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TOPICAL REVIEW

The standard model prediction of the muon anomalous magnetic moment

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Abstract
This paper reviews and updates the standard model prediction of the muon g-2. QED, electroweak and hadronic contributions are presented, and open questions discussed. The theoretical prediction deviates from the present experimental value by 2–3 standard deviations, if e^+e^- annihilation data are used to evaluate the leading hadronic term.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The evaluation of the standard model (SM) prediction for the anomalous magnetic moment of the muon $a_\mu \equiv (g_\mu - 2)/2$ has occupied many physicists for over 50 years. Schwinger's 1948 calculation [1] of its leading contribution, equal to the one of the electron, was one of the very first results of QED, and its agreement with the experimental value of the anomalous magnetic moment of the electron, a_e, provided one of the early confirmations of this theory.

While a_e is rather insensitive to strong and weak interactions, hence providing a stringent test of QED and leading to the most precise determination to date of the fine-structure constant α, a_μ allows to test the entire SM, as each of its sectors contribute in a significant way to the total prediction. Compared with a_e, a_μ is also much better suited to unveil or constrain 'new physics' effects. For a lepton l, their contribution to a_l is generally proportional to $m_l^2\big/\Lambda^2$, where m_l is the mass of the lepton and Λ the scale of 'new physics', thus leading to an $(m_\mu/m_e)^2 \sim 4 \times 10^4$ relative enhancement of the sensitivity of the muon versus the electron anomalous magnetic moment. The anomalous magnetic moment of the τ would thus offer the best opportunity to detect 'new physics', but the very short lifetime of this lepton makes such a measurement very difficult at the moment.

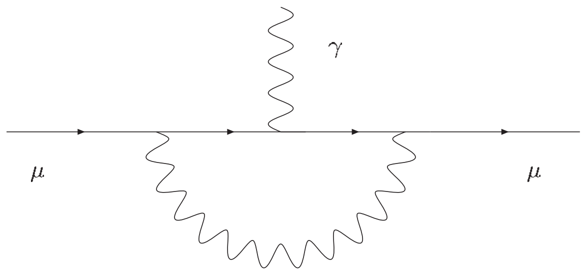

Figure 1. Lowest-order QED contribution to a_μ.

In a sequence of increasingly more precise measurements [2–5], the E821 collaboration at the Brookhaven Alternating Gradient Synchrotron has reached a fabulous relative precision of 0.5 parts per million (ppm) in the determination of a_μ, providing a very stringent test of the SM. Even a tiny statistically significant discrepancy from the SM prediction could be the harbinger for 'new physics' [6].

Several excellent reviews exist on the topic presented here. Among them, I refer the interested reader to [7–15]. In this paper I will provide an update and a review of the theoretical prediction for a_μ in the SM, analyzing in detail the three contributions into which a_μ^{SM} is usually split: QED, electroweak (EW) and hadronic. They are respectively discussed in sections 2, 3 and 4. A numerical re-evaluation of the two- and three-loop QED contributions employing recently updated values for the lepton masses is presented in sections 2.2 and 2.3. Comparisons between a_μ^{SM} results and the current experimental determination a_μ^{EXP} are given in section 5. Conclusions are drawn in section 6.

2. The QED contribution to a_μ

The QED contribution to the anomalous magnetic moment of the muon is defined as the contribution arising from the subset of SM diagrams containing only leptons (e, μ, τ) and photons. As a dimensionless quantity, it can be cast in the following general form [7, 16]

$$a_\mu^{\text{QED}} = A_1 + A_2(m_\mu/m_e) + A_2(m_\mu/m_\tau) + A_3(m_\mu/m_e, m_\mu/m_\tau), \tag{1}$$

where m_e, m_μ and m_τ are the masses of the electron, muon and tau, respectively. The term A_1, arising from diagrams containing only photons and muons, is mass independent (and is therefore same for the QED contribution to the anomalous magnetic moment of all three charged leptons). In contrast, the terms A_2 and A_3 are functions of the indicated mass ratios, and are generated by graphs containing also electrons and taus. The renormalizability of QED guarantees that the functions $A_i (i = 1, 2, 3)$ can be expanded as power series in α/π and computed order-by-order

$$A_i = A_i^{(2)}\left(\frac{\alpha}{\pi}\right) + A_i^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + A_i^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + A_i^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + A_i^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \cdots. \tag{2}$$

2.1. One-loop contribution

Only one diagram, shown in figure 1, is involved in the evaluation of the lowest-order contribution (second-order in the electric charge); it provides the famous result by Schwinger [1], $A_1^{(2)} = 1/2\left(A_2^{(2)} = A_3^{(2)} = 0\right)$.

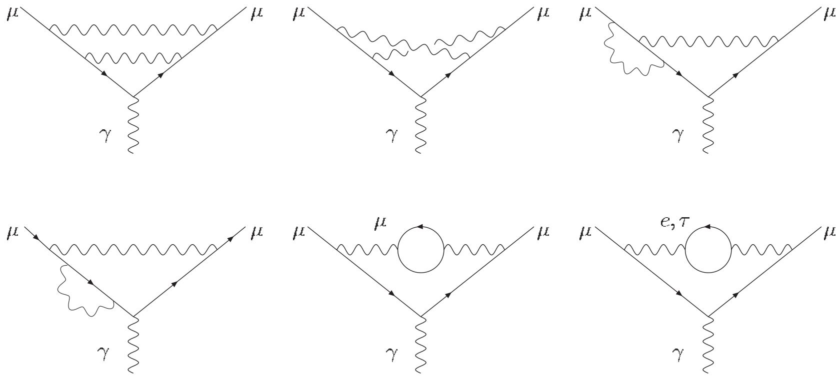

Figure 2. The QED diagrams contributing to the muon g-2 in order α^2. The mirror reflections (not shown) of the third and fourth diagrams must be included as well.

2.2. Two-loop contribution

At fourth order, seven diagrams contribute to $A_1^{(4)}$, one to $A_2^{(4)}(m_\mu/m_e)$ and one to $A_2^{(4)}(m_\mu/m_\tau)$. They are depicted in figure 2. The coefficient $A_1^{(4)}$ has been known for almost 50 years [17, 18]:

$$A_1^{(4)} = \frac{197}{144} + \frac{\pi^2}{12} + \frac{3}{4}\zeta(3) - \frac{\pi^2}{2}\ln 2 = -0.328\,478\,965\,579\ldots, \tag{3}$$

where $\zeta(s)$ is the Riemann zeta function of argument s. The mass-dependent coefficient

$$A_2^{(4)}(1/x) = \int_0^1 \mathrm{d}u \int_0^1 \mathrm{d}v \frac{u^2(1-u)v^2(1-v^2/3)}{u^2(1-v^2)+4x^2(1-u)}, \tag{4}$$

where $x = m_l/m_\mu$ and m_l is the mass of the virtual lepton in the vacuum polarization subgraph, was also computed in the late 1950s [19] for $m_l = m_e$ and neglecting terms of $O(m_e/m_\mu)$. Its exact expression was calculated in 1966 [20]. Actually, the full analytic result of [20] can be greatly simplified by taking advantage of the properties of the dilogarithm $\mathrm{Li}_2(z) = -\int_0^z \mathrm{d}t \ln(1-t)/t$. As a result of this simplification I obtain

$$\begin{aligned} A_2^{(4)}(1/x) = &-\frac{25}{36} - \frac{\ln x}{3} + x^2(4+3\ln x) + x^4\left[\frac{\pi^2}{3} - 2\ln x \ln\left(\frac{1}{x} - x\right) - \mathrm{Li}_2(x^2)\right] \\ &+ \frac{x}{2}(1-5x^2)\left[\frac{\pi^2}{2} - \ln x \ln\left(\frac{1-x}{1+x}\right) - \mathrm{Li}_2(x) + \mathrm{Li}_2(-x)\right]. \end{aligned} \tag{5}$$

