積層膜からの和周波発生(SFG) — 表式

準備

原点:光線の入射点に取る。(広がりがある場合にはその中央)

z軸:媒質1から媒質2に向けた法線。

x 軸:界面上でみた光の進行方向にとる。(k ベクトルの界面への射影に沿わせる)

次のように用語を定義する。

1/m 界面:媒質-1 と膜の界面。

m/2 界面:媒質-1 と膜の界面。

n₁, n₂, n_m: 媒質-1、媒質-2、膜 m の屈折率。

 k_1, k_2, k_m : 媒質-1、媒質-2、膜 m の中での波動ベクトル。

 $\theta_1, \theta_2, \theta_m$: 媒質-1、媒質-2、膜 m の中を進む光の光路が法線に対してなす角。

t_{1m}: 媒質-1 側から膜側に透過する光の電場に対する透過係数。

tm1: 膜側から媒質-1 側に透過する光の電場に対する透過係数。

 $r_{\rm lm}$: 媒質-1 側から来て 1/m 界面で反射する光の電場に対する反射係数。

 r_{m1} : 膜側から来て m/1 界面で反射する光の電場に対する反射係数。

 t_{m2} : 膜側から媒質-2 側に透過する光の電場に対する透過係数。

 t_{2m} : 媒質-2 側から膜側に透過する光の電場に対する透過係数。

 r_{m2} : 膜側から来て m/2 界面で反射する光の電場に対する反射係数。

 r_{2m} : 媒質-2 側から来て 1/m 界面で反射する光の電場に対する反射係数。

上向き(-) 光:m/2 界面から m/1 界面に向かう光。反射光と同じく、-z 側に進む。

下向き(+)光: m/1 界面から m/2 界面に向かう光。入射光と同じく、+z 側に進む。

(反射係数)

$$r_{12}^{s} = E_{r}^{s}/E_{i}^{s} = (k_{1z} - k_{2z})/(k_{1z} + k_{2z}) = (n_{1}\cos\theta_{1} - n_{2}\cos\theta_{2})/(n_{1}\cos\theta_{1} + n_{2}\cos\theta_{2})$$

$$= -\sin(\theta_{1} - \theta_{2})/\sin(\theta_{1} + \theta_{2})$$

$$r_{12}^{p} = E_{r}^{p}/E_{i}^{p} = (\varepsilon_{1}k_{2z} - \varepsilon_{2}k_{1z})/(\varepsilon_{2}k_{1z} + \varepsilon_{1}k_{2z}) = (n_{1}\cos\theta_{2} - n_{2}\cos\theta_{1})/(n_{2}\cos\theta_{1} + n_{1}\cos\theta_{2})$$

$$= -\tan(\theta_{1} - \theta_{2})/\tan(\theta_{1} + \theta_{2})$$

(透過係数)

$$\begin{split} t_{12}^{\ \ s} &= E_{1}^{s}/E_{i}^{s} = 2k_{1z}/(k_{1z} + k_{2z}) = 2n_{1}\text{cos}\theta_{1}/(n_{1}\text{cos}\theta_{1} + n_{2}\text{cos}\theta_{2}) \\ &= 2\text{cos}\theta_{1}\text{sin}\theta_{2}/\text{sin}(\theta_{1} + \theta_{2}) \\ t_{12}^{\ \ p} &= E_{1}^{p}/E_{i}^{p} = 2\varepsilon_{1}k_{2z}\text{cos}\theta_{1}/(\varepsilon_{2}k_{1z} + \varepsilon_{1}k_{2z}) = 2n_{1}\text{cos}\theta_{1}/(n_{2}\text{cos}\theta_{1} + n_{1}\text{cos}\theta_{2}) \\ &= 2\text{cos}\theta_{1}\text{sin}\theta_{2}/\text{sin}(\theta_{1} + \theta_{2})\text{cos}(\theta_{1} - \theta_{2}) \\ r_{21}^{\ \ s} &= -r_{12}^{\ \ s}, \ r_{21}^{\ \ p} = -r_{12}^{\ \ p} \\ t_{21}^{\ \ s} &= (n_{2}\text{cos}\theta_{2}/n_{1}\text{cos}\theta_{1})t_{12}^{\ \ s}, \ t_{21}^{\ \ p} = (n_{2}\text{cos}\theta_{2}/n_{1}\text{cos}\theta_{1})t_{12}^{\ \ p} \end{split}$$

界面の内向き法線を z 軸に、s 偏光の電場の方向を y 軸に、光の進行方向が正になるように x 軸を定義して、電場の軸方向成分に対する係数を求めると、次のようになる。

$$\begin{split} E_{\text{x,i}} &= E^{\text{p}}_{\text{i}} \text{cos} \theta_{\text{l}}, & E_{\text{y,i}} &= E^{\text{s}}_{\text{ri}}, & E_{\text{z,i}} &= -E^{\text{p}}_{\text{i}} \text{sin} \theta_{\text{l}} \\ E_{\text{x,r}} &= E^{\text{p}}_{\text{r}} \text{cos} \theta_{\text{l}}, & E_{\text{y,r}} &= E^{\text{s}}_{\text{r}}, & E_{\text{z,r}} &= E^{\text{p}}_{\text{r}} \text{sin} \theta_{\text{l}} \\ E_{\text{x,t}} &= E^{\text{p}}_{\text{t}} \text{cos} \theta_{\text{2}}, & E_{\text{y,t}} &= E^{\text{s}}_{\text{t}}, & E_{\text{z,t}} &= -E^{\text{p}}_{\text{t}} \text{sin} \theta_{\text{2}} \end{split}$$

上の表式より、下の振幅反射係数および振幅等価係数の表式を得る。

$$\begin{aligned} &r_{12}^{x} = r_{12}^{p}, & r_{12}^{y} = r_{12}^{s}, & r_{12}^{z} = -r_{12}^{p} \\ &r_{21}^{x} = -r_{12}^{p}, & r_{21}^{y} = -r_{12}^{s}, & r_{21}^{z} = -r_{12}^{p} \\ &t_{21}^{x} = (\cos\theta_{2}/\cos\theta_{1})t_{12}^{p}, & t_{12}^{y} = t_{12}^{s}, & t_{12}^{z} = (\sin\theta_{2}/\sin\theta_{1})t_{12}^{p} = (n_{1}/n_{2}) \ t_{12}^{p} \\ &t_{21}^{x} = (n_{2}/n_{1})t_{12}^{p}, & t_{12}^{y} = t_{12}^{s}, & t_{12}^{z} = (\cos\theta_{2}/\cos\theta_{1}) \ (n_{2}/n_{1})^{2}t_{12}^{p} \\ &r_{1mx} = -r_{m1x} = r_{1mp}, & r_{1my} = -r_{m1y} = r_{1ms}, & r_{1mz} = -r_{m1z} = -r_{1mp} \\ &r_{2mx} = -r_{m2x} = r_{2mp}, & r_{2my} = -r_{m2y} = r_{2ms}, & r_{2mz} = -r_{m2z} = -r_{2mp} \\ &t_{1mx} = (\cos\theta_{1}/\cos\theta_{1})t_{1mp}, & t_{1my} = t_{1ms}, & t_{1mz} = (\sin\theta_{1}/\sin\theta_{1})t_{1mp} \\ &t_{m1x} = (\cos\theta_{1}/\cos\theta_{m})t_{m1p}, & t_{m1y} = t_{m1s}, & t_{m1z} = (\sin\theta_{1}/\sin\theta_{m})t_{m1p} \end{aligned}$$

$$t_{2m,x} = (\cos\theta_{m}/\cos\theta_{2})t_{2m,p}, \quad t_{2m,y} = t_{2m,s}, \quad t_{2m,z} = (\sin\theta_{m}/\sin\theta_{2})t_{2m,p}$$

 $t_{m2,x} = (\cos\theta_{2}/\cos\theta_{m})t_{m2,p}, \quad t_{m2,y} = t_{m2,s}, \quad t_{m2,z} = (\sin\theta_{2}/\sin\theta_{m})t_{m2,p}$

