Syllabus of Probability and Statistics

Chapter 1: Fundamental Concepts of Probability

1.1 Fundamental Contents

1.1.1 Sample space, random events, relationship, and operation of events.

1.1.2 Definition and properties of frequency and probability.

1.1.3 Classical probability, geometric probability.

1.1.4 Conditional probability, multiplication formula, total probability formula, Bayesian formula.

1.1.5 Independence of events, Bernoulli probability type.

1.2 Basic Requirements

1.2.1 Understand the concepts of random events and sample space, and master the relationships and operations between events.

1.2.2 Understand the definitions of frequency and probability, grasp the basic properties of probability, and use them in calculations.

1.2.3 Master the definition and operation of classical probability, and understand the model and operation of geometric probability.

1.2.4 Proficiency in conditional probability formula, multiplication formula, total probability formula, and Bayesian formula. Can apply these formulas to calculate probabilities and solve applied problems.

1.2.5 Understand the concept of event independence and apply it to calculate probabilities. Learn about independent experiments and master the Bernoulli concept.

1.3 Class Schedule (8 credit hours)

1.3.1 Sample space, random events, frequency (2 credit hours)

1.3.2 Probability, equal probability models (2 credit hours)

1.3.3 Conditional probability and derived multiplication, total probability, Bayesian formulas (2 credit hours)

1.3.4 Independence and Bernoulli Probability Models (1 credit hour)

1.3.5 Review (1 credit hour)

Chapter 2: Random Variables and Probability Distributions

2.1 Fundamental Contents

2.1.1 Definition of random variables, definition and properties of distribution functions.

2.1.2 Discrete random variables and probability mass function, including geometric distribution, hypergeometric distribution, binomial distribution, Poisson distribution, and Poisson's theorem.

2.1.3 Continuous random variables and probability density functions, including uniform distributions, exponential distributions, Γ distributions.

2.1.4 Distributions of functions of random variables.

2.2 Basic Requirements

2.2.1 Understand the concept of random variables and master the definition and properties of distribution functions.

2.2.2 Calculate the probability distribution of discrete random variables,

and master the relationship between probability distribution and distribution function.

2.2.3 Understand geometric distribution, hypergeometric distribution, master binomial distribution and calculation, Poisson's theorem, and Poisson distribution, and use Poisson distribution to approximate the probability of binomial distribution.

2.2.4 Master the properties and probability calculation of the density function, the relationship between the density function and the distribution function, and obtain the distribution function from the density function.

2.2.5 Master uniform, exponential distribution, and calculation probability, master Γ function and properties, and understand Γ distribution.

2.2.6 Find the probability distribution of a function of a discrete random variable or the density function of a function of a continuous random variable (such as Y=g(X)).

2.3 Class Schedule (8 credit hours)

2.3.1 Random variables and their distribution functions (1 credit hour)

2.3.2 Discrete random variables and their distributions (2 credit hours)

2.3.3 Continuous random variables and their distributions (2 credit hours)

2.3.4 Distribution of a function of a random variable (1 credit hour)

2.3.5 Review and Unit Testing (2 credit hours)

Chapter 3: Multivariate Random Variable and Its Distribution

3.1 Fundamental Contents

3.1.1 Bivariate random variable definition, bivariate distribution function, and properties.

3.1.2 Bivariate discrete random variable and probability distribution, including trinomial distribution.

3.1.3 Bivariate continuous random variable and density function, including bivariate uniform distribution.

3.1.4 Marginal distribution and independence of random variables, conditional distributions, and conditional densities.

3.1.5 Distribution of functions of bivariate random variables.

3.1.6 Multivariate random variable definition and properties.

3.2 Basic Requirements

3.2.1 Master the definition of a bivariate random variable and its distribution functions.

3.2.2 Calculate bivariate discrete probability distribution, understand the trinomial distribution, master the definition and properties of bivariate continuous density function, and calculate related probabilities.

3.2.3 Calculate marginal distribution, and marginal density, and master the necessary and sufficient conditions for X and Y to be independent.

3.2.4 Calculate conditional distribution, conditional density, and related conditional probability.

3.2.5 Find the probability distribution or density function of a function of bivariate random variables (such as Z=g(X, Y)).

3.2.6 Understand the definition of multivariate random variables and their distribution functions; understand and master the additivity of distributions; the largest and smallest distributions of independent and identically distributed random variables.

3.3 Class Schedule (11 credit hours)

3.3.1 Bivariate random variable and its distribution (2.5 credit hours)

3.3.2 Marginal distributions and independence (2.5 credit hours)

3.3.3 Conditional distribution and conditional density (2 credit hours)

3.3.4 Distribution of a function of bivariate random variable (2 credit hours)

3.3.5 Multivariate random variable (1 credit hour)

3.3.6 Review (1 credit hour)

Chapter 4: Numerical Characteristics of Random Variables

4.1 Fundamental Contents

4.1.1 Expectation, variance properties, and calculations.

4.1.2 Moments and coefficients of variation.

4.1.3 Covariance and correlation coefficients and their properties and calculations.

4.2 Basic Requirements

4.2.1 Master the definition, properties, and operations of expectation,

variance, covariance, and correlation coefficient.

4.2.2 Compute the expectation E[g(X)] of a function of a single random variable and the expectation E[g(X, Y)] of a function of a two-dimensional random vector.

