} + Z_m m_0) + \Sigma(p)$$

$$S^{-1}(\zeta) = i\not{\zeta}[Z_2 + A'(\zeta^2) - 1] + Z_2 Z_m m_0 + B'(\zeta^2)$$

$$Z_2 = 2 - A'(\zeta^2), \qquad Z_2 Z_m m_0 = m_f - B'(\zeta^2)$$

- Subtraction scheme (II):

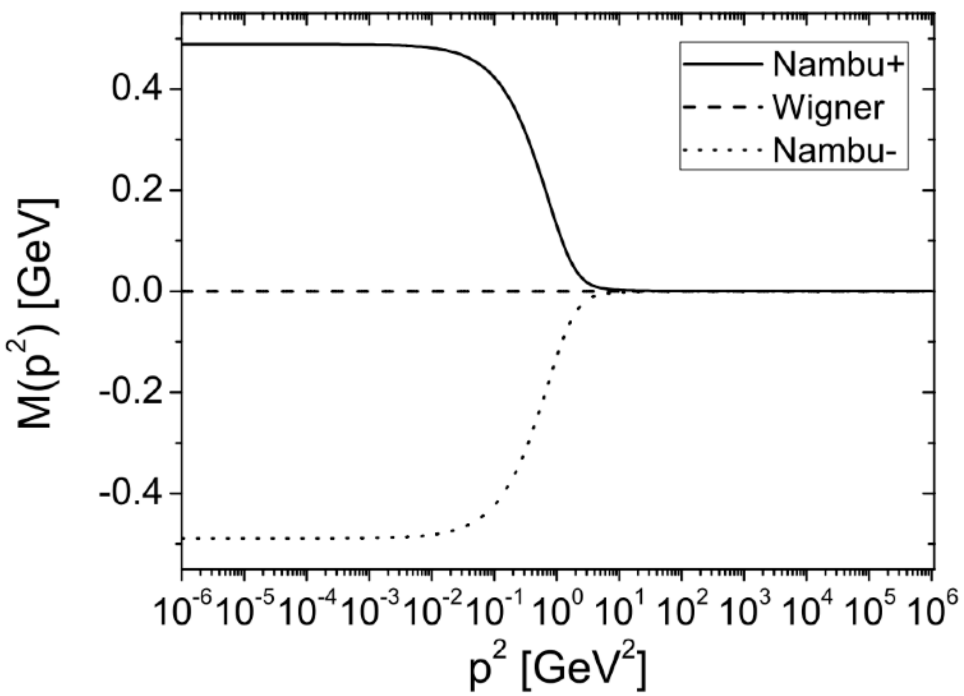
$$S^{-1}(p) = Z_2(i\not{p} + Z_m m_0) + Z_2\Sigma(p)$$

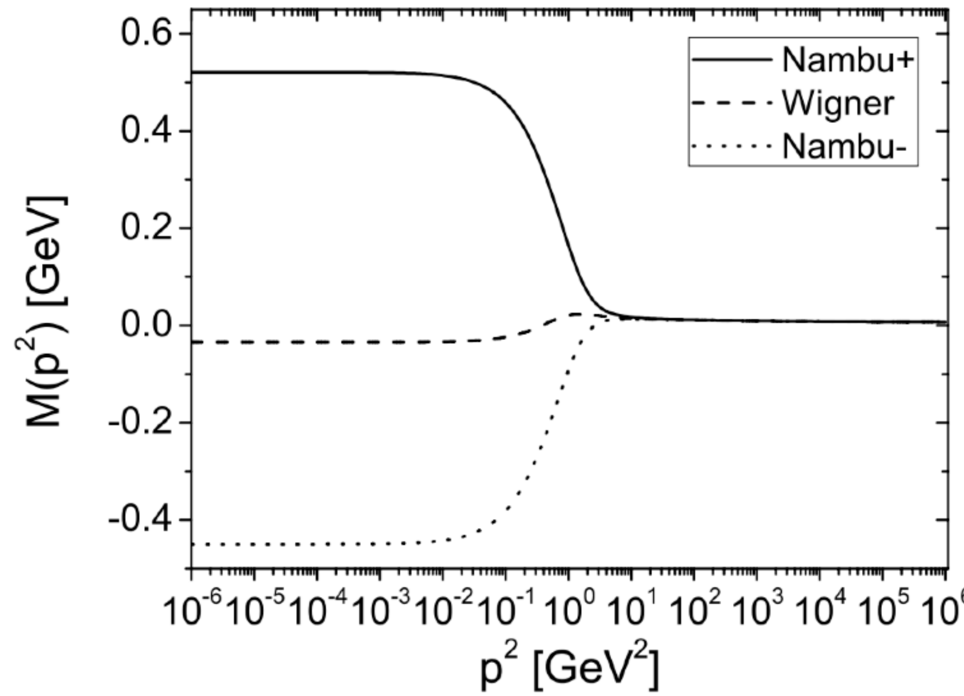
$$S^{-1}(\zeta) = i\not{\zeta}[Z_2 A'(\zeta^2)] + Z_2 Z_m m_0 + Z_2 B'(\zeta^2)$$