 $t_{\text{m2x}} = (coso_2 coso_m n_{\text{m2y}}, t_{\text{m2y}} - t_{\text{m2x}}), t_{\text{m2z}} = (oso_2 coso_m n_{\text{m2p}}, t_{\text{m2z}}), t_{\text{m2z}} = (oso_2 coso_m n_{\text{m2p}}, t_{\text{m2p}}), t_{\text{m2p}} = (oso_2 coso_m n_{\text{m2p}}, t_{\text{m2p}}, t_{\text{m2p}}), t_{\text{m2p}} = (oso_2 coso_m n_{\text{m2p}}, t_{$

上式は次のような関係式を満足する。

$$1 + r_{12}^{\ \ x} = t_{12}^{\ \ x}$$
, $1 + r_{12}^{\ \ y} = t_{12}^{\ \ y}$, $1 + r_{12}^{\ \ z} = (n_2/n_1)^2 t_{12}^{\ \ z}$ (電場に対する境界条件) $1 + r_{12}^{\ \ s} r_{21}^{\ \ s} = 1 - (r_{12}^{\ \ s})^2 = t_{12}^{\ \ s} t_{21}^{\ \ s}$, $1 + r_{12}^{\ \ p} r_{21}^{\ \ p} = 1 - (r_{12}^{\ \ p})^2 = t_{12}^{\ \ p} t_{21}^{\ \ p}$

因みに、光の強度は $E^2n\cos\theta$ に比例するので、 $r_{12}^2n_1\cos\theta_1+t_{12}^2n_2\cos\theta_2=n_1\cos\theta_1$ となり、光の強度は保存される。

L係数、電場振幅

$$E_{\rm SF, \, \alpha} = \sum_{\beta} L_{\alpha\beta} P_{\beta}^{\rm SF}$$

共通因子である $4\pi i\omega_{SF}/c$ (屈折率の代わりに波数ベクトルを使うときには $4\pi i\omega_{SF}^2/c^2$) を省略する。

L係数の表記法; $L_{ijs(p),\alpha}$: 分極シート m' が i 層とj 層に挟まれているときに、分極の α 成分 ($\alpha=x,y,z$) が作る光の s 偏光成分又は p 偏光成分の間の係数(x,y,z 成分ではないことに注意)。上向き (-) 光、上向き (+) 光の区別を上付き -、+ で示す。

媒質1と積層膜 m の間の分極シート m' からの光生成に対しては、

$$\begin{split} L_{1/\text{m,p,x}} &= \cos \theta_{\text{m,SF}} / (n_{1,\text{SF}} \cos \theta_{\text{m,SF}} + n_{\text{m,SF}} \cos \theta_{1,\text{SF}}) \\ L_{1/\text{m,s,y}} &= 1 / (n_{1,\text{SF}} \cos \theta_{1,\text{SF}} + n_{\text{m,SF}} \cos \theta_{\text{m,SF}}) \\ L_{1/\text{m,p,z}} &= (n_{\text{m,SF}} / n_{\text{m'SF}}) \sin \theta_{\text{m'SF}} / (n_{1,\text{SF}} \cos \theta_{\text{m,SF}} + n_{\text{m,SF}} \cos \theta_{1,\text{SF}}) \end{split}$$

$$= (n_{mSF}/n_{m'SF})^{2} \sin\theta_{mSF}/(n_{1,SF}\cos\theta_{mSF} + n_{mSF}\cos\theta_{1,SF})$$

$$L^{+}_{1/m,p,x} = \cos\theta_{1,SF}/(n_{1,SF}\cos\theta_{mSF} + n_{mSF}\cos\theta_{1,SF})$$

$$L^{+}_{1/m,s,y} = 1/(n_{1,SF}\cos\theta_{1,SF} + n_{mSF}\cos\theta_{m,SF})$$

$$L^{+}_{1/m,p,z} = -(n_{1,SF}/n_{m'SF})\sin\theta_{m'SF}/(n_{1,SF}\cos\theta_{mSF} + n_{mSF}\cos\theta_{1,SF})$$

$$= -(n_{1,SF}/n_{m'SF})^{2} \sin\theta_{1,SF}/(n_{1,SF}\cos\theta_{mSF} + n_{mSF}\cos\theta_{1,SF})$$

媒質 2 と積層膜 m の間の分極シート m'' からの光生成に対しては、

$$\begin{split} L_{2/\text{m,p,x}} &= \cos\theta_{2,\text{SF}}/(n_{2,\text{SF}}\cos\theta_{\text{m,SF}} + n_{\text{m,SF}}\cos\theta_{2,\text{SF}}) \\ L_{2/\text{m,s,y}} &= 1/(n_{2,\text{SF}}\cos\theta_{2,\text{SF}} + n_{\text{m,SF}}\cos\theta_{\text{m,SF}}) \\ L_{2/\text{m,p,z}} &= (n_{2,\text{SF}}/n_{\text{m'',SF}})\sin\theta_{\text{m'',SF}}/(n_{2,\text{SF}}\cos\theta_{\text{m,SF}} + n_{\text{m,SF}}\cos\theta_{2,\text{SF}}) \\ &= (n_{2,\text{SF}}/n_{\text{m'',SF}})^2\sin\theta_{2,\text{SF}}/(n_{2,\text{SF}}\cos\theta_{\text{m,SF}} + n_{\text{m,SF}}\cos\theta_{2,\text{SF}}) \\ L_{2/\text{m,p,z}}^+ &= \cos\theta_{\text{m,SF}}/(n_{2,\text{SF}}\cos\theta_{\text{m,SF}} + n_{\text{m,SF}}\cos\theta_{2,\text{SF}}) \\ L_{2/\text{m,p,z}}^+ &= 1/(n_{2,\text{SF}}\cos\theta_{2,\text{SF}} + n_{\text{m,SF}}\cos\theta_{\text{m,SF}}) \\ L_{2/\text{m,p,z}}^+ &= -(n_{\text{m,SF}}/n_{\text{m'',SF}})\sin\theta_{\text{m'',SF}}/(n_{2,\text{SF}}\cos\theta_{\text{m,SF}} + n_{\text{m,SF}}\cos\theta_{2,\text{SF}}) \\ &= -(n_{\text{m,SF}}/n_{\text{m'',SF}})^2\sin\theta_{\text{m,SF}}/(n_{2,\text{SF}}\cos\theta_{\text{m,SF}} + n_{\text{m,SF}}\cos\theta_{2,\text{SF}}) \end{split}$$

積層膜内部の分極シートからの光生成に対しては、

$$\begin{split} L_{\text{m/m,p,x}} &= L_{\text{m/m,p,x}}^{+} = \cos\theta_{\text{m,SF}}/(2n_{\text{m,SF}}\cos\theta_{\text{m,SF}}) \\ L_{\text{m/m,s,y}} &= L_{\text{m/m,s,y}}^{+} = 1/(2n_{\text{m,SF}}\cos\theta_{\text{m,SF}}) \\ L_{\text{m/m,p,z}} &= -L_{\text{m/m,p,z}}^{+} = \sin\theta_{\text{m,SF}}/(2n_{\text{m,SF}}\cos\theta_{\text{m,SF}}) \end{split}$$

よって、各部位からの SFG 光の電場振幅は次のように表される。

P(0) に由来するもの: (P(0) は媒質 1 側の電場による分極、、P(0) は膜側の電場による分極)

$$E_{1,p}(z=0^{\circ}) = L_{1/m,p,x}P_{x}(0^{\circ}) + L_{1/m,p,z}P_{z}(0^{\circ})$$

$$E_{1,s}(z=0^{\circ}) = L_{1/m,s,y}P_{y}(0^{\circ})$$

$$E_{m,p}^{+}(z=0^{\circ}) = L_{1/m,p,x}^{+}P_{x}(0^{\circ}) + L_{1/m,p,z}^{+}P_{z}(0^{\circ})$$

$$E_{m,s}^{+}(z=0^{\circ}) = L_{1/m,s,y}^{+}P_{y}(00^{\circ})$$

 $P(h_m)$ に由来するもの: $(P(h_m^+))$ は媒質 2 側の電場による分極,、 $P(h_m^-)$ は膜側の電場による分極)

$$\begin{split} E_{\text{m,p}}(z = h_{\text{m}}) &= L_{2/\text{m,p,x}} P_{\text{x}}(h_{\text{m}}) + L_{2/\text{m,p,z}} P_{\text{z}}(h_{\text{m}}) \\ E_{\text{m,s}}(z = h_{\text{m}}) &= L_{2/\text{m,s,y}} P_{\text{y}}(h_{\text{m}}) \\ E_{2,\text{p}}^{+}(z = h_{\text{m}}^{+}) &= L_{2/\text{m,p,x}}^{+} P_{\text{x}}(h_{\text{m}}^{+}) + L_{2/\text{m,p,z}}^{+} P_{\text{z}}(h_{\text{m}}^{+}) \\ E_{2,\text{s}}^{+}(z = h_{\text{m}}^{+}) &= L_{2/\text{m,s,y}}^{+} P_{\text{y}}(h_{\text{m}}^{+}) \end{split}$$

