3 Subject content

The mathematical content for each component is detailed below. You can teach the topics in any order you find appropriate.

Information about calculator use and information about the relationships between syllabus components can be found in 4 Details of the assessment.

Notes and examples are included to clarify the subject content. Please note that these are examples only and examination questions may differ from the examples given.

Prior knowledge

Knowledge of the content of the Cambridge IGCSE® Mathematics 0580 (Extended curriculum), or Cambridge International O Level (4024/4029), is assumed. Candidates should be familiar with scientific notation for compound units, e.g. $5\,\mathrm{m\,s^{-1}}$ for 5 metres per second.

In addition, candidates should:

- be able to carry out simple manipulation of surds (e.g. expressing $\sqrt{12}$ as $2\sqrt{3}$ and $\frac{6}{\sqrt{2}}$ as $3\sqrt{2}$),
- know the shapes of graphs of the form $y = kx^n$, where k is a constant and n is an integer (positive or negative) or $\pm\frac{1}{2}$.

1 Pure Mathematics 1 (for Paper 1)

1.1 Quadratics

Candidates should be able to:	Notes and examples
• carry out the process of completing the square for a quadratic polynomial $ax^2 + bx + c$ and use a completed square form	e.g. to locate the vertex of the graph of $y = ax^2 + bx + c$ or to sketch the graph
• find the discriminant of a quadratic polynomial $ax^2 + bx + c$ and use the discriminant	e.g. to determine the number of real roots of the equation $ax^2 + bx + c = 0$. Knowledge of the term 'repeated root' is included.
• solve quadratic equations, and quadratic inequalities, in one unknown	By factorising, completing the square and using the formula.
• solve by substitution a pair of simultaneous equations of which one is linear and one is quadratic	e.g. $x + y + 1 = 0$ and $x^2 + y^2 = 25$, $2x + 3y = 7$ and $3x^2 = 4 + 4xy$.
• recognise and solve equations in x which are quadratic in some function of x.	e.g. $x^4 - 5x^2 + 4 = 0$, $6x + \sqrt{x} - 1 = 0$, $\tan^2 x = 1 + \tan x$.

1 Pure Mathematics 1

1.2 Functions

Candidates should be able to:	Notes and examples
• understand the terms function, domain, range, one-one function, inverse function and composition of functions	
• identify the range of a given function in simple cases, and find the composition of two given functions	e.g. range of $\mathrm{f}: x \mapsto \frac{1}{x}$ for $x \geqslant 1$ and range of $\mathrm{g}: x \mapsto x^2 + 1$ for $x \in \mathbb{R}$. Including the condition that a composite function gf can only be formed when the range of f is within the domain of g.
• determine whether or not a given function is one-one, and find the inverse of a one-one function in simple cases	e.g. finding the inverse of $\mathrm{h}: x \mapsto (2x+3)^2 - 4$ for $x < -\frac{3}{2}$.
• illustrate in graphical terms the relation between a one-one function and its inverse	Sketches should include an indication of the mirror line $y = x$.
• understand and use the transformations of the graph of $y = \mathrm{f}(x)$ given by $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = a\mathrm{f}(x)$, $y = \mathrm{f}(ax)$ and simple combinations of these.	Including use of the terms ‘translation’, ‘reflection’ and ‘stretch’ in describing transformations. Questions may involve algebraic or trigonometric functions, or other graphs with given features.

1.3 Coordinate geometry

Candidates should be able to:	Notes and examples
• find the equation of a straight line given sufficient information	e.g. given two points, or one point and the gradient.
• interpret and use any of the forms $y = mx + c$, $y - y_1 = m(x - x_1)$, $ax + by + c = 0$ in solving problems	Including calculations of distances, gradients, midpoints, points of intersection and use of the relationship between the gradients of parallel and perpendicular lines.
• understand that the equation $(x-a)^2 + (y-b)^2 = r^2$ represents the circle with centre (a, b) and radius r	Including use of the expanded form $x^2 + y^2 + 2gx + 2fy + c = 0$.
• use algebraic methods to solve problems involving lines and circles	Including use of elementary geometrical properties of circles, e.g. tangent perpendicular to radius, angle in a semicircle, symmetry. Implicit differentiation is not included.
• understand the relationship between a graph and its associated algebraic equation, and use the relationship between points of intersection of graphs and solutions of equations.	e.g. to determine the set of values of k for which the line $y = x + k$ intersects, touches or does not meet a quadratic curve.

1 Pure Mathematics 1

1.4 Circular measure

Candidates should be able to:	Notes and examples
• understand the definition of a radian, and use the relationship between radians and degrees	
• use the formulae $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ in solving problems concerning the arc length and sector area of a circle.	Including calculation of lengths and angles in triangles and areas of triangles.

