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Muon anomalous magnetic moment: A harbinger for ''new physics''

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Standard model loop contributions to the muon anomalous magnetic moment, $a_\mu \equiv (g_\mu - 2)/2$, and their theoretical uncertainties are scrutinized. The status and implications of the recently reported 2.6 sigma experiment versus theory deviation $a_\mu^{\rm expt} - a_\mu^{\rm SM} = 426(165)\times 10^{-11}$ are discussed. Possible explanations due to supersymmetric loop effects with $m_{\rm SUSY} \simeq 55^{+16}_{-8}\sqrt{\tan\beta}$ GeV, radiative mass mechanisms at the 1–2 TeV scale and other ''new physics'' scenarios are examined.

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I. INTRODUCTION

Leptonic anomalous magnetic moments $a_l \equiv (g_l - 2)/2$ have traditionally been used for precision tests of the standard model (SM) and stringent constraints on potential ''new physics'' effects [1]. Indeed, agreement between measurements of $a_e^{\rm expt}$ and Schwinger's one-loop calculation $a_e = \alpha/2\pi$ provided beautiful early confirmation of quantum electrodynamics (QED). Currently, $a_e^{\rm expt}$ is used in conjunction with a four-loop QED calculation to obtain the most precise determination of α, the fine structure constant [2]:

$$\alpha^{-1}(a_e) = 137.035\,999\,58(52). \tag{1}$$

The muon's anomalous magnetic moment, a_μ, is not nearly as good as a_e at testing QED, but is much better suited to constrain or unveil ''new physics'' effects. Such effects are generally proportional to m_l^2/Λ^2 where Λ is the scale of ''new physics.'' The $m_\mu^2/m_e^2 \simeq 43\,000$ relative enhancement for the muon more than compensates for the factor of 370 current experimental precision advantage of a_e over a_μ. Furthermore, testing the $a_e^{\rm SM}$ prediction is currently limited by lack of a correspondingly precise independent determination of α.

Recently, a new measurement of a_{μ^+} based on 1999 data was announced by the E821 Collaboration at Brookhaven [3]:

$$a_{\mu^+}^{\rm expt} = 116\,592\,020(160)\times 10^{-11} \quad \text{(BNL99)}. \tag{2}$$

When averaged with the previous world average based on earlier CERN and BNL data,

$$a_\mu^{\rm expt} = 116\,592\,050(460)\times 10^{-11} \quad \text{(CERN77 + BNL98)}, \tag{3}$$

one obtains a new world average

$$a_\mu^{\rm expt}(\text{average}) = 116\,592\,023(151) \times 10^{-11} \quad \text{(CERN77 + BNL98 and 99)}. \tag{4}$$

Existing data from the 2000 run of E821 are expected to further reduce the error in a_{μ^+} in Eq. (2) by more than another factor of 2. Currently, the experiment is taking data with μ^- with the expectation of eventually obtaining similar precision for a_{μ^-}. If CPT is assumed, $a_{\mu^+} = a_{\mu^-}$, the collective data should yield an uncertainty close to the proposed experimental goal of $\pm 40\times 10^{-11}$ for a_μ.

Already, the current precision in Eq. (4) is very effective in probing for ''new physics.'' Indeed, as we later demonstrate, a deviation from standard model theory at that level of sensitivity can quite naturally occur and easily be interpreted as the appearance of supersymmetry at 100–450 GeV or other even higher scale phenomena, exciting prospects. Of course, before making such an interpretation, one must have a reliable theoretical prediction for $a_\mu^{\rm SM}$ with which to compare, an issue that we address in the next section.

II. STANDARD MODEL PREDICTION FOR a_μ

A. QED effects

The QED contribution to a_μ has been computed (or estimated) through 5 loops [4,5]:

$$a_\mu^{\rm QED} = \frac{\alpha}{2\pi} + 0.765\,857\,376(27)\left(\frac{\alpha}{\pi}\right)^2 + 24.050\,508\,98(44) \times\left(\frac{\alpha}{\pi}\right)^3 + 126.07(41)\left(\frac{\alpha}{\pi}\right)^4 + 930(170)\left(\frac{\alpha}{\pi}\right)^5. \tag{5}$$

Growing coefficients in the α/π expansion reflect the presence of large $\ln(m_\mu/m_e) \simeq 5.3$ terms coming from electron loops and the anomalously large coefficient of the light-by-light electron loop [6–8] which contributes about 21 of the 24 in the $(\alpha/\pi)^3$ coefficient. Employing the value of α from a_e in Eq. (1) (which effectively normalizes $a_\mu^{\rm QED}$ by $a_e^{\rm QED}$) gives

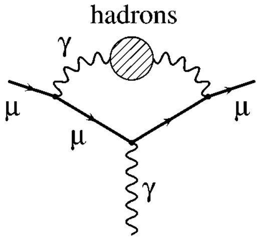

FIG. 1. Leading hadronic vacuum polarization corrections to a_μ.

$$a_\mu^{\rm QED}=116\,584\,705.7(2.9)\times 10^{-11}. \tag{6}$$

The quoted uncertainty is well below the $\pm 40\times 10^{-11}$ ultimate experimental error anticipated from E821 and should, therefore, play no essential role in the confrontation between theory and experiment.