$$Z_2 = \frac{1}{A'(\zeta^2)}, \quad Z_2 Z_m m_0 = m_f - \frac{B'(\zeta^2)}{A'(\zeta^2)}$$

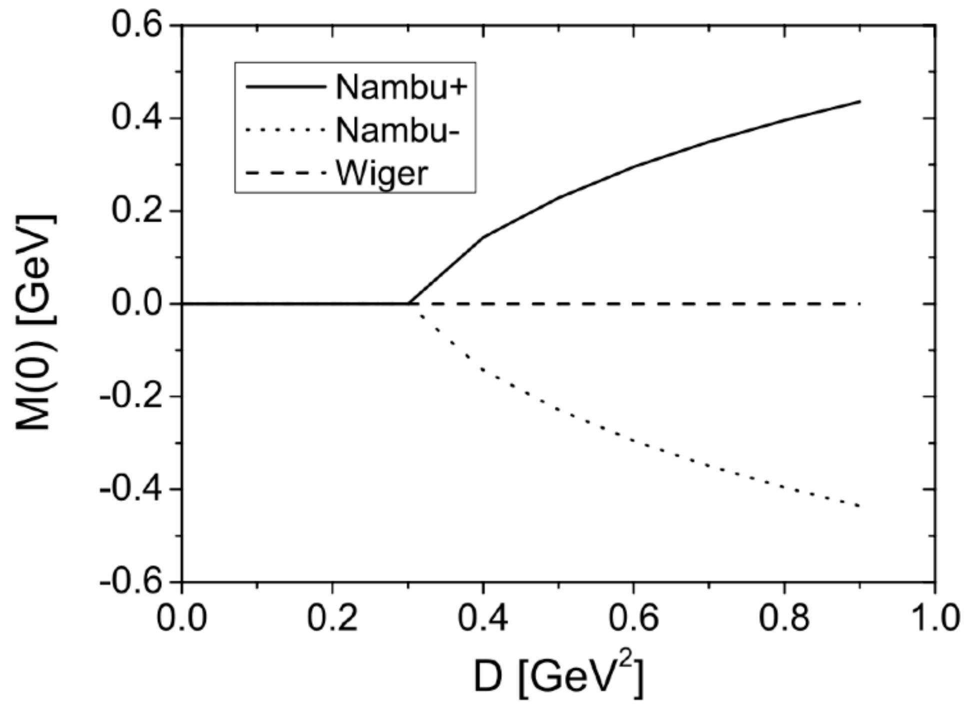
Trial: Quark solution

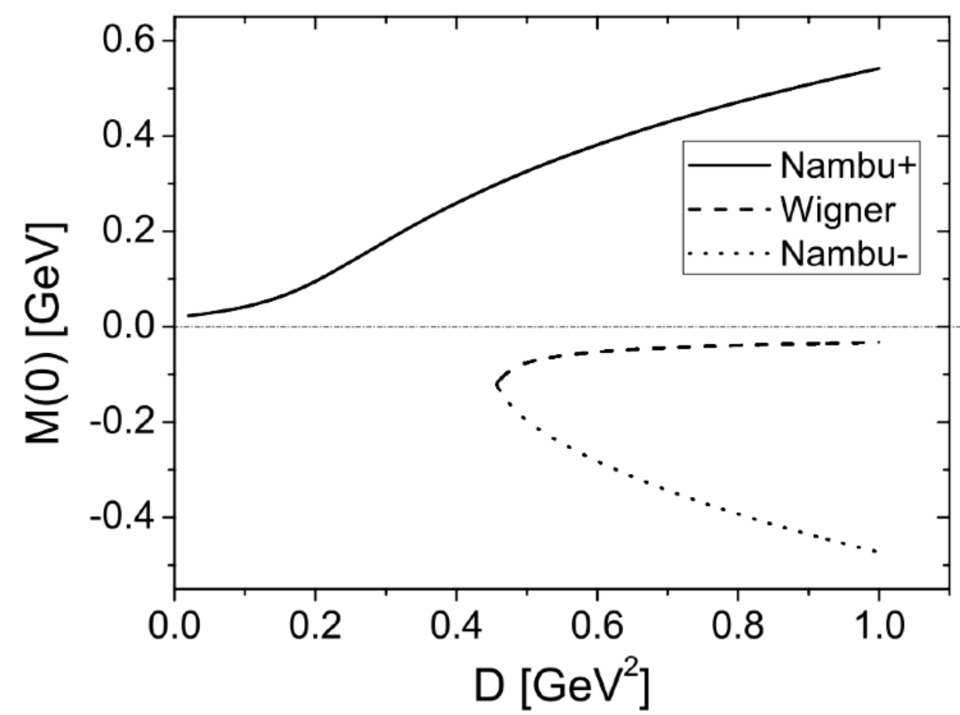
- Mass function





- Critical coupling





Trial: Quark solution