1.5 Trigonometry

Candidates should be able to:	Notes and examples
• sketch and use graphs of the sine, cosine and tangent functions (for angles of any size, and using either degrees or radians)	Including e.g. $y = 3\sin x$, $y = 1 - \cos 2x$, $y = \tan\left(x + \frac{1}{4}\pi\right)$.
• use the exact values of the sine, cosine and tangent of 30°, 45°, 60°, and related angles	e.g. $\cos 150° = -\frac{1}{2}\sqrt{3}$, $\sin \frac{3}{4}\pi = \frac{1}{\sqrt{2}}$.
• use the notations $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ to denote the principal values of the inverse trigonometric relations	No specialised knowledge of these functions is required, but understanding of them as examples of inverse functions is expected.
• use the identities $\frac{\sin\theta}{\cos\theta} \equiv \tan\theta$ and $\sin^2\theta + \cos^2\theta \equiv 1$	e.g. in proving identities, simplifying expressions and solving equations.
• find all the solutions of simple trigonometrical equations lying in a specified interval (general forms of solution are not included).	e.g. solve $3\sin 2x + 1 = 0$ for $-\pi < x < \pi$, $3\sin^2\theta - 5\cos\theta - 1 = 0$ for $0° \leqslant \theta \leqslant 360°$.

1.6 Series

Candidates should be able to:	Notes and examples
• use the expansion of $(a + b)^n$, where n is a positive integer	Including the notations $\binom{n}{r}$ and $n!$ Knowledge of the greatest term and properties of the coefficients are not required.
• recognise arithmetic and geometric progressions	
• use the formulae for the nth term and for the sum of the first n terms to solve problems involving arithmetic or geometric progressions	Including knowledge that numbers a, b, c are 'in arithmetic progression' if $2b = a + c$ (or equivalent) and are 'in geometric progression' if $b^2 = ac$ (or equivalent). Questions may involve more than one progression.
• use the condition for the convergence of a geometric progression, and the formula for the sum to infinity of a convergent geometric progression.	

1 Pure Mathematics 1

1.7 Differentiation

Candidates should be able to:	Notes and examples
• understand the gradient of a curve at a point as the limit of the gradients of a suitable sequence of chords, and use the notations $f'(x)$, $f''(x)$, $\frac{dy}{dx}$, and $\frac{d^2y}{dx^2}$ for first and second derivatives	Only an informal understanding of the idea of a limit is expected. e.g. includes consideration of the gradient of the chord joining the points with x coordinates 2 and $(2+h)$ on the curve $y = x^3$. Formal use of the general method of differentiation from first principles is not required.
• use the derivative of x^n (for any rational n), together with constant multiples, sums and differences of functions, and of composite functions using the chain rule	e.g. find $\frac{dy}{dx}$, given $y = \sqrt{2x^3+5}$.
• apply differentiation to gradients, tangents and normals, increasing and decreasing functions and rates of change	Including connected rates of change, e.g. given the rate of increase of the radius of a circle, find the rate of increase of the area for a specific value of one of the variables.
• locate stationary points and determine their nature, and use information about stationary points in sketching graphs.	Including use of the second derivative for identifying maxima and minima; alternatives may be used in questions where no method is specified. Knowledge of points of inflexion is not included.

1.8 Integration

Candidates should be able to:	Notes and examples
• understand integration as the reverse process of differentiation, and integrate $(ax+b)^n$ (for any rational n except –1), together with constant multiples, sums and differences	e.g. $\int (2x^3 - 5x + 1)\,dx$, $\int \frac{1}{(2x+3)^2}\,dx$.
• solve problems involving the evaluation of a constant of integration	e.g. to find the equation of the curve through $(1, -2)$ for which $\frac{dy}{dx} = \sqrt{2x+1}$.
• evaluate definite integrals	Including simple cases of 'improper' integrals, such as $\int_0^1 x^{-\frac{1}{2}}\,dx$ and $\int_1^\infty x^{-2}\,dx$.
• use definite integration to find – the area of a region bounded by a curve and lines parallel to the axes, or between a curve and a line or between two curves – a volume of revolution about one of the axes.	A volume of revolution may involve a region not bounded by the axis of rotation, e.g. the region between $y = 9 - x^2$ and $y = 5$ rotated about the x-axis.

2 Pure Mathematics 2 (for Paper 2)

Knowledge of the content for Paper 1: Pure Mathematics 1 is assumed, and candidates may be required to demonstrate such knowledge in answering questions.