P(z) に由来するもの:

$$E_{\mathrm{m,p}}(z) = L_{\mathrm{m/m,p,x}} P_{\mathrm{x}}(z) + L_{\mathrm{m/m,p,z}} P_{\mathrm{z}}(z)$$

$$E_{\mathrm{m,s}}(z) = L_{\mathrm{m/m,s,y}} P_{\mathrm{y}}(z)$$

$$E^{+}_{m,p}(z) = L^{+}_{m/m,p,x} P_{x}(z) + L^{+}_{m/m,p,z} P_{z}(z)$$

$$E^{+}_{m,s}(z) = L^{+}_{m/m,s,y} P_{y}(z)$$

1/m 界面の外側 $(x = -2nh_{m}\tan\theta_{mSF}, z = 0)$ からの SFG

反射 SFG

$$E_{1p}(0^{-}) = \sum_{\beta\gamma} E^{0}_{vis}_{\beta} E^{0}_{ir\gamma} \frac{(1 + r_{1m,vis}_{\beta})(1 + r_{m2,vis}_{\beta} e^{2i\beta_{m,vis}h_{m}})(1 + r_{1mir,\gamma})(1 + r_{m2,ir,\gamma} e^{2i\beta_{m,ir}h_{m}})}{(1 + r_{1m,vis}_{\beta} r_{m2,vis}_{\beta} e^{2i\beta_{m,vis}h_{m}})(1 + r_{1mir,\gamma} r_{m2,ir,\gamma} e^{2i\beta_{m,ir}h_{m}})} \times \frac{(1 + r_{m2,SFp} e^{2i\beta_{m,SF}h_{m}})L_{1/m,px} \chi_{x\beta\gamma} + (1 - r_{m2,SF,p} e^{2i\beta_{m,SF}h_{m}})L_{1/m,pz} \chi_{z\beta\gamma}}{(1 + r_{1m,SF} r_{m2,SF,p} e^{2i\beta_{m,SF}h_{m}})}$$
(1a)

$$E_{1s}(0^{-}) = \sum_{\beta\gamma} E^{0}_{vis\beta} E^{0}_{ir\gamma} \frac{(1 + r_{1m,vis\beta})(1 + r_{m2,vis\beta} e^{2\beta_{m,vis}h_{m}})(1 + r_{1m,ir\gamma})(1 + r_{m2,ir\gamma} e^{2i\beta_{m,ir}h_{m}})}{(1 + r_{1m,vis\beta} r_{m2,vis\beta} e^{2\beta_{m,vis}h_{m}})(1 + r_{1m,ir\gamma} r_{m2,ir\gamma} e^{2i\beta_{m,ir}h_{m}})} \times \frac{(1 + r_{m2,SF,s} e^{2i\beta_{m,SF}h_{m}})}{(1 + r_{1m,SF,s} r_{m2,SF,s} e^{2i\beta_{m,SF}h_{m}})} L^{-1/m,sy} \chi_{y\beta\gamma}$$
(1b)

透過 SFG

$$E^{+}_{2p}(h_{m}^{+}) = \sum_{\beta\gamma} E^{0}_{vis}_{\beta} E^{0}_{ir\gamma} \frac{(1 + r_{1m,vis}_{\beta})(1 + r_{n2,vis}_{\beta} e^{2\beta_{m,vis}h_{m}})(1 + r_{1m,ir\gamma})(1 + r_{m2,ir\gamma} e^{2i\beta_{m,ir}h_{m}})}{(1 + r_{1m,vis}_{\beta} r_{n2,vis}_{\beta} e^{2\beta_{m,vis}h_{m}})(1 + r_{1m,ir\gamma} r_{n2,ir\gamma} e^{2i\beta_{m,ir}h_{m}})} \times \frac{t_{1m,SF,p}t_{m2,SF,p}}{1 + r_{1m,SF,p}t_{m2,SF,p}} e^{\beta_{m,SF}h_{m}} \left[\chi_{x\beta\gamma} L^{+}_{m1,px} + \chi_{z\beta\gamma} L^{+}_{m/1,pz}\right]$$
(2a)

$$E^{+}_{2s}(h_{m}^{+}) = \sum_{\beta,\gamma} E^{0}_{vis\beta} E^{0}_{ir\gamma} \frac{(1 + r_{1m,vis,\beta})(1 + r_{m2,vis,\beta} e^{2i\beta_{m,vis}h_{m}})(1 + r_{1m,ir,\gamma})(1 + r_{m2,ir,\gamma} e^{2i\beta_{m,vi}h_{m}})}{(1 + r_{1m,vis,\beta} r_{m2,vis,\beta} e^{2i\beta_{m,vis}h_{m}})(1 + r_{1m,ir,\gamma} r_{m2,ir,\gamma} e^{2i\beta_{m,v}h_{m}})} \times \frac{t_{1m,SF,s} t_{m2,SF,s} e^{\beta_{m,SF}h_{m}}}{1 + r_{1m,SF,s} r_{m2,SF,s} e^{2i\beta_{m,SF}h_{m}}} \chi_{y\beta\gamma} L^{+}_{m/l,sy}$$
(2b)

表面が薄膜として完成しているときの薄膜(屈折率 m')を形成する分子からの SFG は、内部反射を 考慮する必要がある。

反射 SFG

$$E^{-1p}(0^{-}) = \sum_{\beta,\gamma} E^{0}_{vis\beta} E^{0}_{ir\gamma} \frac{t_{1m',vis\beta} (1 + r_{m'm,vis\beta})(1 + r_{m2,vis\beta} e^{2i\beta_{m,vis}h_{m}})}{1 + r_{1m',vis\beta} r_{m'm,vis\beta} + (r_{m'm,vis\beta} + r_{1m',vis\beta}) r_{m2,vis\beta} e^{2i\beta_{m,vil}h_{m}}}$$

$$\times \frac{t_{1m',ir,\gamma} (1 + r_{m'm,ir,\gamma})(1 + r_{m2,ir,\gamma} e^{2i\beta_{m,ir}h_{m}})}{1 + r_{1m',ir,\gamma} r_{m'm,ir,\gamma} + (r_{m'm,ir,\gamma} + r_{1m',ir,\gamma}) r_{m2,ir,\gamma} e^{2i\beta_{m,ir}h_{m}}}$$

$$\times \frac{(1 + r_{1m',SF,p} r_{m'm,SF,p})[(1 + r_{m2,SF,p} e^{2i\beta_{m,SF}h_{m}}) \chi_{x\beta\gamma} L_{1/m,px} + (1 - r_{m2,SF,p} e^{2i\beta_{m,SF}h_{m}}) \chi_{z\beta\gamma} L_{1/m,pz}]}{1 + r_{1m',SF,p} r_{m'm,SF,p} + r_{m2,SF,p} (r_{1m',SF,p} + r_{m'm,SF,p}) e^{2i\beta_{m,SF}h_{m}}}$$
(3a)

$$E^{-1s}(0^{-}) = \sum_{\beta \gamma} E^{0}_{vis\beta} E^{0}_{ir\gamma} \frac{t_{1m',vis\beta} (1 + r_{m'm,vis\beta})(1 + r_{m2,vis\beta} e^{2i\beta_{m,vis}h_{m}})}{1 + r_{1m',vis\beta} r_{m'm,vis\beta} + (r_{m'm,vis\beta} + r_{1m',vis\beta}) r_{m2,vis\beta} e^{2i\beta_{m,vis}h_{m}}}$$

$$\times \frac{t_{1m',ir\gamma} (1 + r_{m'm,ir\gamma})(1 + r_{m2,ir\gamma} e^{2i\beta_{m,ir}h_{m}})}{1 + r_{1m',ir\gamma} r_{m'm,ir\gamma} + (r_{m'm,ir\gamma} + r_{1m',ir\gamma}) r_{m2,ir\gamma} e^{2i\beta_{m,ir}h_{m}}}$$

$$\times \frac{(1 + r_{1m',SF,s} r_{m'm,SF,s})(1 + r_{m2,SF,s} e^{2i\beta_{m,sF}h_{m}})}{1 + r_{1m',SF,s} r_{m'm,SF,s} + r_{m2,SF,s} (r_{1m',SF,s} + r_{m',mSF,s}) e^{2i\beta_{m,sF}h_{m}}} \chi_{\beta\gamma} L^{-1/m,sy}$$
(3b)