Note that this simple formula, in contrast to the one in [20], can be directly used both for $0 < x < 1$ (the case of the electron loop) and for $x \geqslant 1$ (tau loop). In the latter case, the imaginary parts developed by the dilogarithms $\mathrm{Li}_2(x)$ and $\mathrm{Li}_2(x^2)$ are exactly canceled by the corresponding ones arising from the logarithms. For $x = 1$ (muon loop), equation (5) gives $A_2^{(4)}(1) = 119/36 - \pi^2/3$; of course, this contribution is already part of $A_1^{(4)}$ in equation (3). Evaluation of equation (5) with the latest recommended values for the muon–electron mass ratio $m_\mu/m_e = 206.768\,2838(54)$ [21], and the ratio of $m_\mu = 105.658\,3692$ (94) MeV [21] and $m_\tau = 1776.99$ (29) MeV [22] yields

$$A_2^{(4)}(m_\mu/m_e) = 1.094\,258\,3111\,(84) \tag{6}$$

$$A_2^{(4)}(m_\mu/m_\tau) = 0.000\,078\,064\,(25), \tag{7}$$

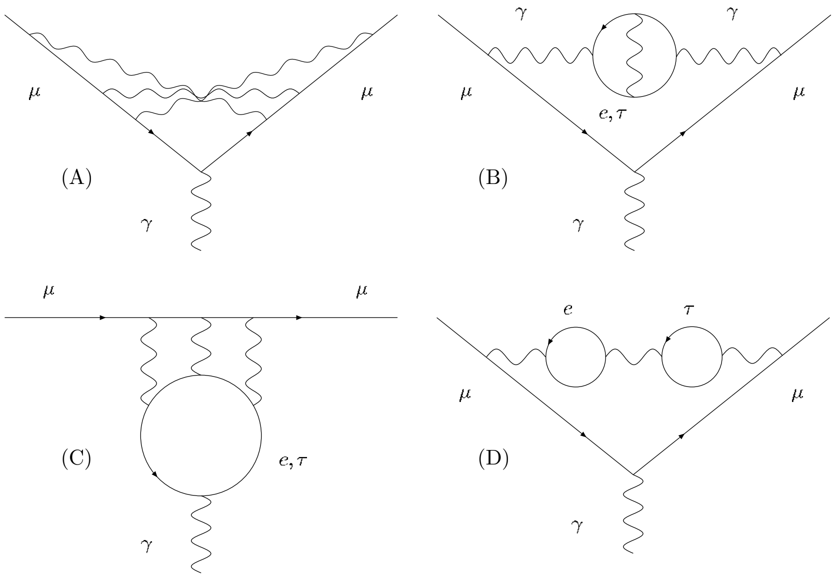

Figure 3. Examples of QED diagrams contributing to the muon g-2 in order α^3: (a) a 'triple-cross' diagram, (b) and (c) sixth-order muon vertex obtained by insertion of an electron or τ vacuum polarization (light-by-light) subdiagram and (d) graph with e and τ loops in the photon propagator.

where the standard uncertainties are only caused by the measurement uncertainties of the lepton mass ratios. Equations (6) and (7) provide the first re-evaluation of these coefficients with the recently updated CODATA and PDG mass ratios of [21, 22]. These new values differ visibly from older ones (see [8, 10]) based on previous measurements of the mass ratios, but the change induces only a negligible shift in the total QED prediction. Note that the τ contribution in equation (7) provides a $\sim$42 $\times$ 10^{-11} contribution to a_μ^{QED}. As there are no two-loop diagrams containing both virtual electrons and taus, $A_3^{(4)}(m_\mu/m_e, m_\mu/m_\tau) = 0$. Adding up equations (3), (6) and (7) I get the new two-loop QED coefficient

$$C_2 = A_1^{(4)} + A_2^{(4)}(m_\mu/m_e) + A_2^{(4)}(m_\mu/m_\tau) = 0.765\,857\,410\,(27). \tag{8}$$

The uncertainties in $A_2^{(4)}(m_\mu/m_e)$ and $A_2^{(4)}(m_\mu/m_\tau)$ have been added in quadrature. The resulting error $\delta C_2 = 2.7 \times 10^{-8}$ leads to a tiny 0.01×10^{-11} uncertainty in a_μ^{QED}.

2.3. Three-loop contribution

More than one hundred diagrams are involved in the evaluation of the three-loop (sixth-order) QED contribution. Their analytic computation required approximately three decades, ending in the late 1990s.

The coefficient $A_1^{(6)}$ arises from 72 diagrams. Its calculation in closed analytic form is mainly due to Remiddi and his collaborators [23, 24], who completed it in 1996 with the evaluation of the last class of diagrams, the non-planar 'triple cross' topologies (see, for example, figure 3(a)) [24]. The result reads

$$\begin{aligned} A_1^{(6)} = {} & \frac{83}{72}\pi^2\zeta(3) - \frac{215}{24}\zeta(5) + \frac{100}{3}\left[a_4 + \frac{1}{24}(\ln^2 2 - \pi^2)\ln^2 2\right] - \frac{239}{2160}\pi^4 \\ & + \frac{139}{18}\zeta(3) - \frac{298}{9}\pi^2\ln 2 + \frac{17101}{810}\pi^2 + \frac{28259}{5184} = 1.181\,241\,4566\ldots, \end{aligned} \tag{9}$$

where $a_4 = \sum_{n=1}^{\infty} 1/(2^n n^4) = \mathrm{Li}_4(1/2) = 0.517\,479\,061\,674\ldots.$

The calculation of the exact expression for the coefficient $A_2^{(6)}(m/M)$ for arbitrary values of the mass ratio (m/M) was completed in 1993 by Laporta and Remiddi [25, 26] (earlier works include [27, 28]). For our analysis, $m = m_\mu$, and $M = m_e$ or m_τ. This coefficient can be further split into two parts: the first one, $A_2^{(6)}(m/M, \text{vp})$, receives contributions from 36 diagrams containing electron or τ vacuum polarization loops (see, for example, figure 3(b)) [25], whereas the second one, $A_2^{(6)}(m/M, \text{lbl})$, is due to 12 light-by-light scattering diagrams with electron or τ loops (like the graph of figure 3(c)) [26]. The exact expression for $A_2^{(6)}(m/M)$ in closed analytic form is quite complicated, containing hundreds of polylogarithmic functions up to fifth degree (for the light-by-light diagrams) and complex arguments (for the vacuum polarization contribution). It also includes harmonic polylogarithms [29]. As it is very lengthy, it was not listed in the original papers [25, 26] which provided, however, useful series expansions in the mass ratio (m/M) for the cases of physical relevance. Using the exact expressions in closed analytic form kindly provided to me by the authors, and the latest values for the mass ratios mentioned above, I obtain the following values:

