# R7

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#### 1 9.7

Let  $X_1, \ldots, X_n$  be a sample from a Poisson distribution. Find the likelihood ratio for testing  $H_0$ :  $\lambda = \lambda_0$  versus  $H_A : \lambda = \lambda_1$ , where  $\lambda_1 > \lambda_0$ . Use the fact that the sum of independent Poisson random variables follows a Poisson distribution to explain how to determine a rejection region for a test at level  $\alpha$ .

 $\mathbf{2}$ 

$$X_1, \dots, X_n \sim \mathcal{N}(\theta, 1).$$
  $\Theta_0 = \{\theta \le 0\}, \ \Theta_1 = \{\theta > 0\}.$ 

Construct level- $\alpha(0.05)$  test with statistic  $T = \bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$ .

## 3 9.20

Consider two probability density functions on [0,1]:  $f_0(x)=1$ , and  $f_1(x)=2x$ . Among all tests of the null hypothesis  $H_0: X \sim f_0(x)$  versus the alternative  $X \sim f_1(x)$ , with significance level  $\alpha=0.10$ , how large can the power possibly be?

#### 4 9.12

Let  $X_1, \ldots, X_n$  be a random sample from an exponential distribution with the density function  $f(x \mid \theta) = \theta \exp[-\theta x]$ . Derive a likelihood ratio test of  $H_0: \theta = \theta_0$  versus  $H_A: \theta \neq \theta_0$ , and show that the rejection region is of the form  $\{\bar{X} \exp\left[-\theta_0 \bar{X}\right] \leq c\}$ .