透過 SFG

$$E^{+}_{2p}(h_{m}^{+}) = \sum_{\beta\gamma} E^{0}_{vis\beta} E^{0}_{ir\gamma} \frac{t_{1m',vis\beta} (1 + r_{m'm,vis\beta})(1 + r_{m2,vis\beta} e^{2\beta_{m,vis}h_{m}})}{1 + r_{1m',vis\beta} r_{m'm,vis\beta} + (r_{m'm,vis\beta} + r_{1m',vis\beta}) r_{m2,vis\beta} e^{2\beta_{m,vis}h_{m}}}$$

$$\times \frac{t_{1m',ir\gamma} (1 + r_{m'm,ir\gamma})(1 + r_{m2,ir\gamma} e^{2i\beta_{m,ir}h_{m}})}{1 + r_{1m',ir\gamma} r_{m'm,ir\gamma} + (r_{m'm,ir\gamma} + r_{1m',ir\gamma}) r_{m2,ir\gamma} e^{2i\beta_{m,ir}h_{m}}}$$

$$\times \frac{t_{m2,SF,p} (1 + r_{1m',SF,p} r_{m'm,SF,p}) e^{i\beta_{m,SF}h_{m}}}{1 + r_{1m',SF,p} r_{m'm,SF,p} + r_{m2,SF,p} (r_{mm',SF,p} + r_{m1,SF,p}) e^{2i\beta_{m,SF}h_{m}}}) (L^{+}_{1/m,px} \chi_{x\beta\gamma} + L^{+}_{1/m,pz} \chi_{z\beta\gamma})$$

$$(4a)$$

$$E^{+}_{2s}(h_{m}^{+}) = \sum_{\beta,\gamma} E^{0}_{vis\beta} E^{0}_{ir\gamma} \frac{t_{1m',vis\beta} (1 + r_{m'm,vis\beta})(1 + r_{m2,vis\beta} e^{2i\beta_{m,vi}h_{m}})}{1 + r_{1m',vis\beta} r_{mim,vis\beta} + (r_{m'm,vis\beta} + r_{1m',vis\beta}) r_{m2,vis\beta} e^{2i\beta_{m,vi}h_{m}}} \times \frac{t_{1m',ir,\gamma} (1 + r_{m'm,ir,\gamma})(1 + r_{m2,ir,\gamma} e^{2i\beta_{m,v}h_{m}})}{1 + r_{1m',ir,\gamma} r_{m'm,ir,\gamma} + (r_{m'm,ir,\gamma} + r_{1m',ir,\gamma}) r_{m2,ir,\gamma} e^{2i\beta_{m,v}h_{m}}} \times \frac{t_{m2,SF,s} (1 + r_{1m',SF,s} r_{m'm,SF,s}) e^{i\beta_{m,sF}h_{m}}}{1 + r_{1m',SF,s} r_{m'm,SF,s} + r_{m2,SF,s} (r_{mm',SF,s} + r_{m1,SF,s}) e^{2\beta_{m,SF}h_{m}}}) L^{+}_{1/m,sy} \chi_{\beta\gamma}$$

$$(4b)$$

1/m 界面の内側 $(x = -2nh_{m} \tan \theta_{mSF}, z = 0^{+})$ からの SFG

反射 SFG

$$E^{-1p}(0^{-}) = \sum_{\beta,\gamma} E^{0}_{vis\beta} E^{0}_{ir\gamma} \frac{t_{1m,vis\beta} t_{1m,ir,\gamma} (1 + r_{m2,vis\beta} e^{2i\beta_{m,vis}h_{m}}) (1 + r_{m2,ir\gamma} e^{2i\beta_{m,ir}h_{m}})}{(1 + r_{1m,vis\beta} r_{m2,vis\beta} e^{2i\beta_{m,vis}h_{m}}) (1 + r_{1m,ri,\gamma} r_{m2,ir,\gamma} e^{2i\beta_{m,ir}h_{m}})} \times \frac{[(1 + r_{m2,SF,p} e^{2\beta_{m,SF}h_{m}}) L^{-}_{1/m,px} \chi_{x\beta\gamma} + (1 - r_{m2,SF,p} e^{2\beta_{m,SF}h_{m}}) L^{-}_{1/m,pz} \chi_{z\beta\gamma}]}{1 + r_{m,vis\beta} r_{m2,vis\beta} e^{2i\beta_{m,sr}h_{m}}}$$
(5a)

$$E^{-1s}(0^{-}) = \sum_{\beta \gamma} E^{0}_{vis\beta} E^{0}_{ir\gamma} \frac{t_{1m,vis\beta} t_{1m,ir\gamma} (1 + r_{m2,vis\beta} e^{2i\beta_{m,vis}h_{m}})(1 + r_{m2,ir\gamma} e^{2i\beta_{m,ir}h_{m}})}{(1 + r_{1m,vis\beta} r_{m2,vis\beta} e^{2i\beta_{m,vis}h_{m}})(1 + r_{1m,ir\gamma} r_{m2,ir\gamma} e^{2i\beta_{m,ir}h_{m}})} \times \frac{(1 + r_{m2,SF,s} e^{2i\beta_{m,SF}h_{m}})L^{-}_{1/m,s\gamma} \chi_{y\beta\gamma}}{1 + r_{m-sF,F} r_{m2,sF,g} e^{2i\beta_{m,SF}h_{m}}}$$
(5b)

透過 SFG

$$E^{+}_{2p}(h_{m}^{+}) = \sum_{\beta\gamma} E^{0}_{vis\beta} E^{i}_{ir\gamma} \frac{t_{1m,vis\beta} t_{1m,ir\gamma} (1 + r_{m2,vis\beta} e^{2i\beta_{m,vis}h_{m}}) (1 + r_{m2,ir\gamma} e^{2\beta_{m,ir}h_{m}})}{(1 + r_{1m,vis\beta} r_{m2,vis\beta} e^{2i\beta_{m,vis}h_{m}}) (1 + r_{1m,ir\gamma} r_{m2,ir\gamma} e^{2i\beta_{m,ir}h_{m}})} \times \frac{t_{m2,SF,p} e^{\beta_{m,SF}h_{m}}}{1 + r_{1m,SE,p} r_{m2,SE,p} e^{2i\beta_{m,SF}h_{m}}} (L^{+}_{1/m,px} \chi_{x\beta\gamma} + L^{+}_{1/m,pz} \chi_{z\beta\gamma})$$
(6a)

$$E^{+}_{2s}(h_{m}^{+}) = \sum_{\beta,\gamma} E^{0}_{vis\beta} E^{0}_{ir\gamma} \frac{t_{1m,vis\beta} t_{1m,ir\gamma} (1 + r_{m2,vis\beta} e^{2i\beta_{m,vis}h_{m}}) (1 + r_{m2,ir\gamma} e^{2i\beta_{m,ir}h_{m}})}{(1 + r_{1m,vis\beta} r_{m2,vis\beta} e^{2i\beta_{m,vis}h_{m}}) (1 + r_{1m,ir\gamma} r_{m2,ir\gamma} e^{2i\beta_{m,ir}h_{m}})} \times \frac{t_{m2,SF,s} e^{i\beta_{m,SF}h_{m}}}{1 + r_{1mSF,s} r_{m2,SF,s} e^{2i\beta_{m,SF}h_{m}}} L^{+}_{1/m,sy} \chi_{y\beta\gamma}$$
(6b)