$$A_2^{(6)}(m_\mu/m_e, \text{vp}) = 1.920\,455\,130\,(33), \tag{10}$$

$$A_2^{(6)}(m_\mu/m_e, \text{lbl}) = 20.947\,924\,89\,(16), \tag{11}$$

$$A_2^{(6)}(m_\mu/m_\tau, \text{vp}) = -0.001\,782\,33\,(48), \tag{12}$$

$$A_2^{(6)}(m_\mu/m_\tau, \text{lbl}) = 0.002\,142\,83\,(69). \tag{13}$$

The sums of equations (10)–(11) and equations (12)–(13) are

$$A_2^{(6)}(m_\mu/m_e) = 22.868\,380\,02\,(20), \tag{14}$$

$$A_2^{(6)}(m_\mu/m_\tau) = 0.000\,360\,51\,(21); \tag{15}$$

to determine the uncertainties, the correlation of the addends has been taken into account. Equations (10)–(15) provide the first re-evaluation of these coefficients with the recently updated CODATA and PDG mass ratios of [21, 22]. These new values differ visibly from older ones (see [8, 10]) based on previous measurements of the mass ratios, but the change induces only a negligible shift in the total QED prediction. Note the large contribution from the electron light-by-light diagrams, equation (15)—its leading term is $(2/3)\pi^2 \ln(m_\mu/m_e)$ [30]. More generally, it was shown in [31] that the $O(\alpha^{2n+1})$ contribution to a_μ^{QED}, from diagrams in which the electron light-by-light subgraph is connected with $2n + 1$ photons to the muon, contains a large $\pi^{2n} \ln(m_\mu/m_e)$ term with a coefficient of $O(1)$.

The analytic calculation of the three-loop diagrams with both electron and τ loop insertions in the photon propagator (see figure 3(d)) became available in 1999 [32]. This analytic result yields the numerical value

$$A_3^{(6)}(m_\mu/m_e, m_\mu/m_\tau) = 0.000\,527\,66\,(17) \tag{16}$$

providing a small 0.7×10^{-11} contribution to a_μ^{QED}. The error, 1.7×10^{-7}, is caused by the uncertainty of the ratio m_μ/m_τ. Combining the three-loop results presented above, I obtain the new sixth-order QED coefficient

$$\begin{aligned} C_3 &= A_1^{(6)} + A_2^{(6)}(m_\mu/m_e) + A_2^{(6)}(m_\mu/m_\tau) + A_3^{(6)}(m_\mu/m_e, m_\mu/m_\tau) \\ &= 24.050\,509\,64\,(43). \end{aligned} \tag{17}$$

The error $\delta C_3 = 4.3 \times 10^{-7}$, due to the measurement uncertainties of the lepton masses, has been determined considering the correlation of the addends. It induces a negligible $O(10^{-14})$ uncertainty in a_μ^{QED}.

In parallel to these analytic results, numerical methods were also developed, mainly by Kinoshita and his collaborators, for the evaluation of the full set of three-loop diagrams [33, 7].

2.4. Four-loop contribution

More than one thousand diagrams enter the evaluation of the four-loop QED contribution to a_μ. As only few of them are known analytically [34], this eighth-order term has thus far been evaluated only numerically. This formidable task was first accomplished by Kinoshita and his collaborators in the early 1980s [35, 36]. Since then, they made a major effort to continuously improve this result [7, 9, 16, 37, 38, 39], also benefiting from fast advances in computing power. The latest analysis appeared in [40]. One should realize that this eighth-order QED contribution, being about six times larger than the present experimental uncertainty of a_μ, is crucial for the comparison between the SM prediction of a_μ and its experimental determination.

There are 891 four-loop diagrams contributing to the mass-independent coefficient $A_1^{(8)}$. Its latest published value is $A_1^{(8)} = -1.7502\,(384)$ [39], where the error is caused by the numerical procedure. This coefficient has undergone a small revision in [39]. In September 2004, Kinoshita reported a new preliminary updated result [41],

$$A_1^{(8)} = -1.7093\,(42). \tag{18}$$

Note the small shift in the central value and the significant reduction of the numerical uncertainty of this new result. I will adopt it for the value of $A_1^{(8)}$. The latest value of the coefficient $A_2^{(8)}(m_\mu/m_e)$, arising from 469 diagrams, is [40]

$$A_2^{(8)}(m_\mu/m_e) = 132.6823\,(72). \tag{19}$$

This value is significantly higher than the older one, 127.50(41) [9] (its precision is impressively higher too) shifting up the value of a_μ^{QED} by a non-negligible $\sim 15 \times 10^{-11}$. This difference is partly accounted for by the correction of a program error described in [39], but is mostly due to the fact that the computation of the older value suffered from insufficient numerical precision. The term $A_2^{(8)}(m_\mu/m_\tau)$ has been roughly estimated to give an $O(10^{-13})$ contribution to a_μ^{QED} and can be safely ignored for now [40]. The numerical evaluation of the 102 diagrams containing both electron and τ loop insertions yields the three-mass coefficient [40]

$$A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) = 0.037\,594\,(83), \tag{20}$$

which provides a small $O(10^{-12})$ contribution to a_μ^{QED}. Adding up the four-loop results described above, we obtain the eighth-order QED coefficient

$$C_4 \simeq A_1^{(8)} + A_2^{(8)}(m_\mu/m_e) + A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) = 131.011\,(8). \tag{21}$$

Note that this expression does not contain the term $A_2^{(8)}(m_\mu/m_\tau)$, which has been roughly estimated to be of the same order of magnitude of the uncertainty on the rhs of (21). However, this uncertainty, 0.008, causes only a tiny 0.02×10^{-11} error in a_μ^{QED}.

2.5. Five-loop contribution

The evaluation of the five-loop QED contribution is in progress [41]. The existing estimates are mainly based on the experience accumulated computing the sixth- and eighth-order terms, and include only specific contributions enhanced by powers of $\ln(m_\mu/m_e)$ times powers of π. The first estimate, $C_5 = 570\,(140)$, provided by Kinoshita and collaborators in 1990 [16], considered the contribution of graphs containing an electron light-by-light subdiagram with one-loop vacuum polarization insertions. A few other predictions for C_5 exist, and classes of diagrams were computed or estimated with various methods [31, 42–48]. In September 2004, Kinoshita reported a very preliminary new result [41],

$$C_5 \simeq A_2^{(10)}(m_\mu/m_e) = 677\,(40), \tag{22}$$

(9080 diagrams contribute to $A_2^{(10)}(m_\mu/m_e)$!) corresponding to a $4.6(0.3)\times10^{-11}$ contribution to a_μ^{QED}. This is the value of C_5 I will employ. The uncertainty in this new estimate of the tenth-order term (0.3×10^{-11}) no longer dominates the error of the total QED prediction (see next section). Efforts to improve upon the evaluation of C_5 are presently being pursued by Kinoshita and Nio.

2.6. The numerical value of a_μ^{QED}

Adding up all the above contributions and using the latest CODATA recommended value for the fine-structure constant [21], known to 3.3 ppb,

$$\alpha^{-1} = 137.035\,999\,11\,(46), \tag{23}$$

I obtain the following value for the QED contribution to the muon g-2:

$$a_\mu^{\text{QED}} = 116\,584\,718.8\,(0.3)\,(0.4)\times10^{-11}. \tag{24}$$

The first error is due to the uncertainties of the $O(\alpha^2)$, $O(\alpha^4)$ and $O(\alpha^5)$ terms, and is strongly dominated by the last of them. (The uncertainty of the $O(\alpha^3)$ term is negligible.) The second error is caused by the 3.3 ppb uncertainty of the fine-structure constant α. When combined in quadrature, these uncertainties yield $\delta a_\mu^{\text{QED}} = 0.5\times10^{-11}$. The value of a_μ^{QED} in equation (24) is close to that presented by Kinoshita in [41], $a_\mu^{\text{QED}} = 116\,584\,717.9\,(0.3)\,(0.9)\times10^{-11}$, and has a smaller error. This latter result was in fact derived using the value of α determined from atom interferometry measurements [49], $\alpha^{-1} = 137.036\,000\,3\,(10)$ (7.4 ppb), which has a larger uncertainty than the latest CODATA value employed for equation (24).