- Chiral condensate

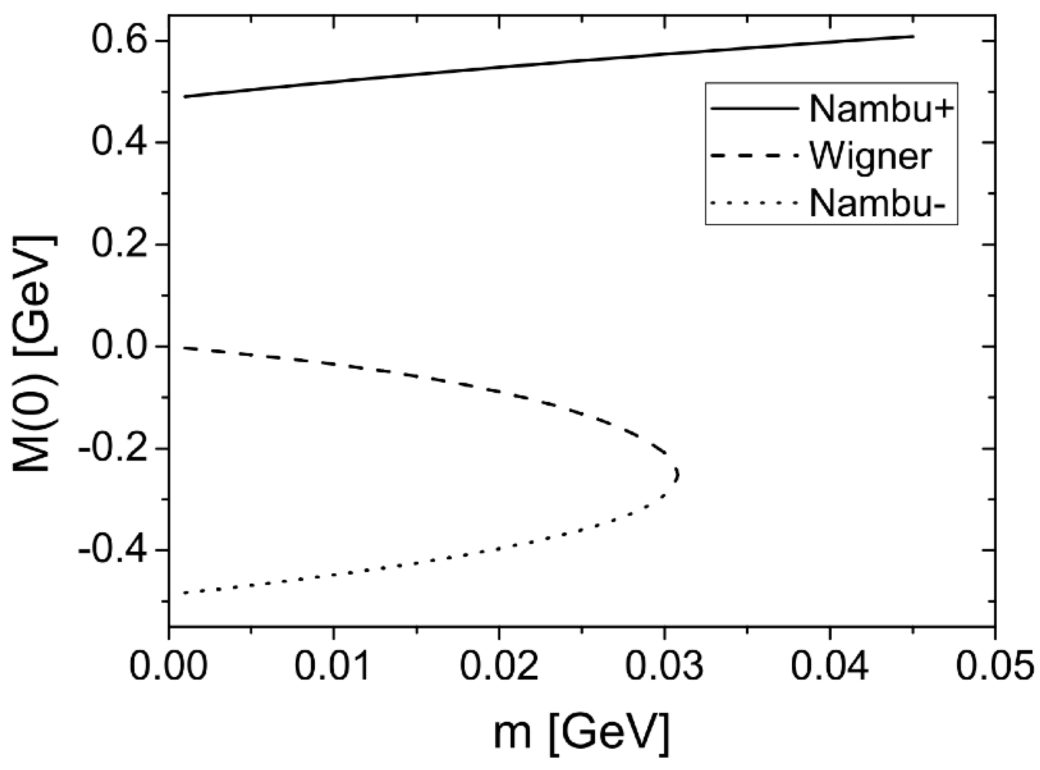
$$-\langle\bar{q}q\rangle^0 = tr\langle q\bar{q}\rangle = \lim_{\Lambda\to\infty}\int_q^\Lambda trS(q) = N_c \lim_{\Lambda\to\infty}\int_q^\Lambda \frac{Z(q^2)M(q^2)}{q^2+M^2(q^2)}$$

