3) Measure the renormalized mass, m_R, and coupling, g_R

Using the relations derived in the lecture,

$$(am_R)^2 = \frac{8\chi_2}{\mu_2}, \; g_R = \frac{64\chi_4}{\mu_2^2} \tag{1}$$

$$\chi_n = \sum_{i_1,\dots,i_{n-1}} \langle \sigma_0 \sigma_{i_1} \cdots \sigma_{i_{n-1}} \rangle_c, \; \mu_2 = \sum_x \langle \sigma_0 x^2 \sigma_x \rangle_c, \tag{2}$$

measure the renormalized mass and coupling. Take advantage of the so called improved estimators available from the cluster updates. Convince yourself that $\langle \sigma_x \sigma_y \rangle$ is one if x, y belong to the same cluster and zero otherwise. Use this to show that in the symmetric phase where $\langle \sigma \rangle = 0$:

$$\chi_2 = \frac{1}{N} \sum_{\mathcal{C}} |\mathcal{C}|^2 \tag{3}$$

$$\chi_4 = \frac{1}{N} \left(3 \sum_{\mathcal{C}_1, \mathcal{C}_2} |\mathcal{C}_1|^2 \, |\mathcal{C}_2|^2 - 2 \sum_{\mathcal{C}} |\mathcal{C}|^4 - 3 \left(\sum_{\mathcal{C}} |\mathcal{C}|^2 \right)^2 \right). \tag{4}$$

Plot (am_R) vs g_R and verify that both go to zero as you approach the critical coupling, $J_c \approx 0.1497$. A lattice size of 16^4 should be appropriate and do not forget to include errorbars in your results.