Note on the quark propagator on 2D pure U(1) gauge

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I. DEFINITION

The definition of the 2D Wilson fermion action can be expressed as,

$$\begin{aligned}
\mathcal{S}(x,t) =& \bar{\psi}(x,t)D(x,t;x',t')\psi(x',t'),\\
D(x,t;x',t') =& \frac{1}{2}\big\{(1+\sigma_2)[U_t(x,t)\delta_{x',x}\delta_{t',t+1} + (1-\sigma_2)U_t^*(x,t-1)\delta_{x',x}\delta_{t',t-1}\\
&+ (1+\sigma_1)U_x(x,t)\delta_{x',x+1}\delta_{t',t} + (1-\sigma_1)U_x^*(x-1,t)\delta_{x',x-1}\delta_{t',t}\big\}\\
&- (2+m)\delta_{x',x}\delta_{t',t},
\end{aligned} \tag{1}$$

where $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $\sigma_2 = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$.

For the 2D Wilson fermion on a lattice $L \times T$, D is a matrix with dimension $2 \times L \times T$. Even though obtaining the entire propagator from any point to any point is not impossible, but we can assume the translation invariance for the vacuum expectation value and then just consider the propagator from $(x,t) = (0,0)$ to any point. It is equivalent to solve the linear equations $D \cdot \xi = \eta$ with given source position (likes (0,0)) for both the spinor indices.

The biCGstab algorithm is the following: we can start from a random initial guess x_0, $\rho_0 = \alpha_0 = \omega_0 = 1.0$, $r_0 = b - Ax_0$, and $s_0 = p_0 = 0$, then do the the following iterations

$$\begin{aligned}
\rho_{i+1} &= r_0 \cdot r_i;\\
p_{i+1} &= r_i + \frac{\rho_{i+1}}{\rho_i}\frac{\alpha_i}{\omega_i}(p_i + \omega_i s_i);\\
s_{i+1} &= A \cdot p_{i+1};\\
\alpha_{i+1} &= \frac{\rho_{i+1}}{r_0 \cdot s_{i+1}}(= \frac{r_0 \cdot r_i}{r_0 \cdot s_{i+1}});\\
r' &= r_i - \alpha_{i+1}s_{i+1};\\
s' &= A \cdot r';\\
\omega_{i+1} &= \frac{s' \cdot r'}{s' \cdot s'};\\
x_{i+1} &= x_i + \alpha_{i+1}p_{i+1} + \omega_{i+1}r';\\
r_{i+1} &= r' - \omega_{i+1}s'
\end{aligned} \tag{2}$$

until the residual $|r_i|$ ($r_i = b - A \cdot x_i$) smaller the required residual (e.g., 1e-5).