m/2 界面の外側 $(x = -(2n + 1)h_m \tan \theta_{mSF}, z = h_m^+)$ からの SFG

反射 SFG

$$E^{-1p}(0^{-}) = \sum_{\beta,\gamma} E^{0}_{vis\beta} E^{0}_{ir\gamma} \frac{t_{1m,vis\beta} t_{m2,vis\beta} t_{1mjr,\gamma} t_{m2jr,\gamma} e^{i(\beta_{m,ir} + \beta_{m,vis})h_{m}}}{(1 + r_{1m,vis\beta} r_{m2,vvis\beta} e^{2\beta_{m,vis}h_{m}})(1 + r_{1mjr\gamma} r_{m2,jr\gamma} e^{2\beta_{m,ir}h_{m}})}$$

$$\times \frac{t_{m1p} e^{i\beta_{m,sF}h_{m}}}{1 + r_{1m,r} r_{m2,p} 2e^{2\beta_{m,sF}h_{m}}} [L^{-2/m,px} \chi_{x\beta\gamma} + L^{-2/m,pz} \chi_{x\beta\gamma}]$$
(7a)

$$E^{-1s}(0^{-}) = \sum_{\beta \gamma} E^{0}_{vis\beta} E^{0}_{ir\gamma} \frac{t_{1m,vis\beta} t_{m2,vis\beta} t_{1m,ir\gamma} t_{m2,ir\gamma} e^{i(\beta_{m,ir} + \beta_{m,vis})h_{m}}}{(1 + r_{1m,vis\beta} r_{m2,vvis\beta} e^{2i\beta_{m,vis}h_{m}})(1 + r_{1m,ir\gamma} r_{m2,ir\gamma} e^{2i\beta_{m,ir}h_{m}})} \times \frac{t_{m1,s} e^{i\beta_{m,sr}h_{m}}}{1 + r_{s} r_{s} 2 e^{2i\beta_{m,sr}h_{m}}} L^{-2/m,sy} \chi_{y\beta\gamma}$$

$$(7b)$$

透過 SFG

$$E^{+}_{2p}(h_{m}^{+}) = \sum_{\beta,\gamma} E^{0}_{vis}_{\beta} E^{0}_{ir,\gamma} \frac{t_{1m,vis}_{\beta} t_{n2,vis}_{\beta} t_{1m,ir,\gamma} t_{m2,ir,\gamma} e^{i(\beta_{m,ir}^{+}\beta_{m,vis}^{+})h_{m}}}{(1 + r_{1m,vis}_{\beta} r_{m2,vvis}_{\beta} e^{2i\beta_{m,vis}h_{m}})(1 + r_{1m,ir,\gamma} r_{m2,ir,\gamma} e^{2i\beta_{m,ir}h_{m}})} \times \frac{(1 - r_{1m,SF,p} e^{2i\beta_{m,SF}h_{m}})L^{+}_{2/m,px} \chi_{x\beta\gamma} + (1 + r_{1m,SF,p} e^{2i\beta_{m,SF}h_{m}})L^{+}_{2/m,pz} \chi_{z\beta\gamma}}{1 + r_{1m,SF,p} r_{m2,SF,p} e^{2i\beta_{m,SF}h_{m}}}$$

$$E^{+}_{2s}(h_{m}^{+}) = \sum_{\beta,\gamma} E^{0}_{vis}_{\beta} E^{0}_{ir\gamma} \frac{t_{1m,vis}_{\beta} t_{m2,vis}_{\beta} t_{1m,ir\gamma} t_{m2,ir\gamma} e^{i(\beta_{m,ir}^{+}\beta_{m,vis}^{-})h_{m}}}{(1 + r_{1m,vis}_{\beta} r_{m2,vvis}_{\beta} e^{2i\beta_{m,vis}^{+}h}})(1 + r_{1m,ir\gamma} r_{m2,ir\gamma} e^{2i\beta_{m,ir}^{-}h_{m}})}$$

$$(8a)$$

$$\times \frac{(1 - r_{1m,SF,s}e^{2i\beta_{m,SF}h_m})L^{+}_{2/m,sy}\chi_{y\beta\gamma}}{1 + r_{m,cr}r_{2,crs}e^{2i\beta_{m,SF}h_m}}$$
(8b)

SFG を出す界面が屈折率 m'' の薄膜の時には、

反射 SFG

$$E^{-1p}(0^{-}) = \sum_{\beta,\gamma} E^{0}_{vis\beta} E^{0}_{ir\gamma} \frac{t_{1m,vis\beta} t_{mm^{"},vis\beta} (1 + r_{m^{"}2,vis\beta}) e^{i\beta_{m,vis}h_{m}}}{(1 + r_{mm^{"},vis\beta} r_{m^{"}2,vis\beta}) + r_{1m,vis\beta} (r_{mm^{"},vis\beta} + r_{m^{"}2,vis\beta}) e^{2i\beta_{m,vis}h_{m}}}$$

$$\times \frac{t_{1m,ir\gamma} t_{mm^{"},ir\gamma} (1 + r_{m^{"}2,ir\gamma}) e^{i\beta_{m,ir}h_{m}}}{(1 + r_{mm^{"},ir\gamma} r_{m^{"}2,ir\gamma}) + r_{1m,ir\gamma} (r_{mm^{"},ir\gamma} + r_{m^{"}2,ir\gamma}) e^{2i\beta_{m,ir}h_{m}}}$$

$$\times \frac{t_{m1SF,p} e^{i\beta_{m,SF}h_{m}} (1 + r_{mn^{"}SF,p} r_{m^{"}2,SF,p})}{1 + r_{mm^{"}SF,p} r_{m^{"}2,SF,p} + r_{m^{"}2,SF,p}) e^{2i\beta_{m,SF}h_{m}}} [L^{-}2/m px \chi_{x\beta\gamma} + L^{-}2/m pz \chi_{x\beta\gamma}]$$

$$(9a)$$

$$E^{-1s}(0^{-}) = \sum_{\beta \gamma} E^{0}_{vis\beta} E^{0}_{ir\gamma} \frac{t_{1m,vis\beta} t_{mm'',vis\beta} (1 + r_{m''2,vis\beta}) e^{4\beta_{m,vis}h_{m}}}{(1 + r_{mm'',vis\beta} r_{m''2,vis\beta}) + r_{1m,vis\beta} (r_{mm'',vis\beta} + r_{m''2,vis\beta}) e^{2i\beta_{m,vis}h_{m}}} \times \frac{t_{1m,ir\gamma} t_{mm'',ir\gamma} (1 + r_{m''2,ir\gamma}) e^{i\beta_{m,ir}h_{m}}}{(1 + r_{mm'',ir\gamma} r_{m'''2,ir\gamma}) + r_{1m,ir\gamma} (r_{mm'',ir\gamma} + r_{m'''2,ir\gamma}) e^{2i\beta_{m,ir}h_{m}}}} \times \frac{t_{ml,SFS} e^{i\beta_{m,SF}h_{m}} (1 + r_{mm'',SFS} r_{m'''2,SFS})}{1 + r_{mm'',SFS} r_{m'''2,SFS} + r_{1m,SFS} (r_{mm'',SFS} + r_{m'''2,SFS}) e^{2i\beta_{m,SF}h_{m}}} L^{-2/m,sy} \chi_{\beta\beta\gamma}$$

$$(9b)$$

透過 SFG

$$E_{2p}(h_{m}^{+}) = \sum_{\beta \gamma} E^{0}_{vis\beta} E^{0}_{ir\gamma} \frac{t_{1m,vis\beta} t_{mm'',vis\beta} (1 + r_{m''2,vis\beta}) e^{\beta_{m,vis}h_{m}}}{(1 + r_{mm'',vis\beta} r_{m''2,vis\beta}) + r_{1m,vis\beta} (r_{mm'',vis\beta} + r_{m''2,vis\beta}) e^{2i\beta_{m,vis}h_{m}}}$$

