QUADROTOR CONTROL USING ADAPTIVE ADRC

Adaptive Filtering

INTRODUCTION

In this work, the altitude control of a quadrotor unmanned aerial vehicle is treated using its altitude dynamics in hover mode.

Due to persistent aerodynamic disturbances, the control of a quadrotor unit is cumbersome; especially, the ground wake effect which comes into play during landing.

In this work, an Adaptive Active disturbance rejection control (AADRC) is proposed to compensate for undesirable effects resulting in smoother control.

The adaptation of the ADRC controller is governed by an LMS algorithm on the extended state observer (ESO).

QUADROTOR MODEL

The nominal altitude model of a quadrotor is given as:

$$\ddot{z} = \frac{U_z}{m}\cos(\phi)\cos(\theta) - g + D_z$$

z is the altitude from ground level, U_z is the control input, m is the mass of the quadrotor, ϕ is the roll angle from the roll subsystem, θ is the pitch angle from the pitch subsystem, D_z is the aerodynamic drag and g is the gravity constant.

$$\ddot{z} = \frac{U_1}{m\left(1 - \rho\left(\frac{r}{4z_r}\right)^2\right)}\cos(\phi)\cos(\theta) - g + D_z$$

In state space form: Let $x_1 = z$ and $x_2 = \dot{z}$ with output $y = x_1$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = G\frac{U_1}{m}\cos(\phi)\cos(\theta) - g + D_z$$

ADRC CONTROL

Active disturbance rejection control ($ADRC$) is a relatively new control technique that has gained attention of the control community.

The novel $ADRC$ technique developed by Jinqing Han was first introduced to English literature in 2001.

$ADRC$ control technique is based on a unique type of disturbance observer known as extended state observer (ESO).

State observers, also known as estimators, are crucial in the design of modern control systems.

They estimate the internal variables of a physical plant (sometimes immeasurable with sensors) using the plant’s input and output data only.

An obvious need for observers arise in flux estimation of A.C. induction motors.

ADRC CONTROL

$$\ddot{x} = f(x_1, x_2, \omega(t), t) + bu$$

In state space form:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = f(x_1, x_2, \omega(t), t) + bu$$

$$y = x_1$$

where $y \in \mathbb{R}$ is the plant output, measurable and to be controlled, $u \in \mathbb{R}$ is the input, and $f(x_1, x_2, \omega(t), t) = F(t)$ is a function of the plant's states: $x_i \in \mathbb{R}$, external disturbances ω, and time t. $F(t)$ is regarded to as the total disturbance and assumed to be differentiable. The goal is usually to make y track a desired signal or reference by manipulating u. Taking $F(t)$ as an additional state variable $x_3 = F(t)$ and denoting $\dot{F}(t) = \bar{G}(t)$, with $\bar{G}(t)$ unknown, the original plant in (5) is now described in extended or augmented form as

ADRC: PLANT + ESO

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3 + bu$$
$$\dot{x}_3 = \bar{G}(t)$$
$$y = x_1$$

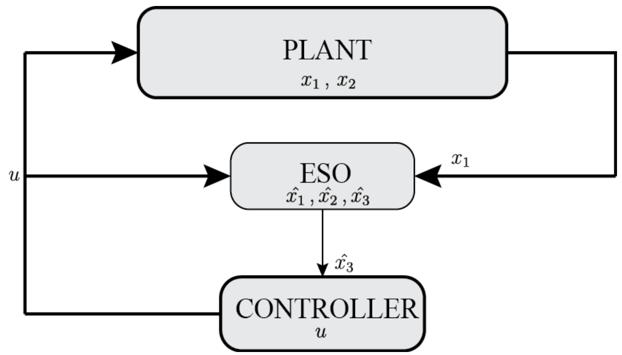
$$\dot{\hat{x}}_1 = \hat{x}_2 + \boldsymbol{p_1}(x_1 - \hat{x}_1)$$
$$\dot{\hat{x}}_2 = \hat{x}_3 + \boldsymbol{p_2}(x_1 - \hat{x}_1) + \hat{b}u$$
$$\dot{\hat{x}}_3 = \boldsymbol{p_3}(x_1 - \hat{x}_1)$$