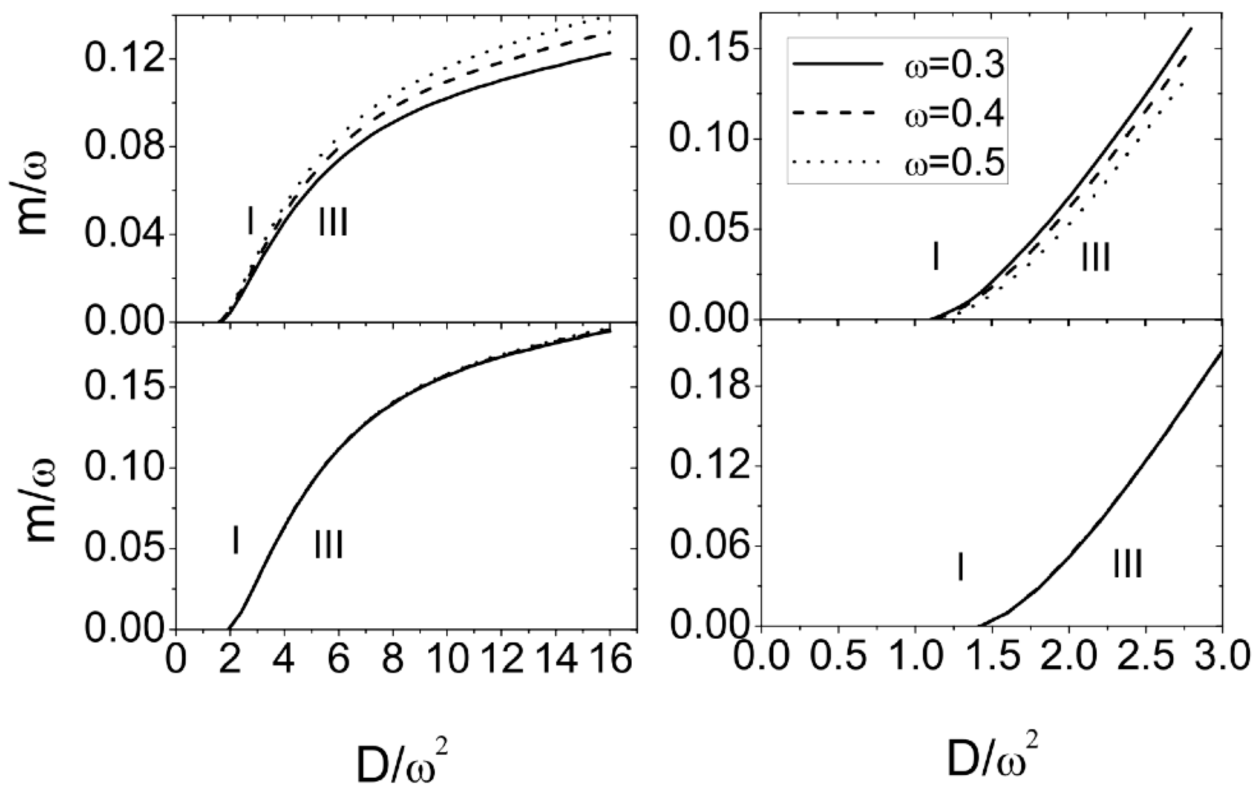
$$\langle\Omega|\bar{q}q|\Omega\rangle = \langle\Omega|\bar{q}_L q_R|\Omega\rangle + \langle\Omega|\bar{q}_R q_L|\Omega\rangle$$

$$q(x)\to q'(x) = e^{i\gamma_5\theta}q(x), \quad \bar{q}(x)\to\bar{q}'(x) = \bar{q}(x)e^{i\gamma_5\theta}$$

$$S'(p) = \int d^4x e^{ip\cdot(x-y)}\langle\Omega'|q(x)\bar{q}(y)|\Omega'\rangle = -i\not{p}\sigma_v(p^2) + e^{2i\gamma_5\theta}\sigma_s(p^2)$$

- Critical current mass





Trial: Scattering kernel

- Dirac structure

$$K^{\rho\sigma}_{\alpha\beta}(q,k,P) = K^{L}_{\rho\sigma}(q,k,P) \otimes K^{R}_{\alpha\beta}(q,k,P)$$