$$\times \frac{t_{1m,ir\gamma} t_{mm'',ir\gamma} (1 + r_{m''2,ir\gamma}) e^{i\beta_{m,ir}h_{m}}}{(1 + r_{mm'',ir\gamma} r_{m'''2,ir\gamma}) + r_{1m,ir\gamma} (r_{mm'',ir\gamma} + r_{m'''2,ir\gamma}) e^{2i\beta_{m,ir}h_{m}}}$$

$$\times \frac{(1 + r_{mm'',SF,p} r_{m'''2,SF,p}) [(1 - r_{1m,SF,p} e^{2\beta_{m,SF}h_{m}}) L^{+}_{2/m,px} \chi_{\beta\gamma} + (1 + r_{1m,SF,p} e^{2i\beta_{m,SF}h_{m}}) L^{+}_{2/m,pz} \chi_{\beta\gamma}]}{1 + r_{mm'',SF,p} r_{m'''2,SF,p} + r_{1m,SF,p} (r_{mm'',SF,p} + r_{m''''2,SF,p}) e^{2i\beta_{m,SF}h_{m}}}$$

$$(10a)$$

$$E_{2s}(h_{m}^{+}) = \sum_{\beta,\gamma} E^{0}_{vis\beta} E^{0}_{ir\gamma} \frac{t_{1m,vis\beta} t_{mm^{n},vis\beta} (1 + r_{m^{n}2,vis\beta}) e^{i\beta_{m,vis}h_{m}}}{(1 + r_{mm^{n},vis\beta} r_{m^{n}2,vis\beta}) + r_{1m,vis\beta} (r_{mm^{n},vis\beta} + r_{m^{n}2,vis\beta}) e^{2i\beta_{m,vis}h_{m}}}$$

$$\times \frac{t_{1m,ir\gamma} t_{mm^{n},ir\gamma} (1 + r_{m^{n}2,ir\gamma}) e^{i\beta_{m,ir}h_{m}}}{(1 + r_{mm^{n},ir\gamma} r_{m^{n}2,ir\gamma}) + r_{1m,ir\gamma} (r_{mm^{n},ir\gamma} + r_{m^{n}2,ir\gamma}) e^{2i\beta_{m,ir}h_{m}}}$$

$$\times \frac{(1 + r_{mm^{n},SF,s} r_{m^{n}2,SF,s})(1 - r_{1m,SF,s} e^{2\beta_{m,SF}h_{m}}) L^{+}_{2/m,s\gamma} \chi_{\gamma\beta\gamma}}{1 + r_{mm^{n},SF,s} r_{m^{n}2,SF,s} + r_{m,SF,s} (r_{mm^{n},SF,s} + r_{m^{n}2,SF,s}) e^{2i\beta_{m,SF}h_{m}}}$$

$$(10b)$$

m/2 界面の内側 $(x = -(2n + 1)h_m \tan \theta_{m,SF}, z = h_m)$ からの反射 SFG

反射 SFG

$$E^{-1p}(0^{-}) = \sum_{\beta,\gamma} E^{0}_{vis\beta} E^{0}_{ir\gamma} \frac{t_{1m,vis\beta} t_{1m,ir\gamma} (1 + r_{m2,vis\beta}) (1 + r_{m2,ir\gamma}) e^{i(\beta_{m,ir} + \beta_{m,vis})h_{m}}}{(1 + r_{1m,vis\beta} r_{m2,vis\beta} e^{2\beta_{m,vis}h_{m}}) (1 + r_{1m,ir\gamma} r_{m2,ir\gamma} e^{2\beta_{m,ir}h_{m}})} \times \frac{t_{m1,SF,p} e^{i\beta_{m,SF}h_{m}}}{1 + r_{1m,er\beta} r_{m2,er\beta} e^{2i\beta_{m,SF}h_{m}}} (L^{2/m,px} \chi_{x\beta\gamma} + L^{2/m,pz} \chi_{z\beta\gamma})$$
(11a)

$$E^{-1s}(0^{-}) = \sum_{\beta \gamma} E^{0}_{vis\beta} E^{0}_{ir\gamma} \frac{t_{1m,vis\beta} t_{1m,ir\gamma} (1 + r_{m2,vis\beta})(1 + r_{m2,ir\gamma}) e^{i(\beta_{m,ir} + \beta_{m,vis})h_{m}}}{(1 + r_{1m,vis\beta} r_{m2,vis\beta} e^{2i\beta_{m,vis}h_{m}})(1 + r_{1m,ir\gamma} r_{m2,ir\gamma} e^{2i\beta_{m,ir}h_{m}})} \times \frac{t_{m1,SF,s} e^{i\beta_{m,SF}h_{m}}}{1 + r_{1m,SF,s} r_{m2,SF,s} e^{2i\beta_{m,SF}h_{m}}} L^{-2/m,sy} \chi_{y\beta\gamma}$$

$$(11b)$$

透過 SFG

$$E^{+}_{2p}(h_{m}^{+}) = \sum_{\beta\gamma} E^{0}_{vis\beta} E^{0}_{ir\gamma} e^{\frac{t_{1mvis\beta} t_{1mjr\gamma} (1 + r_{m2,vis\beta})(1 + r_{m2,ir\gamma}) e^{\frac{i(\beta_{m,ir} + \beta_{m,vis})h_{m}}{h_{m}}}}{(1 + r_{1m,vis\beta} r_{m2,vis\beta} e^{2i\beta_{m,vis}h_{m}})(1 + r_{1m,ir\gamma} r_{m2,ir\gamma} e^{2i\beta_{m,ir}h_{m}})} \times \frac{(1 + r_{ml,SF,p} e^{2i\beta_{m,SF}h_{m}}) L^{+}_{2/m,px} \chi_{x\beta\gamma} + (-1 + r_{ml,SF,p} e^{2i\beta_{m,SF}h_{m}}) L^{+}_{2/m,pz} \chi_{x\beta\gamma}}{1 + r_{1mSF,p} r_{m2,SF,p} e^{2i\beta_{m,SF}h_{m}}}$$

$$(12a)$$

$$E^{+}_{2s}(h_{m}^{+}) = \sum_{\beta,\gamma} E^{0}_{vis\beta} E^{0}_{ir\gamma} e^{\frac{t_{1m,vis\beta}}{(1 + r_{1m,vis\beta} r_{n2,vis\beta})(1 + r_{m2,ir\gamma})} e^{i(\beta_{m,ir} + \beta_{m,vis})h_{m}}}{(1 + r_{1m,vis\beta} r_{m2,vis\beta} e^{2i\beta_{m,vis}h_{m}})(1 + r_{1m,ir\gamma} r_{m2,ir\gamma} e^{2i\beta_{m,ir}h_{m}})} \times \frac{(1 + r_{ml,SF,s} e^{2i\beta_{m,SF}h_{m}})L^{+}_{2/m,sy} \chi_{y\beta\gamma}}{1 + r_{1mSF,s} r_{m2,SF,s} e^{2i\beta_{m,SF}h_{m}}}$$
(12b)

膜パルク $[0 < z < h_{\rm m}, \quad A_{\rm n=0,\,1,\,2}: x = -2nh_{\rm m} tan\theta_{\rm m,SF} - z tan\theta_{\rm m,SF}, \quad B_{\rm n=-1,\,0,\,1,\,2}: x = -(2n+1)h_{\rm m} tan\theta_{\rm m,SF} + z tan\theta_{\rm m,SF}]$ からの SFG