B. Hadronic effects

Beginning at $\mathcal{O}(\alpha^2)$, hadronic loops contribute to a_μ via vacuum polarization (see Fig. 1). A pure QCD calculation of that effect is currently not possible. Fortunately, it is possible to evaluate those corrections via the dispersion integral [9]

$$a_\mu^{\rm Had}(\text{vac. pol.})=\frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} \mathrm{d}s K(s)\sigma^0(s)_{e^+e^-\to\text{hadrons}}, \tag{7}$$

where $\sigma^0(s)_{e^+e^-\to\text{hadrons}}$ means QED vacuum polarization and some other extraneous radiative corrections (e.g. initial state radiation) have been subtracted from measured $e^+e^- \to$ hadrons cross sections, and

$$K(s)=x^2\left(1-\frac{x^2}{2}\right)+(1+x)^2\left(1+\frac{1}{x^2}\right)\left[\ln(1+x)-x+\frac{x^2}{2}\right]$$
$$+\frac{1+x}{1-x}x^2\ln x$$
$$x=\frac{1-\sqrt{1-4m_\mu^2/s}}{1+\sqrt{1-4m_\mu^2/s}}. \tag{8}$$

Detailed evaluations of Eq. (7) have been carried out by various authors [10–21]. The most precise published analysis to date, by Davier and Höcker [11–13], found

$$a_\mu^{\rm Had}(\text{vac. pol.})=6924(62)\times 10^{-11}. \tag{9}$$

It employed experimental e^+e^- data, hadronic tau decays, perturbative QCD and sum rules to minimize the uncertainty in that result. The contributions coming from different energy regions are illustrated in Table I.

Table I nicely illustrates that the final result and its uncertainty are dominated by the low energy region. In fact, the $\rho(770\ \text{MeV})$ resonance region provides the bulk of the total hadronic contribution to $a_\mu^{\rm Had}(\text{vac. pol.})$.

To reduce the uncertainty in the low-energy region, Davier and Höcker employed $\Gamma(\tau\to\nu_\tau\pi^-\pi^0)/\Gamma(\tau\to\nu_\tau\bar{\nu}_e e^-)$ data to supplement $e^+e^-\to\pi^+\pi^-$ cross sections. In the $I=1$ channel they are related by isospin. Currently, tau decay data is experimentally more precise and in principle has the advantage of being self-normalizing if $\tau\to\nu_\tau\pi^-\pi^0$ and $\tau\to\nu_\tau\bar{\nu}_e e$ are both measured in the same experiment.

TABLE I. Contributions to $a_\mu^{\rm Had}(\text{vac. pol.})$ from different energy regions as found by Davier and Höcker [11–13].

$\sqrt{s}$ (GeV)	$a_\mu^{\rm Had}(\text{vac. pol.})\times 10^{11}$
$2m_\pi-1.8$	6343 ± 60
$1.8-3.7$	338.7 ± 4.6
$3.7-5+\psi(1S,2S)$	143.1 ± 5.4
$5-9.3$	68.7 ± 1.1
$9.3-12$	12.1 ± 0.5
$12-\infty$	18.0 ± 0.1
Total	6924 ± 62

An issue in the use of tau decay data is the magnitude of isospin violating corrections due to QED and the m_d-m_u mass difference. A universal short-distance QED correction [22] of about -2% was applied to the hadronic tau decay data and isospin violating effects such as $m_{\pi^\pm}$-m_{π^0} phase space, $\rho^\pm$-ρ^0 differences and effects due to ρ^0-ω interference have been accounted for. Other uncorrected distinctions are estimated to be about $\pm 0.5\%$ and are included in the quoted hadronic uncertainties of Table I.

Although the error assigned to the use of tau decay data appears reasonable, it has been questioned [23,24]. More recent preliminary $e^+e^-\to\pi^+\pi^-$ data from Novosibirsk [23] seem to suggest a potential difference with corrected hadronic tau decays which could compromise somewhat the estimated $a_\mu^{\rm Had}$ in Eq. (9). It is not clear at this time whether the difference is due to additional isospin violating corrections to hadronic tau decays, normalization issues [25], or radiative corrections to $e^+e^-\to$ hadrons data which must be accounted for in any precise comparison [26]. Resolution of this issue is extremely important.

A more conservative approach would be to ignore the tau data and use QCD theory input as little as possible. Such an analysis by Eidelman and Jegerlehner [17] in 1995 found $a_\mu^{\rm Had}(\text{vac. pol.})=7024(153)\times 10^{-11}$. Additional data were later used by those authors to refine their value to $6967(119)\times 10^{-11}$ [24]. In a further update of that work, Jegerlehner obtained

$$a_\mu^{\rm Had}(\text{vac. pol.})=6974.0(104.5)\times 10^{-11}$$
$$(\text{Jegerlehner 2001, [27]}). \tag{10}$$

Within their quoted errors, Eqs. (9) and (10) agree but the central values differ by 50×10^{-11} and the error in Eq. (10) is bigger because in some energy regions $\sigma(e^+e^-\to\text{hadrons})$ data have large uncertainties. The sign of the difference between Eq. (10) and Eq. (9) may be a little misleading, since tau data tend to favor a somewhat larger contribution to $a_\mu^{\rm Had}$

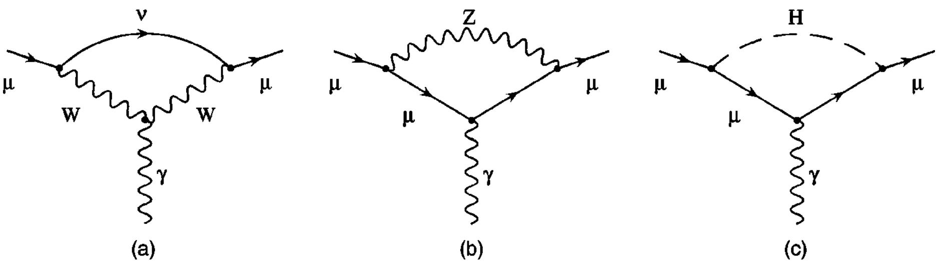

FIG. 2. One-loop electroweak radiative corrections to a_μ.

from the ρ than $e^+e^- \rightarrow$ hadrons [25]. More aggressive use of perturbative QCD and sum rules is largely responsible for the difference between central values in Eqs. (9) and (10). Anticipated new results for $e^+e^- \rightarrow$ hadrons at Novosibirsk (as well as Frascati, Beijing and Cornell) should reduce the uncertainty in Eq. (10) by at least a factor of 2 without requiring tau data or as much dependence on perturbative QCD. It will be interesting to see what happens to its central value.