The system is now presented as:

$$f(x_1, x_2, \omega(t), t) - F(t) \approx 0.$$

$$u = \frac{u_0 - F(t)}{\hat{b}}$$

$$\ddot{x} = (f(x_1, x_2, \omega(t), t) - F(t)) + u_0$$



$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = u_0$$
$$\ddot{\boldsymbol{x}} = u_0, y = x_1, \boldsymbol{x} = [x_1, x_2]^{\mathrm{T}}$$

- $u_0 = k_1(x_{1d} - x_1) + k_2(\dot{x}_{1d} - x_2)$

STATEMENT OF PROBLEM

- We have a system with Gains/Parameters/Weights that need to be chosen.
- The choice of these Gains affect the performance of the Quadrotor's states
- Apart from using static gains, dynamic gains can be utilized
- Dynamic gains result in an adaptive system that accommodates unknown and unwanted factors while maintaining robustness due to a preset policy (cost-function)
- The LMS algorithm is proposed

ASSUMPTIONS AS USUAL …

- Continuous-time system
  - Thus; use continuous-time algorithms (CT-LMS)

- Deterministic System
  - Thus, ignore the 'expectation' ($E[*]$) operator

- Cost function
  - $e = x_1 - \widehat{x_1}$ results in complex integral functions
  - A smart choice: $e = \dot{x_1} - \dot{\widehat{x_1}}$
  - $J = [e]^2$

FORMULATION OF THE LMS

Define the error: $e = \dot{x}_1 - \dot{\hat{x}}_1$

$$e = (x_2) - (\hat{x}_2 + p_1(x_1 - \hat{x}_1))$$

Set Cost-function: $J = [e]^2$

$$J = (x_2 - \hat{x}_2 - p_1 x_1 + p_1 \hat{x}_1)^2$$

Gradient of Cost-function: $\nabla_{p_1} J = 2(x_2 - \hat{x}_2 - p_1 x_1 + p_1 \hat{x}_1)(x_1 - \hat{x}_1)$

$$\nabla_{p_1} J = 2(x_2 - \hat{x}_2)(x_1 - \hat{x}_1) - 2p_1(x_1 + \hat{x}_1)(x_1 - \hat{x}_1)$$

LMS ADAPTATION

$$e = (x_2 - \hat{x}_2) - p_1(x_1 - \hat{x}_1)$$

$$\nabla_{p_1} J = 2(x_2 - \hat{x}_2 - p_1 x_1 + p_1 \hat{x}_1)(x_1 - \hat{x}_1)$$