反射 SFG

$$\begin{split} E^{-1\mu}(0^{-}) &= \frac{i}{\delta} \sum_{\beta,\gamma} E^{0}_{vis} E^{0}_{ir\gamma} \\ &\times \frac{I_{mlSE,\rho} I_{msio} \beta^{1}_{lmir\gamma}}{(1 + r_{lmSE,\rho} r_{m2SE,\rho} e^{2\beta_{nm}h_{n}})(1 + r_{lmsis} \beta^{1}_{lmir\gamma}} \\ &\times \frac{I_{mlSE,\rho} I_{msi} E^{0}_{nmh}}{(1 + r_{lmsis} \beta^{1}_{lmir\gamma})(1 + r_{lmir\gamma} r_{m2jr\gamma} e^{2j\beta_{nm}h_{n}})} \\ &\times \{ \frac{1}{\beta_{mSF} - \beta_{msis} - \beta_{mir}} \\ &\times \{ \chi_{\beta p_{i}} E_{-1m,\mu} [r_{m2sis} \beta^{1}_{m2sri} q^{1}_{i} e^{i(\beta_{nSr} + \beta_{nm} + \beta_{n\rho})h_{n}} - e^{2i(\beta_{nm} + \beta_{n\rho})h_{n}}) \\ &+ r_{m2SE,\rho} e^{i(\beta_{nSr} + \beta_{nm} + \beta_{n\rho})h_{n}} - e^{2\delta_{nSr}h_{n}})] \\ &+ \chi_{\beta p_{i}} E_{-1m,pi} [r_{m2sis} \beta^{1}_{m2sri} q^{1}_{i} e^{i(\beta_{nSr} + \beta_{nm} + \beta_{n\rho})h_{n}} - e^{2i(\beta_{nm} + \beta_{n\rho})h_{n}})] \} \\ &+ \frac{1}{\beta_{mSF} - \beta_{msis} + \beta_{mir}} \\ &\times \{ \chi_{\beta p_{i}} E_{-1m,pis} [r_{m2sis} \beta^{1}_{i} e^{i(\beta_{nSr} + \beta_{nm} + \beta_{n\rho})h_{n}} - e^{2j\beta_{nm}h_{n}})] \\ &+ \chi_{\beta p_{i}} E_{-1m,pis} [r_{m2sis} \beta^{1}_{i} e^{i(\beta_{nSr} + \beta_{nm} + \beta_{n\rho})h_{n}} - e^{2j\beta_{nm}h_{n}})] \\ &+ \chi_{\beta p_{i}} E_{-1m,pis} [r_{m2sis} \beta^{1}_{i} e^{i(\beta_{nSr} + \beta_{nm} + \beta_{n\rho})h_{n}} - e^{2j\beta_{nm}h_{n}})] \} \\ &+ \frac{1}{\beta_{mSF} + \beta_{msis} - \beta_{mir}} \\ &\times \{ \chi_{\alpha p_{i}} E_{-1mnpi} [r_{m2sis} \beta^{1}_{i} e^{i(\beta_{nSr} + \beta_{nm} + \beta_{n\rho})h_{n}} - e^{2i(\beta_{nSr} + \beta_{nm} h_{n})h_{n}})] \} \\ &+ \frac{1}{\beta_{mSF} + \beta_{msis} - \beta_{mir}} \\ &\times \{ \chi_{\alpha p_{i}} E_{-1mmpi} [r_{m2sis} \beta^{1}_{i} e^{i(\beta_{nSr} + \beta_{nm} + \beta_{n\rho})h_{n}} - e^{2i(\beta_{nSr} + \beta_{nm} h_{n})h_{n}})] \} \\ &+ \frac{1}{\beta_{mSF} + \beta_{msis} - \beta_{mir}} \\ &\times \{ \chi_{\alpha p_{i}} E_{-1mmpi} [r_{m2ir} q^{1}_{i} e^{i(\beta_{nSr} + \beta_{nm} + \beta_{n\rho})h_{n}} - e^{2i(\beta_{nSr} + \beta_{nm} h_{n})h_{n}})] \\ &+ \chi_{\alpha p_{i}} E_{-1mmpi} [r_{m2ir} q^{1}_{i} e^{i(\beta_{nSr} + \beta_{nm} + \beta_{n\rho})h_{n}} - e^{2i(\beta_{nSr} + \beta_{nm} h_{n})h_{n}})] \\ &+ \chi_{\alpha p_{i}} E_{-1mmpi} [r_{m2ir} q^{1}_{i} e^{i(\beta_{nSr} + \beta_{nm} + \beta_{n\rho})h_{n}} - e^{2i(\beta_{nSr} + \beta_{nm} h_{n})h_{n}})] \\ &+ \mu_{m2SF,p} r_{m2sip} p^{1}_{m2ir} q^{1}_{i} e^{i(\beta_{nSr} + \beta_{nm} + \beta_{n\rho})h_{n}} - e^{2i(\beta_{nSr} + \beta_{nm} + \beta_{n\rho})h_{n}})] \\ &+ r_{m2SF,p} r_{m2sip} p^{1}_{m2ir} q^{1}_{i} e^{i(\beta_{nSr} + \beta_{nm} + \beta_{n\rho})h_{n}} - e$$

$$E^{-1s}(0^{-}) = \frac{i}{\delta} \sum_{\beta \gamma} E^{0}_{vis\beta} E^{0}_{ir\gamma}$$

$$\times \frac{t_{m1SF_{S}}t_{1m,sis,\beta}t_{1m,jr,\gamma}}{(1 + r_{1mSF_{S}}r_{m2SF_{S}}e^{2i\beta_{m,si}h_{m}})(1 + r_{1m,sis,\beta}r_{m2sjis,\beta}e^{2i\beta_{m,si}h_{m}})(1 + r_{1mjr,\gamma}r_{m2jr,\gamma}e^{2i\beta_{m,si}h_{m}})}$$

$$\times \{\frac{1}{\beta_{mSF} - \beta_{m,sis} - \beta_{m,jr}}$$

$$\times \{\chi_{y\beta\gamma} L_{1/mxy}[r_{m2sjis,\beta}r_{m2,ir\gamma}(e^{i(\beta_{mSF}+\beta_{m,ii}+\beta_{m,ir})h_{m}} - e^{2i(\beta_{m,sir}+\beta_{m,ir})h_{m}})$$

$$+ r_{m2SF_{S}}(e^{i(\beta_{mSF}+\beta_{m,ii}+\beta_{m,ir})h_{m}} - e^{2i\beta_{m,sir}h_{m}})]$$

$$+ \frac{1}{\beta_{mSF} - \beta_{m,yis} + \beta_{mjr}}$$

$$\times \{\chi_{y\beta\gamma} L_{1/mxy}[r_{m2sjis,\beta}(e^{i(\beta_{mSF}+\beta_{m,ii}+\beta_{m,ir})h_{m}} - e^{2i(\beta_{m,sr}+\beta_{m,ir})h_{m}})]$$

$$+ r_{m2SF_{S}}r_{m2jr,\gamma}(e^{i(\beta_{mSF}+\beta_{m,ii}+\beta_{m,ir})h_{m}} - e^{2i(\beta_{mSF}+\beta_{m,ii})h_{m}})]$$

$$+ \frac{1}{\beta_{mSF} + \beta_{m,yis} - \beta_{mjr}}$$

$$\times \{\chi_{y\beta\gamma} L_{m/m,sy}[r_{m2jr,\gamma}(e^{i(\beta_{mSF}+\beta_{m,ii}+\beta_{m,ir})h_{m}} - e^{2i(\beta_{m,sr}+\beta_{m,ii})h_{m}})]$$

$$+ r_{m2SF_{S}}r_{m2jxis,\beta}(e^{i(\beta_{m,SF}+\beta_{m,sir}+\beta_{m,ir})h_{m}} - e^{2i(\beta_{m,ir}+\beta_{m,ir})h_{m}})]$$

$$+ \frac{1}{\beta_{mSF} + \beta_{m,yis} + \beta_{mjr}}$$

$$\times \{\chi_{y\beta\gamma} L_{m/m,sy}[(e^{i(\beta_{m,SF}+\beta_{m,sir}+\beta_{m,ir})h_{m}} - e^{2i(\beta_{m,ir}+\beta_{m,ir})h_{m}})]$$

$$+ r_{m2SF_{S}}r_{m2jxis,\beta}(e^{i(\beta_{m,SF}+\beta_{m,sir}+\beta_{m,ir})h_{m}} - e^{2i(\beta_{m,ir}+\beta_{m,ir})h_{m}})]$$
(13b)