2.1 Algebra

Candidates should be able to:	Notes and examples
• understand the meaning of $\lvert x \rvert$, sketch the graph of $y = \lvert ax + b \rvert$ and use relations such as $\lvert a \rvert = \lvert b \rvert \Leftrightarrow a^2 = b^2$ and $\lvert x - a \rvert < b \Leftrightarrow a - b < x < a + b$ when solving equations and inequalities	Graphs of $y = \lvert \mathrm{f}(x) \rvert$ and $y = \mathrm{f}(\lvert x \rvert)$ for non-linear functions f are not included. e.g. $\lvert 3x - 2 \rvert = \lvert 2x + 7 \rvert$, $2x + 5 < \lvert x + 1 \rvert$
• divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero)	
• use the factor theorem and the remainder theorem.	e.g. to find factors and remainders, solve polynomial equations or evaluate unknown coefficients. Including factors of the form $(ax + b)$ in which the coefficient of x is not unity, and including calculation of remainders.

2.2 Logarithmic and exponential functions

Candidates should be able to:	Notes and examples
• understand the relationship between logarithms and indices, and use the laws of logarithms (excluding change of base)	
• understand the definition and properties of e^x and $\ln x$, including their relationship as inverse functions and their graphs	Including knowledge of the graph of $y = \mathrm{e}^{kx}$ for both positive and negative values of k.
• use logarithms to solve equations and inequalities in which the unknown appears in indices	e.g. $2^x < 5$, $3 \times 2^{3x-1} < 5$, $3^{x+1} = 4^{2x-1}$.
• use logarithms to transform a given relationship to linear form, and hence determine unknown constants by considering the gradient and/or intercept.	e.g. $y = kx^n$ gives $\ln y = \ln k + n \ln x$ which is linear in $\ln x$ and $\ln y$ $y = k(a^x)$ gives $\ln y = \ln k + x \ln a$ which is linear in x and $\ln y$.

2 Pure Mathematics 2

2.3 Trigonometry

Candidates should be able to:	Notes and examples
• understand the relationship of the secant, cosecant and cotangent functions to cosine, sine and tangent, and use properties and graphs of all six trigonometric functions for angles of any magnitude	
• use trigonometrical identities for the simplification and exact evaluation of expressions, and in the course of solving equations, and select an identity or identities appropriate to the context, showing familiarity in particular with the use of – $\sec^2\theta \equiv 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta \equiv 1 + \cot^2\theta$ – the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ – the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$ – the expression of $a\sin\theta + b\cos\theta$ in the forms $R\sin(\theta \pm \alpha)$ and $R\cos(\theta \pm \alpha)$.	e.g. simplifying $\cos(x - 30°) - 3\sin(x - 60°)$. e.g. solving $\tan\theta + \cot\theta = 4$, $2\sec^2\theta - \tan\theta = 5$, $3\cos\theta + 2\sin\theta = 1$.

2.4 Differentiation

Candidates should be able to:	Notes and examples
• use the derivatives of e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$, together with constant multiples, sums, differences and composites	
• differentiate products and quotients	e.g. $\frac{2x-4}{3x+2}$, $x^2 \ln x$, xe^{1-x^2}.
• find and use the first derivative of a function which is defined parametrically or implicitly.	e.g. $x = t - e^{2t}$, $y = t + e^{2t}$. e.g. $x^2 + y^2 = xy + 7$. Including use in problems involving tangents and normals.

2.5 Integration

Candidates should be able to:	Notes and examples
• extend the idea of 'reverse differentiation' to include the integration of e^{ax+b}, $\frac{1}{ax+b}$, $\sin(ax+b)$, $\cos(ax+b)$ and $\sec^2(ax+b)$	Knowledge of the general method of integration by substitution is not required.
• use trigonometrical relationships in carrying out integration	e.g. use of double-angle formulae to integrate $\sin^2 x$ or $\cos^2(2x)$.
• understand and use the trapezium rule to estimate the value of a definite integral.	Including use of sketch graphs in simple cases to determine whether the trapezium rule gives an over-estimate or an under-estimate.

2 Pure Mathematics 2

2.6 Numerical solution of equations

Candidates should be able to:	Notes and examples
• locate approximately a root of an equation, by means of graphical considerations and/or searching for a sign change	e.g. finding a pair of consecutive integers between which a root lies.
• understand the idea of, and use the notation for, a sequence of approximations which converges to a root of an equation	
• understand how a given simple iterative formula of the form $x_{n+1} = \mathrm{F}(x_n)$ relates to the equation being solved, and use a given iteration, or an iteration based on a given rearrangement of an equation, to determine a root to a prescribed degree of accuracy.	Knowledge of the condition for convergence is not included, but an understanding that an iteration may fail to converge is expected.

3 Pure Mathematics 3 (for Paper 3)

Knowledge of the content of Paper 1: Pure Mathematics 1 is assumed, and candidates may be required to demonstrate such knowledge in answering questions.

