Syllabus of Linear Algebra

Chapter 1: Systems of Linear Equations

1.1 Fundamental Concepts

1.1.1 Concepts of homogeneous and non-homogeneous linear systems, solutions, and general solutions.

1.1.2 Application of Gaussian elimination, preliminary assessment of solution existence and uniqueness.

1.1.3 Linear systems, coefficient matrices, and augmented matrices.

1.1.4 Elementary operations on linear systems, elementary row operations on matrices.

1.2 Basic Requirements

1.2.1 Understanding the concepts of homogeneous and non-homogeneous linear systems, solutions, and general solutions.

1.2.2 Understanding linear systems, coefficient matrices, and elementary operations. Mastery of elementary row operations and their effect on augmented matrices.

1.2.3 Mastery of the methods for transforming matrices into echelon forms and further into reduced row-echelon forms.

1.2.4 Mastery of determining the solvability and uniqueness of solutions based on the echelon form or reduced row-echelon form of augmented matrices.

1.2.5 Mastery of determining the relationship between the rank of the augmented matrix and the number of pivot columns in the coefficient matrix.

1.3 Recommended Study Credit hours (4 credit hours)

1.3.1 Gaussian elimination and matrices (2 credit hours).

1.3.2 Row reduction, echelon forms, and solution existence and uniqueness (2 credit hours).

Chapter 2: Matrix Algebra

2.1 Fundamental Concept

2.1.1 Matrix addition, scalar multiplication, and multiplication.

2.1.2 Matrix powers, the inverse of a matrix, transposition, and block matrices.

2.2 Basic Requirements

2.2.1 Mastery of matrix linear operations, multiplication, and their properties, understanding matrix powers.

2.2.2 Understanding the concept of matrix inverses, mastery of properties of inverse matrices, and the necessary and sufficient conditions for matrix invertibility.

2.2.3 Mastery of elementary matrix operations, understanding properties of elementary matrices, and the concept of matrix equivalence. Mastery of methods for finding inverse matrices using elementary operations.

2.2.4 Understanding transposition, symmetric matrices, skew-symmetric matrices, and their properties.

2.2.5 Understanding block matrices and operations involving them.

2.3 Recommended Study Credit hours (12 credit hours)

2.3.1 Matrix algebraic operations (3 credit hours).

2.3.2 Inverse matrices and elementary transformations (4 credit hours).

2.3.3 Transposed matrices and important square matrices (1 credit hour).

2.3.4 Block matrices (2 credit hours).

2.3.5 Review and summary of the entire chapter (2 credit hours).

Chapter 3: Determinants of Square Matrices

3.1 Fundamental Concepts

3.1.1 Definition of the determinant of a square matrix, basic properties.

3.1.2 Expansion theorem of determinants along rows or columns, applications.

3.2 Basic Requirements

3.2.1 Understanding the concept of the determinant of a square matrix, mastering properties of determinants.

3.2.2 Mastery of the expansion theorem along rows or columns; using determinants to determine invertibility of matrices; application of Cramer's rule; calculating determinants of matrix products.

3.3 Recommended Study Credit hours (8 credit hours)

3.3.1 Determinants of square matrices (2 credit hours).

3.3.2 Basic properties of determinants (2.5 credit hours).

3.3.3 Applications of determinants (1.5 credit hours).

3.3.4 Review and unit test (2 credit hours).

Chapter 4: Vector Spaces

4.1 Fundamental Concepts

4.1.1 Concepts of vectors, linear combinations, and linear representations.

4.1.2 Linear independence, linear dependence, vector spaces, subspaces, spans of vector sets, maximal linearly independent sets.

4.2 Basic Requirements

4.2.1 Understanding n-dimensional vectors, concepts of linear combinations, and linear representations.

4.2.2 Understanding linear independence, linear dependence, and equivalent vector sets. Mastery of properties and criteria for linear independence and dependence.

4.2.3 Understanding equivalent vector sets, concepts of maximal linearly independent sets, and vector space rank. Mastery of finding maximal linearly independent sets and vector space rank.

4.2.4 Understanding the basis and dimension of vector spaces and transition matrices.

4.3 Recommended Study Credit hours (18 credit hours)

4.3.1 Vector spaces (1 hour).

4.3.2 Linear independence of vectors (2 credit hours).

4.3.3 Maximal linearly independent sets and vector space rank (4 credit hours).

4.3.4 Subspaces (1 hour).

4.3.5 Basis and dimension of subspaces (4 credit hours).

4.3.6 Matrix rank (4 credit hours).

4.3.7 Review and summary of the entire chapter (2 credit hours).

Chapter 5: Eigenvalues, Eigenvectors, and Matrix Similarity

5.1 Fundamental Concepts

5.1.1 Concepts of matrix eigenvalues and eigenvectors, and their properties.

5.1.2 Matrix similarity, conditions for diagonalization, diagonal matrices.

5.2 Basic Requirements

5.2.1 Understanding concepts and properties of matrix eigenvalues and eigenvectors, mastery of methods for finding them.

5.2.2 Understanding matrix similarity, conditions for diagonalization, and methods for diagonalizing matrices.

5.3 Recommended Study Credit hours (10 credit hours)

5.3.1 Matrix eigenvalues and eigenvectors (2 credit hours).

5.3.2 Matrix similarity and diagonalization (3 credit hours).

5.3.3 Diagonalization of real symmetric matrices (3 credit hours).

5.3.4 Review and summary of the entire chapter (2 credit hours).

Chapter 6: Quadratic Forms

6.1 Fundamental Concepts

6.1.1 Definition and matrix representation of quadratic forms.

6.1.2 Congruent transformations and congruence matrices, rank of quadratic forms, inertia theorem.

6.1.3 Standard and canonical forms of quadratic forms, reduction by orthogonal transformations.

6.2 Basic Requirements

6.2.1 Mastery of quadratic forms and their matrix representations. Understanding the concept of the rank of quadratic forms.

6.2.2 Understanding congruent transformations, congruence matrices, and the inertia theorem.

6.2.3 Mastery of methods for reducing quadratic forms to standard and canonical forms using orthogonal transformations. Understanding positive definite quadratic forms and positive definite matrices.

6.3 Recommended Study Credit hours (12 credit hours)

6.3.1 Positive definite quadratic forms, positive definite matrices (4 credit hours).

6.3.2 Discriminant, signature, and Sylvester's law of inertia (6 credit hours).

6.3.3 Review and summary of the entire chapter (2 credit hours).