4.2.3 Compute E(X), D(X), and E[g(X)] based on the joint probability distribution or joint density function of (X, Y).

4.2.4 Understand the definitions and properties of moments and coefficients of variation.

4.3 Class Schedule (8 credit hours)

4.3.1 Expectations and their nature (2.5 credit hours)

4.3.2 Variance and moments (1.5 credit hours)

4.3.3 Covariance and Correlation Coefficient (2.5 credit hours)

4.3.4 Review and Unit Testing (2 credit hours)

Chapter 5: Normal Distribution and Natural Exponential Distribution Family

5.1 Fundamental Contents

5.1.1 Definition of normal distribution and its probability density function, the numerical characteristics of normal distribution, normal distribution additivity.

5.1.2 Two-dimensional normal distribution.

5.2 Basic Requirements

5.2.1 Master the definition of normal distribution and the calculation of density and probability.

5.2.2 Master the numerical characteristics of normal distribution and the linear properties of the normal distribution.

5.2.3 Master the relevant results of the two-dimensional normal distribution.

5.3 Class Schedule (4 credit hours)

5.3.1 Normal distribution and its probability calculation (1 credit hour)

5.3.2 Numerical characteristics and linear properties of normal distribution (1 credit hour)

5.3.3 Bivariate normal distribution (1 credit hour)

5.3.4 Review (1 credit hour)

Chapter 6: Limit Theorem

6.1 Fundamental Contents

6.1.1 Chebyshev Inequality, Convergence by Probability, Law of Large Numbers, Central Limit Theorem.

6.2 Basic Requirements

6.2.1 Master Chebyshev's inequality and simple applications.

6.2.2 Understand the definitions and conclusions of probability convergence and the law of large numbers, master the independent and identically distributed law of large numbers, and understand the meaning

of frequency convergence to probability.

6.2.3 Master the application form of the Central Limit Theorem and be able to solve simple application problems.

6.3 Class Schedule (4 credit hours)

6.3.1 Chebyshev's Inequality and the Law of Large Numbers (1 credit hour)

6.3.2 Central Limit Theorem (1 credit hour)

6.3.3 Review and Unit Testing (2 credit hours)

Chapter 7: Basic Knowledge of Mathematical Statistics

7.1 Fundamental Contents

7.1.1 Concepts of population, individual, and sample.

7.1.2 Distributions, t-distributions, F-distributions, and percentiles.

7.1.3 Common statistical measures and sampling distribution theorems.

7.2 Basic Requirements

7.2.1 Understand the concepts of population, individual, and sample.

7.2.2 Familiarity with distributions, t-distributions, and F-distributions; ability to find percentiles.

7.2.3 Calculate the sample mean and sample standard deviation.

7.2.4 Mastery of sampling distribution theorems for one or two normal populations.

7.3 Class Schedule (4 credit hours)

7.3.1 Concepts of population, individual, and sample (1 credit hour)

7.3.2 Distributions, t-distributions, F-distributions, and percentiles (2 credit hours)

7.3.3 Common Statistical Measures and sampling distribution theorems (1 credit hour)

7.3.4 Review and Unit Testing (2 credit hours)

Chapter 8: Parameter Estimation

8.1 Fundamental Contents

8.1.1 Concepts of point estimation, method of moments, and maximum likelihood estimation.

8.1.2 Criteria for evaluating estimators.

8.1.3 Interval estimation for parameters of a normal distribution.

8.2 Basic Requirements

8.2.1 Understand the concept of point estimation; grasp the basic idea of maximum likelihood estimation.

8.2.2 Mastery of unbiasedness and efficiency criteria; awareness of consistency and mean squared error criteria; ability to judge whether an estimator is unbiased; ability to compare the efficiency or mean squared error efficiency of two simple estimators.

8.2.3 Understand the concept and significance of interval estimation; ability to calculate confidence intervals for a parameter of a normal distribution (including one-sided confidence limits); ability to calculate

confidence intervals for the difference in means and the ratio of variances of two normal populations.

8.3 Class Schedule (5 credit hours)

8.3.1 Concepts and methods of point estimation (1 credit hour)

8.3.2 Criteria for evaluating estimators (2 credit hours)

8.3.3 Interval estimation and applications (1 credit hour)

8.3.4 Review and Unit Testing (2 credit hours)

Chapter 9: Hypothesis Testing

9.1 Fundamental Contents

9.1.1 Basic principles of hypothesis testing, fundamental steps, and two types of errors.

9.1.2 Hypothesis testing for one or two parameters of a normal distribution.

9.2 Basic Requirements

9.2.1 Understand the basic idea of significance testing; grasp the basic steps of hypothesis testing.

9.2.2 Awareness of the two types of errors that may occur during hypothesis testing.

9.2.3 Conduct hypothesis tests for one parameter of a normal distribution; conduct hypothesis tests for the means and variances of two normal populations.

9.3 Class Schedule (4 credit hours)

9.3.1 Concepts of hypothesis testing (1 credit hour)

9.3.2 Two types of errors and hypothesis testing for a single normal population (2 credit hours)

9.3.3 Hypothesis testing for two normal populations (1 credit hour)

9.3.4 Review and Unit Testing (2 credit hours)

Chapter 10: Regression Analysis and Analysis of Variance

10.1 Fundamental Contents

10.1.1 Simple linear regression and significance testing.

10.2 Basic Requirements

10.2.1 Ability to find the equation of a simple linear regression from specific problems.

10.2.2 Ability to perform significance tests on the obtained regression equation.

10.3 Class Schedule (2 credit hours)

10.3.1 Simple linear regression and significance testing (2 credit hours)

10.3.2 Review and Unit Testing (2 credit hours)