3.1 Algebra

Candidates should be able to:	Notes and examples
• understand the meaning of $\lvert x\rvert$, sketch the graph of $y = \lvert ax + b\rvert$ and use relations such as $\lvert a\rvert = \lvert b\rvert \Leftrightarrow a^2 = b^2$ and $\lvert x - a\rvert < b \Leftrightarrow a - b < x < a + b$ when solving equations and inequalities	Graphs of $y = \lvert \mathrm{f}(x)\rvert$ and $y = \mathrm{f}(\lvert x\rvert)$ for non-linear functions f are not included. e.g. $\lvert 3x - 2\rvert = \lvert 2x + 7\rvert$, $2x + 5 < \lvert x + 1\rvert$.
• divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero)	
• use the factor theorem and the remainder theorem	e.g. to find factors and remainders, solve polynomial equations or evaluate unknown coefficients. Including factors of the form $(ax + b)$ in which the coefficient of x is not unity, and including calculation of remainders.
• recall an appropriate form for expressing rational functions in partial fractions, and carry out the decomposition, in cases where the denominator is no more complicated than – $(ax + b)(cx + d)(ex + f)$ – $(ax + b)(cx + d)^2$ – $(ax + b)(cx^2 + d)$	Excluding cases where the degree of the numerator exceeds that of the denominator
• use the expansion of $(1 + x)^n$, where n is a rational number and $\lvert x\rvert < 1$.	Finding the general term in an expansion is not included. Adapting the standard series to expand e.g. $\left(2 - \frac{1}{2}x\right)^{-1}$ is included, and determining the set of values of x for which the expansion is valid in such cases is also included.

3 Pure Mathematics 3

3.2 Logarithmic and exponential functions

Candidates should be able to:	Notes and examples
• understand the relationship between logarithms and indices, and use the laws of logarithms (excluding change of base)	
• understand the definition and properties of e^x and $\ln x$, including their relationship as inverse functions and their graphs	Including knowledge of the graph of $y = \mathrm{e}^{kx}$ for both positive and negative values of k.
• use logarithms to solve equations and inequalities in which the unknown appears in indices	e.g. $2^x < 5$, $3 \times 2^{3x-1} < 5$, $3^{x+1} = 4^{2x-1}$.
• use logarithms to transform a given relationship to linear form, and hence determine unknown constants by considering the gradient and/or intercept.	e.g. $y = kx^n$ gives $\ln y = \ln k + n \ln x$ which is linear in $\ln x$ and $\ln y$. $y = k(a^x)$ gives $\ln y = \ln k + x \ln a$ which is linear in x and $\ln y$.

3.3 Trigonometry

Candidates should be able to:	Notes and examples
• understand the relationship of the secant, cosecant and cotangent functions to cosine, sine and tangent, and use properties and graphs of all six trigonometric functions for angles of any magnitude	
• use trigonometrical identities for the simplification and exact evaluation of expressions, and in the course of solving equations, and select an identity or identities appropriate to the context, showing familiarity in particular with the use of – $\sec^2\theta \equiv 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta \equiv 1 + \cot^2\theta$ – the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ – the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$ – the expression of $a\sin\theta + b\cos\theta$ in the forms $R\sin(\theta \pm \alpha)$ and $R\cos(\theta \pm \alpha)$.	e.g. simplifying $\cos(x - 30°) - 3\sin(x - 60°)$. e.g. solving $\tan\theta + \cot\theta = 4$, $2\sec^2\theta - \tan\theta = 5$, $3\cos\theta + 2\sin\theta = 1$.

3 Pure Mathematics 3

3.4 Differentiation

Candidates should be able to:	Notes and examples
• use the derivatives of e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$, $\tan^{-1} x$, together with constant multiples, sums, differences and composites	Derivatives of $\sin^{-1} x$ and $\cos^{-1} x$ are not required.
• differentiate products and quotients	e.g. $\dfrac{2x-4}{3x+2}$, $x^2 \ln x$, xe^{1-x^2}.
• find and use the first derivative of a function which is defined parametrically or implicitly.	e.g. $x = t - e^{2t}$, $y = t + e^{2t}$. e.g. $x^2 + y^2 = xy + 7$. Including use in problems involving tangents and normals.

3.5 Integration

Candidates should be able to:	Notes and examples
• extend the idea of 'reverse differentiation' to include the integration of e^{ax+b}, $\dfrac{1}{ax+b}$, $\sin(ax+b)$, $\cos(ax+b)$, $\sec^2(ax+b)$ and $\dfrac{1}{x^2+a^2}$	Including examples such as $\dfrac{1}{2+3x^2}$.
• use trigonometrical relationships in carrying out integration	e.g. use of double-angle formulae to integrate $\sin^2 x$ or $\cos^2(2x)$.
• integrate rational functions by means of decomposition into partial fractions	Restricted to types of partial fractions as specified in topic 3.1 above.
• recognise an integrand of the form $\dfrac{kf'(x)}{f(x)}$, and integrate such functions	e.g. integration of $\dfrac{x}{x^2+1}$, $\tan x$.
• recognise when an integrand can usefully be regarded as a product, and use integration by parts	e.g. integration of $x \sin 2x$, $x^2 e^{-x}$, $\ln x$, $x \tan^{-1} x$.
• use a given substitution to simplify and evaluate either a definite or an indefinite integral.	e.g. to integrate $\sin^2 2x \cos x$ using the substitution $u = \sin x$.

