R7

Xuanxi Zhang

1 9.7

Let $X_1, \ldots, X_n$ be a sample from a Poisson distribution. Find the likelihood ratio for testing H_0 : $\lambda = \lambda_0$ versus $H_A : \lambda = \lambda_1$, where $\lambda_1 > \lambda_0$. Use the fact that the sum of independent Poisson random variables follows a Poisson distribution to explain how to determine a rejection region for a test at level α.

2

$X_1, \ldots, X_n \sim \mathcal{N}(\theta, 1)$.

$$\Theta_0 = \{\theta \leq 0\}, \quad \Theta_1 = \{\theta > 0\}.$$

Construct level-$\alpha(0.05)$ test with statistic $T = \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.

3 9.20

Consider two probability density functions on $[0, 1] : f_0(x) = 1$, and $f_1(x) = 2x$. Among all tests of the null hypothesis $H_0 : X \sim f_0(x)$ versus the alternative $X \sim f_1(x)$, with significance level $\alpha = 0.10$, how large can the power possibly be?

4 9.12

Let $X_1, \ldots, X_n$ be a random sample from an exponential distribution with the density function $f(x \mid \theta) = \theta \exp[-\theta x]$. Derive a likelihood ratio test of $H_0 : \theta = \theta_0$ versus $H_A : \theta \neq \theta_0$, and show that the rejection region is of the form $\left\{\bar{X} \exp\left[-\theta_0 \bar{X}\right] \leq c\right\}$.