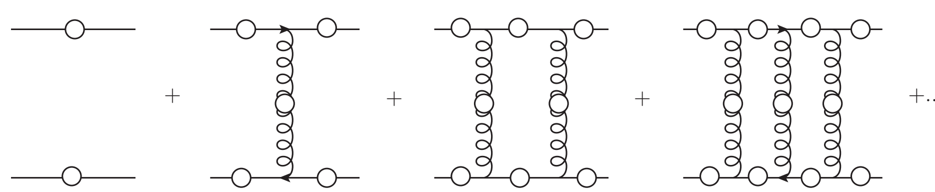
$256 \to 64$

- Simple model

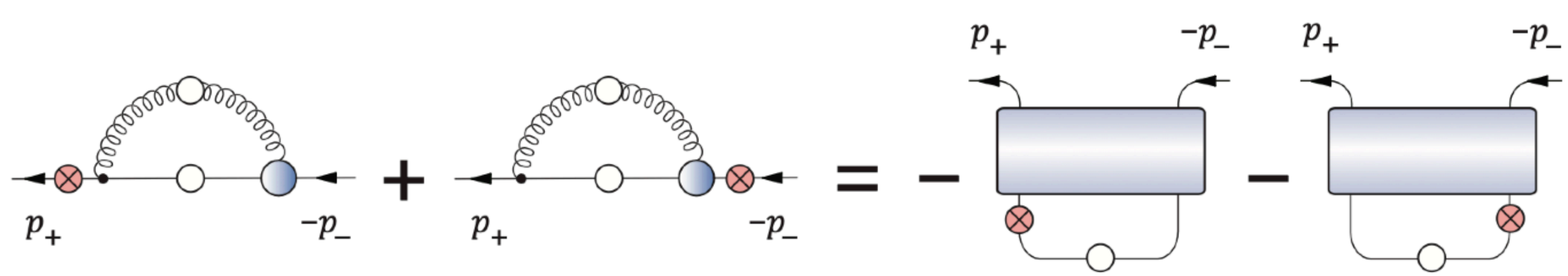
$$K^{\rho\sigma,AC}_{\alpha\beta,BD}(q,k,P) = -g^2 D_{\mu\nu}(k-q)\left[\frac{\lambda^a}{2}\right]_{AC}\left[\frac{\lambda^a}{2}\right]_{BD}[\gamma_\mu]_{\alpha\rho}[\gamma_\nu]_{\beta\sigma}$$

- Ladder approximation

$$\Gamma^{fg}_X(k,P) = -\int_q^{\Lambda} g^2 D_{\mu\nu}(k-q)\frac{\lambda^a}{2}\gamma_\mu S_f(q_+)\Gamma^{fg}_X(q,P)S_g(q_-)\frac{\lambda^a}{2}\gamma_\nu$$