3 Pure Mathematics 3

3.6 Numerical solution of equations

Candidates should be able to:	Notes and examples
• locate approximately a root of an equation, by means of graphical considerations and/or searching for a sign change	e.g. finding a pair of consecutive integers between which a root lies.
• understand the idea of, and use the notation for, a sequence of approximations which converges to a root of an equation	
• understand how a given simple iterative formula of the form $x_{n+1} = \mathrm{F}(x_n)$ relates to the equation being solved, and use a given iteration, or an iteration based on a given rearrangement of an equation, to determine a root to a prescribed degree of accuracy.	Knowledge of the condition for convergence is not included, but an understanding that an iteration may fail to converge is expected.

3.7 Vectors

Candidates should be able to:	Notes and examples
• use standard notations for vectors, i.e. $\begin{pmatrix} x \\ y \end{pmatrix}$, $x\mathbf{i} + y\mathbf{j}$, $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, $\overrightarrow{AB}$, $\mathbf{a}$	
• carry out addition and subtraction of vectors and multiplication of a vector by a scalar, and interpret these operations in geometrical terms	e.g. '$OABC$ is a parallelogram' is equivalent to $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{OC}$. The general form of the ratio theorem is not included, but understanding that the midpoint of AB has position vector $\frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})$ is expected.
• calculate the magnitude of a vector, and use unit vectors, displacement vectors and position vectors	In 2 or 3 dimensions.
• understand the significance of all the symbols used when the equation of a straight line is expressed in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, and find the equation of a line, given sufficient information	e.g. finding the equation of a line given the position vector of a point on the line and a direction vector, or the position vectors of two points on the line.
• determine whether two lines are parallel, intersect or are skew, and find the point of intersection of two lines when it exists	Calculation of the shortest distance between two skew lines is not required. Finding the equation of the common perpendicular to two skew lines is also not required.
• use formulae to calculate the scalar product of two vectors, and use scalar products in problems involving lines and points.	e.g. finding the angle between two lines, and finding the foot of the perpendicular from a point to a line; questions may involve 3D objects such as cuboids, tetrahedra (pyramids), etc. Knowledge of the vector product is not required.

3 Pure Mathematics 3

3.8 Differential equations

Candidates should be able to:	Notes and examples
• formulate a simple statement involving a rate of change as a differential equation	The introduction and evaluation of a constant of proportionality, where necessary, is included.
• find by integration a general form of solution for a first order differential equation in which the variables are separable	Including any of the integration techniques from topic 3.5 above.
• use an initial condition to find a particular solution	
• interpret the solution of a differential equation in the context of a problem being modelled by the equation.	Where a differential equation is used to model a 'real-life' situation, no specialised knowledge of the context will be required.

3.9 Complex numbers

Candidates should be able to:	Notes and examples
• understand the idea of a complex number, recall the meaning of the terms real part, imaginary part, modulus, argument, conjugate, and use the fact that two complex numbers are equal if and only if both real and imaginary parts are equal	Notations $\mathrm{Re}\,z$, $\mathrm{Im}\,z$, $\lvert z\rvert$, $\arg z$, z^* should be known. The argument of a complex number will usually refer to an angle θ such that $-\pi < \theta \leqslant \pi$, but in some cases the interval $0 \leqslant \theta < 2\pi$ may be more convenient. Answers may use either interval unless the question specifies otherwise.
• carry out operations of addition, subtraction, multiplication and division of two complex numbers expressed in Cartesian form $x + \mathrm{i}y$	For calculations involving multiplication or division, full details of the working should be shown.
• use the result that, for a polynomial equation with real coefficients, any non-real roots occur in conjugate pairs	e.g. in solving a cubic or quartic equation where one complex root is given.
• represent complex numbers geometrically by means of an Argand diagram	
• carry out operations of multiplication and division of two complex numbers expressed in polar form $r(\cos\theta + \mathrm{i}\sin\theta) \equiv r\mathrm{e}^{\mathrm{i}\theta}$	Including the results $\lvert z_1 z_2\rvert = \lvert z_1\rvert\lvert z_2\rvert$ and $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$, and corresponding results for division.
• find the two square roots of a complex number	e.g. the square roots of $5 + 12\mathrm{i}$ in exact Cartesian form. Full details of the working should be shown.
• understand in simple terms the geometrical effects of conjugating a complex number and of adding, subtracting, multiplying and dividing two complex numbers	
• illustrate simple equations and inequalities involving complex numbers by means of loci in an Argand diagram	e.g. $\lvert z - a\rvert < k$, $\lvert z - a\rvert = \lvert z - b\rvert$, $\arg(z - a) = \alpha$.

