Default Title

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1 Unbiased Estimation

For sampling without replacement, is $\bar{X}^2$ an unbiased estimator for μ^2? If not, what is the bias?

2 8.5

Suppose that X is a discrete random variable with $P(X = 1) = \theta$ and $P(X = 2) = 1 - \theta$. We draw n independent observations and find that n_1 of them take the value 1 and n_2 take the value 2.

1. Find the method of moments estimator of θ.
2. What is the likelihood function?
3. What is the maximum likelihood estimator of θ?

3 8.6

Suppose $X \sim \text{Bin}(n, p)$.

1. Show that the MLE of p is $\hat{p} = \frac{X}{n}$.
2. Show that the MLE from part (a) attains the Cramer-Rao bound.

4 True or False?

1. MLE estimators are always unbiased.
2. The square of the MLE estimator of a parameter θ converges to θ^2 in probability, for any parameter and any observations $X_1, \ldots, X_n$.
3. The MOM estimator always exists.
4. The likelihood function integrates to 1, i.e.,

$$\int_{-\infty}^{\infty} \mathcal{L}_n(\theta) d\theta = 1$$