$$\nabla_{p_1} J = 2e(x_1 - \hat{x}_1)$$

Continuous-time LMS Algorithm

$$\frac{dW(t)}{dt} = -\frac{\rho}{2} \nabla_w J$$

$$\frac{dp_1(t)}{dt} = -\rho e(x_1 - \hat{x}_1)$$

The optimum p_1 is obtained by evaluating $\nabla_{p_1} J = 0$

$$(x_2 - \hat{x}_2) - p_1(x_1 - \hat{x}_1) = 0$$

$$p_1 = \frac{(x_2 - \hat{x}_2)}{(x_1 - \hat{x}_1)}$$

CT-LMS ALGORITHM FOR ADAPTIVE ADRC

Final Algorithm

- Initialize ESO gain p_1 and adaptive gain ρ

$$p_1 > \epsilon \text{ and } \rho > 0$$

- Calculate the Error

$$e = (x_2 - \hat{x}_2) - p_1(x_1 - \hat{x}_1)$$

- Update the ESO gain

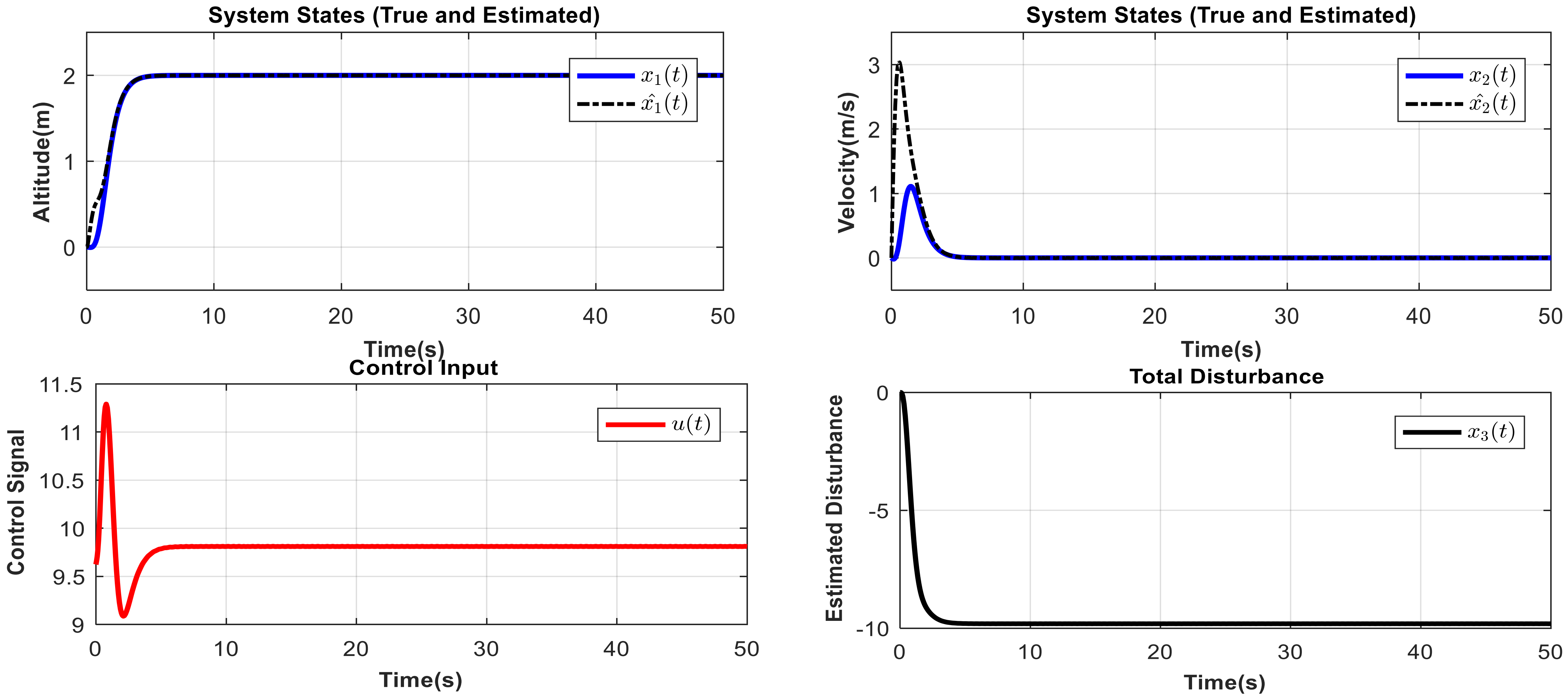
$$\dot{p}_1 = -\rho e(x_1 - \hat{x}_1)$$

SIMULATION & PARAMETERS

> Matlab Simulation

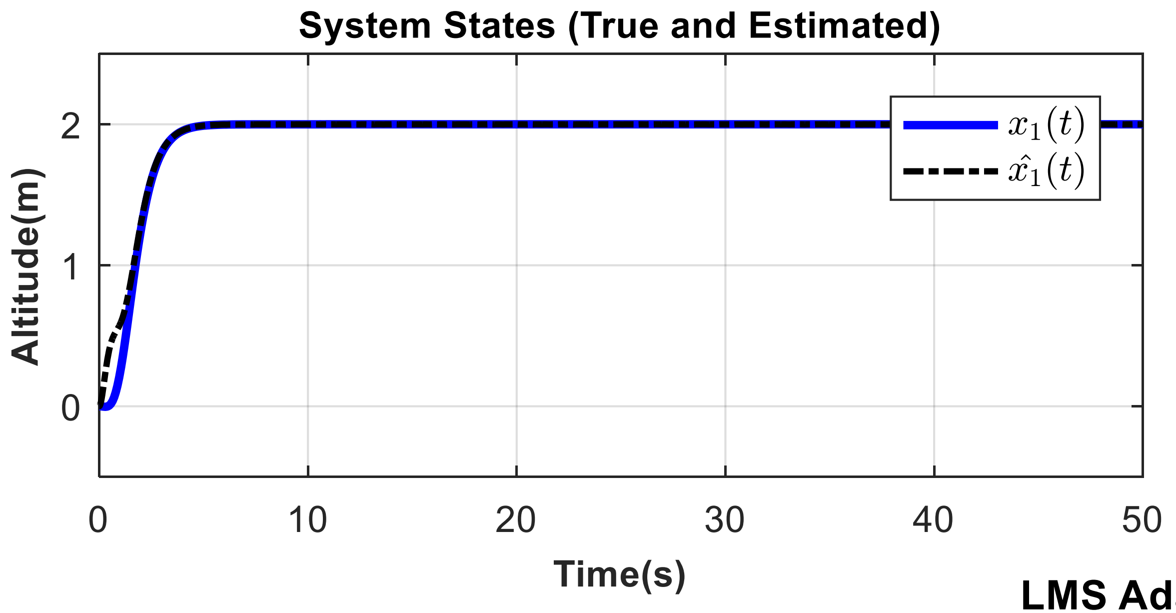
No.	Quadrotor Parameters		
	Parameter	Symbol	Value
1	Mass	$m(kg)$	2
2	Rotor radius	$r(m)$	0.1905
5	Altitide sensor noise	$\sigma_z^2(m)$	0.14^2
3	Aerodynamic friction coefficient	k_z	0.3729
4	Ground effect coefficient	ρ	8.6
5	Gravitational constant	$g(ms^{-2})$	9.81
6	Initial p_1	p_1	50
7	Adaptive gain	ρ	1