4 Mechanics (for Paper 4)

Questions set will be mainly numerical, and will aim to test mechanical principles without involving difficult algebra or trigonometry. However, candidates should be familiar in particular with the following trigonometrical results:

$$\sin(90^\circ - \theta) \equiv \cos\theta,\ \cos(90^\circ - \theta) \equiv \sin\theta,\ \tan\theta \equiv \frac{\sin\theta}{\cos\theta},\ \sin^2\theta + \cos^2\theta \equiv 1.$$

Knowledge of algebraic methods from the content for Paper 1: Pure Mathematics 1 is assumed.

This content list refers to the equilibrium or motion of a 'particle'. Examination questions may involve extended bodies in a 'realistic' context, but these extended bodies should be treated as particles, so any force acting on them is modelled as acting at a single point.

Vector notation will not be used in the question papers.

4.1 Forces and equilibrium

Candidates should be able to:	Notes and examples
• identify the forces acting in a given situation	e.g. by drawing a force diagram.
• understand the vector nature of force, and find and use components and resultants	Calculations are always required, not approximate solutions by scale drawing.
• use the principle that, when a particle is in equilibrium, the vector sum of the forces acting is zero, or equivalently, that the sum of the components in any direction is zero	Solutions by resolving are usually expected, but equivalent methods (e.g. triangle of forces, Lami's Theorem, where suitable) are also acceptable; these other methods are not required knowledge, and will not be referred to in questions.
• understand that a contact force between two surfaces can be represented by two components, the normal component and the frictional component	
• use the model of a 'smooth' contact, and understand the limitations of this model	
• understand the concepts of limiting friction and limiting equilibrium, recall the definition of coefficient of friction, and use the relationship $F = \mu R$ or $F \leqslant \mu R$, as appropriate	Terminology such as 'about to slip' may be used to mean 'in limiting equilibrium' in questions.
• use Newton's third law.	e.g. the force exerted by a particle on the ground is equal and opposite to the force exerted by the ground on the particle.

4 Mechanics

4.2 Kinematics of motion in a straight line

Candidates should be able to:	Notes and examples
• understand the concepts of distance and speed as scalar quantities, and of displacement, velocity and acceleration as vector quantities	Restricted to motion in one dimension only. The term 'deceleration' may sometimes be used in the context of decreasing speed.
• sketch and interpret displacement–time graphs and velocity–time graphs, and in particular appreciate that – the area under a velocity–time graph represents displacement, – the gradient of a displacement–time graph represents velocity, – the gradient of a velocity–time graph represents acceleration	
• use differentiation and integration with respect to time to solve simple problems concerning displacement, velocity and acceleration	Calculus required is restricted to techniques from the content for Paper 1: Pure Mathematics 1.
• use appropriate formulae for motion with constant acceleration in a straight line.	Questions may involve setting up more than one equation, using information about the motion of different particles.

4.3 Momentum

Candidates should be able to:	Notes and examples
• use the definition of linear momentum and show understanding of its vector nature	For motion in one dimension only.
• use conservation of linear momentum to solve problems that may be modelled as the direct impact of two bodies.	Including direct impact of two bodies where the bodies coalesce on impact. Knowledge of impulse and the coefficient of restitution is not required.

4 Mechanics

4.4 Newton's laws of motion

Candidates should be able to:	Notes and examples
• apply Newton's laws of motion to the linear motion of a particle of constant mass moving under the action of constant forces, which may include friction, tension in an inextensible string and thrust in a connecting rod	If any other forces resisting motion are to be considered (e.g. air resistance) this will be indicated in the question.
• use the relationship between mass and weight	$W = mg$. In this component, questions are mainly numerical, and use of the approximate numerical value 10 ($\mathrm{m\,s^{-2}}$) for g is expected.
• solve simple problems which may be modelled as the motion of a particle moving vertically or on an inclined plane with constant acceleration	Including, for example, motion of a particle on a rough plane where the acceleration while moving up the plane is different from the acceleration while moving down the plane.
• solve simple problems which may be modelled as the motion of connected particles.	e.g. particles connected by a light inextensible string passing over a smooth pulley, or a car towing a trailer by means of either a light rope or a light rigid tow-bar.