3. The electroweak contribution

The electroweak (EW) contribution to the anomalous magnetic moment of the muon is suppressed by a factor $(m_\mu/M_W)^2$ with respect to the QED effects. The one-loop part was computed in 1972 by several authors [50]. Back then, the experimental uncertainty of a_μ was one or two orders of magnitude larger than this one-loop contribution. Today it is less than one-third as large.

3.1. One-loop contribution

The analytic expression for the one-loop EW contribution to a_μ, due to the diagrams in figure 4, reads

$$a_\mu^{\text{EW}}(\text{one loop}) = \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1+\frac{1}{5}\left(1-4\sin^2\theta_W\right)^2+O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right], \tag{25}$$

where $G_\mu = 1.16637(1)\times10^{-5}\ \text{GeV}^{-2}$ is the Fermi coupling constant, M_Z, M_W and M_H are the masses of the Z, W and Higgs bosons, and θ_W is the weak mixing angle. Closed analytic expressions for $a_\mu^{\text{EW}}(\text{one loop})$ taking exactly into account the m_μ^2/M_B^2 dependence ($B = Z$, W,

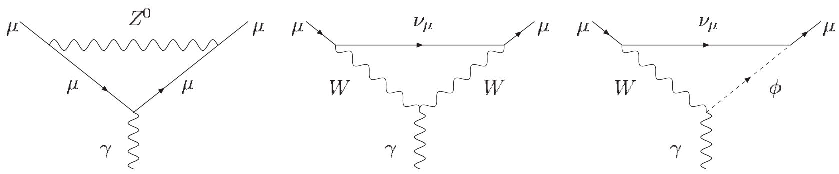

Figure 4. One-loop electroweak contributions to a_μ. The diagram with a W and a Goldstone boson (ϕ) must be counted twice. The diagrams with the Higgs boson loop and with two Goldstone boson couplings to the muon are suppressed by a factor $m_\mu^2/M_{Z,W,H}^2$ and are not drawn.

Higgs, or other hypothetical bosons) can be found in [51]. Following [52], I employ for $\sin^2\theta_W$ the on-shell definition $\sin^2\theta_W = 1 - M_W^2/M_Z^2$ [53], where $M_Z = 91.1875\,(21)$ GeV and M_W is the theoretical SM prediction of the W mass. The latter can be easily derived from the simple analytic formulae of [54],

$$M_W = \left[80.4077 - 0.05738\ln\left(\frac{M_H}{100\ \mathrm{GeV}}\right) - 0.00892\ln^2\left(\frac{M_H}{100\ \mathrm{GeV}}\right)\right]\mathrm{GeV}, \tag{26}$$

(on-shell scheme II with $\Delta\alpha_{\mathrm{h}}^{(5)} = 0.02761\,(36), \alpha_s(M_Z) = 0.118\,(2)$ and $M_{\mathrm{top}} = 178.0\,(4.3)$ GeV [55]), leading to $M_W = 80.383$ GeV for $M_H = 150$ GeV, compared with the direct experimental value $M_W = 80.425\,(38)$ GeV [22], which corresponds to a small M_H [56]. For $M_H = 150$ GeV, equation (25) thus gives

$$a_\mu^{\mathrm{EW}}\ (\text{one loop}) = 194.8 \times 10^{-11}, \tag{27}$$

but this value encompasses the predictions derived from a wide range of values of M_H varying from 114.4 GeV, the current lower bound at 95% confidence level [57], up to a few hundred GeV.

The contribution of the Higgs diagram alone, part of the $O\left(m_\mu^2/M_{Z,W,H}^2\right)$ terms of equation (25), is [7, 51]

$$a_\mu^{\mathrm{EW,H}}\ (\text{one loop}) = \frac{G_\mu m_\mu^2}{4\sqrt{2}\pi^2}\left[\frac{\log R_H}{R_H} - \frac{7}{6R_H} + O\left(\frac{1}{R_H^2}\right)\right], \tag{28}$$

where $R_H = M_H^2/m_\mu^2$. Given the current lower bound $M_H > 114.4$ GeV (95% CL), $a_\mu^{\mathrm{EW,H}}$(one loop) is smaller than 3×10^{-14} and can be safely neglected.

3.2. Higher-order contributions

The two-loop EW contribution to a_μ was computed in 1995 by Czarnecki *et al* [58, 59]. This remarkable calculation, probably the first (and still one of the very few) complete two-loop electroweak computation, leads to a significant reduction of the one-loop prediction. Naïvely one would expect the two-loop EW contribution a_μ^{EW}(two loop) to be of order $(\alpha/\pi) \times a_\mu^{\mathrm{EW}}$(one loop), and thus negligible, but this turns out not to be so. As first noticed in 1992 [60], a_μ^{EW}(two loop) is actually quite substantial because of the appearance of terms enhanced by a factor of $\ln(M_{Z,W}/m_f)$, where m_f is a fermion mass scale much smaller than M_W.

The two-loop contributions to a_μ^{EW} can be divided into fermionic and bosonic parts; the former includes all two-loop EW corrections containing closed fermion loops, whereas all other contributions are grouped into the latter. The full two-loop calculation involves 1678 diagrams in the linear 't Hooft–Feynman gauge [61]. As a check, the authors of [58, 59] employed both this gauge and a nonlinear one in which the vertex of the photon, the W and

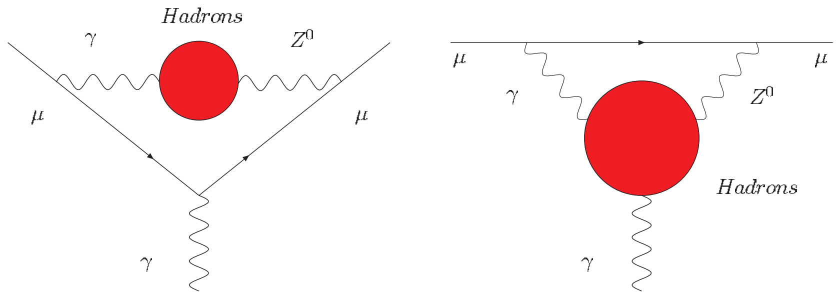

Figure 5. Hadronic loops in two-loop EW contributions.

the unphysical charged scalar vanishes. Their result for $M_H = 150$ GeV (obtained in the approximation $M_H \gg M_{W,Z}$ computing the first two terms in the expansion in $M_{W,Z}^2/M_H^2$) was $a_\mu^{\mathrm{EW}}(\text{two loop}) = -42.3(2.0)(1.8) \times 10^{-11}$, leading to a significant reduction of a_μ^{EW}. The first error is meant to roughly reflect low momentum hadronic uncertainties (more below), whereas the second allows for a range of M_H values from 114 GeV to about 250 GeV. Note that the contribution from γ–Z mixing diagrams is not included in this result: as it is suppressed by $(1 - 4\sin^2\theta_W) \sim 0.1$ for quarks and $(1 - 4\sin^2\theta_W)^2$ for leptons, it was neglected in this early calculation. It was later studied in [52]: together with small contributions proportional to $(1 - 4\sin^2\theta_W)(m_t^2/M_W^2)$ induced by the renormalization of $\sin^2\theta_W$, it shifts the above value of a_μ^{EW} down by a tiny 0.4×10^{-11}.