Evaluation of the 3-loop hadronic vacuum polarization contribution to a_μ has been updated to [28,19]

$$\Delta a_\mu^{\rm Had}(\text{vac. pol.}) = -100(6)\times 10^{-11}. \tag{11}$$

Light-by-light hadronic diagrams have been evaluated using low-energy hadronic models and/or form factors. An average [11–13] of two recent detailed studies [29,30] gives

$$\Delta a_\mu^{\rm Had}(\text{light by light}) = -85(25)\times 10^{-11}. \tag{12}$$

Adding those contributions to Eqs. (9) leads to the total hadronic contribution

$$a_\mu^{\rm Had} = 6739(67)\times 10^{-11} \quad (\text{Davier and Höcker, 1998}) \tag{13}$$

which we will subsequently use in comparison of theory and experiment. However, we note that a more conservative approach might employ a larger uncertainty such as found using Jegerlehner's (mainly data driven) unpublished result in Eq. (10),

$$a_\mu^{\rm Had} = 6789(108)\times 10^{-11} \quad (\text{Jegerlehner 2001}). \tag{14}$$

At the very least, one should be mindful of the difference between the two and the need to further justify the use of tau decay data and low-energy perturbative QCD. The uncertainties in those results represent the main theoretical error in $a_\mu^{\rm SM}$. It would be very valuable to supplement the above evaluation of $a_\mu^{\rm Had}$ with lattice calculations (for the light-by-light contribution) and further improved e^+e^- data (beyond ongoing experiments). An ultimate goal of $\pm 40\times 10^{-11}$ or smaller appears to be within reach and is well matched to the prospectus of experiment E821 at Brookhaven which aims for a similar level of accuracy.

C. Electroweak effects

At the one-loop level, standard model electroweak radiative corrections to a_μ (see Fig. 2) are predicted to be [31–37]

$$a_\mu^{\rm EW}(1\text{ loop}) = \frac{5}{3}\frac{G_\mu m_\mu^2}{8\sqrt{2}\pi^2}\left[1+\frac{1}{5}(1-4\sin^2\theta_W)^2 + \mathcal{O}\left(\frac{m_\mu^2}{M^2}\right)\right] \approx 195\times 10^{-11} \tag{15}$$

where $G_\mu = 1.16637(1)\times 10^{-5}$ GeV^{-2}, $\sin^2\theta_W \equiv 1 - M_W^2/M_Z^2 \simeq 0.223$, and $M = M_W$ or $M_{\rm Higgs}$. The original goal of E821 was to measure that predicted effect at about the 5 sigma level. Subsequently, it was pointed out [38] that two-loop electroweak contributions are relatively large due to the presence of $\ln m_Z^2/m_\mu^2 \simeq 13.5$ terms. A full two-loop calculation [39,40], including low-energy hadronic electroweak loops [41,40], found, for $m_H \simeq 150$ GeV,

$$a_\mu^{\rm EW}(2\text{ loop}) = -43(4)\times 10^{-11}, \tag{16}$$

where the quoted error is a conservative estimate of hadronic electroweak loop, Higgs boson mass, and higher-order uncertainties. Combining Eqs. (15) and (16) gives the current electroweak contribution

$$a_\mu^{\rm EW} = 152(4)\times 10^{-11}. \tag{17}$$

Higher-order leading logarithms of the form $(\alpha \ln m_Z^2/m_\mu^2)^n$, $n = 2,3,\ldots$, can be computed via renormalization group techniques [42]. As a result of cancellations between the running of α and anomalous dimension effects, they give a relatively small $+0.5\times 10^{-11}$ contribution to $a_\mu^{\rm EW}$ which is included in the uncertainty of Eq. (17).

D. Comparison between theory and experiment

The complete standard model prediction for a_μ is given by

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm Had} + a_\mu^{\rm EW}. \tag{18}$$

Combining Eqs. (6), (13) and (17) leads to

$$a_\mu^{\rm SM} = 116\,591\,597(67)\times 10^{-11} \tag{19}$$

or, using Eq. (6), along with the more conservative Eqs. (14) and (17), $a_\mu^{\rm SM} = 116\,591\,647(108)\times 10^{-11}$. Comparing Eq. (19) with the current experimental average in Eq. (4) gives

$$a_\mu^{\rm expt} - a_\mu^{\rm SM} = 426 \pm 165\times 10^{-11}. \tag{20}$$

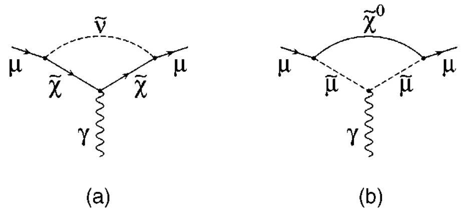

FIG. 3. Supersymmetric loops contributing to the muon anomalous magnetic moment.

The implied roughly 2.6σ difference is very exciting. It may be an indicator or harbinger of contributions from ''new physics'' beyond the standard model. At 90% C.L., one finds

$$215\times 10^{-11} \leqslant a_\mu(\text{new physics}) \leqslant 637\times 10^{-11}, \tag{21}$$

which suggests a relatively large ''new physics'' effect, even larger than the predicted electroweak contribution, may be appearing. As we show in the next section, several realistic examples of ''new physics'' could quite easily lead to $a_\mu(\text{new physics}) \sim \mathcal{O}(426\times 10^{-11})$ and might be responsible for the apparent deviation. If that is the case, the difference in Eq. (20) should increase to a 6 or more sigma effect as E821 is completed and the hadronic uncertainties in a_μ^{SM} are further reduced. Note, however, that the more conservative but less precise estimate in Eq. (14) leads to a smaller deviation of about 2 sigma due to the larger theory uncertainty and shift by $+50\times 10^{-11}$ in a_μ^{Had}. Use of any of the more recent [43] up-to-date studies of a_μ^{Had} will yield similar disagreement between experiment and theory at about the 2–2.6 sigma level. So the deviation appears real, but not yet statistically compelling.