透過 SFG

$$E^{+2p}(h_{m}^{+}) = \frac{i}{\delta} \sum_{\beta \gamma} E^{0} vis \beta E^{0} ir \gamma$$

$$\times \frac{t_{1m,vis} \beta t_{1m,ir\gamma} e^{i\beta_{m,SF}h_{m}}}{(1 + r_{1m,SFp} r_{m2,SF,p} e^{2i\beta_{m,SF}h_{m}})(1 + r_{1m,vis} \beta r_{m2,vis} \beta e^{2i\beta_{m,vi}h_{m}})(1 + r_{1m,ir\gamma} r_{m2,ir\gamma} e^{2i\beta_{m,vi}h_{m}})}$$

$$\times \{L^{+} 2/m_{PX} \chi_{\beta \gamma} [\frac{e^{i(-\beta_{m,SF} + \beta_{m,vi} + \beta_{m,ir})h_{m}} - 1] + r_{ml,SFp} r_{m2,vis} \beta r_{m2ir\gamma} [e^{2i(\beta_{m,vi} + \beta_{m,ir})h_{m}} - e^{i(\beta_{m,SF} + \beta_{m,vi} + \beta_{m,ir})h_{m}}]}$$

$$+ \frac{r_{m2ir\gamma} [e^{i(-\beta_{m,SF} + \beta_{m,vi} + \beta_{m,ir})h_{m}} - e^{2i\beta_{m,i}h_{m}}] + r_{m1,SFp} r_{m2,vis} \beta [e^{2i\beta_{m,vi}h_{m}} - e^{i(\beta_{m,SF} + \beta_{m,vi} + \beta_{m,ir})h_{m}}]}$$

$$+ \frac{r_{m2,vis} \beta [e^{i(-\beta_{m,SF} + \beta_{m,vi} + \beta_{m,ir})h_{m}} - e^{2i\beta_{m,vi}h_{m}}] + r_{m1,SFp} r_{m2,ir\gamma} [e^{2i\beta_{m,vi}h_{m}} - e^{i(\beta_{m,SF} + \beta_{m,vi} + \beta_{m,ir})h_{m}}]}$$

$$+ \frac{r_{m2,vis} \beta r_{m2,ir\gamma} [e^{i(-\beta_{m,SF} + \beta_{m,vi} + \beta_{m,vi})h_{m}} - e^{2i\beta_{m,vi}h_{m}}] + r_{m1,SFp} r_{m2,ir\gamma} [e^{2i(\beta_{m,vi} + \beta_{m,vi} + \beta_{m,vi})h_{m}}]}$$

$$+ \frac{r_{m2,vis} \beta r_{m2,ir\gamma} [e^{i(-\beta_{m,SF} + \beta_{m,vi} + \beta_{m,vi})h_{m}} - e^{2i(\beta_{m,vi} + \beta_{m,vi})h_{m}}] + r_{m1,SFp} r_{m2,vis} \beta [e^{2i(\beta_{m,vi} + \beta_{m,vi})h_{m}} - e^{i(\beta_{m,SF} + \beta_{m,vi} + \beta_{m,vi})h_{m}}]}$$

$$+ L^{+} 2/m_{pz} \chi_{z\beta\gamma} [e^{i(-\beta_{m,SF} + \beta_{m,vi} + \beta_{m,vi})h_{m}} - 1] - r_{m1,SFp} r_{m2,vis} \beta r_{m2,vis} \beta [e^{2i(\beta_{m,vi} + \beta_{m,vi})h_{m}} - e^{i(\beta_{m,SF} + \beta_{m,vi} + \beta_{m,vi})h_{m}}]}$$

$$+ L^{+} 2/m_{pz} \chi_{z\beta\gamma} [e^{i(-\beta_{m,SF} + \beta_{m,vi} + \beta_{m,vi})h_{m}} - 1] - r_{m1,SFp} r_{m2,vis} \beta r_{m2,vis} \beta [e^{2i(\beta_{m,vi} + \beta_{m,vi})h_{m}} - e^{i(\beta_{m,SF} + \beta_{m,vi} + \beta_{m,vi})h_{m}}]}$$

$$+ L^{+} 2/m_{pz} \chi_{z\beta\gamma} [e^{i(-\beta_{m,SF} + \beta_{m,vi} + \beta_{m,vi})h_{m}} - 1] - r_{m1,SFp} r_{m2,vis} \beta r_{m2,vis} \beta [e^{2i(\beta_{m,vi} + \beta_{m,vi})h_{m}} - e^{i(\beta_{m,SF} + \beta_{m,vi} + \beta_{m,vi})h_{m}}]}$$

$$+ R_{m,SF} - R_{m,vis} + R_{m,vi} + R_{m,vi}$$

$$+\frac{r_{m2,vis,\beta}\left[e^{i(-\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_{m}}-e^{2i\beta_{m,vi}h_{m}}\right]-r_{m1,SF,p}r_{m2,ir,\gamma}\left[e^{2i\beta_{m,ir}h_{m}}-e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_{m}}\right]}{\beta_{m,SF}+\beta_{m,vis}-\beta_{m,ir}} + \frac{r_{m2,vis,\beta}r_{m2,ir,\gamma}\left[e^{i(-\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_{m}}-e^{2i(\beta_{m,vis}+\beta_{m,ir})h_{m}}\right]-r_{m1,SF,p}\left[1-e^{i(\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir})h_{m}}\right]}{\beta_{m,SF}+\beta_{m,vis}+\beta_{m,ir}}\right]\}$$

$$(14a)$$

$$E^{+}_{2s}(h_{m}^{+}) = \frac{i}{\delta} \sum_{\beta,\gamma} E^{0}_{vis}_{\beta} E^{0}_{ir\gamma}$$

$$\times \frac{t_{1 m, vis,\beta} t_{1 m, ir,\gamma} e^{i\beta_{m,sr}h_{m}}}{(1 + r_{1 m,SF,s} r_{m2,SF,s} e^{2i\beta_{m,sr}h_{m}})(1 + r_{1 m,vis,\beta} r_{m2,vis,\beta} e^{2i\beta_{m,vis}h_{m}})(1 + r_{1 m,ir,\gamma} r_{m2,ir,\gamma} e^{2i\beta_{m,ir}h_{m}})}$$

$$\times L^{+}_{2 / m, sy} \chi_{j\beta\gamma} \left\{ \frac{[e^{i(-\beta_{m,SF} + \beta_{m,vis} + \beta_{m,ir}})h_{m} - 1] + r_{m1,SF,s} r_{m2,vis,\beta} r_{m2ir,\gamma} [e^{2i\beta_{m,vis} + \beta_{m,ir}})h_{m} - e^{i(\beta_{m,SF} + \beta_{m,vis} + \beta_{m,ir}})h_{m}]}{\beta_{m,SF} - \beta_{m,vis} - \beta_{m,ir}} + \frac{r_{m2,ir,\gamma} [e^{i(-\beta_{m,SF} + \beta_{m,vis} + \beta_{m,ir}})h_{m} - e^{2i\beta_{m,vi}h_{m}}] + r_{m1,SF,s} r_{m2,vis,\beta} [e^{2i\beta_{m,vis}h_{m}} - e^{i(\beta_{m,SF} + \beta_{m,vis} + \beta_{m,ir}})h_{m}]}{\beta_{m,SF} - \beta_{m,vis} + \beta_{m,ir}} + \frac{r_{m2,vis,\beta} [e^{i(-\beta_{m,SF} + \beta_{m,vis} + \beta_{m,ir}})h_{m} - e^{2i\beta_{m,vis}h_{m}}] + r_{m1,SF,s} r_{m2,vis,\beta} [e^{2i\beta_{m,vis}h_{m}} - e^{i(\beta_{m,SF} + \beta_{m,vis} + \beta_{m,ir}})h_{m}]}{\beta_{m,SF} + \beta_{m,vis} - \beta_{m,ir}}} + \frac{r_{m2,vis,\beta} r_{m2,ir,\gamma} [e^{i(-\beta_{m,SF} + \beta_{m,vis} + \beta_{m,ir}})h_{m} - e^{2i\beta_{m,vis}h_{m}}] + r_{m1,SF,s} r_{m2,ir,\gamma} [e^{i(-\beta_{m,SF} + \beta_{m,vis} + \beta_{m,ir}})h_{m}]}{\beta_{m,SF} + \beta_{m,vis} + \beta_{m,ir}}} \}$$

$$(14b)$$