The hadronic uncertainties, above estimated to be $\sim 2 \times 10^{-11}$, arise from two types of two-loop diagrams: hadronic photon–Z mixing, and quark triangle loops with the external photon, a virtual photon and a Z attached to them (see figure 5). The tiny hadronic γ–Z mixing terms can be evaluated either in the free quark approximation or via a dispersion relation using data from e^+e^- annihilation into hadrons; the difference was shown to be numerically insignificant [52]. The contribution from the second type of diagrams (the quark triangle ones), calculated in [58] in the free quark approximation, is numerically more important. The question of how to treat properly the contribution of the light quarks was originally addressed in [62] within a low-energy effective field theory approach and was further investigated in the detailed analyses of [52, 63, 64]. These refinements significantly improved the reliability of the fermionic part of $a_\mu^{\mathrm{EW}}(\text{two loop})$ and increased it by 2×10^{-11} relative to the free quark calculation, leading, for $M_H = 150$ GeV, to [52]

$$a_\mu^{\mathrm{EW}} = 154\,(1)\,(2) \times 10^{-11}, \tag{29}$$

where the first error corresponds to hadronic loop uncertainties and the second to an allowed Higgs mass range of $114\,\text{GeV} < M_H < 250\,\text{GeV}$, the current top mass uncertainty[1] and unknown three-loop effects.

The leading-logarithm three-loop contribution to a_μ^{EW} was first studied via a renormalization group analysis in [65]. Such analysis was revisited and refined in [52], where this contribution was found to be extremely small (indeed, consistent with zero to a level of accuracy of 10^{-12}). An uncertainty of 0.2×10^{-11}, included in equation (29), has been conservatively assigned to a_μ^{EW} for uncalculated three-loop nonleading-logarithm terms.

Lastly, I would like to point out that until recently only one evaluation existed of the two-loop bosonic part of a_μ^{EW}, i.e. [59]. The recent calculation of [66], performed without the

[1] Indeed, although the result in equation (29) was computed for $m_t = 174.3$ GeV, I checked that the shift induced in a_μ^{EW} by the latest experimental value $m_t = 178.0\,(4.3)$ GeV [55] is well within the quoted error.

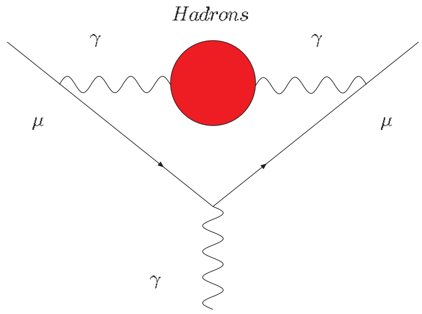

Figure 6. Leading hadronic contribution to a_μ.

approximation of large Higgs mass previously employed, agrees with the result of [59]. Work is also in progress for an independent recalculation based on the numerical methods of [67].

4. The hadronic contribution

In this section I will analyze the contribution to the muon g-2 arising from QED diagrams involving hadrons. Hadronic effects in (two-loop) EW contributions are already included in a_μ^{EW} (see the previous section).

4.1. Leading-order hadronic contribution

The leading hadronic contribution to the muon g-2, a_μ^{HLO}, is due to the hadronic vacuum polarization correction to the internal photon propagator of the one-loop diagram (see diagram in figure 6). The evaluation of this $O(\alpha^2)$ diagram involves long-distance QCD for which perturbation theory cannot be employed. However, using analyticity and unitarity (the optical theorem), Bouchiat and Michel [68] showed long ago that this contribution can be computed from hadronic e^+e^- annihilation data via the dispersion integral [68, 69]

$$a_\mu^{\text{HLO}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} \mathrm{d}s\, K(s)\sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} \frac{\mathrm{d}s}{s} K(s)R(s), \tag{30}$$

where $\sigma^{(0)}(s)$ is the experimental total cross section for e^+e^- annihilation into any hadronic state, with extraneous QED radiative corrections subtracted off (more later), and $R(s)$ is the ratio of $\sigma^{(0)}(s)$ and the high-energy limit of the Born cross section for μ-pair production: $R(s) = \sigma^{(0)}(s)/(4\pi\alpha^2/3s)$. The kernel $K(s)$ is a well-known function

$$K(s) = \int_0^1 \mathrm{d}x \frac{x^2(1-x)}{x^2 + (1-x)s/m_\mu^2} \tag{31}$$

(see [70] for some of its explicit representations and their suitability for numerical evaluations). It decreases monotonically for increasing s, and for large s it behaves as $m_\mu^2/3s$ to a good approximation. For this reason the low-energy region of the dispersive integral is enhanced by $\sim 1/s^2$. About 91% of the total contribution to a_μ^{HLO} is accumulated at centre-of-mass energies $\sqrt{s}$ below 1.8 GeV and 73% of a_μ^{HLO} is covered by the two-pion final state which is dominated by the $\rho(770)$ resonance [71]. Exclusive low-energy e^+e^- cross sections have been mainly measured by experiments running at e^+e^- colliders in Novosibirsk (OLYA, TOF, ND, CMD, CMD-2, SND) and Orsay (M3N, DM1, DM2), while at higher energies the total cross-section ratio $R(s)$ has been measured inclusively by the experiments $\gamma\gamma 2$, MARK I, DELCO, DASP, PLUTO, LENA, Crystal Ball, MD-1, CELLO, JADE, MARK-J, TASSO, CLEO, CUSB, MAC, and BES. Perturbative QCD becomes applicable at higher loop momenta, so that at some energy scale one can switch from data to QCD [72–74].

Detailed evaluations of the dispersive integral in equation (30) have been carried out by several authors [70, 71, 73–93]. A prominent role among all datasets is played by the precise measurements by the CMD-2 detector at the VEPP-2M collider in Novosibirsk [94–96] of the cross section for $e^+e^- \to \pi^+\pi^-$ at values of $\sqrt{s}$ between 0.61 and 0.96 GeV (i.e. $s \in [0.37, 0.93]$ GeV^2). The quoted systematic error of these data is 0.6% [96], dominated by the uncertainties in the radiative corrections (0.4%). In July 2004, also the KLOE experiment at the DAΦNE collider in Frascati presented the final analysis [97] of the 2001 data for the precise measurement of $\sigma(e^+e^- \to \pi^+\pi^-)$ via the radiative return method [98] from the ϕ resonance. In this case the machine is operating at a fixed centre-of-mass energy $W \simeq 1.02$ GeV, the mass of the ϕ meson, and initial-state radiation is used to reduce the invariant mass of the $\pi^+\pi^-$ system. In [97] the cross-section $\sigma(e^+e^- \to \pi^+\pi^-)$ was extracted for the range $s \in [0.35, 0.95]$ GeV^2 with a systematic error of 1.3% (0.9% experimental and 0.9% theoretical) and a negligible statistical one. The study of the $e^+e^- \to \pi^+\pi^-$ process via the initial-state radiation method is also in progress at the BABAR detector at the PEP-II collider in SLAC [99]. This analysis will be important to further assess the consistency of the e^+e^- data. The BABAR collaboration has already presented data for the $\pi^+\pi^-\pi^0$ final state [100], and preliminary ones for the process $e^+e^- \to 2\pi^+2\pi^-$ [99]. On the theoretical side, the properties of analyticity, unitarity and chiral symmetry provide strong constraints for the pion form factor $F_\pi(s)$ in the low-energy region [101–105, 92]. They can lead to further improvements. Perhaps, also lattice QCD computations of a_μ^{HLO}, although not yet competitive with the precise results of the dispersive method, may eventually rival that precision [106].