III. ''NEW PHYSICS'' CONTRIBUTIONS

Since the anomalous magnetic moment comes from a dimension 5 operator, ''new physics'' (i.e. beyond the standard model expectations) will contribute to a_μ via induced quantum loop effects (rather than the tree level). Whenever a new model or standard model extension is proposed, such effects are computed and $a_\mu^{\text{expt}} - a_\mu^{\text{SM}}$ is often employed to constrain or rule it out.

In this section we discuss examples of interesting ''new physics'' probed by $a_\mu^{\text{expt}} - a_\mu^{\text{SM}}$. Rather than attempting to be inclusive, we concentrate primarily on two general scenarios: (1) Supersymmetric loop effects which can be substantial and would be heralded as the most likely explanation if the deviation in a_μ^{expt} is confirmed and (2) models of radiative muon mass generation which generically predict $a_\mu(\text{new physics}) \sim m_\mu^2/M^2$ where M is the scale of ''new physics.'' Either case is capable of explaining the apparent deviation in $a_\mu^{\text{expt}} - a_\mu^{\text{SM}}$ exhibited in Eq. (20). Several other examples of potential ''new physics'' contributions to a_μ are only briefly discussed.

A. Supersymmetry

The supersymmetric contributions to a_μ stem from smuon-neutralino and sneutrino-chargino loops (see Fig. 3). They include 2 chargino and 4 neutralino states and could in principle entail 3 generation slepton mixing and complex phases. Depending on the supersymmetry (SUSY) masses, mixing and other parameters, the resulting contribution of a_μ^{SUSY} can span a broad range of possibilities. Studies have been carried out for a variety of models where the parameters are constrained or specified. Here we give a generic discussion primarily intended to illustrate the strong likelihood that evidence for supersymmetry can be inferred from a_μ^{expt} and may in fact be the natural explanation for the deviation from SM theory reported by E821.

Extensive studies of the supersymmetric contributions a_μ^{SUSY} have been carried out [44–46]. An important observation was made in [45], namely that some of the contributions are enhanced by the ratio of Higgs' vacuum expectation values, $\tan\beta \equiv \langle\Phi_2\rangle/\langle\Phi_1\rangle$, which in some models is large (in some cases of order $m_t/m_b \approx 40$). For large $\tan\beta$, the dominant contribution is generally due to the chargino-sneutrino diagram (Fig. 3a) and approximately given (in the large $\tan\beta$ limit) by

$$|a_\mu^{\text{SUSY}}| \simeq \frac{\alpha(M_Z)}{8\pi\sin^2\theta_W}\frac{m_\mu^2}{\tilde{m}^2}\tan\beta\left(1-\frac{4\alpha}{\pi}\ln\frac{\tilde{m}}{m_\mu}\right), \tag{22}$$

where $\tilde{m} = m_{\text{SUSY}}$ represents a typical SUSY loop mass. (Chargino and sneutrino masses are actually assumed degenerate in that expression [46]; otherwise, $\tilde{m}$ is approximately the heavier mass scale.) Also, we have included a 7–8% suppression factor due to leading 2-loop EW effects. Like most ''new physics'' effects, SUSY loops contribute directly to the dimension 5 magnetic dipole operator. In that case, they are subject to the same EW suppression factor as the W loop contribution to a_μ^{EW}. From the calculation in Refs. [39,42], one finds a leading logarithmic suppression factor

$$1-\frac{4\alpha}{\pi}\ln\frac{M}{m_\mu} \tag{23}$$

where M is the characteristic ''new physics'' scale. For $M \sim 200$ GeV, that factor corresponds to about a 7% reduction.

Numerically, one expects, in the large $\tan\beta$ regime (after a small negative contribution from Fig. 3b is included, again assuming degenerate masses),

$$a_\mu^{\text{SUSY}} \simeq (\text{sgn}\,\mu)\times 130\times 10^{-11}\left(\frac{100\ \text{GeV}}{\tilde{m}}\right)^2\tan\beta, \tag{24}$$

where a_μ^{SUSY} generally has the same sign as the μ parameter $(\text{sgn}\,\mu)$ in SUSY models.

Rather than focusing on a specific model, we simply employ for illustration the large $\tan\beta$ approximate formula in Eq. (24) with degenerate SUSY masses and the current constraint in Eq. (20). Then we find [for positive $\text{sgn}(\mu)$], from comparison with Eq. (20),

$$\tan\beta\left(\frac{100\ \text{GeV}}{\tilde{m}}\right)^2 \simeq 3.3\pm 1.3 \tag{25}$$

or

$$\tilde{m}\simeq(55^{+16}_{-8}\ \text{GeV})\sqrt{\tan\beta},\quad \text{sgn}\,\mu=+1. \tag{26}$$

(Of course, in specific models with non-degenerate gauginos and sleptons, a more detailed analysis is required, but here we only want to illustrate roughly the scale of supersymmetry being probed.) Negative μ models give the opposite sign contribution to a_μ than the apparent deviation and are strongly disfavored. The muon anomalous magnetic moment measurement may have effectively eliminated half (i.e. $\text{sgn}\,\mu=-1$) of all SUSY models.