RESULTS

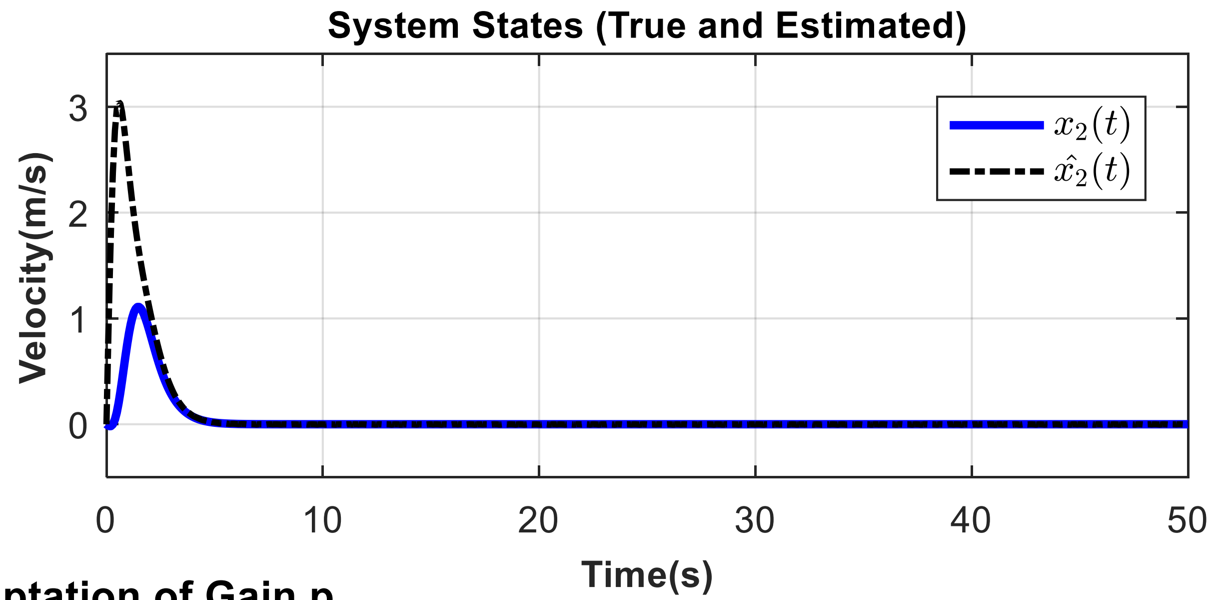


System States (True and Estimated)
Altitude(m)
Time(s)
$x_1(t)$
$\hat{x}_1(t)$
0
1
2
0
10
20
30
40
50
System States (True and Estimated)
Velocity(m/s)
Time(s)
$x_2(t)$
$\hat{x}_2(t)$
0
1
2
3
0
10
20
30
40
50
Control Input
Control Signal
Time(s)
$u(t)$
9
9.5
10
10.5
11
11.5
0
10
20
30
40
50
Total Disturbance
Estimated Disturbance
Time(s)
$x_3(t)$
-10
-5
0
0
10
20
30
40
50