The hadronic contribution a_μ^{HLO} is of order 7000×10^{-11}. Of course, this is a small fraction of the total SM prediction for a_μ, but is very large compared with the current experimental uncertainty $\delta a_\mu^{\text{EXP}} = 60 \times 10^{-11}$. Indeed, as $\delta a_\mu^{\text{EXP}}$ is less than 1% of a_μ^{HLO}, precision analyses of this hadronic term as well as full treatment of its higher-order corrections are clearly warranted. Normally, the 'bare' cross-section $\sigma^{(0)}(s)$ is used in the evaluation of the dispersive integral and the higher-order hadronic corrections (see section 4.2) are addressed separately. But what does 'bare' really mean?

The extraction of $\sigma^{(0)}(s)$ from the observed hadronic cross-section $\sigma(s)$ requires the subtraction of several radiative corrections (RC) which, at the level of precision we are aiming at, have a substantial impact on the result. To start with, RC must be applied to the luminosity determination, which is based on large-angle Bhabha scattering and muon-pair production in low-energy experiments, and small-angle Bhabha scattering at high energies. The first step to derive $\sigma^{(0)}(s)$ consists then in subtracting the initial-state radiative (ISR) corrections (virtual and real, described by pure QED) from $\sigma(s)$. The resulting cross section still contains the effects of the photon vacuum polarization corrections (VP), which can be simply undressed by multiplying it by $\alpha^2/\alpha(s)^2$, where $\alpha(s)$ is the effective running coupling (obviously depending on nonperturbative contributions itself). The problem with data from old experiments is that it is difficult to find out if (and which of) these corrections have been included (see [86]). The latest analysis from CMD-2 [96] is explicitly corrected for both ISR and VP (leptonic as well as hadronic) effects, whereas the preliminary data [94] of the same experiment were only corrected for ISR. For a thorough analysis of these problems, I refer the reader to [86, 91, 107] and references therein.

All hadronic final states should be incorporated in the hadronic contribution to the muon g-2, in particular final states including photons. These final-state radiation (FSR) effects, although of higher-order (α^3), are normally included in the leading-order hadronic contribution

$a_\mu^{\rm HLO}$. I will stick to this time-honored convention. The precise CMD-2 data for the cross-section $e^+e^- \to \pi^+\pi^-$ (quoted systematic error of 0.6% dominated by the uncertainties in the RC) are corrected for FSR effects using *scalar* QED. I find this worrisome. The following is done: their experimental analysis imposes cuts to isolate the two-pion final states. These cuts exclude a large fraction of the $\pi^+\pi^-\gamma$ states, in particular those where the photon is radiated off at a relatively large angle [12]. The fraction left is then removed using the Monte Carlo simulation based on point-like pions. Finally, the full FSR contribution is added *back* using an analytic expression computed in scalar QED for point-like pions [108], shifting up the value of $a_\mu^{\rm HLO}$ by $\sim 50 \times 10^{-11}$ [12, 84, 92, 107, 109, 110]. (This full scalar-QED FSR contribution is also added to older $\pi^+\pi^-$ data.) This procedure is less than perfect, as it introduces a model dependence which could be avoided by a direct measurement of the cross section into hadronic states inclusive of photons. Any calculation that invokes scalar QED probably falls short of what is needed.

The 2001 final analysis [95] of the precise CMD-2 $\pi^+\pi^-$ data taken in 1994–1995 substantially differed from the preliminary one [94] released two years earlier (based on the same data sample). The difference mostly consisted in the treatment of RC, resulting in a reduction of the cross section by about 1% below the ρ peak and 5% above. A second significant change occurred during the summer of 2003, when the CMD-2 collaboration discovered an error in the Monte Carlo program for Bhabha scattering that was used to determine the luminosity [96]. As a result, the luminosity was overestimated by 2–3%, depending on energy. (Another problem was found in the RC for μ-pairs production.) Overall, the pion-pair cross section increased by 2.1–3.8% in the measured energy range [71], a non-negligible shift. The 2004 results of the KLOE collaboration, obtained via the radiative return method from the ϕ resonance, are in fair agreement with the latest energy scan data from CMD-2 [93, 96, 97]. Here I will only report the evaluations of the dispersive integral in equation (30) based on the latest CMD-2 reanalysis, as it supersedes all earlier ones. These evaluations are in very good agreement:[2]

$$[93] \qquad a_\mu^{\rm HLO} = 6934\,(53)_{\rm exp}(35)_{\rm rad} \times 10^{-11}, \tag{32}$$

$$[88] \qquad a_\mu^{\rm HLO} = 6948\,(86) \times 10^{-11}, \tag{33}$$

$$[89] \qquad a_\mu^{\rm HLO} = 6934\,(92) \times 10^{-11}, \tag{34}$$

$$[91] \qquad a_\mu^{\rm HLO} = 6924\,(59)_{\rm exp}(24)_{\rm rad} \times 10^{-11}, \tag{35}$$

$$[92] \qquad a_\mu^{\rm HLO} = 6944\,(48)_{\rm exp}(10)_{\rm rad} \times 10^{-11}. \tag{36}$$

The preliminary result in equation (32) already includes KLOE's 2004 data analysis and updates the one of [71], shifting it down by 29×10^{-11}; two-thirds of this shift are due to the inclusion of KLOE's data. The preliminary new result in equation (34) updates the value $a_\mu^{\rm HLO} = 6996\,(85)_{\rm exp}(19)_{\rm rad}(20)_{\rm proc} \times 10^{-11}$ previously obtained by the same authors [90]. Their central value decreased because of an improvement of their integration procedure.

The authors of [81] pioneered the idea of using vector spectral functions derived from the study of hadronic τ decays [111] to improve the evaluation of the dispersive integral in equation (30). Indeed, assuming isospin invariance to hold, the isovector part of the cross section for $e^+e^- \to$ hadrons can be calculated via the Conserved Vector Current (CVC) relations from τ-decay spectra. An updated analysis is presented in [71], where τ spectral functions are obtained from the results of ALEPH [112], CLEO [113] and OPAL [114], and

[2] I have translated the results of [92] into the notation of the present paper.