For large $\tan\beta$ in the range 4–40, where the approximate results given above should be valid, one finds (assuming $\tilde{m}>100$ GeV from other experimental constraints)

$$\tilde{m}\simeq 100\text{–}450\ \text{GeV}, \tag{27}$$

precisely the range where SUSY particles are often expected. Note also that $\tan\beta\gtrsim 3$ is currently favored in SUSY models because of the failure of CERN e^+e^- collider LEP II to find the Higgs boson and the implied bound $m_H\gtrsim 113$ GeV. If supersymmetry in the mass range of Eq. (27) with relatively large $\tan\beta$ is responsible for the apparent $a_\mu^{\text{expt}}-a_\mu^{\text{SM}}$ difference, it will have many dramatic consequences. Besides expanding the known symmetries of nature and our fundamental notion of space-time, it will impact other new exploratory experiments. Indeed, for $\tilde{m}\simeq 100$–450 GeV, one can expect a plethora of new SUSY particles to be discovered soon, either at the Fermilab 2 TeV $p\bar{p}$ collider or certainly at the Large Hadron Collider (LHC) 14 TeV pp collider which is expected to start running in 2006.

Large $\tan\beta$ supersymmetry can also have other interesting loop–induced low energy consequences beyond a_μ. For example, it can affect $b\to s\gamma$. For $\text{sgn}\,\mu=+1$ as suggested by $g_\mu-2$, destructive interference SUSY effects should lead to $\Gamma(b\to s\gamma)$ below the SM prediction [47]. Confirmation of such a reduction in the $b\to s\gamma$ decay rate would significantly reinforce the hint of SUSY in $g_\mu-2$. Even for the muon, ''new physics'' in a_μ is likely to suggest potentially observable $\mu\to e\gamma$, $\mu^- N\to e^- N$ and a muon electric dipole moment (EDM), depending on the degree of flavor mixing and CP violating phases. Searches for these phenomena are now entering an exciting era, with a new generation of experiments being proposed or constructed. The decay $\mu\to e\gamma$ will be searched for with 2×10^{-14} single event sensitivity (SES) at the Paul Scherrer Institute [48]. The MECO experiment at BNL [49] will search for the muon-electron conversion, $\mu^-\text{Al}\to e^-\text{Al}$, with 2×10^{-17} SES. A proposal is also being made [50] to search for the muon's electric dipole moment with sensitivity of about 10^{-24} e cm at the BNL muon storage ring. Certainly, the hint of supersymmetry suggested by a_μ^{expt} will provide strong additional motivation to extend such studies both theoretically and experimentally.

B. Radiative muon mass scenarios

The relatively light masses of the muon and most other known fundamental fermions might suggest that they are radiatively loop induced by ''new physics'' beyond the standard model. Although no compelling model exists, the concept is very attractive as a natural scenario for explaining the flavor mass hierarchy, i.e. why most fermion masses are so much smaller than the electroweak scale ~ 250 GeV.

The basic idea is to start off with a naturally zero bare fermion mass due to an underlying chiral symmetry. The symmetry is broken in the fermion 2-point function by quantum loop effects. They lead to a finite calculable mass which depends on the mass scales, coupling strengths and dynamics of the underlying symmetry breaking mechanism. In such a scenario, one generically expects, for the muon mass,

$$m_\mu \propto \frac{g^2}{16\pi^2}M_F, \tag{28}$$

where g is some new interaction coupling strength and $M_F\sim 100$–1000 GeV is a heavy scale associated with chiral symmetry breaking and perhaps electroweak symmetry breaking. Of course, there may be additional suppression factors at work in Eq. (28) that keep the muon mass small.

Whatever source of chiral symmetry breaking is responsible for generating the muon's mass will also give rise to non–standard model contributions in a_μ. Indeed, fermion masses and anomalous magnetic moments are intimately connected chiral symmetry breaking operators. Remarkably, in such radiative scenarios, the additional contribution to a_μ is quite generally given by [51,52]

$$a_\mu(\text{new physics})\simeq C\frac{m_\mu^2}{M^2},\quad C\simeq\mathcal{O}(1), \tag{29}$$

where M is some physical high mass scale associated with the ''new physics'' and C is a model-dependent number roughly of order 1 (it can even be larger). M need not be the same scale as M_F in Eq. (28). In fact, M is usually a somewhat larger gauge or scalar boson mass responsible for mediating the chiral symmetry breaking interaction. The result in Eq. (29) is remarkably simple in that it is largely independent of coupling strengths, dynamics, etc. Furthermore, rather than exhibiting the usual $g^2/16\pi^2$ loop suppression factor, $a_\mu(\text{new physics})$ is related to m_μ^2/M^2 by a (model dependent) constant, C, roughly of $\mathcal{O}(1)$.

1. Toy model

To demonstrate how the relationship in Eq. (29) arises, we first consider a simple toy model example [52] for muon mass generation which is graphically depicted in Fig. 4.

If the muon is massless in lowest order (i.e. no bare m_μ^0 is possible due to a symmetry), but couples to a heavy fermion F via scalar, S, and pseudoscalar, P, bosons with couplings g and $g\gamma_5$ respectively, then the diagrams give rise to

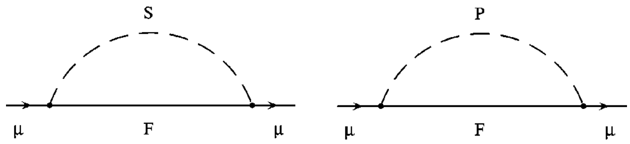

FIG. 4. Example of a pair of one-loop diagrams which can induce a finite radiative muon mass.