Ward identities

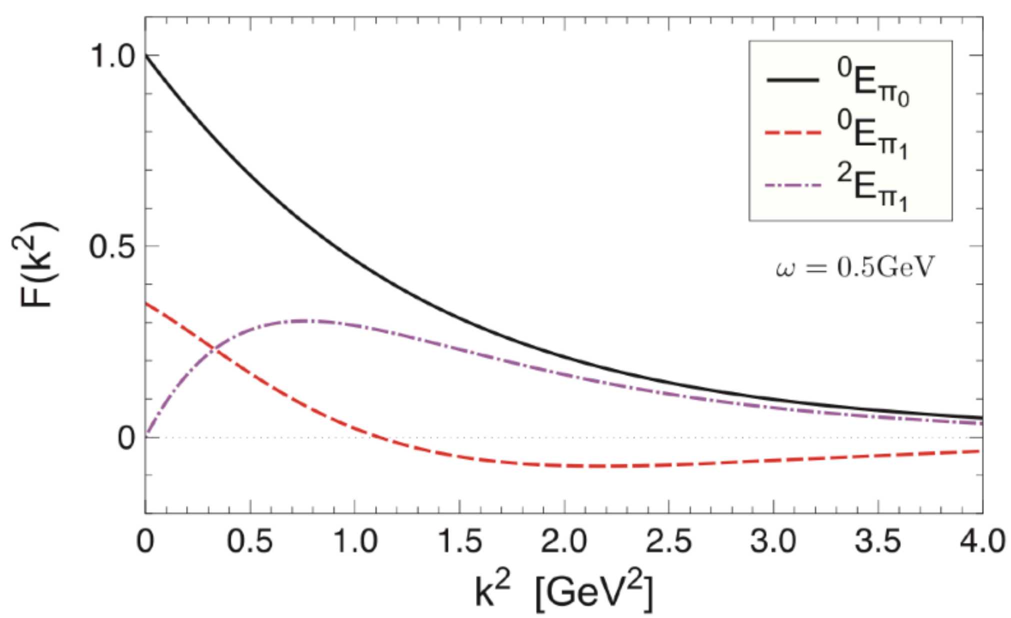


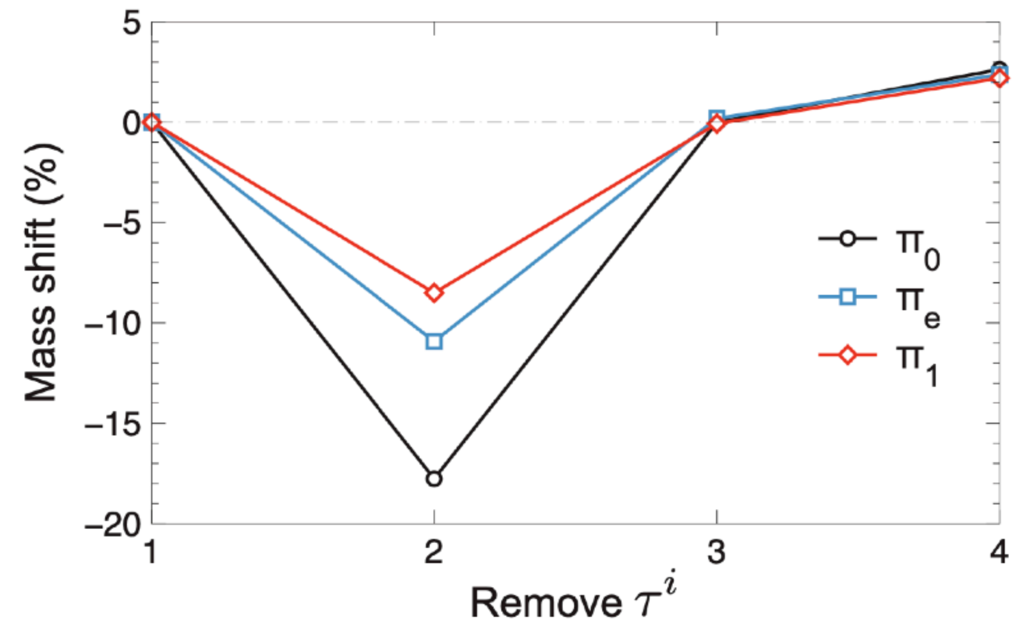
Trial: Meson solution

- BSE amplitude

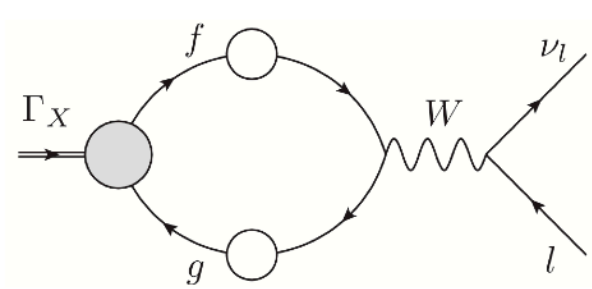
$$\gamma_5 E(k,P) + \gamma_5 \not{P} F(k,P) + \gamma_5 \not{k} G(k,P) + \gamma_5 [\not{k}, \not{P}] H(k,P)$$

