R6

Xuanxi Zhang

1

Suppose random variable $X \sim \mathcal{U}[0, \theta]$, where $\theta > 0$ is an unknown parameter. $\{X_1, X_2, \ldots, X_n\}$ are i.i.d. samples drawn from the distribution of X. Consider two estimators of θ:

$$\hat{\theta}_1 = 2\bar{X}, \quad \hat{\theta}_2 = \max_i X_i,$$

where $\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$ is the sample mean.

1. Find the bias and variance of $\hat{\theta}_1$ and $\hat{\theta}_2$.

2. Find the mean squared error (MSE) of $\hat{\theta}_1$ and $\hat{\theta}_2$.

3. Which estimator would you prefer? Explain your answer.

4. Can we apply the cramer-rao lower bound to this problem? Why or why not?

2 Maximum a Posteriori

$\{X_1, X_2, \ldots, X_n\}$ are i.i.d. samples drawn from $\mathcal{N}(\theta, 1)$.

1. what is the maximum likelihood estimator (MLE) of θ?

2. If we have a prior distribution $\theta \sim \mathcal{N}(\theta_0, \sigma^2)$, what is the maximum a posteriori (MAP) estimator of θ?

3 likelihood ratio test

$X = x$ is a single observation with PDF $f_\theta(x) = \theta x^{\theta-1}, \quad 0 < x < 1$
Find the most powerful level- α (level 5%) test for

$$H_0 : \theta = 3$$
$$H_1 : \theta = 2$$