$$m_\mu \simeq \frac{g^2}{16\pi^2} M_F \left(\frac{M_S^2}{M_S^2 - M_F^2} \ln \frac{M_S^2}{M_F^2} - \frac{M_P^2}{M_P^2 - M_F^2} \ln \frac{M_P^2}{M_F^2} \right) \tag{30}$$

$$\to \frac{g^2}{16\pi^2} M_F \ln \left(\frac{M_S^2}{M_P^2} \right) \quad (M_{S,P} \gg M_F). \tag{31}$$

Note that short-distance ultraviolet divergences have canceled (as they must since there is no mass counterterm) and the induced mass vanishes in the chirally symmetric limit $M_S = M_P$. If we attach a photon to the heavy internal fermion, F, or boson S or P (assumed to carry fractions Q_F and $1 - Q_F$ of the muon charge, respectively), then a new contribution to a_μ is also induced (see Fig. 5). One obtains

$$a_\mu(\text{new physics}) = \frac{g^2}{16\pi^2} \{ Q_F [f_N(M_P) - f_N(M_S)] + (1 - Q_F)[f_C(M_P) - f_C(M_S)] \}, \tag{32}$$

with

$$f_N(M_X) = \frac{m_\mu M_F}{(M_X^2 - M_F^2)^3} \times \left(3M_X^4 + M_F^4 - 4M_X^2 M_F^2 - 2M_X^4 \ln \frac{M_X^2}{M_F^2} \right), \tag{33}$$

$$f_C(M_X) = \frac{m_\mu M_F}{(M_X^2 - M_F^2)^3} \left(M_X^4 - M_F^4 - 2M_F^2 M_X^2 \ln \frac{M_X^2}{M_F^2} \right). \tag{34}$$

In the limit $M_{S,P} \gg M_F$ and $Q_F = 1$, one finds [52]

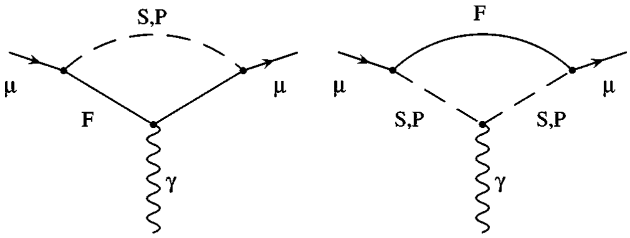

FIG. 5. Potential diagrams that can contribute to the anomalous magnetic moment in radiative muon mass models.

$$a_\mu(\text{new physics}) \simeq \frac{g^2}{8\pi^2} \frac{m_\mu M_F}{M_P^2} \left(\frac{M_P^2}{M_S^2} \ln \frac{M_S^2}{M_F^2} - \ln \frac{M_P^2}{M_F^2} \right), \tag{35}$$

while for $Q_F = 0$

$$a_\mu(\text{new physics}) \simeq \frac{g^2}{8\pi^2} \frac{m_\mu M_F}{M_P^2} \left(1 - \frac{M_P^2}{M_S^2} \right). \tag{36}$$

The induced a_μ(new physics) also vanishes in the $M_S = M_P$ chiral symmetry limit. Interestingly, a_μ(new physics) exhibits a linear rather than quadratic dependence on m_μ at this point.

Although Eqs. (31) and (35) both depend on unknown parameters such as g and M_F, those quantities largely cancel when we combine both expressions. One finds

$$a_\mu(\text{new physics}) \simeq C \frac{m_\mu^2}{M_P^2},$$

$$C = 2 \left[1 - \left(1 - \frac{M_P^2}{M_S^2} \right) \ln \frac{M_S^2}{M_F^2} \Big/ \ln \frac{M_S^2}{M_P^2} \right]$$

$$(\text{for } Q_F = 1)$$

$$C = \left(1 - \frac{M_P^2}{M_S^2} \right) \Big/ \ln \frac{M_S^2}{M_P^2} \quad (\text{for } Q_F = 0), \tag{37}$$

where C is very roughly $\mathcal{O}(1)$. It can actually span a broad range and take on either sign, depending on the M_S/M_P ratio and Q_F. A loop produced a_μ(new physics) effect that started out at $\mathcal{O}(g^2/16\pi^2)$ has effectively been promoted to $\mathcal{O}(1)$ by absorbing the couplings and M_F factor into m_μ. Along the way, the linear dependence on m_μ has been replaced by a more natural quadratic dependence, such that $a_\mu(\text{new physics})/m_\mu$ vanishes as $m_\mu \to 0$ as one would more naturally expect.

2. Dynamical mass generation

An alternative prescription for radiatively generating fermion masses involves new strong dynamics, e.g. extended technicolor. In such scenarios, technifermions acquire, via new strong dynamics, dynamical self-energies

$$\Sigma_F(p) \simeq m_F \left(\frac{\Lambda^2}{\Lambda^2 - p^2} \right)^{1 - \gamma/2}, \tag{38}$$

where $0 < \gamma < 2$ is an anomalous dimension, $m_F \simeq \mathcal{O}(300\ \text{GeV})$, and Λ is the new strong interaction scale $\sim \mathcal{O}(1\ \text{TeV})$.

Ordinary fermions such as the muon can receive loop induced masses via the diagram in Fig. 6.

The extended gauge boson X_μ links μ and F via the nonchiral coupling

$$g \gamma_\mu \left(a \frac{1 - \gamma_5}{2} + b \frac{1 + \gamma_5}{2} \right) \tag{39}$$

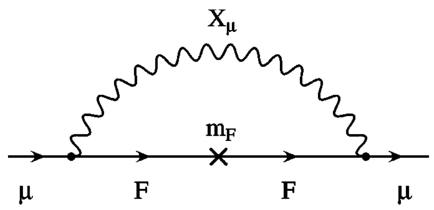

FIG. 6. Extended technicolor-like diagram responsible for generating the muon mass.

and gives rise to a mass [51,52]

$$m_\mu \simeq \frac{g^2 ab}{4\pi^2} m_F \left(\frac{\Lambda}{m_{X_\mu}}\right)^{2-\gamma} \Gamma\left(\frac{\gamma}{2}\right)\Gamma\left(1-\frac{\gamma}{2}\right), \tag{40}$$

where $\Gamma(x)$ is the gamma function. Notice the short-distance ultraviolet divergence at $\gamma=2$ which corresponds to a non-dynamical m_F scenario.