$$P \neq (-1)^{L+1}$$

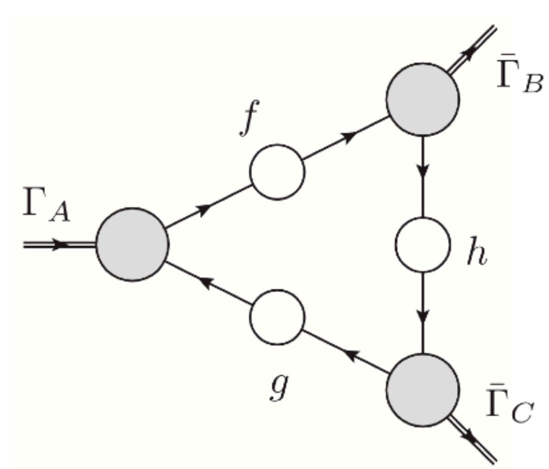




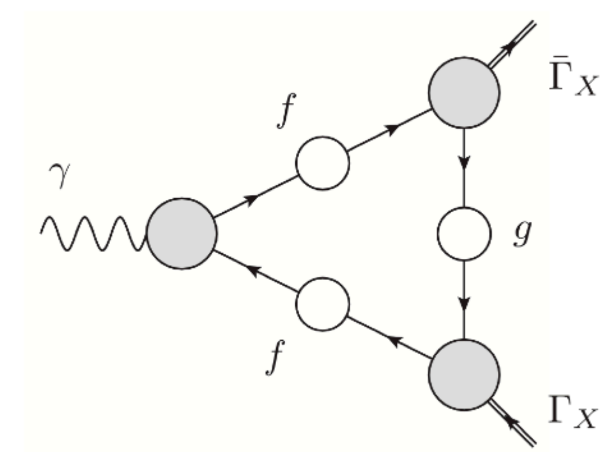
- Meson observable



leptonic decay



strong decay



EM structure

Trial: Meson solution

- Comparison with experiments

Summary of light meson results

$m_{u=d} = 5.5$ MeV, $m_s = 125$ MeV at $\mu = 1$ GeV

Pseudoscalar (PM, Roberts, PRC56, 3369)

	expt.	calc.
$-\langle \bar{q}q \rangle_\mu^0$	(0.236 GeV)^3	$(0.241^\dagger)^3$
m_π	0.1385 GeV	$0.138^\dagger$
f_π	0.0924 GeV	$0.093^\dagger$
m_K	0.496 GeV	$0.497^\dagger$
f_K	0.113 GeV	0.109

Charge radii (PM, Tandy, PRC62, 055204)

r_π^2	0.44 fm^2	0.45
$r_{K^+}^2$	0.34 fm^2	0.38
$r_{K^0}^2$	-0.054 fm^2	-0.086

$\gamma\pi\gamma$ transition (PM, Tandy, PRC65, 045211)

$g_{\pi\gamma\gamma}$	0.50	0.50
$r_{\pi\gamma\gamma}^2$	0.42 fm^2	0.41

Weak K_{l3} decay (PM, Ji, PRD64, 014032)

$\lambda_+(e3)$	0.028	0.027
$\Gamma(K_{e3})$	$7.6 \cdot 10^6$ s^{-1}	7.38
$\Gamma(K_{\mu 3})$	$5.2 \cdot 10^6$ s^{-1}	4.90

Vector mesons (PM, Tandy, PRC60, 055214)

$m_{\rho/\omega}$	0.770 GeV	0.742
$f_{\rho/\omega}$	0.216 GeV	0.207
m_{K^*}	0.892 GeV	0.936
f_{K^*}	0.225 GeV	0.241
m_ϕ	1.020 GeV	1.072
f_ϕ	0.236 GeV	0.259

Strong decay (Jarecke, PM, Tandy, PRC67, 035202)

$g_{\rho\pi\pi}$	6.02	5.4
$g_{\phi KK}$	4.64	4.3
$g_{K^*K\pi}$	4.60	4.1

Radiative decay (PM, nucl-th/0112022)

$g_{\rho\pi\gamma}/m_\rho$	0.74	0.69
$g_{\omega\pi\gamma}/m_\omega$	2.31	2.07
$(g_{K^*K\gamma}/m_K)^+$	0.83	0.99
$(g_{K^*K\gamma}/m_K)^0$	1.28	1.19

Scattering length (PM, Cotanch, PRD66, 116010)

a_0^0	0.220	0.170
a_0^2	0.044	0.045
a_1^1	0.038	0.036

Tandy @ Beijing Lectures 2010

Summary

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✦ **Frontier**: theoretical and experimental topics, non-perturbative approaches

✦ **Framework**: QFT equation of motion, QCD's DSE, advantages and disadvantages

✦ **Formalism**: one-body, two-body, and three-body equations, basic ingredients

✦ **Trial**: simple approximations for basic ingredients, quark solutions, meson solutions