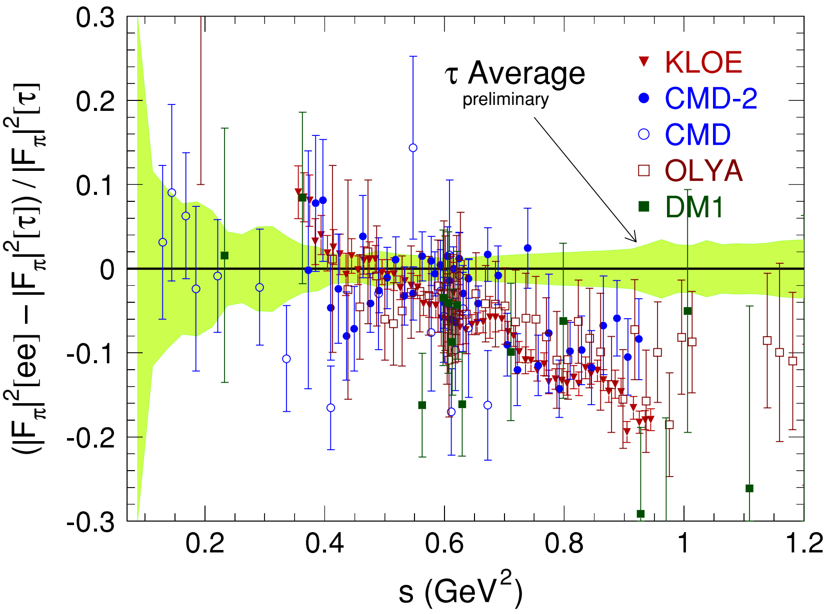

Figure 7. Relative comparison of the $\pi^+\pi^-$ spectral functions from e^+e^- and isospin-breaking-corrected τ data, expressed as a ratio to the τ spectral functions. The band shows the uncertainty of the latter. This figure is from [93].

isospin-breaking corrections are applied [115–117]. In this τ-based evaluation, the 2π and the two 4π channels are taken from τ data up to 1.6 GeV and complemented by e^+e^- data above (the QCD prediction for $R(s)$ is employed above 5 GeV). Note that τ decay experiments measure decay rates which are inclusive with respect to radiative photons. Their result is

$$[71] \qquad a_\mu^{\mathrm{HLO}} = 7110\,(50)_{\mathrm{exp}}\,(8)_{\mathrm{rad}}\,(28)_{SU(2)} \times 10^{-11}, \tag{37}$$

where the quoted uncertainties are experimental, missing radiative corrections to some e^+e^- data, and isospin violation. This value must be compared with their e^+e^--based determination in equation (32). Also the analysis of [92] includes information from τ decay. They obtain

$$[92] \qquad a_\mu^{\mathrm{HLO}} = 7027(47)_{\mathrm{exp}}(10)_{\mathrm{rad}} \times 10^{-11}, \tag{38}$$

to be compared with their determination in equation (36).

Although the latest CMD-2 $e^+e^- \to \pi^+\pi^-$ data are consistent with τ data for the energy region below 850 MeV, there is an unexplained discrepancy for larger energies. This is clearly visible in figure 7, from [93], where the relative comparison of the $\pi^+\pi^-$ spectral functions from e^+e^- and isospin-breaking-corrected τ data is illustrated. The same figure also shows the $\pi^+\pi^-$ spectral functions derived from KLOE's 2004 e^+e^- analysis. They are in fair agreement with those of CMD-2 and confirm the discrepancy with the τ data.

Among the possible causes of this discrepancy, one may wonder about inconsistencies in the e^+e^- data, in the τ data, or in the isospin-breaking corrections applied to the τ spectral functions. Given the good consistency of the ALEPH and CLEO datasets, and the confirmation by KLOE of the trend exhibited by other e^+e^- data, further careful investigations of the isospin-violating effects are clearly warranted—see the interesting studies in [92, 93, 103, 118–120], in particular the discussion of the possible difference between the masses and the widths of neutral and charged ρ-mesons. Until we reach a better understanding of this problem, it is probably safer to discard information from τ decays for the evaluation of a_μ^{HLO} [93].

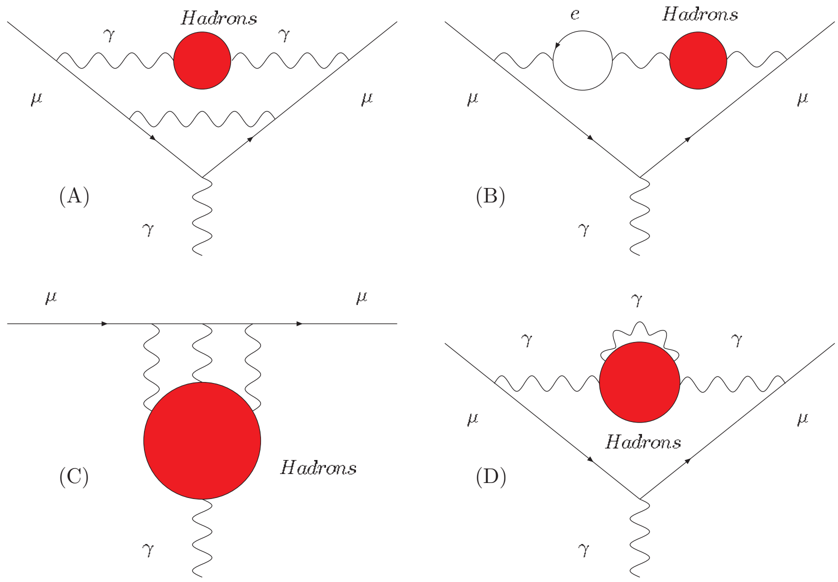

Figure 8. Some of the higher-order hadronic diagrams contributing to a_μ.

4.2. Higher-order hadronic contributions

We will now briefly discuss the $O(\alpha^3)$ hadronic contribution to the muon g-2, a_μ^{HHO}, which can be divided into two parts:

$$a_\mu^{\text{HHO}} = a_\mu^{\text{HHO}}(\text{vp}) + a_\mu^{\text{HHO}}(\text{lbl}). \tag{39}$$

The first term is the $O(\alpha^3)$ contribution of diagrams containing hadronic vacuum polarization insertions, including, among others, those depicted in figures 8(a) and (b). The second one is the light-by-light contribution, shown in figure 8(c). Note that the $O(\alpha^3)$ diagram in figure 8(d) has already been included in the leading-order hadronic contribution a_μ^{HLO} although, unsatisfactorily, using *scalar* QED (see discussion in section 4.1). In recent years, $a_\mu^{\text{HHO}}(\text{vp})$ was evaluated by Krause [121] and slightly updated in [81]. Its latest value is [91]

$$a_\mu^{\text{HHO}}(\text{vp}) = -97.9\,(0.9)_{\text{exp}}(0.3)_{\text{rad}} \times 10^{-11}. \tag{40}$$

This result was obtained using the same hadronic e^+e^- annihilation data described in section 4.1. It changes by about -3×10^{-11} if hadronic τ-decay data (again, see section 4.1) are used instead [15].

The hadronic light-by-light contribution changed sign already three times in its troubled life. Contrary to $a_\mu^{\text{HHO}}(\text{vp})$, it cannot be expressed in terms of experimental observables determined from data and its evaluation therefore relies on purely theoretical considerations. The estimate of the authors of [13, 122, 123], who uncovered in 2001 a sign error in earlier evaluations, is

$$[35] \qquad a_\mu^{\text{HHO}}(\text{lbl}) = +80\,(40) \times 10^{-11}. \tag{41}$$

Earlier determinations now agree with this result [124, 125]. Further studies include [126, 127]. At the end of 2003 a higher value was reported in [128],

$$[128] \qquad a_\mu^{\text{HHO}}(\text{lbl}) = +136\,(25) \times 10^{-11}. \tag{42}$$

It was obtained by including short-distance QCD constraints previously overlooked. Further independent calculations would provide an important check of this result for $a_\mu^{\text{HHO}}(\text{lbl})$, a

Table 1. SM predictions for a_μ compared with the current measured world average value. See text for details.