If we attach a photon to the internal fermion line in Fig. 6 (assumed here for illustration to have charge -1), an anomalous magnetic moment contribution is induced. One finds

$$a_\mu(\text{new dynamics}) \simeq \frac{g^2 ab}{2\pi^2} \frac{m_\mu m_F}{m_{X_\mu}^2} \left(\frac{\Lambda}{m_{X_\mu}}\right)^{2-\gamma} \frac{\Gamma\left(2-\frac{\gamma}{2}\right)\Gamma\left(\frac{\gamma}{2}\right)}{1+\frac{\gamma}{2}}. \tag{41}$$

Again we see only a linear dependence on m_μ. However, when Eqs. (40) and (41) are combined, one finds [51]

$$a_\mu(\text{new dynamics}) \simeq 2\left(\frac{2-\gamma}{2+\gamma}\right)\frac{m_\mu^2}{m_{X_\mu}^2}, \tag{42}$$

i.e. the generic result $\mathcal{O}(1)m_\mu^2/M^2$ where M is the ''new physics'' scale (here the extended-techniboson mass) emerges.

A similar relationship, $a_\mu(\text{new physics}) \simeq C m_\mu^2/M^2$, can be found in more realistic multi-Higgs-boson models [53], SUSY with soft masses [54], etc. It is also a natural expectation in composite models [55–57] as well as some models with large extra dimensions [58,59] or chiral violating leptoquark interactions, although studies of such cases have not necessarily made that connection. Basically, the requirement that m_μ remain relatively small in the presence of new chiral symmetry breaking interactions forces $a_\mu(\text{new physics})$ to effectively exhibit a quadratic m_μ^2 dependence.

For models of the above variety, where $|a_\mu(\text{new physics})| \simeq m_\mu^2/M^2$, the current constraint in Eq. (21) suggests (very roughly) that they can accommodate the deviation in a_μ^{expt} if

$$M \simeq 1\text{–}2 \text{ TeV}. \tag{43}$$

Of course, for a specific model, one must check that the sign of the induced a_μ^{NP} is in accord with experiment (i.e. it should be positive).

Such a scale of ''new physics'' could be quite natural in multi-Higgs-boson radiative mass and soft SUSY mass scenarios as well as some leptoquark models. It would appear to be somewhat low for dynamical symmetry breaking, compositeness and perhaps extra dimension models; however, confirmation of an a_μ^{expt} deviation will certainly lead to all possibilities being revisited.

C. Other ''new physics'' examples

1. Anomalous W boson properties

Anomalous W boson magnetic dipole and electric quadrupole moments can also lead to a deviation in a_μ from SM expectations. We generalize the γWW coupling such that the W boson magnetic dipole moment is given by

$$\mu_W = \frac{e}{2m_W}(1+\kappa+\lambda) \tag{44}$$

and electric quadrupole moment by

$$Q_W = -\frac{e}{2m_W}(\kappa-\lambda) \tag{45}$$

where $\kappa=1$ and $\lambda=0$ in the standard model, i.e. the gyromagnetic ratio $g_W=\kappa+1=2$. For non-standard couplings, one obtains the additional one loop contribution to a_μ given by [60–64]

$$a_\mu(\kappa,\lambda) \simeq \frac{G_\mu m_\mu^2}{4\sqrt{2}\pi^2}\left[(\kappa-1)\ln\frac{\Lambda^2}{m_W^2} - \frac{1}{3}\lambda\right], \tag{46}$$

where Λ is the high momentum cutoff required to give a finite result. It presumably corresponds to the onset of ''new physics'' such as the W compositeness scale or new strong dynamics. Higher order electroweak loop effects reduce that contribution by roughly the suppression in Eq. (23), i.e. $\sim 9\%$.

For $\Lambda \simeq 1$ TeV, the deviation in Eq. (20) corresponds to

$$\kappa - 1 = 0.37 \pm 0.14. \tag{47}$$

Such a large deviation from standard model expectations, $\kappa=1$, is already ruled out by $e^+e^- \to W^+W^-$ data at LEP II which gives [65,66]

$$\kappa - 1 = 0.04 \pm 0.08 \quad \text{(LEP II)}. \tag{48}$$

One could reduce the requirement in Eq. (47) somewhat by assuming a much larger Λ cutoff in Eq. (46). However, it is generally felt that $\kappa-1$ and Λ should be inversely correlated. For example $\kappa-1 \sim m_W/\Lambda$ or $(m_W/\Lambda)^2$. So the rather substantial $\kappa-1$ needed to accommodate a_μ^{expt} would argue against a much larger Λ. Similarly, the large value of the anomalous W electric quadrupole moment $\lambda \simeq -6$ needed to reconcile $a_\mu^{\text{expt}} - a_\mu^{\text{SM}}$ is also ruled out by collider data (which implies $|\lambda|\lesssim 0.1$). Hence, it appears that anomalous W boson properties cannot be the primary source of the discrepancy in $a_\mu^{\rm expt}$.

2. New gauge bosons

The local $SU(3)_C\times SU(2)_L\times U(1)_Y$ symmetry of the standard model can be easily expanded to a larger gauge group with additional charged and neutral gauge bosons. Here, we consider effects due to a charged $W_R^\pm$ which couples to right-handed charged currents in generic left-right symmetric models and a neutral gauge boson, Z', which can naturally arise in higher rank grand unified theory (GUT) models such as $SO(10)$ or E_6. A general analysis of one-loop contributions to a_μ from extra gauge bosons has been carried out by Leveille [67] and the specific examples considered here were illustrated in [68]. We will only discuss the likelihood of such bosons being the source of the apparent $a_\mu^{\rm expt}-a_\mu^{\rm SM}$ discrepancy.

