Lattice QCD - Fall 2017 ETH Zürich

Lecturer:	Philippe de Forcrand	Office: K22.1	forcrand@phys.ethz.ch
Tutor:	Liam Keegan	Office: K31.3	keeganl@phys.ethz.ch

Exercise Nr. 4

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1) ϕ^4 on the Lattice

Show explicitly that starting from the Euclidean continuum action

$$S_E = \int d^4x \left\{ \frac{1}{2}(\partial_\mu \phi_0)^2 + \frac{1}{2} m_0^2 \phi_0^2 + \frac{g_0}{4!}\phi_0^4 \right\}$$

one can obtain after discretization the conventional notation

$$S_{\text{Lat}} = \sum_x \left\{ -2\kappa \sum_\mu \phi(x)\phi(x+\hat{\mu}) + \phi(x)^2 + \lambda(\phi(x)^2 - 1)^2 - \lambda \right\}.$$

Explain the conversion relations between the bare and lattice couplings. If you set $g_0 = 0$ (free theory), what is the critical value κ_c at which the correlation length $\xi \sim (m_0 a)^{-1}$ will diverge? Why does this correspond to the continuum limit? Why is the free theory not defined for $\kappa > \kappa_c$.

2) Preparations for Simulating ϕ^4

Modify your Metropolis code on QM and replace the quantum mechanical action by the action of the ϕ^4 model, by also generalizing your program to 4 dimensions.

- Make use of the above conventional notation. Set $\lambda = 0.01$ (such that we are close to the free case) and take some values for the hopping parameter κ in the interval $[0, 0.3]$. Note that there is no β involved now, as well as there is no lattice spacing. Simply set the interval from which to draw the new $\phi(x)$ as $[\phi(x) - 0.5, \phi(x) + 0.5]$ and $\bar{n} = 1$ (no repetitions on the same site).
- Do simulations for the lattice volumina 8^4 (about 10^7 updates) and 12^4 (about 10^8 updates) and measure the approximate order parameter

$$\langle |\phi| \rangle = \langle | \sum_x \phi(x) | \rangle$$

as a function of κ. Can you locate where $\langle |\phi| \rangle$ has the steepest rise?

Your Metropolis algorithm will not perform very well on the above task due to critical slowing down. You will have to increase the number of Metropolis updates drastically as you increase the volume. But this program will serve as a basis for future projects, which involve new algorithms, and which will allow to determine κ_c.