4.5 Energy, work and power

Candidates should be able to:	Notes and examples
• understand the concept of the work done by a force, and calculate the work done by a constant force when its point of application undergoes a displacement not necessarily parallel to the force	$W = Fd\cos\theta$; Use of the scalar product is not required.
• understand the concepts of gravitational potential energy and kinetic energy, and use appropriate formulae	
• understand and use the relationship between the change in energy of a system and the work done by the external forces, and use in appropriate cases the principle of conservation of energy	Including cases where the motion may not be linear (e.g. a child on a smooth curved 'slide'), where only overall energy changes need to be considered.
• use the definition of power as the rate at which a force does work, and use the relationship between power, force and velocity for a force acting in the direction of motion	Including calculation of (average) power as $\dfrac{\text{Work done}}{\text{Time taken}}$. $P = Fv$.
• solve problems involving, for example, the instantaneous acceleration of a car moving on a hill against a resistance.	

5 Probability & Statistics 1 (for Paper 5)

Questions set will be mainly numerical, and will test principles in probability and statistics without involving knowledge of algebraic methods beyond the content for Paper 1: Pure Mathematics 1.

Knowledge of the following probability notation is also assumed: $\mathrm{P}(A)$, $\mathrm{P}(A \cup B)$, $P(A \cap B)$, $\mathrm{P}(A|B)$ and the use of A' to denote the complement of A.

5.1 Representation of data

Candidates should be able to:	Notes and examples
• select a suitable way of presenting raw statistical data, and discuss advantages and/or disadvantages that particular representations may have	
• draw and interpret stem-and-leaf diagrams, box-and-whisker plots, histograms and cumulative frequency graphs	Including back-to-back stem-and-leaf diagrams.
• understand and use different measures of central tendency (mean, median, mode) and variation (range, interquartile range, standard deviation)	e.g. in comparing and contrasting sets of data.
• use a cumulative frequency graph	e.g. to estimate medians, quartiles, percentiles, the proportion of a distribution above (or below) a given value, or between two values.
• calculate and use the mean and standard deviation of a set of data (including grouped data) either from the data itself or from given totals Σx and Σx^2, or coded totals $\Sigma(x-a)$ and $\Sigma(x-a)^2$, and use such totals in solving problems which may involve up to two data sets.	

5.2 Permutations and combinations

Candidates should be able to:	Notes and examples
• understand the terms permutation and combination, and solve simple problems involving selections	
• solve problems about arrangements of objects in a line, including those involving – repetition (e.g. the number of ways of arranging the letters of the word 'NEEDLESS') – restriction (e.g. the number of ways several people can stand in a line if two particular people must, or must not, stand next to each other).	Questions may include cases such as people sitting in two (or more) rows. Questions about objects arranged in a circle will not be included.

5 Probability & Statistics 1

5.3 Probability

Candidates should be able to:	Notes and examples
• evaluate probabilities in simple cases by means of enumeration of equiprobable elementary events, or by calculation using permutations or combinations	e.g. the total score when two fair dice are thrown. e.g. drawing balls at random from a bag containing balls of different colours.
• use addition and multiplication of probabilities, as appropriate, in simple cases	Explicit use of the general formula $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ is not required.
• understand the meaning of exclusive and independent events, including determination of whether events A and B are independent by comparing the values of $\mathrm{P}(A \cap B)$ and $\mathrm{P}(A) \times \mathrm{P}(B)$	
• calculate and use conditional probabilities in simple cases.	e.g. situations that can be represented by a sample space of equiprobable elementary events, or a tree diagram. The use of $\mathrm{P}(A\|B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ may be required in simple cases.

5.4 Discrete random variables

Candidates should be able to:	Notes and examples
• draw up a probability distribution table relating to a given situation involving a discrete random variable X, and calculate $\mathrm{E}(X)$ and $\mathrm{Var}(X)$	
• use formulae for probabilities for the binomial and geometric distributions, and recognise practical situations where these distributions are suitable models	Including the notations $\mathrm{B}(n, p)$ and $\mathrm{Geo}(p)$. $\mathrm{Geo}(p)$ denotes the distribution in which $p_r = p(1-p)^{r-1}$ for $r = 1, 2, 3, \ldots$.
• use formulae for the expectation and variance of the binomial distribution and for the expectation of the geometric distribution.	Proofs of formulae are not required.

5 Probability & Statistics 1

5.5 The normal distribution

Candidates should be able to:	Notes and examples
• understand the use of a normal distribution to model a continuous random variable, and use normal distribution tables	Sketches of normal curves to illustrate distributions or probabilities may be required.
• solve problems concerning a variable X, where $X \sim N(\mu, \sigma^2)$, including – finding the value of $P(X > x_1)$, or a related probability, given the values of x_1, μ, σ. – finding a relationship between x_1, μ and σ given the value of $P(X > x_1)$ or a related probability	For calculations involving standardisation, full details of the working should be shown. e.g. $Z = \dfrac{(X - \mu)}{\sigma}$
• recall conditions under which the normal distribution can be used as an approximation to the binomial distribution, and use this approximation, with a continuity correction, in solving problems.	n sufficiently large to ensure that both $np > 5$ and $nq > 5$.