For the case of a W_R coupled to μ_R and a (very light) ν_R with gauge coupling g_R, one finds

$$a_\mu(W_R)\simeq(390\times10^{-11})\frac{g_R^2}{g_2^2}\frac{m_W^2}{m_{W_R}^2}, \tag{49}$$

where g_2 is the usual $SU(2)_L$ gauge coupling. To accommodate the discrepancy in Eq. (20) requires $m_{W_R}\simeq m_W=80.4$ GeV for $g_R\simeq g_2$, which is clearly ruled out by direct searches and precision measurements which give $m_{W_R}\gtrsim 715$ GeV. Hence, $W_R^\pm$ is not a viable candidate for explaining the $a_\mu^{\rm expt}$ discrepancy.

In the case of a Z' boson with diagonal $\bar\mu\mu$ couplings described by the Lagrangian

$$\mathcal{L}=-\sqrt{\frac{3}{8}}g_2\tan\theta_W Z'_\alpha(Q_R\bar\mu_R\gamma^\alpha\mu_R+Q_L\bar\mu_L\gamma^\alpha\mu_L) \tag{50}$$

(where the normalization is based on GUT models), one finds [68]

$$a_\mu(Z')=\frac{G_\mu m_\mu^2}{4\sqrt{2}\pi^2}\sin^2\theta_W\frac{m_Z^2}{m_{Z'}^2}(3Q_LQ_R-Q_L^2-Q_R^2)$$
$$\simeq(54\times10^{-11})\frac{m_Z^2}{m_{Z'}^2}(3Q_LQ_R-Q_L^2-Q_R^2). \tag{51}$$

For $SO(10)$ models, $Z'=Z_\chi$, $Q_L^\chi=3Q_R^\chi=1$, one finds

$$a_\mu(Z_\chi)\simeq-6\times10^{-11}\left(\frac{m_Z^2}{m_{Z'}^2}\right). \tag{52}$$

The contribution is very small and negative. Collider constraints, $m_{Z_\chi}\gtrsim 600$ GeV, imply that the potential effect in Eq. (52) is completely unobservable.

To make a Z' contribution to $a_\mu(Z')$ maximally positive, one should have $Q_L=Q_R$, i.e. vector-like interactions. Even then, one would require very strong coupling $Q_{L,R}$ or a relatively light Z' to generate an appreciable contribution to $a_\mu(Z')$. In such a vector-like coupling scenario the Z' does not violate parity and one might be able to avoid low energy constraints on Z' bosons from atomic parity violation and polarized scattering (applicable if $e-\mu$ universality is assumed). However, an effort to explain $a_\mu^{\rm expt}$ with such a strongly coupled light Z' appears contrived and not particularly well motivated.

An exception to the small effects from gauge bosons illustrated above is provided by non-chiral coupled bosons which connect μ and a heavy fermion F. In those cases, $\Delta a_\mu\simeq(g^2/16\pi^2)m_\mu m_F/M^2$, where M is the gauge boson mass. However, loop effects then give $\delta m_\mu\sim g^2 m_F$ (see the discussion in Sec. III B) and we have argued that in such scenarios Δa_μ should actually turn out to be $\sim m_\mu^2/M^2$. As previously pointed out in Eq. (43), $a_\mu^{\rm expt}-a_\mu^{\rm SM}$ then corresponds to $M\sim 1$–2 TeV. Such a scenario, if made consistent, would be similar to extended technicolor efforts to explain light fermion mass generation.

Many other examples of ''new physics'' contributions to a_μ have been considered in the literature. General analyses in terms of effective interactions were presented in [69,70]. Specific other considerations include effects due to muon compositeness [57], extra Higgs [71] bosons, leptoquarks [72–75], bileptons [76], 2-loop pseudoscalar effects [77], compact extra dimensions [78,79], etc. Given the provocative hint of a deviation in experiment from theory, all will certainly be revisited.

IV. OUTLOOK

After many years of experimental and theoretical toil, studies of the muon anomalous magnetic moment have entered an exciting new phase. Experiment E821 at Brookhaven has reported a 2.6 sigma difference between $a_\mu^{\rm expt}$ and the standard model prediction, $a_\mu^{\rm SM}$. That difference could be a strong hint of supersymmetry in roughly the $\tan\beta\simeq4$, $m_{\rm SUSY}\simeq100$ GeV$-\tan\beta\simeq40$, $m_{\rm SUSY}\simeq450$ GeV region or perhaps an indication of radiative muon mass generation from ''new physics'' in the 1–2 TeV range. Either case represents an exciting prospect with interesting implications for future experiments.

Of course, before the assertion of ''new physics'' can be taken seriously, the values of $a_\mu^{\rm expt}$ and $a_\mu^{\rm SM}$ should be further scrutinized and refined. In that regard, it is fortunate that ongoing analysis of existing μ^+ data should reduce the uncertainty in $a_\mu^{\rm expt}$ by about another factor of 2.5 and similar statistical accuracy is expected from ongoing μ^- studies, which will further reduce the uncertainty or can be used to test CPT [80]. In addition, ongoing analysis of $e^+e^-\to\pi^+\pi^-$ data in the ρ resonance region and future experimental studies at higher energy should significantly reduce the uncertainty in a_μ^{SM} and enhance its credibility. Should a significant difference between theory and experiment persist after these improvements, it will rightfully be heralded as a harbinger of ''new physics.'' We look forward to the anticipated confrontation.

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