$a_\mu^{\rm SM} \times 10^{11}$	$(a_\mu^{\rm EXP} - a_\mu^{\rm SM}) \times 10^{11}$	σ	HLO reference
116 591 845 (69)	235 (91)	2.6 (3.0)	[93] (e^+e^-)
116 591 859 (90)	221 (108)	2.1 (2.5)	[88] (e^+e^-)
116 591 845 (95)	235 (113)	2.1 (2.5)	[89] (e^+e^-)
116 591 835 (69)	245 (91)	2.7 (3.1)	[91] (e^+e^-)
116 591 855 (55)	225 (81)	2.8 (3.2)	[92] (e^+e^-)
116 592 018 (63)	62 (87)	0.7 (1.3)	[71] (τ)
116 591 938 (54)	142 (81)	1.8 (2.3)	[92] (e^+e^-, τ)

contribution whose uncertainty may become the ultimate limitation of the SM prediction of the muon g-2.

5. The standard model prediction versus measurement

We now have all the ingredients to derive the SM prediction for a_μ:

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm HLO} + a_\mu^{\rm HHO}({\rm vp}) + a_\mu^{\rm HHO}({\rm lbl}). \tag{43}$$

For convenience, I collect here the values of each term from equations (24), (29), (32)–(38), (40)–(42):

[this paper]		$a_\mu^{\rm QED}$	=	116 584 718.8 (0.5)	$\times 10^{-11}$
[52]		$a_\mu^{\rm EW}$	=	154(1)(2)	$\times 10^{-11}$
[93]	(e^+e^-)	$a_\mu^{\rm HLO}$	=	$6934\,(53)_{\rm exp}(35)_{\rm rad}$	$\times 10^{-11}$
[88]	(e^+e^-)	$a_\mu^{\rm HLO}$	=	6948 (86)	$\times 10^{-11}$
[89]	(e^+e^-)	$a_\mu^{\rm HLO}$	=	6934 (92)	$\times 10^{-11}$
[91]	(e^+e^-)	$a_\mu^{\rm HLO}$	=	$6924\,(59)_{\rm exp}(24)_{\rm rad}$	$\times 10^{-11}$
[92]	(e^+e^-)	$a_\mu^{\rm HLO}$	=	$6944\,(48)_{\rm exp}(10)_{\rm rad}$	$\times 10^{-11}$
[71]	(τ)	$a_\mu^{\rm HLO}$	=	$7110\,(50)_{\rm exp}(8)_{\rm rad}(28)_{SU(2)}$	$\times 10^{-11}$
[92]	(e^+e^-, τ)	$a_\mu^{\rm HLO}$	=	$7027\,(47)_{\rm exp}(10)_{\rm rad}$	$\times 10^{-11}$
[91]	(e^+e^-)	$a_\mu^{\rm HHO}({\rm vp})$	=	$-97.9\,(0.9)_{\rm exp}(0.3)_{\rm rad}$	$\times 10^{-11}$
[15]	(τ)	$a_\mu^{\rm HHO}({\rm vp})$	=	$-101\,(1)$	$\times 10^{-11}$
[13]		$a_\mu^{\rm HHO}({\rm lbl})$	=	80 (40)	$\times 10^{-11}$
[128]		$a_\mu^{\rm HHO}({\rm lbl})$	=	136 (25)	$\times 10^{-11}$

The values I obtain for $a_\mu^{\rm SM}$ are shown in the first column of table 1. The values employed for $a_\mu^{\rm HLO}$ are indicated by the reference in the last column. I used the latest value available for the hadronic light-by-light contribution $a_\mu^{\rm HHO}({\rm lbl}) = 136\,(25) \times 10^{-11}$ [128]. Errors were added in quadrature.

The latest measurement of the anomalous magnetic moment of negative muons by the experiment E821 at Brookhaven is [5]

$$a_{\mu^-}^{\rm EXP} = 116\,592\,140\,(80)\,(30) \times 10^{-11}, \tag{44}$$

where the first uncertainty is statistical and the second is systematic. This result is in good agreement with the average of the measurements of the anomalous magnetic moment of positive muons [2–4, 129], as predicted by the CPT theorem [130]. The present world average experimental value is [5]

$$a_\mu^{\mathrm{EXP}} = 116\,592\,080\,(60) \times 10^{-11} \quad (0.5 \text{ ppm}). \tag{45}$$

The comparison of the SM results with the present experimental average in equation (45) gives the discrepancies ($a_\mu^{\mathrm{EXP}} - a_\mu^{\mathrm{SM}}$) listed in the second column of table 1. The number of standard deviations, shown in the third column, spans a wide range from 0.7 to 2.8. Somewhat higher discrepancies, shown in parentheses in the third column, are obtained if the hadronic light-by-light contribution $a_\mu^{\mathrm{HHO}}(\mathrm{lbl}) = 80(40) \times 10^{-11}$ [13] is used instead of $a_\mu^{\mathrm{HHO}}(\mathrm{lbl}) = 136(25) \times 10^{-11}$ [128], with the number of standard deviations spanning the range [1.3–3.2] instead of [0.7–2.8]. Note that the entries of the first row in table 1 are based on the preliminary result for a_μ^{HLO} of [93], which already includes the recent data from KLOE and updates the one of [71], shifting it down by 29×10^{-11}. As two-thirds of this shift are due to the inclusion of the KLOE data, it is possible that eventually also the a_μ^{HLO} results of [88, 89, 91, 92] will undergo some decrease as a consequence of this inclusion, thus increasing the corresponding $a_\mu^{\mathrm{EXP}} - a_\mu^{\mathrm{SM}}$ discrepancies.

6. Conclusions

In the previous sections I presented an update and a review of the contributions to the SM prediction for the muon g-2. What should we conclude from the wide spectrum of results obtained in section 5? The discrepancies in table 1 between recent SM predictions and the current world average experimental value range from 0.7 to 3.2 standard deviations, according to the values used for the leading-order and light-by-light hadronic contributions. In particular, the contribution of the hadronic vacuum polarization depends on which of the two datasets, e^+e^- collisions or τ decays, are employed.

This puzzling discrepancy between the $\pi^+\pi^-$ spectral functions from e^+e^- and isospin-breaking-corrected τ data could be caused by inconsistencies in the e^+e^- data, in the τ data, or in the isospin-breaking corrections applied to the latter. Given the fair agreement between the CMD-2 and KLOE e^+e^- data, and the good consistency of the ALEPH and CLEO τ spectral functions, it is clear that further careful investigations of the isospin violations are highly warranted. Indeed, the question remains whether all possible isospin-breaking effects have been properly taken into account. Until we reach a better understanding of this problem, it is probably safer to discard information from hadronic τ decays [93]. (Of course, discarding τ data information still leaves us with the problem of their discrepancy, a troublesome issue on its own, independent of the calculation of the muon g-2.) If e^+e^- annihilation data are used to evaluate the leading hadronic contribution, the SM prediction of the muon g-2 deviates from the present experimental value by 2–3 standard deviations.

The measurement of the muon g-2 by the E821 experiment at the Brookhaven Alternating Gradient Synchrotron, with an impressive relative precision of 0.5 ppm, is still limited by statistical errors rather than systematic ones. A new experiment, E969, has been approved (but not yet funded) at Brookhaven in September 2004 [131]. Its goal would be to reduce the present experimental uncertainty by a factor of 2.5 to about 0.2 ppm ($\pm 23 \times 10^{-11}$). A letter of intent for an even more precise g-2 experiment was submitted to J-PARC with the proposal to reach a precision below 0.1 ppm [132]. While the theoretical predictions for the QED and EW contributions appear to be ready to rival these precisions, much effort will be needed in the hadronic sector to test a_μ^{SM} at an accuracy comparable to the experimental one. Such an effort is certainly well motivated by the excellent opportunity the muon g-2 is providing us to unveil or constrain 'new physics' effects.

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