Lattice QCD - Fall 2017 ETH Zürich

Lecturer:	Philippe de Forcrand	Office: K22.2	forcrand@phys.ethz.ch
Tutor:	Liam Keegan	Office: K31.3	keeganl@phys.ethz.ch

Exercise Nr. 9

Discussion on November 28th, 14:45

1) Literature

Read the paper by M. Creutz [1] on the Monte Carlo simulation of SU(2) gauge theory. Be prepared to explain how the importance sampling works, in particular how the Haar measure is treated and how the random numbers for a_0 are generated according to the exponential distribution

$$P(a_0) \sim (1 - a_0^2)^{1/2} \exp(\beta k a_0).$$

[1] M. Creutz, Phys. Rev. D21 (1980) 2308

2) SU(2) Heatbath Algorithm

Implement the heatbath algorithm for the pure SU(2) gauge theory in four dimensions. It is useful to remember that the sum of two SU(2) elements is proportional to an SU(2) element. Apply again the strategy of sweeping thorough the lattice via bushes. The pseudo-code for the update of a single link variable is as follows:

procedure `heatbath-su2`
input $\{U_\rho(z)\}_{z,\rho}$, x, μ
$S_\mu \leftarrow \sum_{\nu\neq\mu}\{U_\nu(x)U_\mu(x+\hat\nu)U_\nu^\dagger(x+\hat\mu) + U_\nu^\dagger(x-\hat\nu)U_\mu(x-\hat\nu)U_\nu(x+\hat\mu-\hat\nu)\}$
$k \leftarrow (\det S_\mu)^{1/2}$
do
    $z \leftarrow$ ran($e^{-\beta k}$,$e^{\beta k}$)
    $a_0 \leftarrow \log(z)/(\beta k)$
    $r \leftarrow (1 - a_0^2)^{1/2}$
while ($r <$ran(0,1))
$\cos(\theta) \leftarrow$ ran(-1,1)
$\phi \leftarrow$ ran(0,2π)
$V \leftarrow (a_0, r\sin(\theta)\cos(\phi), r\sin(\theta)\sin(\phi), r\cos(\theta)) \in$ SU(2)
$\hat{S}_\mu \leftarrow (S_\mu/\mathrm{k}) \in$ SU(2)
$U_\mu(x) \leftarrow V\hat{S}_\mu$
output $\{U_\rho(z)\}_{z,\rho}$

Measure the average plaquette and reproduce Figs. 1 and 2 of Ref. [1].

3) SU(2) overrelaxation

Generalize the overrelaxation algorithm you used in your U(1) simulation to the SU(2) Wilson action. To do this, consider the local action of one link and find the value that minimizes the action and reflect the link around this minimum.

A) Some useful formulas

Wilson Action: $S = -\beta/N \sum_p \mathrm{ReTr} U_p$

Vector rep. of SU(2): $U = u_\mu \sigma^\mu \in \mathrm{SU}(2)$, $|u|^2 = 1$, $\sigma^\mu = (\mathbf{1}, i\sigma^1, i\sigma^2, i\sigma^3)$

Sum of two SU(2) elements: $U, V \in \mathrm{SU}(2)$, $U + V = (u+v)_\mu \sigma^\mu = |u+v|\hat{S}$, $\hat{S} \in \mathrm{SU}(2)$

Local action: $S_{x,\mu} = -\beta k/N \,\mathrm{ReTr} U_\mu(x)\hat{S}_\mu$, $k = \sqrt{\det(S_\mu)}$, S_μ is sum over staples around link pointing in μ-direction at x

Invariance of real trace: $\mathrm{ReTr} US = \mathrm{ReTr} S^\dagger U^\dagger$