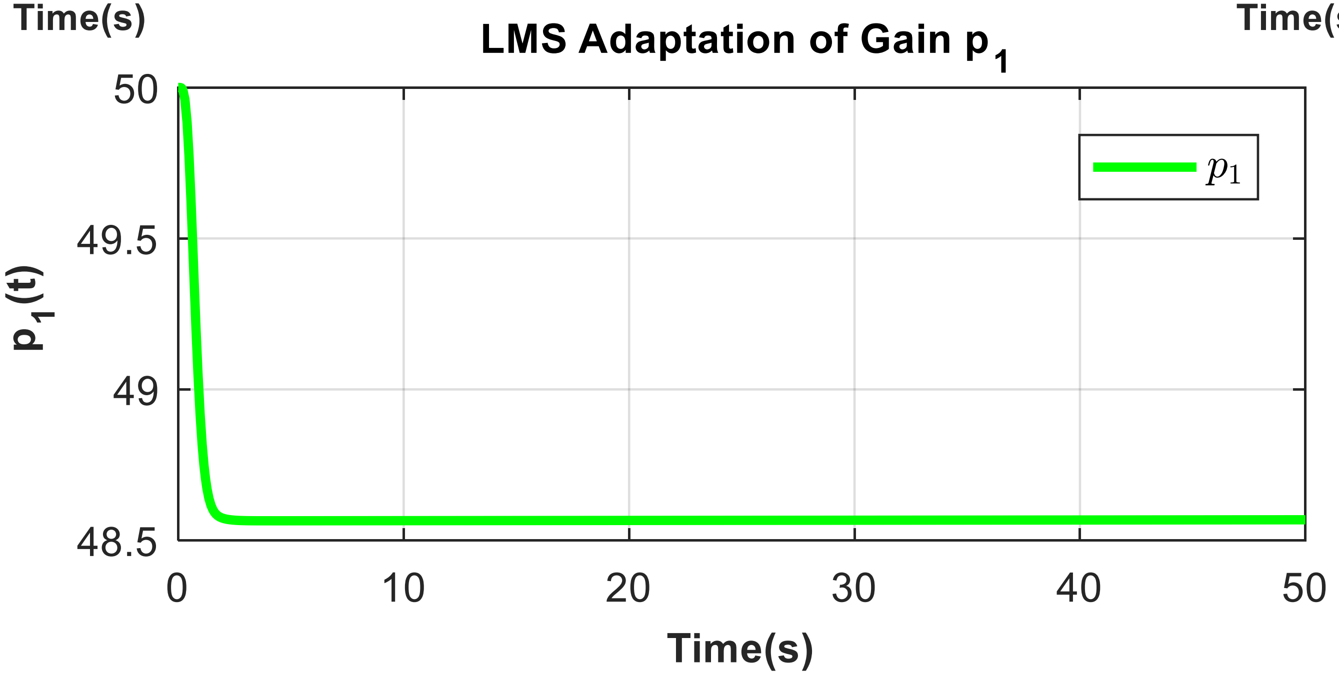
RESULTS



System States (True and Estimated)
$x_1(t)$
$\hat{x}_1(t)$
Altitude(m)
Time(s)
0
1
2
0
10
20
30
40
50



System States (True and Estimated)
$x_2(t)$
$\hat{x}_2(t)$
Velocity(m/s)
Time(s)
0
1
2
3
0
10
20
30
40
50



LMS Adaptation of Gain p_1
p_1
$p_1(t)$
Time(s)
48.5
49
49.5
50
0
10
20
30
40
50

SUMMARY & CONCLUSION

- In summary, the quadrotor altitude control was solved using a technique which observes the total disturbances in the system and actively compensates for it.
- An LMS algorithm for the adaptation of the observer gain was presented and successfully implemented in Matlab.

$$\text{Optimal weight } \boldsymbol{p_1} \text{ via LMS} = 48.4$$

$$\text{Optimal weight } \boldsymbol{p_1} \text{ via SQP} = 55$$

- Results are pretty.
- Need for further analysis (stability, transients, multiobjective).

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