6 Probability & Statistics 2 (for Paper 6)

Knowledge of the content of Paper 5: Probability & Statistics 1 is assumed, and candidates may be required to demonstrate such knowledge in answering questions. Knowledge of calculus within the content for Paper 3: Pure Mathematics 3 will also be assumed.

6.1 The Poisson distribution

Candidates should be able to:	Notes and examples
• use formulae to calculate probabilities for the distribution $\mathrm{Po}(\lambda)$	
• use the fact that if $X \sim \mathrm{Po}(\lambda)$ then the mean and variance of X are each equal to λ	Proofs are not required.
• understand the relevance of the Poisson distribution to the distribution of random events, and use the Poisson distribution as a model	
• use the Poisson distribution as an approximation to the binomial distribution where appropriate	The conditions that n is large and p is small should be known; $n > 50$ and $np < 5$, approximately.
• use the normal distribution, with continuity correction, as an approximation to the Poisson distribution where appropriate.	The condition that λ is large should be known; $\lambda > 15$, approximately.

6.2 Linear combinations of random variables

Candidates should be able to:	Notes and examples
• use, when solving problems, the results that – $\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$ and $\mathrm{Var}(aX + b) = a^2\,\mathrm{Var}(X)$ – $\mathrm{E}(aX + bY) = a\mathrm{E}(X) + b\mathrm{E}(Y)$ – $\mathrm{Var}(aX + bY) = a^2\,\mathrm{Var}(X) + b^2\,\mathrm{Var}(Y)$ for independent X and Y – if X has a normal distribution then so does $aX + b$ – if X and Y have independent normal distributions then $aX + bY$ has a normal distribution – if X and Y have independent Poisson distributions then $X + Y$ has a Poisson distribution.	Proofs of these results are not required.

6 Probability & Statistics 2

6.3 Continuous random variables

Candidates should be able to:	Notes and examples
• understand the concept of a continuous random variable, and recall and use properties of a probability density function	For density functions defined over a single interval only; the domain may be infinite, e.g. $\frac{3}{x^4}$ for $x \geqslant 1$.
• use a probability density function to solve problems involving probabilities, and to calculate the mean and variance of a distribution.	Including location of the median or other percentiles of a distribution by direct consideration of an area using the density function. Explicit knowledge of the cumulative distribution function is not included.

6.4 Sampling and estimation

Candidates should be able to:	Notes and examples
• understand the distinction between a sample and a population, and appreciate the necessity for randomness in choosing samples	
• explain in simple terms why a given sampling method may be unsatisfactory	Including an elementary understanding of the use of random numbers in producing random samples. Knowledge of particular sampling methods, such as quota or stratified sampling, is not required.
• recognise that a sample mean can be regarded as a random variable, and use the facts that $\mathrm{E}(\overline{X}) = \mu$ and that $\mathrm{Var}(\overline{X}) = \frac{\sigma^2}{n}$	
• use the fact that $(\overline{X})$ has a normal distribution if X has a normal distribution	
• use the Central Limit Theorem where appropriate	Only an informal understanding of the Central Limit Theorem (CLT) is required; for large sample sizes, the distribution of a sample mean is approximately normal.
• calculate unbiased estimates of the population mean and variance from a sample, using either raw or summarised data	Only a simple understanding of the term ‘unbiased’ is required, e.g. that although individual estimates will vary the process gives an accurate result ‘on average’.
• determine and interpret a confidence interval for a population mean in cases where the population is normally distributed with known variance or where a large sample is used	
• determine, from a large sample, an approximate confidence interval for a population proportion.	

6 Probability & Statistics 2

6.5 Hypothesis tests

Candidates should be able to:	Notes and examples
• understand the nature of a hypothesis test, the difference between one-tailed and two-tailed tests, and the terms null hypothesis, alternative hypothesis, significance level, rejection region (or critical region), acceptance region and test statistic • formulate hypotheses and carry out a hypothesis test in the context of a single observation from a population which has a binomial or Poisson distribution, using – direct evaluation of probabilities – a normal approximation to the binomial or the Poisson distribution, where appropriate • formulate hypotheses and carry out a hypothesis test concerning the population mean in cases where the population is normally distributed with known variance or where a large sample is used • understand the terms Type I error and Type II error in relation to hypothesis tests • calculate the probabilities of making Type I and Type II errors in specific situations involving tests based on a normal distribution or direct evaluation of binomial or Poisson probabilities.	Outcomes of hypothesis tests are expected to be interpreted in terms of the contexts in which questions are set.